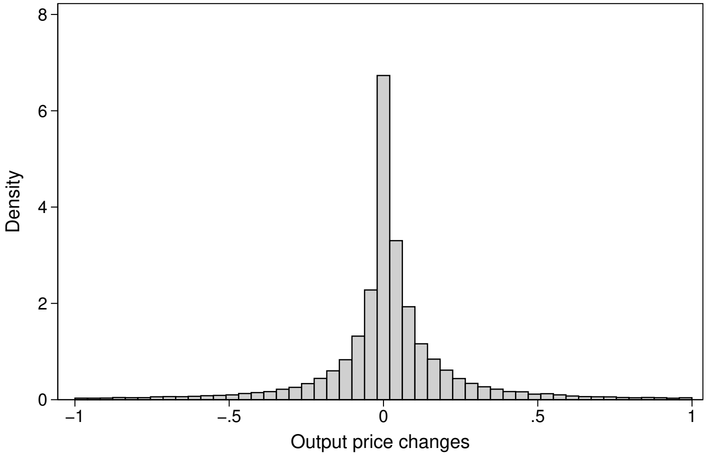
Price Updating with Production Networks

Cedric Duprez — NBB

Glenn Magerman — ECARES, ULB; CEPR

ChaMP ESCB Research Network, Brussels
October 25, 2023

Year-on-year producer price changes (Belgium, pooled 2002-2014)



- symmetric
- large variance: p10/p90: $\pm$ .20
- robust by year, within product, within firm, other datasets (micro PPI, ...)

Why do firms adjust their output prices?

With per-period cost minimization, firm j changes its output price as

$$\underbrace{d\ln p_{jt}}_{\text{price}} = \underbrace{d\ln c_{jt}}_{\text{marginal cost}} + \underbrace{d\ln \mu_{jt}}_{\text{markup}}$$

Canonical models with complete pass-through

- no/constant markups: $d\ln \mu_{jt} = 0$
- e.g. perfect competition, monop. comp. with CES preferences (hom/het firms)

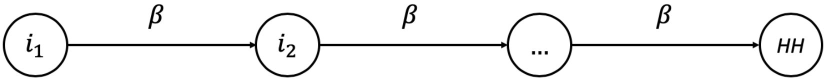
Inconsistent with repeated empirical evidence of incomplete pass-through

- variable markups: $d\ln \mu_{jt} \neq 0$
- macro: exchange rate disconnect; imports vs CPI variance
- micro: sector-specific (partial equilibrium) studies

CPI response depends critically on passthrough rate

Thought experiment: line network with n producers

- How much of initial shock to i_1 ends up with the final consumer (HH)?



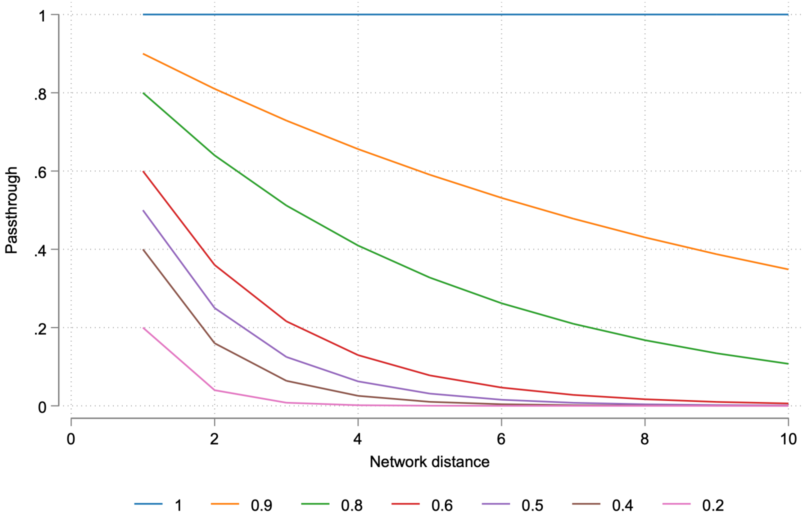
Passthrough models

- Complete: shock fully borne by HH, even as $n \to \infty$
- Incomplete: shock decays at rate β^n, and $\beta^n \to 0$ as $n \to \infty$
- For $n = 4$: $\beta = 1 \Rightarrow \beta^n = 1$ but $\beta = 0.5 \Rightarrow \beta^n = 1/16$

More generally, ultimate CPI response depends on

- Input-output structure of production network
- Location of the initial shock (Hulten, 1978; Gabaix, 2011; Acemoglu et al., 2012; Baqaee, 2018; Baqaee and Farhi, 2019a,b; 2020)

CPI response depends critically on passthrough rate



This paper

Questions

- How do firms change their prices in production networks?
- What is the impact of firm-level price changes on aggregate prices?

