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Gradualism and liquidity traps

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Abstract

Modifying the objective function of a discretionary central bank to include an interest-rate smoothing objective increases the welfare of an economy where large contractionary shocks occasionally force the central bank to lower the policy rate to its effective lower bound. The central bank with an interest-rate smoothing objective credibly keeps the policy rate low for longer than the central bank with the standard objective function does. Through expectations, the temporary overheating of the economy associated with such low-for-long interest rate policy mitigates the declines in inflation and output when the lower bound constraint is binding. In a calibrated model, we find that the introduction of an interest-rate smoothing objective can reduce the welfare costs associated with the lower bound constraint by more than half.

Keywords: Gradualism, Inflation Targeting, Interest-Rate Smoothing, Liquidity Traps, Zero Lower Bound

JEL-Codes: E52, E61

Non-technical summary

Central banks typically tend to adjust the policy rate gradually, in a series of small steps. While there is likely a myriad of factors behind this partial adjustment in the policy rate, some evidence suggests that such observed inertia in the policy rate reflects central banks' deliberate desire to smooth the interest-rate path, in addition to what the intrinsic inertia in economic conditions calls for. Several theoretical studies argue that such desire by central banks for interest-rate smoothing can improve society's welfare in various environments.

In this paper, we revisit the desirability of interest-rate smoothing in an economy where large contractionary shocks occasionally force the central bank to lower the policy rate to the zero lower bound (ZLB). Monetary policy is delegated to a discretionary central bank whose objectives are stipulated by the benevolent government. Using a stochastic New Keynesian model we ask how modifying the central bank's standard objective function to include an interest-rate smoothing objective affects stabilization policy and society's welfare.

Our main finding is that adding an interest-rate smoothing objective to central banks' standard inflation and output gap stabilization objectives can go a long way in mitigating the adverse consequences of the ZLB constraint. In the aftermath of a deep recession involving a binding ZLB constraint, a gradualist central bank increases the policy rate more slowly than a central bank without an interest-rate smoothing objective would do. Such a slow increase of the policy rate generates a temporary overheating of the economy, which mitigates the declines in inflation and output while the ZLB constraint is binding by raising expectations of future inflation and real activity. A smaller contraction at the ZLB, in turn, alleviates the deflationary bias—the systematic undershooting of the inflation target—away from the ZLB via expectations. In equilibrium, interest-rate smoothing increases society's welfare by improving stabilization outcomes not only when the policy rate is at the ZLB, but also when the policy rate is away from it.

Interest-rate smoothing, however, does not provide a free lunch. In particular, interest-rate gradualism prevents the central bank from responding sufficiently strongly to less severe shocks that could be neutralized by an appropriate policy rate adjustment without hitting the ZLB. From a normative perspective, when the policy rate is away from the ZLB, the central bank should reduce the policy rate one-for-one to a downward shift in aggregate demand so as to completely offset the effect of the demand shock. A gradualist central bank will reduce the policy rate by less on impact, thus failing to keep inflation and the output gap fully stabilized. The optimal degree of interest-rate gradualism balances this cost against the aforementioned benefits. The welfare gains from interest-rate smoothing are quantitatively important. In our baseline calibration, a central bank with an optimized weight on its interest-rate smoothing objective improves society's welfare by more than 50 percent.

1 Introduction

As a general rule, the Federal Reserve tends to adjust interest rates incrementally, in a series of small or moderate steps in the same direction.

Ben S. Bernanke, on May 20, 2004

Gradual adjustment in the federal funds rate has been a key feature of monetary policy in the United States. Over the two decades prior to December 2008—the beginning of the most recent lower bound episode—the Federal Open Market Committee (FOMC) changed its target for the federal funds rate at 89 out of 191 meetings. At these 89 meetings, the FOMC adjusted the federal funds target rate on average by just 33 basis points in absolute terms. More recently, when announcing the first increase in its target range for the federal funds rate in December 2015 after seven years of zero-interest-rate policy, the FOMC emphasized that it expects the policy rate to increase only gradually (Federal Open Market Committee (2015)). Indeed, as of June 2016, the federal funds target range has remained unchanged since December 2015.

While there is likely a myriad of factors behind this gradual adjustment in the policy rate, some evidence suggests that such observed inertia in the policy rate reflects the central bank's deliberate desire to smooth the interest-rate path, in addition to what the intrinsic inertia in economic conditions calls for (Coibion and Gorodnichenko (2012); Givens (2012)). As reviewed below, several theoretical studies suggest that such a desire by central banks for interest-rate smoothing can improve society's welfare in various environments.

In this paper, we revisit the desirability of interest-rate smoothing in an economy where large contractionary shocks occasionally force the central bank to lower the policy rate to the zero lower bound (ZLB). Society designs the central bank's objective function. The central bank, in turn, acts under discretion and sets the policy rate in accordance with its objective.[1] Using a stochastic New Keynesian model, we ask how modifying the central bank's objective function to include an interest-rate smoothing objective affects stabilization policy and society's welfare, as measured by the expected lifetime utility of the representative household.

Our main finding is that adding an interest-rate smoothing objective to central banks' standard inflation and output gap stabilization objectives can go a long way in mitigating the adverse consequences of the ZLB constraint. In the aftermath of a deep recession involving a binding ZLB constraint, a gradualist central bank increases the policy rate more slowly than a central bank with the standard objective would do. Such a slow increase of the policy rate generates a temporary overheating of the economy, which mitigates the declines in inflation and output while the ZLB constraint is binding by raising expectations of future inflation and real activity. A smaller contraction at the ZLB, in turn, alleviates the deflationary bias—the systematic undershooting of the inflation target—away from the ZLB via expectations. In equilibrium, interest-rate smoothing

[1] Rogoff (1985), Persson and Tabellini (1993), Walsh (1995, 2003), and Svensson (1997) are prominent examples adopting the policy delegation approach to the design of the central bank's objective. See Persson and Tabellini (1999) for a literature review.

increases society's welfare by improving stabilization outcomes not only when the policy rate is at the ZLB, but also when the policy rate is away from it.

Interest-rate smoothing, however, does not provide a free lunch. In particular, interest-rate gradualism prevents the central bank from responding sufficiently strongly to less severe shocks that could be neutralized by an appropriate policy rate adjustment without hitting the ZLB. From a normative perspective, when the policy rate is away from the ZLB, the central bank should reduce the policy rate one-for-one to a downward shift in aggregate demand so as to completely offset the effect of the demand shock. A gradualist central bank will reduce the policy rate by less on impact, thus failing to keep inflation and the output gap fully stabilized.[2] The optimal degree of interest-rate gradualism balances this cost against the aforementioned benefits. The welfare gains from interest-rate smoothing are quantitatively important. In our baseline calibration, a central bank with an optimized weight on its interest-rate smoothing objective improves society's welfare by more than 50 percent.

We also explore two simple refinements to our baseline interest-rate smoothing objective function that enhance the welfare gains from interest-rate gradualism. The first refinement is to allow the central bank's preference for interest-rate smoothing to be asymmetric, making the central bank more averse to policy-rate increases than to policy-rate cuts. This refinement improves society's welfare by attenuating the disadvantages of the baseline interest-rate smoothing objective function described in the previous paragraph. The second refinement is to let the central bank be concerned with smoothing of the shadow policy rate—the policy rate that it would like to set given the current state of the economy if the ZLB were not a constraint for nominal interest rates. If the policymaker aims to smooth the shadow rate, the lagged shadow rate becomes an endogenous state variable that remembers the history of inflation rates and output gaps. In particular, the larger the economic downturn in a liquidity trap, the lower the shadow rate and the longer the actual policy rate remains low. This history dependence induced by the shadow interest rate is akin to that observed under optimal commitment policy or discretionary price-level targeting, and increases the welfare gains from interest-rate smoothing.

Our paper is related to a body of work that has examined various motives for gradualist monetary policy.[3] The strand of the literature closest to our paper emphasizes the benefit of interest-rate smoothing arising from its ability to steer private sector expectations by inducing history dependence in the policy rate (Woodford (2003b); Giannoni and Woodford (2003)). Another strand of the literature emphasizes the benefit of interest-rate smoothing arising from its ability to better manage uncertainties about data, parameter values or the structure of the economy facing the central bank, (Sack (1998); Orphanides and Williams (2002); Levin, Wieland, and Williams (2003); Orphanides and Williams (2007)). Yet another strand of the literature emphasizes the costs and

[2]Interest-rate gradualism also prevents the central bank from neutralizing expansionary demand shocks, thereby allowing for above-target inflation rates and output gaps. As described in Section 3.3, while such transitory overshootings are by themselves associated with lower welfare, they can be welfare-improving in an economy with an occasionally binding ZLB constraint, as they raise inflation and output gap expectations in states where aggregate demand is low.

[3]See Sack and Wieland (2000) for an early literature overview.

benefits of interest-rate smoothing arising from its effects on financial stability (Cukierman (1991) and Stein and Sunderam (2015)). None of these studies, however, accounts for the ZLB on nominal interest rates. Our contribution is to show that the presence of the ZLB provides a novel rationale for why monetary policy should be guided by gradualist principles.

Our work is also closely related to a set of papers that explores ways to mitigate the adverse consequences of the ZLB constraint while preserving time consistency. In particular, several approaches try to mimic the prescription of the optimal commitment policy for liquidity traps to keep the policy rate low for long and thus generating a temporary overheating of the economy. Eggertsson (2006) and Burgert and Schmidt (2014) show that in models with non-Ricardian fiscal policy and nominal government debt, discretionary policymakers can incentivize future policymakers to keep policy rates low for long by means of expansionary fiscal policy that raises the nominal level of government debt. Jeanne and Svensson (2007), Berriel and Mendes (2015), and Bhattarai, Eggertsson, and Gafarov (2015) find that central banks' balance-sheet policies can, under certain conditions, operate as a commitment device for discretionary policymakers that facilitates the use of low-for-longer policies. Finally, Billi (2016) explores policy delegation schemes where the discretionary central bank's standard inflation and output gap stabilization objectives are replaced by either a price-level or a nominal-income stabilization objective. He finds that these delegation schemes can generate low-for-long policies and thereby improve welfare.[4] Compared to these approaches, the relative appeal of our approach is that it neither requires an additional policy instrument nor does it represent a fundamental departure from the inflation-targeting framework currently embraced by many central banks.

The paper is organized as follows. Section 2 describes the model. Section 3 presents the main results on the effect of interest-rate smoothing. Section 4 explores two more sophisticated forms of interest-rate smoothing that help to further mitigate the welfare costs associated with the ZLB. Section 5 compares interest-rate gradualism with price-level targeting, and extends the baseline model to include both demand and supply shocks. A final section concludes.

2 The model

This section presents the model, lays down the policy problem of the central bank, and defines the equilibrium.

2.1 Private sector

The private sector of the economy is given by the standard New Keynesian structure formulated in discrete time with infinite horizon as developed in detail in Woodford (2003a) and Gali (2008). A continuum of identical infinitely-living households consumes a basket of differentiated goods and

[4] Nakata and Schmidt (2014) show that the appointment of an inflation-conservative central banker improves welfare by mitigating the deflationary bias associated with discretionary policy in the presence of the ZLB. However, an inflation-conservative central banker does not generate low-for-long policies.

supplies labor in a perfectly competitive labor market. The consumption goods are produced by firms using (industry-specific) labor. Firms maximize profits subject to staggered price-setting as in Calvo (1983). Following the majority of the literature on the ZLB, we put all model equations except for the ZLB constraint in semi-loglinear form.

The equilibrium conditions of the private sector are given by the following two equations

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} \tag{1}$$

$$y_t = E_t y_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t \right) \tag{2}$$

where π_t is the inflation rate between periods $t-1$ and t, y_t denotes the output gap, i_t is the level of the nominal interest rate between periods t and $t+1$ and r_t is the exogenous natural real rate of interest. Equation (1) is a standard New Keynesian Phillips curve and equation (2) is the consumption Euler equation. The parameters are defined as follows. $\beta \in (0,1)$ denotes the representative household's subjective discount factor, $\sigma > 0$ is the intertemporal elasticity of substitution in consumption, and κ represents the slope of the New Keynesian Phillips curve.[5]

The natural rate shock r_t is assumed to follow a stationary autoregressive process of order one

$$r_t = (1-\rho_r) r + \rho_r r_{t-1} + \epsilon_t^r, \tag{3}$$

where $r \equiv \frac{1}{\beta} - 1$ is the steady state level of the natural rate, $\rho_r \in [0,1)$ is the persistence parameter and ϵ_t^r is a *i.i.d.* $N(0, \sigma_r^2)$ innovation.