Why is it important?

- Aggregation: identifying micro origins of aggregate price fluctuations
- Welfare: reallocation of surplus across producers and consumers

Network propagation literature assumes perfect pass-through

- All shocks ultimately end up with the final customer and thus CPI

Existing studies mostly rely on partial data and/or structural assumptions

- Marginal costs have to be estimated or backed out
- Missing link between imports and final consumption price volatility

What we do

1. **Non-parametric framework of price updating**
   - Very light assumptions on market structure, technology, demand
   - Assumptions: per-period cost minimization, CRS wrt. variable inputs
2. **No need to estimate marginal costs**
   - Generally: $d \ln c_{jt} = f(d \ln p_{1jt}, \ldots, d \ln p_{Njt}, d \ln z_{jt})$
   - We observe all $d \ln p_{ijt}$ in the data and estimate $d \ln z_{jt}$
3. **Estimate elasticities: pass-through, productivity, strategic complementarities**
   - Multiple instruments used: TFP shocks of suppliers, import prices, producer prices
4. **Propagation and aggregation to CPI**
   - Depends on nature of shock, IO structure, pass-through, strategic complementarities
5. **Detailed product classification concordance**
   - $m : n$ correspondences in production (PC) and trade (CN)
   - No synthetic "family trees", consistent unitse of measurement

Related literature

Theory on variable markups, incomplete pass-through
Atkeson-Burstein (2008), Melitz-Ottaviano (2008), Weyl-Fabinger (2013), Atkin-Donaldson (2015), Edmond et al. (2015), Parenti et al. (2017), Arkolakis-Morlacco (2018), Amiti et al. (2019)
→ **Include production networks**

Empirics on variable markups, incomplete pass-through
Burstein-Gopinath (2014), Goldberg-Verboven (2001), Campa-Goldberg (2006), Nakamura-Zerom (2010), Berman et al. (2012), Goldberg-Hellerstein (2013), Fabra-Reguant (2014), Garetto (2016), De loecker et al. (2016)
→ **GE model with welfare implications (doing)**

Production networks, pricing and propagation
Acemoglu et al. (2012), Baqaee (2018), Baqaee & Farhi (2019a,b; 2020), Baqaee et al. (2022)
→ **Endogenous markups**

Concordance methods
Pierce-Schott (2012a, 2012b), Bernard et al. (2018)
→ **Exact mapping, no synthetic aggregation**

Today

General framework of price updating

Data and identification

Empirical results on price updating

Propagation and aggregation

Next steps

Production

Cost function for producer j at time t

$$\overbrace{y_{jt}\underbrace{c_{jt}\left(\left(1+\tau_{1j}\right)p_{1t},\ldots,\left(1+\tau_{nj}\right)p_{nt},z_{jt}\right)}_{\text{unit cost}}}^{\text{constant returns}}+\underbrace{F_{jt}}_{\text{fixed costs}}$$

- τ_{ij}: bilateral wedges (e.g. transport costs)
- z_{jt}: productivity
- F_{jt}: fixed costs

Notes

- Embeds network structure of production: inputs i and outputs j
- CRS wrt variable inputs. IRS from fixed costs, DRS: add firms that provide factors
- General technological change (Hicks-neutral in empirics)

Pricing and markups

General pricing equation under static cost minimization

$$\ln p_{jt} = \ln c_{jt}\left(\left(1+\tau_{1j}\right)p_{1t}, ..., \left(1+\tau_{Nj}\right)p_{Nt}, z_{jt}\right) + \ln \mu_{jt}\left(p_{jt}, \mathcal{P}_{-jt}; \xi_{jt}\right)$$

- $\mathcal{P}_{-jt}$: price index of j's environment (e.g. agg P-index, strat. comp.)
- ξ_{jt}: quantity shifter (e.g. price elasticity of demand)