2.2 Society's welfare and the central bank's problem

Society's welfare is represented by a second-order approximation to the representative household's expected lifetime utility

$$V_t = u(\pi_t, y_t) + \beta E_t V_{t+1} \tag{4}$$

where

$$u(\pi, y) = -\frac{1}{2} \left(\pi^2 + \lambda y^2 \right) \tag{5}$$

Society's relative weight on output gap stabilization, λ, is a function of the structural parameters and is given by $\lambda = \frac{\kappa}{\epsilon}$.[6]

The value for the central bank with an interest-rate smoothing objective generically differs from society's welfare and is given by

[5] κ is related to the structural parameters of the economy as follows $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta(1+\eta\epsilon)} \left(\sigma^{-1} + \eta \right)$, where $\theta \in (0,1)$ denotes the share of firms that cannot reoptimize their price in a given period, $\eta > 0$ is the inverse of the elasticity of labor supply, and $\epsilon > 1$ denotes the price elasticity of demand for differentiated goods.

[6] See Woodford (2003a) and Gali (2008).

$$V_t^{CB} = u^{CB}(\pi_t, y_t, \Delta i_t) + \beta E_t V_{t+1}^{CB} \tag{6}$$

where $\Delta i_t = i_t - i_{t-1}$ denotes the change in the one-period nominal interest rate between periods $t-1$ and t. The central bank's contemporaneous objective function, $u^{CB}(\cdot,\cdot)$, is given by

$$u^{CB}(\pi, y, \Delta i) = -\frac{1}{2}\left[(1-\alpha)\left(\pi^2 + \lambda y^2\right) + \alpha \Delta i^2\right] \tag{7}$$

The last term, $\alpha \Delta i^2$, captures the interest-rate smoothing objective, and the parameter $\alpha \in [0,1]$ determines how the smoothing objective weighs against the central bank's inflation and output gap objectives. When $\alpha = 0$, then $u^{CB}(\cdot) = u(\cdot)$.

We assume that the central bank does not have a commitment technology. Each period t, the central bank chooses the inflation rate, the output gap, and the nominal interest rate to maximize its objective function subject to the behavioral constraints of the private sector, with the policy functions at time t+1 taken as given

$$V_t^{CB}(r_t, i_{t-1}) = \max_{\pi_t, y_t, i_t} \quad u^{CB}(\pi_t, y_t, \Delta i_t) + \beta E_t V_{t+1}^{CB}(r_{t+1}, i_t) \tag{8}$$

subject to the zero lower bound constraint

$$i_t \geq 0 \tag{9}$$

and the private sector equilibrium conditions (1) and (2) described above. *A Markov-Perfect equilibrium with an interest-rate smoothing objective* is defined as a set of time-invariant value and policy functions $\{V^{CB}(\cdot), \pi(\cdot), y(\cdot), i(\cdot)\}$ that solves the central bank's problem above together with society's value function $V(\cdot)$ that is consistent with $\pi(\cdot)$ and $y(\cdot)$.

Since units of welfare are not particularly meaningful, we express the social welfare of an economy in terms of the perpetual consumption transfer (as a share of its steady state) that would make the household in the artificial economy without any shocks indifferent to living in the stochastic economy. This is given by

$$W := (1-\beta)\frac{\epsilon}{\kappa}\left(\sigma^{-1} + \eta\right) E[V] \tag{10}$$

where the mathematical expectation is taken with respect to the unconditional distribution of r_t.[7]

2.3 Calibration and model solution

The structural parameters are calibrated using the parameter values from Eggertsson and Woodford (2003), as listed in Table 1, except for the interest rate elasticity σ which we set to 2.[8] We set the

[7]For a derivation of the expression for the welfare-equivalent consumption transfer, see, for instance, Billi (2016).
[8]Eggertsson and Woodford (2003) assume $\sigma = 0.5$.

persistence parameter capturing the law of motion of the natural real rate to 0.85. The standard deviation of the natural rate shock is set so that the probability of being at the ZLB is about 20 percent when the central bank puts no weight on the interest-rate smoothing objective ($\alpha = 0$).

Table 1: Calibration

Parameter	Value	Economic interpretation
β	0.99	Subjective discount factor
σ	2	Intertemporal elasticity of substitution in consumption
η	0.47	Inverse labor supply elasticity
ϵ	10	Price elasticity of demand
θ	0.8106	Share of firms per period keeping prices unchanged
ρ_r	0.85	AR coefficient natural real rate
σ_r	0.4	Standard deviation natural real rate shock

To solve the model, we approximate the policy functions using a projection method. The details of the solution algorithm are described in Appendix B.

3 Results

This section analyzes how the introduction of the interest-rate smoothing objective affects the dynamics of the economy and welfare. We first describe how society's welfare depends on the degree of interest-rate gradualism, captured by α, and then analyze how the interest-rate smoothing objective affects the dynamics of the economy to understand the key forces behind the welfare result.

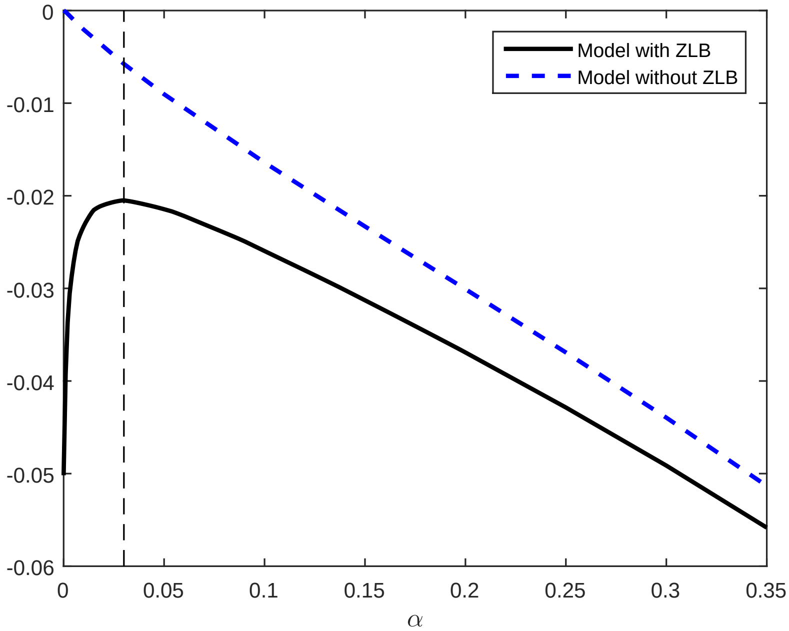
3.1 Welfare effects of policy gradualism

Figure 1 plots the social welfare measure as defined in equation (10) for alternative values of α over $\alpha \in [0, 0.35]$.[9] The black solid line indicates welfare outcomes when accounting for the ZLB whereas the blue dashed line indicates welfare when ignoring the ZLB. In the model without the ZLB, welfare declines monotonically with the degree of interest-rate smoothing α and it is optimal for society if the central bank focuses only on inflation and output gap stabilization. The reason for why welfare declines with interest-rate gradualism is straightforward: The central bank can completely absorb any shock to the natural real rate of interest by setting the policy rate such that in equilibrium the actual real interest rate equals the natural real rate at each point in time. Indeed, if the central bank is not concerned with interest-rate smoothing, the central bank can completely stabilize output and inflation—in other words, the so-called divine coincidence holds—and welfare is at its maximum value.

The welfare effects of interest-rate gradualism change markedly once we account for the ZLB constraint. In the model with the ZLB, welfare depends on the degree of interest-rate smoothing in

[9] For each candidate we conduct 2000 simulations, each consisting of 1050 periods where the first 50 periods are discarded as burn-in periods.

Figure 1: Welfare effects of interest-rate smoothing



Note: The figure shows how welfare as defined in (10) varies with the relative weight α on the interest-rate smoothing objective. The vertical dashed black line indicates the optimal relative weight on the interest-rate smoothing objective in the model with ZLB.

a non-monotonic way; it initially increases with the degree of policy gradualism α before starting to decrease. Under our baseline calibration, the optimal weight on the interest-rate smoothing term is $\alpha = 0.030$, as indicated by the vertical dashed line. Welfare can be lower than under the standard objective function ($\alpha = 0$) when the degree of interest-rate smoothing is sufficiently high, which happens in our model for values of α larger than 0.3.

The welfare effects of interest-rate smoothing are quantitatively important. According to Table 2, modifying the objective function of a central bank acting under discretion to include an interest-rate smoothing objective with a relative weight of 0.03 reduces the welfare costs associated with the presence of the ZLB constraint by more than half (-2.05 in the first row versus -5.02 in the second row).

While the stabilization performance of optimized interest-rate smoothing falls short of the optimal plan under commitment—shown by the third row—this welfare improvement due to interest-rate gradualism is significant. In Section 4, we consider some refinements of the interest-rate smoothing objective function that bring the optimal discretionary policy closer to the optimal commitment policy.

3.2 Why some degree of gradualism is desirable

To understand the benefits of interest-rate smoothing in the model with the ZLB, we consider the following liquidity trap scenario. The economy is initially in the risky steady state. In period 0,

Table 2: Results

Regime	Optimal α	Welfare ($W \times 100$)	ZLB frequency (in %)
Interest-rate smoothing	0.03	-2.05	4
Standard discretion	-	-5.02	22
Commitment	-	-0.23	13
Asymmetric smoothing	0.013	-1.75	4
Shadow-rate smoothing	0.0165	-1.11	7
Price level targeting	1	-0.30	17

Note: The welfare measure is defined in equation (10).

the natural real rate of interest falls into negative territory and stays at the new level for 3 quarters before jumping back to its steady state level. At each point in time, households and firms account for the uncertainty regarding the future path of the natural real rate in making their decisions. The considered scenario is arguably rather extreme given the assumed autoregressive process for the natural real rate but is useful in cleanly illustrating the implications of the interest-rate smoothing objective for monetary policy and stabilization outcomes.

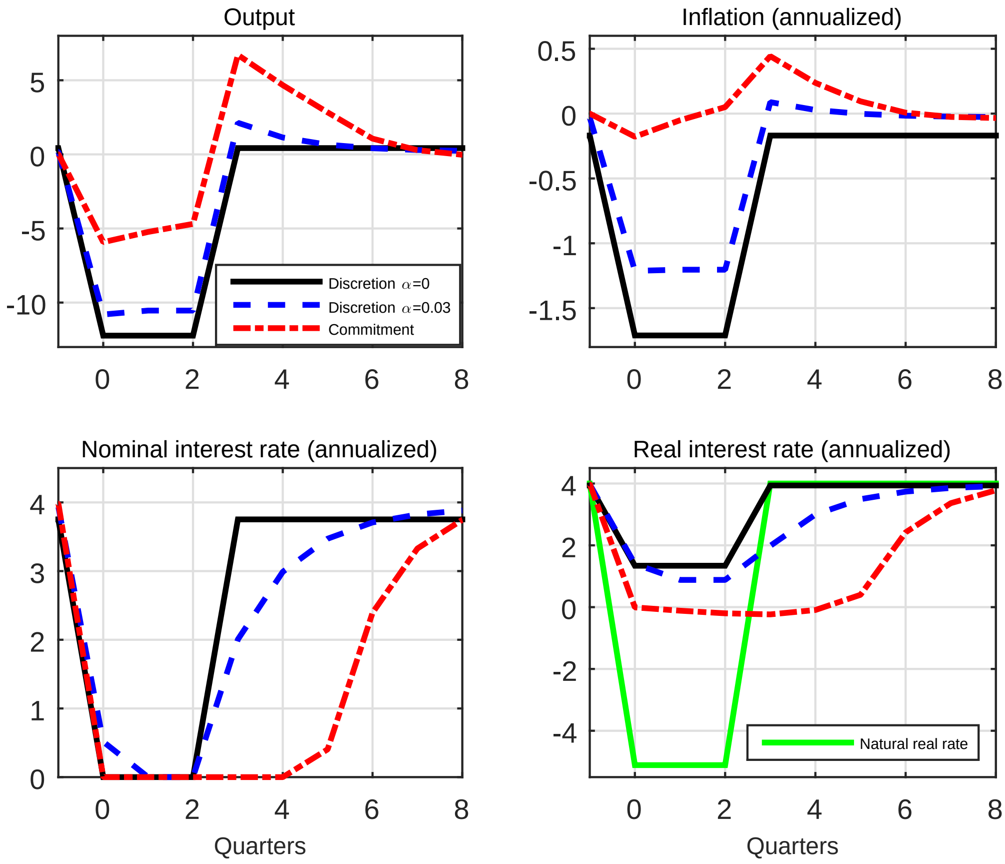
Figure 2 plots the dynamics of the economy in this experiment for three regimes; the standard discretionary regime without an interest-rate smoothing objective (solid black lines), the augmented discretionary regime with an optimally-weighted objective for policy gradualism of $\alpha = 0.03$ (dashed blue lines), and the optimal commitment policy (dash-dotted red lines). The exogenous path of the natural real rate is shown in the lower-right chart (solid green line).