Notes

- Profit maximization not necessary (e.g. cost-plus pricing, price capping)
- Nests no, constant and variable markups
- $\mathcal{P}_{-jt}$ depends on underlying model of price setting
  (e.g. oligopoly, monop. competition, or just responding to news)
- Single-product firms: multi-product firms possible with additional assumptions

Price updating

Totally differentiating the pricing equation

$$d\ln p_{jt} = \underbrace{\sum_{i\in\mathcal{S}_{jt}} \frac{\partial \ln c_{jt}}{\partial \ln p_{it}} d\ln p_{it}}_{\text{total input price shock}} + \underbrace{\frac{\partial \ln c_{jt}}{\partial \ln z_{jt}} d\ln z_{jt}}_{\text{productivity shock}} + \underbrace{\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}} d\ln p_{jt}}_{\text{own price markup effect}}$$

$$+ \underbrace{\frac{\partial \ln \mu_{jt}}{\partial \ln \mathcal{P}_{-jt}} d\ln \mathcal{P}_{-jt}}_{\text{environment price index effect}} + \underbrace{\frac{\partial \ln \mu_{jt}}{\partial \xi_{jt}} d\xi_{jt}}_{\text{shifters}}$$

where cost elasticity is given by its input share (envelope theorem)

$$\frac{\partial \ln c_{jt}}{\partial \ln p_{it}} = \frac{p_{ijt}x_{ijt}}{\sum_{i\in\mathcal{S}_{jt}} p_{ijt}x_{ijt}} \equiv \omega_{ijt}$$

Towards estimation equation

$$d \ln p_{jt} = \beta_{jt} \underbrace{\sum_{i \in \mathcal{S}_{jt}} \omega_{ijt-1} d \ln p_{it}}_{\text{change in input price index}} + \gamma_{jt} d \ln z_{jt} + \delta_{jt} d \ln \mathcal{P}_{-jt} + \eta_{jt} d\xi_{jt}$$

Coefficients have a structural interpretation as elasticities, consistent with many pricing models

$$\begin{cases} \beta_{jt} = \frac{1}{1 - \frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}}} \\ \gamma_{jt} = \frac{1}{1 - \frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}}} \frac{\partial \ln y_{jt}}{\partial \ln z_{jt}} \\ \delta_{jt} = \frac{1}{1 - \frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}}} \frac{\partial \ln \mu_{jt}}{\partial \ln \mathcal{P}_{-jt}} \\ \eta_{jt} = \frac{1}{1 - \frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}}} \frac{\partial \ln \mu_{jt}}{\partial \xi_{jt}} \end{cases}$$

Hypothesis: H_0 : constant/no markups ($\beta_{jt} = 1$); H_a: variable markups ($\beta_{jt} \neq 1$)

Today

General framework of price updating

Data and identification

Empirical results on price updating

Propagation and aggregation

Next steps

Variables

Datasets (2002-2014)

- Production: firm, product (PC8), year, value, quantity, unit
- Int'l trade: firm, product (CN8), country, year, value, quantity, unit
- Domestic production network: seller, buyer, year, value
- Annual accounts: sales, inputs, employment, NACE codes

Estimate productivity shocks

- Estimate *TFPq* (Hicks neutral)

Calculate change in firm's environment prices $\mathcal{P}_{-jt}$

- Depends on your underlying competition model of choice
- Sufficient statistic for many oligopolistic models: market shares and competitors' prices (best response functions)

$$d \ln \mathcal{P}_{-jt} = \sum_{l \neq j \in PC8} \lambda_{ljt-1} d \ln p_{lt}$$

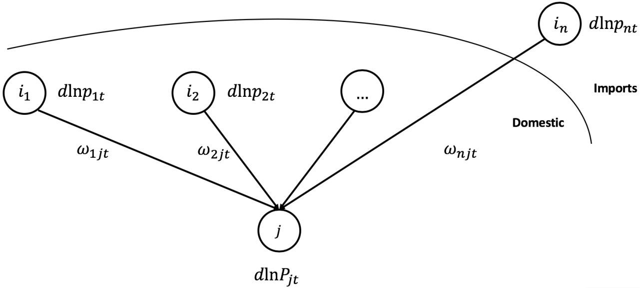
Input and output prices

Change in input price index $d\ln P_{jt} = \sum_{i \in \mathcal{S}_{jt}} \omega_{ijt-1} d\ln p_{it}$