Under the standard discretionary regime, the central bank immediately lowers the nominal interest rate to zero. The real interest rate stays strictly positive, leading to large declines in output and inflation: Output and inflation drop by 12.2 and 1.7 percent, respectively. When the economy exits the liquidity trap in period 3, the nominal interest rate is raised immediately to its risky steady-state level, and the real interest rate closely tracks the natural rate.

Now, consider the interest-rate smoothing regime. Due to its desire for a gradual adjustment in the policy rate, the central bank refrains from lowering the policy rate immediately all the way to zero in period 0. Nevertheless, the declines in output and inflation are smaller (10.5 and 1.2 percent, respectively) than under the standard discretionary regime. In period 1, the policy rate reaches the ZLB and the real interest rate declines further. At the same time, output and inflation slightly rise beyond their previous period's troughs. Upon exiting the liquidity trap in period 3, the policy rate is raised only gradually, resulting in a temporarily negative real rate gap—that is, a real interest rate that is below its natural rate counterpart. This negative real rate gap boosts output and inflation above their longer-run targets. At period 4, output and inflation are at 2.1 and 0.1 percent, respectively. Since households and firms are forward-looking, the anticipated temporary overheating of the economy leads to less deflation and smaller output losses at the outset of the liquidity trap event compared to the standard discretionary regime.

The history dependence just described manifests itself in one of the optimality conditions of the

Figure 2: Liquidity trap scenario



Note: In the considered liquidity trap scenario the economy is initially in the risky steady state. In period 0, the natural real rate falls into negative territory and stays at the new level for 3 quarters before jumping back to its steady state level.

gradualist central bank's maximization problem:

$$
\begin{aligned}
0 =& \alpha(1+\beta)i_t - \alpha i_{t-1} - \beta\alpha E_t i(r_{t+1}, i_t) \\
&+\beta(1-\alpha)\frac{\partial E_t \pi(r_{t+1}, i_t)}{\partial i_t}\pi_t - (1-\alpha)\left(\frac{\partial E_t y(r_{t+1}, i_t)}{\partial i_t} + \sigma\frac{\partial E_t \pi(r_{t+1}, i_t)}{\partial i_t}\right)(\lambda y_t + \kappa\pi_t) \\
&-(1-\alpha)\sigma(\lambda y_t + \kappa\pi_t) - \phi_t^{ZLB}
\end{aligned}
\tag{11}
$$

The optimality condition shows that for given economic conditions, a gradualist central bank aims to set the contemporaneous policy rate such that the deviations from the lagged policy rate as well as from the expected future policy rate are small in equilibrium. Notice that if $\alpha = 0$, i.e. if the central bank has no smoothing objective, then the right-hand-side terms in the first two rows of equation (11) vanish and the equation reduces to the familiar static target criterion (accounting for the ZLB) of the standard discretionary regime.[10]

The policy of keeping the interest rate low for long under gradualism is shared by the optimal commitment policy. Under the commitment policy, the central bank lowers the policy rate immediately all the way to zero and keeps the policy rate at the ZLB even after the natural rate becomes positive. The promise of an extended period of holding the policy rate at the ZLB leads to an even larger overshooting of inflation and the output gap than observed under the gradualist central bank, which in turn results in smaller deflation and output losses during the crisis period.

The benefit of interest-rate gradualism—smaller declines in inflation and output at the ZLB—spills over to the stabilization outcomes when the policy rate is away from the ZLB through expectations. As described in detail in Nakata and Schmidt (2014) and Hills, Nakata, and Schmidt (2016), the standard discretionary regime fails to fully stabilize inflation and output even at the risky steady state—where the policy rate is comfortably above the ZLB—due to the asymmetry in the distribution of future inflation and output induced by the possibility of returning to the ZLB. For our calibration, under the standard discretionary regime, the inflation rate is -0.17 and the output gap is 0.42 at the risky steady state.[11] Since the decline in inflation at the ZLB is smaller under the interest-rate smoothing regime than under the standard discretionary regime, the distribution is less asymmetric and inflation and output away from the ZLB are better stabilized under interest-rate gradualism. With the optimized interest-rate smoothing weight, the inflation rate and the output gap are -0.03 and 0.18 at the risky steady state. Thus, interest-rate smoothing improves stabilization outcomes not only at the ZLB, but also at the risky steady state.

3.3 Why too much gradualism is undesirable

While the introduction of an interest rate smoothing objective is welfare-improving for a wide range of weights α, we have seen that putting too much weight on the smoothing objective delivers lower welfare than the central bank with the standard objective function ($\alpha = 0$) does, see Figure 1. This section takes a closer look at the costs associated with excessive interest-rate gradualism.

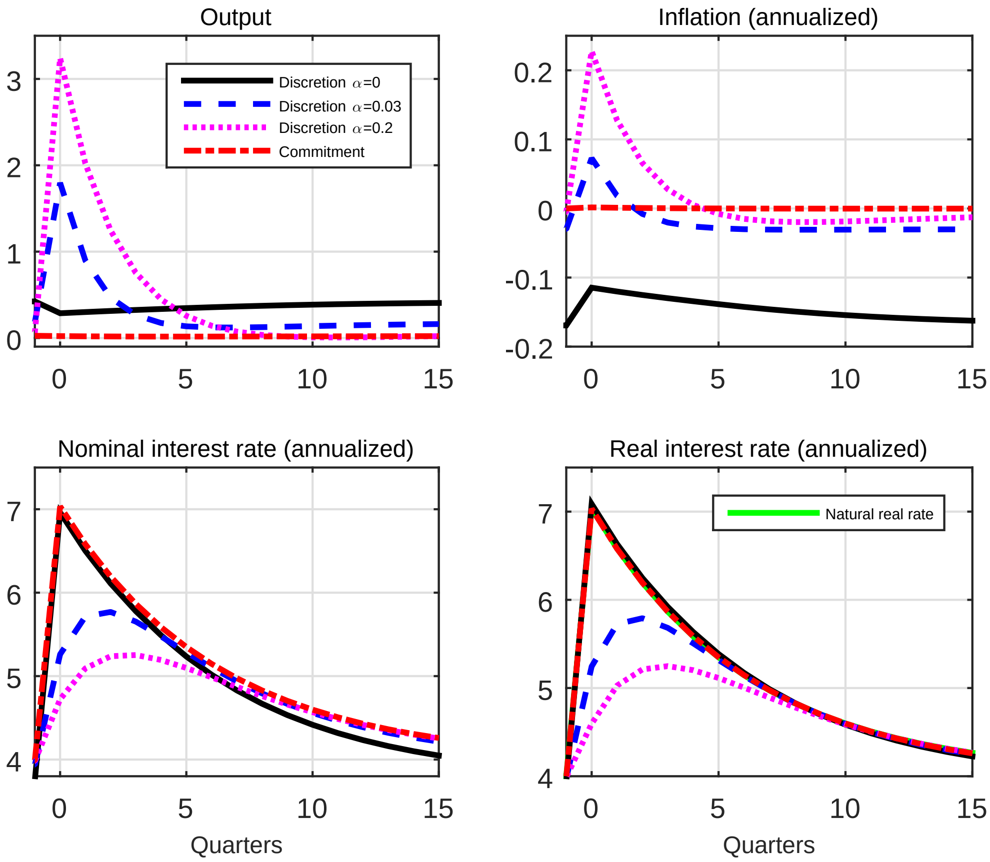
Figure 3 shows impulse responses to a natural real rate shock of one unconditional standard deviation when the economy is initially at the risky steady state for the three regimes considered before as well as for an interest-rate smoothing regime with a higher-than-optimal weight on the gradualism objective, $\alpha = 0.2$ (dotted magenta lines).

Under the standard discretionary regime, the central bank raises the policy rate such that the real interest rate closely tracks the path of the natural real rate, making the latter hardly visible in the lower-right chart. The larger buffer against hitting the ZLB slightly mitigates the downward bias in expected output and inflation, which attenuates the stabilization trade-off for the central bank. Output and inflation move closer to their target levels so long as the shock prevails, albeit

[10]The second row on the right-hand side of equation (11) vanishes if $\alpha = 0$ because the nominal interest rate ceases to be a state variable and hence the partial derivative terms become zero.

[11]The welfare costs associated with this stabilization shortfall are non-negligible. If we take the welfare loss of an economy that stays permanently in the risky steady state associated with the standard discretionary regime as a proxy, they make up 24% of the overall welfare costs.

Figure 3: Impulse responses to a positive natural rate shock



Note: In the considered scenario, the economy is initially in the risky steady state. In period 0, the natural real rate increases by one unconditional standard deviation. The shock recedes in subsequent periods according to its law of motion.

by a small amount.

Under the two interest-rate smoothing regimes—one with the optimal weight and the other with a higher-than-optimal weight—the central bank raises the nominal interest rate only sluggishly so that the path of the real interest rate is temporarily below that of the natural rate. This more accommodative monetary policy stance stimulates output and inflation, and both variables overshoot their targets for a few quarters. The larger the weight on the smoothing objective, the more gradually interest rates respond and the larger the positive deviations of output and inflation from target. Such overshooting, while costly in terms of contemporaneous utility flows, has the desirable effect that it increases inflation expectations in states where the ZLB constraint is binding, since rational agents take into account how the central bank responds to shocks in the future when forming expectations. However, in the case of too much gradualism, the welfare costs of these target overshootings outweigh the gains from improved expectations. The discretionary regime with the optimized weight on the smoothing objective optimally trades the gains from gradual policy rate adjustments off against these costs.

Before closing this section, it is interesting to observe that in this experiment, away from the ZLB, the interest-rate response under the optimal commitment policy is very similar to the one under the standard discretionary regime. Thus, contrary to the casual impression one might get from the liquidity trap scenario, policy inertia is not a generic feature of the optimal commitment policy. Under both, the standard discretionary policy as well as the optimal commitment policy, the central bank wants to adjust the policy rate to neutralize the effects of demand shocks. If there is a sudden change in aggregate demand, both types of policy regimes will adjust the policy rate instantaneously.

4 Two sophisticated interest-rate smoothing regimes

In this section, we consider two refinements of the interest-rate smoothing regime that further increase the welfare gains from interest-rate gradualism. The first refinement consists of replacing the symmetric interest-rate smoothing objective with an asymmetric objective that is more averse to policy-rate increases than to policy-rate cuts. The second refinement consists of smoothing the path of the actual policy rate with respect to the lagged shadow policy rate—the policy rate that the central bank would like to set given current economic conditions if it had not been constrained by the ZLB—as opposed to the actual lagged policy rate.