- We observe all input shares (domestic and imports) $\omega_{ijt} = \frac{p_{ijt}x_{ijt}}{\sum_{i \in \mathcal{S}_{jt}} p_{ijt}x_{ijt}}$
- $d\ln p_{it}$ from Prodcom (domestic) and Comext (imports)

Change in output price $d\ln p_{jt}$

- Identify continuing products year-on-year (own concordances)
- Domestic prices, corrected for re-exports (Prodcom and Comext exports)



Identification

Goal: obtain consistent estimates for parameters $\theta = (\beta, \gamma, \delta)$

$$d \ln p_{jt} = \alpha + \beta d \ln P_{jt} + \gamma d \ln z_{jt} + \delta d \ln \mathcal{P}_{-jt} + \eta d\xi_{st} + \varepsilon_{jt}$$

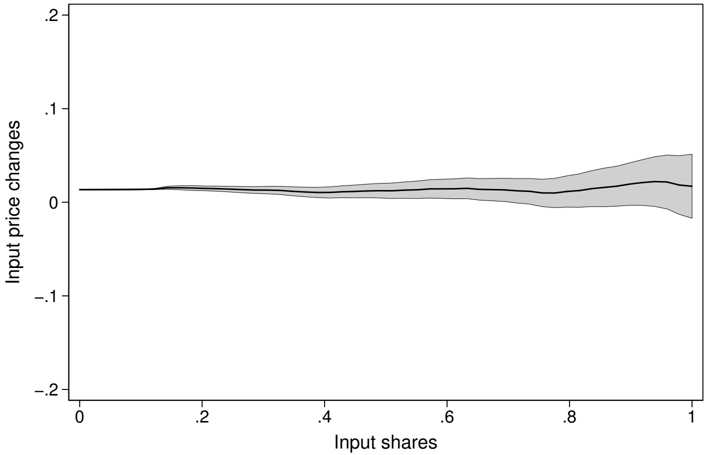
Problem: OLS estimates are biased and inconsistent

- simultaneity of prices (co-movement, best response)
- measurement error in regressors (since use unit values)
- selection bias if $Cov(\omega_{ijt-1}, d \ln p_{it}) \neq 0$ (intensive/extensive margins)

Selection bias: intensive margin

Selection bias if ω_{ijt-1} correlates with $d \ln p_{it}$

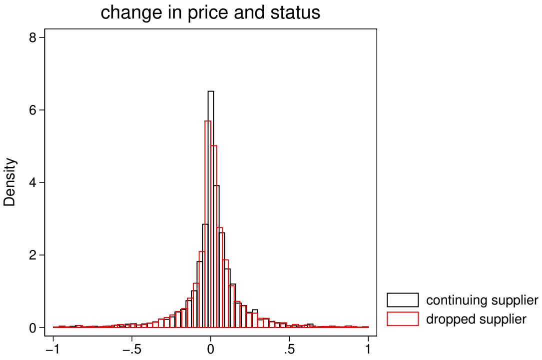
- E.g. price contracts for important inputs
- Surprising: we find no correlation!



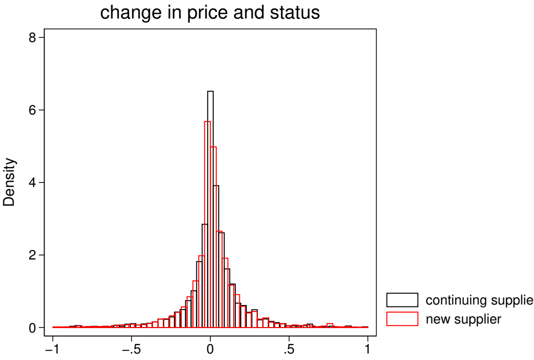
Selection bias: extensive margin

Selection bias if firms add/drop suppliers in response to price shocks

- 90% of value of input bundle is continuing from any year to the next
- Firms do not systematically add/drop suppliers in response to shocks
- Matching on levels (e.g. high productivity or low price) is fine



(a) **Continuing vs dropped**



(b) **Continuing vs added**

Simultaneity: Instruments

Instruments for input price index $d \ln P_{jt}$

$$d \ln P_{jt}^{IV} = \sum_{i \in S_j} \omega_{ijt-1} I_{it}$$

where $I_{it} = \left\{ d \ln z_{it}, d \ln \bar{p}_{-it}^{PC8-EU}, d \ln \bar{p}_{-it}^{CN8-EU} \right\}$
Exclusion restriction: $\mathbb{E}(\sum_{i \in S_j} \omega_{ijt-1} I_{it} \varepsilon_{jt}) = 0$, which collapses to $\mathbb{E}(\omega_{ijt-1} \varepsilon_{jt}) = 0, \forall i$, when $i, j \to \infty$ (GMM).