4.1 Asymmetric interest-rate smoothing

The first refinement aims to improve welfare by eliminating an undesirable feature of interest-rate gradualism under the liquidity trap scenario considered in Section 3.2, namely that the policy rate fails to decline to the ZLB at the time the negative demand shock hits the economy. Under the first refinement featuring an asymmetric interest-rate smoothing (AIRS) objective, the central bank dislikes changes in the policy rate, but dislikes interest-rate increases more than interest-rate cuts. The value of the central bank with an AIRS objective is given by

$$V_t^{CB,AIRS} = u^{CB,AIRS}(\pi_t, y_t, \Delta i_t) + \beta E_t V_{t+1}^{CB,AIRS} \tag{12}$$

where the central bank's contemporaneous objective function, $u^{CB,AIRS}(\cdot,\cdot,\cdot)$, takes the following form

$$u^{CB,AIRS}(\pi_t, y_t, \Delta i_t) = -\frac{1}{2}\left[(1-\alpha)\left(\pi_t^2 + \lambda y_t^2\right) + \alpha\left(\Delta i_t^2 + \max(\Delta i_t, 0)\right)\right] \tag{13}$$

$\max(\Delta i_t, 0) = 0$ if today's interest rate is lower than last period's, while $\max(\Delta i_t, 0) = \Delta i_t \geq 0$ otherwise. That is, interest-rate increases are more expensive in the eyes of the central bank than equivalent interest-rate reductions.

The optimization problem of the central bank with an AIRS objective and the associated Markov-Perfect equilibrium are defined similarly to those with the standard IRS objective. They are relegated to the Appendix for the sake of brevity.

4.2 Shadow interest-rate smoothing

The second refinement—featuring a shadow interest-rate smoothing (SIRS) objective—aims to improve welfare by enhancing the key benefit of interest-rate gradualism, which is its ability to keep the policy rate low for long in the aftermath of a recession. The shadow interest rate keeps track of the severity of the recession and makes the period for which the policy rate is kept at the ZLB depend on the severity of the recession. The value of the central bank with a SIRS objective is given by

$$V_t^{CB,SIRS} = u^{CB,SIRS}(\pi_t, y_t, i_t, i^*_{t-1}) + \beta E_t V_{t+1}^{CB,SIRS} \tag{14}$$

where the central bank's contemporaneous objective function, $u^{CB,SIRS}(\cdot,\cdot,\cdot,\cdot)$, is given by

$$u^{CB,SIRS}(\pi_t, y_t, i_t, i^*_{t-1}) = -\frac{1}{2}\left[(1-\alpha)\left(\pi_t^2 + \lambda y_t^2\right) + \alpha(i_t - i^*_{t-1})^2\right] \tag{15}$$

Each period t, the central bank with a shadow interest-rate smoothing objective first chooses the shadow nominal interest rate in order to maximize the value today subject to the behavioral constraints of the private sector, with the value and policy functions at time $t+1$—$V_{t+1}^{CB,SIRS}(\cdot,\cdot)$, $y_{t+1}(\cdot,\cdot)$, $\pi_{t+1}(\cdot,\cdot)$—taken as given:

$$i_t^* = \text{argmax}_x \qquad u^{CB,SIRS}(\pi(x), y(x), x, i^*_{t-1}) + \beta E_t V_{t+1}^{CB,SIRS}(r_{t+1}, x) \tag{16}$$

with

$$\begin{aligned} y(x) =& E_t y_{t+1}(r_{t+1}, x) - \sigma(x - E_t\pi_{t+1}(r_{t+1}, x) - r_t) \\ \pi(x) =& \kappa y(x) + \beta E_t \pi_{t+1}(r_{t+1}, x) \end{aligned} \tag{17}$$

The actual policy rate i_t is given by

$$i_t = \max(i_t^*, 0) \tag{18}$$

That is, the actual policy rate today is zero when $i_t^* < 0$, and it is equal to i_t^* when $i_t^* \geq 0$.

The central bank's value today is given by

$$V_t^{CB,SIRS}(r_t, i^*_{t-1}) = u^{CB,SIRS}(\pi_t, y_t, i_t, i^*_{t-1}) + \beta E_t V_{t+1}^{CB,SIRS}(r_{t+1}, i_t^*) \tag{19}$$

where inflation and the output gap are given by

$$\begin{aligned} y_t =& E_t y_{t+1}(r_{t+1}, i_t^*) - \sigma(i_t - E_t\pi_{t+1}(r_{t+1}, i_t^*) - r_t) \\ \pi_t =& \kappa y_t + \beta E_t \pi_{t+1}(r_{t+1}, i_t^*) \end{aligned}$$

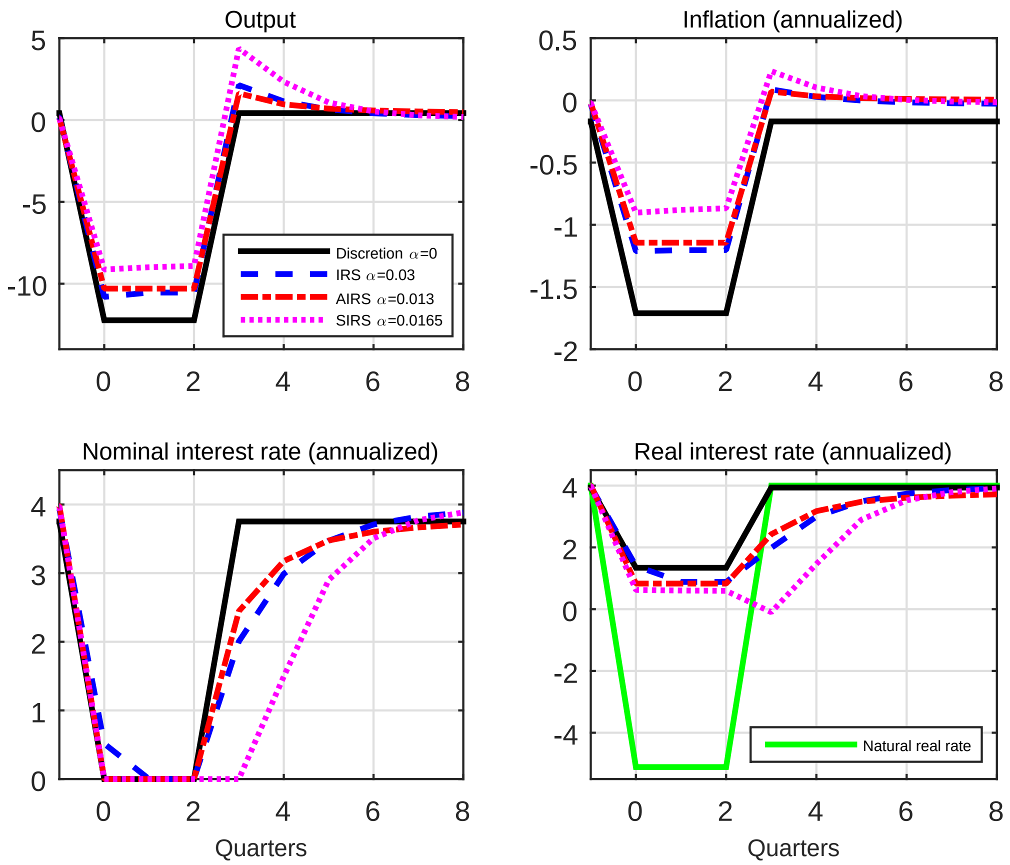
The definition of the Markov-Perfect equilibrium with a SIRS is similar to that with the standard interest-rate smoothing (IRS) objective, and is relegated to the Appendix.