Instruments for environment price index $d \ln \mathcal{P}_{-jt}$

$$d \ln \mathcal{P}_{-jt}^{IV} = \sum_{l \neq j \in PC4} \lambda_{ljt-1} \left(\sum_{m \neq i \in S_{lt}} \omega_{mlt-1} I_{mt} \right)$$

where $I_{mt} = \left\{ d \ln z_{mt}, d \ln \bar{p}_{mt}^{PC8-EU}, d \ln \bar{p}_{mt}^{CN8-EU} \right\}$ and m are other suppliers to competitor l who are not also supplying j
Exclusion restriction: $\mathbb{E}(\omega_{mlt-1} \varepsilon_{jt}) = 0, \forall m$.

Today

General framework of price updating

Data and identification

Empirical results on price updating

Propagation and aggregation

Next steps

Price updating

	OLS			IV		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dep. var.	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$
$d \ln P_{jt}$	0.260*	0.259*	0.256*	0.521***	0.524***	0.531***
	(0.065)	(0.065)	(0.064)	(0.063)	(0.063)	(0.062)
$d \ln z_{jt}$	-0.106*	-0.109**	-0.109**	-0.107***	-0.110***	-0.109***
	(0.023)	(0.023)	(0.023)	(0.005)	(0.005)	(0.005)
$d \ln \mathcal{P}_{-jt}$	0.362**	0.347**	0.345**	0.377***	0.368***	0.403***
	(0.051)	(0.047)	(0.046)	(0.090)	(0.090)	(0.098)
FE	year	year + sector	year×sector	year	year + sector	year×sector
N	33,787	33,787	33,787	33,718	33,718	33,718
J-test χ^2				3.70	3.21	4.72
[p-value]				[.30]	[.36]	[.19]

Note: Columns (i)-(iii) report OLS estimates, columns (iv)-(vi) reports the second stage of IV estimates employing GMM with 5 instruments. All regressions are pooled over the years 2004-2014. The IV specifications pass all validity tests. Hansen's over-identification J-test statistic cannot reject the null hypothesis that the over-identifying restrictions are valid at the 1% level. Robust standard errors, clustered at the aggregated sector level (5 clusters) in parentheses. Significance: * < 5%, ** < 1%, *** <0.1%.

Discussion

Results

- Pass-through is incomplete ($\beta < 1$)
- Strategic complementarities exist ($\delta > 0$)
- Generalizes imports/sector studies to full production network

Identification/robustness

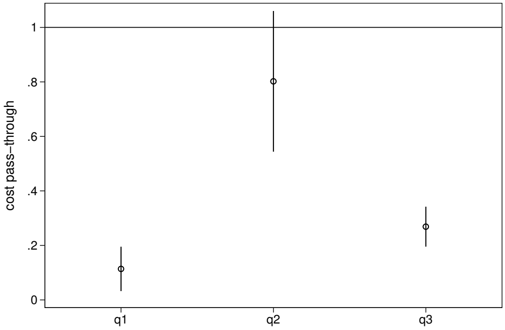
- IV (GMM) passes all over-identification tests
- Robust to alternative estimators (LIML, 2SLS)
- Robust to alternative instruments (Duranton & Turner, 2012)

Heterogeneity: pass-through by sector

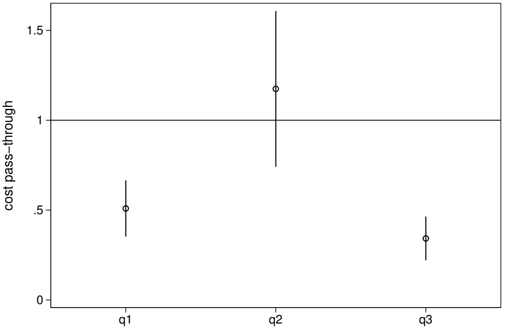
NACE Rev.2 sectors		N	IV β	IV γ	IV δ
8-9	Mining and quarrying	398	.933*	-.050	.387
10-12	Food products and beverages	6,023	.340***	-.059***	.512***
13-15	Textiles and apparel	1,363	.229	-.122***	.232**
16	Wood[...]	1,281	.077	-.100***	.192**
17-18	Paper products and media	1,121	.334**	-.119***	.239*
20	Chemicals and chemical products	1,479	.628***	-.061*	.274**
22	Rubber and plastic products	1,159	.344	-.112**	-.067
23	Other non-metallic minerals[...]	2,179	.459**	-.103***	.218
24	Basic metals	468	.486**	-.042	.695***
25	Fabricated metal products[...]	2,841	.391**	-.095***	.374***
26-27	Computer, electronic and[...]	580	.583	-.162***	.035
28-29	Machinery, motor vehicles[...]	254	-1.86	-.058	1.148
31-32	Furniture and other manufacturing	1,342	.541***	-.139***	-.016
33	Repair and installation of machinery/equipment	63	.381	-.007	1.187**

Heterogeneity: idiosyncratic vs common shocks

Setup: Demean input price index by sector-year average, group in terciles (q3: "large cost increase").



(c) OLS



(d) IV

Today

General framework of price updating

Data and identification

Empirical results on price updating

Propagation and aggregation

Next steps

Propagation

Pricing equation in reduced form

$$d \ln p_{jt} = \alpha + \beta d \ln P_{jt} + \gamma d \ln z_{jt} + \delta d \ln \mathcal{P}_{-jt} + \eta d\xi_{st} + \varepsilon_{jt}$$

$$d \ln \mathbf{p} = \beta \Omega d \ln \mathbf{p} + \gamma d \ln \mathbf{z} + \delta \Lambda d \ln \mathbf{p} + \eta d\xi$$

$$\iff d \ln \mathbf{p} = \underbrace{[I - \beta\Omega - \delta\Lambda]^{-1}}_{\text{network structure}} \underbrace{(\gamma d \ln \mathbf{z} + \eta d\xi)}_{\text{exogenous shocks}}$$

Intuition

- Price shocks accumulate through production network Ω
- Nests other models (e.g. $\beta = 1$ and $\delta = 0$ = no/constant markups)
- Validity of chosen instruments (see proof appendix)

Any shock has an impact on all moments of $d \ln p_{jt}$

- Mean and variance: exchange rate disconnect
- 3rd-4th moments: Symmetric shocks can have asymmetric effects and varying tails

Aggregation

Change in producer price index due to supply shock

$$d \ln \mathbb{P} = \sum_j \nu_j d \ln p_j(\Omega, \beta, \delta, \Lambda; d \ln \mathbf{z}, d\xi)$$

with ν_j some appropriate weight depending on chosen P-index

Provides structural interpretation and micro foundation of PPI

- Shocks can taper off before reaching final consumers
- Function of many dimensions of heterogeneity

With incomplete pass-through, aggregation measures fail

- Solow (1957), Hulten (1978) fail with inefficient economies
- Baqaee and Farhi (2020) fails with variable markups

Today

General framework of price updating

Data and identification

Empirical results on price updating

Propagation and aggregation

Next steps

Counterfactuals & applications

Monetary policy

- Does inflation targeting work as it is intended?
- Need incomplete adjustment to prices to get real short term effects
- Mostly model these as nominal rigidities (e.g. Calvo)
- Alternative explanation: flexible prices with incomplete pass-through

Exchange rate disconnect

- Macro puzzle: high import price volatility cannot be matched with low consumer price volatility
- Example introduction: ultimate impact on final prices is $\beta^4 \simeq 0.06$.

Productivity shocks and incomplete pass-through

- Network shock propagation models mostly assume perfect pass-through
- With incomplete pass-through, aggregate effects are smaller in terms of consumer surplus
- Redistribution between producer and consumer surplus

Conclusions

Takeaways

- Non-parametric model of price updating in production networks
- Cost pass-through is incomplete
- Impact on propagation and aggregation
- Applications in both micro and macro

Next steps

- Quantitative CPI analysis
- Welfare and surplus division
- Counterfactual exercises

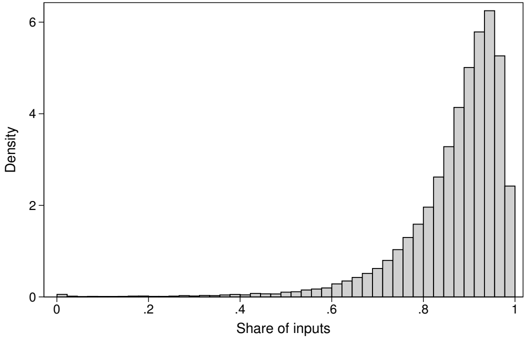
Thank you!

- cedric.duprez@nbb.be
- glenn.magerman@ulb.be

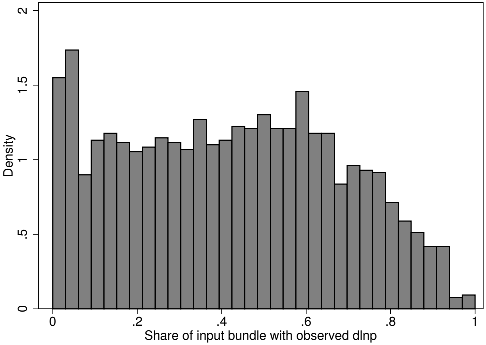
Appendix

Coverage change in input price index

- **Is $\sum_i \omega_{ijt-1} d \ln p_{it}$ a good proxy for $d \ln P_{jt}$?**
  - Continuing inputs from $t-1$ to t account for 90% of input expenditures on average
  - Observed share of input bundle with $d \ln p_{it}$ from micro data: 55% on average



(e) **Continuing inputs.**



(f) **$d \ln p_{it}$ from micro data.**

Identification (cont'd)

- **Estimating equation**

$$d \ln p_{jt} = \alpha + \beta d \ln P_{jt} + \gamma d \ln z_{jt} + \delta d \ln \mathcal{P}_{-jt} + \eta d\xi_{st} + \varepsilon_{jt}$$

  with $\tilde{\varepsilon} \equiv [I_N - \beta\Omega - \delta\Theta]^{-1}\varepsilon$

- **Hence**
  - $d \ln \mathbf{z}$ and $d\xi$ are exogenous
  - valid instruments for $d \ln p_{it}$ and $d \ln p_{lt}$ are exogenous variables of i and l (e.g. their $d \ln z$). (Proof: Bramoulle et al. (2009)).
  - Use reduced form for the setup of counterfactuals + IV.
- **Notes**
  - Suff. cond. for invertibility: If $||\beta + \delta|| < 1$, then $||\beta\Omega + \delta\Theta|| < 1$ (since $\sum_i \omega_{ij} = 1$ and $\sum_{l \neq j} \theta_{lj} = 1$), and so $[I_N - \beta\Omega - \delta\Theta]$ is non-singular.

Robustness – alternative estimators

- Underlying assumptions are different. Under constant effects, point estimates should be similar
- Also test for model mis-specification

	LIML			2SLS		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dep. var.	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$
$d \ln P_{jt}$	0.521***	0.524***	0.531***	0.521***	0.524***	0.531***
	(0.080)	(0.079)	(0.077)	(0.092)	(0.092)	(0.062)
$d \ln z_{jt}$	-0.107***	-0.110***	-0.109***	-0.107***	-0.110***	-0.109***
	(0.010)	(0.010)	(0.010)	(0.021)	(0.021)	(0.005)
$d \ln \mathcal{P}_{-jt}$	0.373**	0.365***	0.401***	0.374***	0.365***	0.402***
	(0.109)	(0.110)	(0.117)	(0.089)	(0.087)	(0.098)
FE	year	year + sector	year×sector	year	year + sector	year×sector

Robustness – correlated shocks

- Different instruments exploit different sources of variation, hence potential sources of endogeneity are also different (e.g. Duranton and Turner (2012))

	(i)	(ii)	(iii)	(iv)	(v)
Dep. var.	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$	$d \ln p_{jt}$
$d \ln P_{jt}$	0.357**	0.522***	0.408*	0.532***	0.529***
	(0.122)	(0.063)	(0.205)	(0.064)	(0.064)
$d \ln z_{jt}$	-0.106***	-0.108***	-0.107***	-0.107***	-0.107***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)
$d \ln \mathcal{P}_{-jt}$	0.778**	0.353***	0.452**	0.371***	0.362***
	(0.269)	(0.091)	(0.158)	(0.090)	(0.091)
$d \ln P_{jt}^{TFP}$	Yes	Yes	Yes	Yes	
$d \ln P_{jt}^{PC}$	Yes	Yes	Yes		Yes
$d \ln P_{jt}^{CN}$	Yes	Yes		Yes	Yes
$d \ln \mathcal{P}_{-jt}^{PC}$	Yes		Yes	Yes	Yes
$d \ln \mathcal{P}_{-jt}^{CN}$		Yes	Yes	Yes	Yes
N	33,718	33,718	33,718	33,718	33,718

First stages

	Year fixed effects		Year + sector fixed effects		Year × sector fixed effects	
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dep. var.	$d \ln P_{jt}$	$d \ln \mathcal{P}_{-jt}$	$d \ln P_{jt}$	$d \ln \mathcal{P}_{-jt}$	$d \ln P_{jt}$	$d \ln \mathcal{P}_{-jt}$
$d \ln z_{jt}$	.010***	-.000	.010***	-.002**	.009***	-.002**
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
$d \ln P_{jt}^{TFP}$	-.084***	.027**	-.087***	.018*	-.087***	.015
	(.020)	(.008)	(.020)	(.008)	(.020)	(.008)
$d \ln P_{jt}^{PC}$	.673***	.220***	.671***	.207***	.653***	.165**
	(.068)	(.058)	(.068)	(.058)	(.068)	(.061)
$d \ln P_{jt}^{CN}$	.831***	.179***	.832***	.181***	.824***	.159***
	(.019)	(.014)	(.019)	(.014)	(.019)	(.014)
$d \ln \mathcal{P}_{-jt}^{PC}$	1.123***	1.605***	1.123***	1.513***	.762***	.727***
	(.171)	(.144)	(.171)	(.144)	(.188)	(.166)
$d \ln \mathcal{P}_{-jt}^{CN}$	.131***	.798***	.133***	.801***	.121***	.768***
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Extension – multi-product firms

- **Extension 1 – Model at firm-product level**

$$\ln p_{jkt} = \ln c_{jkt}\left(\left(1+\tau_{1j}\right)p_{1t}, \ldots, \left(1+\tau_{nj}\right)p_{nt}, z_{jt}\right) + \ln \mu_{jkt}\left(p_{jkt}, \mathcal{P}_{-jkt}; \xi_{jkt}\right)$$

  - Additional assumptions
  - A1: No physical synergies across products within producers
  - A2: Proportionality of inputs to outputs

- **Extension 2 – Model at firm level**
  - Output price index of j

$$d \ln \tilde{P}_{jt} \equiv \sum_{k} \varphi_{jkt} d \ln p_{jkt}$$

  where φ_{jkt} is revenue share of k for j

  - A3: Markup shocks are the same across products within firms

- **If assumptions do not hold, additional cross-elasticities bias structural estimates of price updating**

Extension – multi-product firms (firm-level)

	OLS			IV		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dep. var.	$d \ln \tilde{P}_{jt}$	$d \ln \tilde{P}_{jt}$	$d \ln \tilde{P}_{jt}$	$d \ln \tilde{P}_{jt}$	$d \ln \tilde{P}_{jt}$	$d \ln \tilde{P}_{jt}$
$d \ln P_{jt}$	0.257*	0.256*	0.253*	0.493***	0.496***	0.502***
	(0.063)	(0.063)	(0.062)	(0.059)	(0.059)	(0.059)
$d \ln z_{jt}$	-0.103*	-0.105*	-0.105*	-0.104***	-0.106***	-0.106***
	(0.023)	(0.023)	(0.023)	(0.004)	(0.004)	(0.004)
$d \ln \mathcal{P}_{-jt}$	0.336**	0.321**	0.318**	0.373***	0.363***	0.406***
	(0.049)	(0.046)	(0.047)	(0.085)	(0.086)	(0.093)
FE	year	year + sector	year × sector	year	year + sector	year × sector
N	33,787	33,787	33,787	33,718	33,718	33,718
J-test χ^2				3.99	3.67	4.84
[p-value]				[.26]	[.30]	[.18]