4.3 Results for the sophisticated interest-rate smoothing regimes

The fourth and fifth row of Table 2 report the optimal weights, welfare, and the ZLB frequencies for the AIRS and SIRS regimes, respectively.[12] Under both regimes, the optimal relative weight on the interest-rate gradualism objective in the central bank's objective function is considerably smaller than under the standard IRS regime while welfare is higher than under the standard IRS regime. Relative to welfare under the standard IRS regime, welfare is about 15% higher in case of the AIRS regime and 46% higher in case of the SIRS regime.

Figure 4 compares the dynamics of the economy under the two refined interest-rate smoothing regimes with those under the standard IRS regime and the discretionary regime with zero weight on the interest-rate smoothing objective in the context of the liquidity trap scenario of Section 3.2. As a result of the lower optimized weights on the interest-rate smoothing objective, and in contrast

Figure 4: Liquidity trap scenario: Two refined interest-rate smoothing regimes



Note: In the considered liquidity trap scenario the economy is initially in the risky steady state. In period 0, the natural real rate falls into negative territory and stays at the new level for 3 quarters before jumping back to its steady state level.

to standard IRS, both sophisticated regimes lower the policy rate immediately to its lower bound

[12] The AIRS regime is solved using value function iteration.

when the shock buffets the economy. The SIRS regime also raises the policy rate more slowly when the shock has receded, leading to a more accommodative real interest-rate path. The economic boom upon exiting the liquidity trap is therefore larger under the SIRS regime than under the IRS regime, and as a result the drop in the inflation rate and the output gap during the liquidity trap is smaller. Under the AIRS regime, output and inflation also decline by less than under the IRS regime but the gap is less pronounced than under SIRS.

A key difference between the SIRS and the other two interest-rate smoothing frameworks lies in the endogenous state variable. In the IRS and AIRS regimes, the endogenous state variable is the actual policy rate i_t, while it is the shadow interest rate i_t^* in the SIRS regime. Unlike the actual policy rate, the shadow interest rate can go below zero. This unconstrained nature of the shadow rate has two important implications for interest rate policy. The first implication is that, in the face of large contractionary shocks the policy rate is lowered more aggressively than under standard IRS. This reflects the fact that the shadow rate is anticipated to enter negative territory whereas in the standard IRS regime the policy rate is anticipated not to fall below zero. Since the SIRS regime smooths the shadow rate path, the shadow rate declines faster than the policy rate in the standard IRS regime does. The policy rate path in the SIRS regime then simply mimics the shadow rate path subject to the ZLB constraint.

The second implication is that, as large contractionary shocks dissipate, the policy rate is kept at the ZLB for a longer period under the SIRS regime than under the standard IRS or AIRS regime. The shadow rate remembers the severity of the recession: The larger the downturn, the lower the shadow rate. As the policy rate follows the shadow rate path subject to the ZLB constraint, a larger downturn thus leads to a lower path of interest rates under the SIRS regime, akin to the optimal commitment policy.[13] In contrast, under conventional interest-rate smoothing, history dependence operates via the nominal interest rate, which has a lower bound of zero. Thus, once the ZLB is reached, a further decline in the natural rate has no direct implications for the size of the subsequent monetary stimulus.

5 Additional results

The first part of this section compares our standard symmetric interest-rate smoothing regime to a price-level targeting regime. In the second part of this section, we assess the desirability of interest-rate smoothing in an economy that is subject to both demand and supply shocks.

[13]The conduct of interest-rate policy under SIRS also has similarities with that observed when the interest rate is set according to a truncated inertial Taylor rule with a lagged shadow policy rate (considered in Hills and Nakata (2014), Hills, Nakata, and Schmidt (2016), Gust, Lopez-Salido, and Smith (2012)) or a Reifschneider-Williams (2000) rule (Reifschneider and Williams (2000)). Under these policy rules, how long the central bank keeps the policy rate at the ZLB also depends on the severity of the recession.

5.1 Comparison to price-level targeting

This section assesses how interest-rate smoothing performs relative to price-level targeting. Eggertsson and Woodford (2003) have shown that an output-gap-adjusted targeting rule for the price level performs almost as well as the optimal commitment policy. In order to facilitate the comparison with the interest-rate smoothing regime, we consider price-level targeting in the form of an amendment to the standard central bank objective function.

Under a (partial) price-level targeting regime, the central bank's contemporaneous objective function is given by

$$u^{CB,PLT}(\pi_t, y_t, p_t) = -\frac{1}{2}\left[(1-\alpha)\left(\pi_t^2 + \lambda y_t^2\right) + \alpha p_t^2\right] \tag{20}$$

where $p_t \equiv p_{t-1} + \pi_t$ denotes the price level in period t. If $\alpha = 1$, then the central bank follows a strict price-level targeting regime.

Each period t, the central bank chooses the price level, the output gap, and the nominal interest rate in order to maximize its objective function subject to the behavioral constraints of the private sector and the ZLB constraint, taking the policy functions at time t+1 as given

$$V_t^{CB,PLT}(r_t, p_{t-1}) = \max_{p_t, y_t, i_t} \quad u^{CB,PLT}(\pi_t, y_t, p_t) + \beta E_t V_{t+1}^{CB,PLT}(r_{t+1}, p_t) \tag{21}$$

The details of the optimization problem are relegated to Appendix A.4.

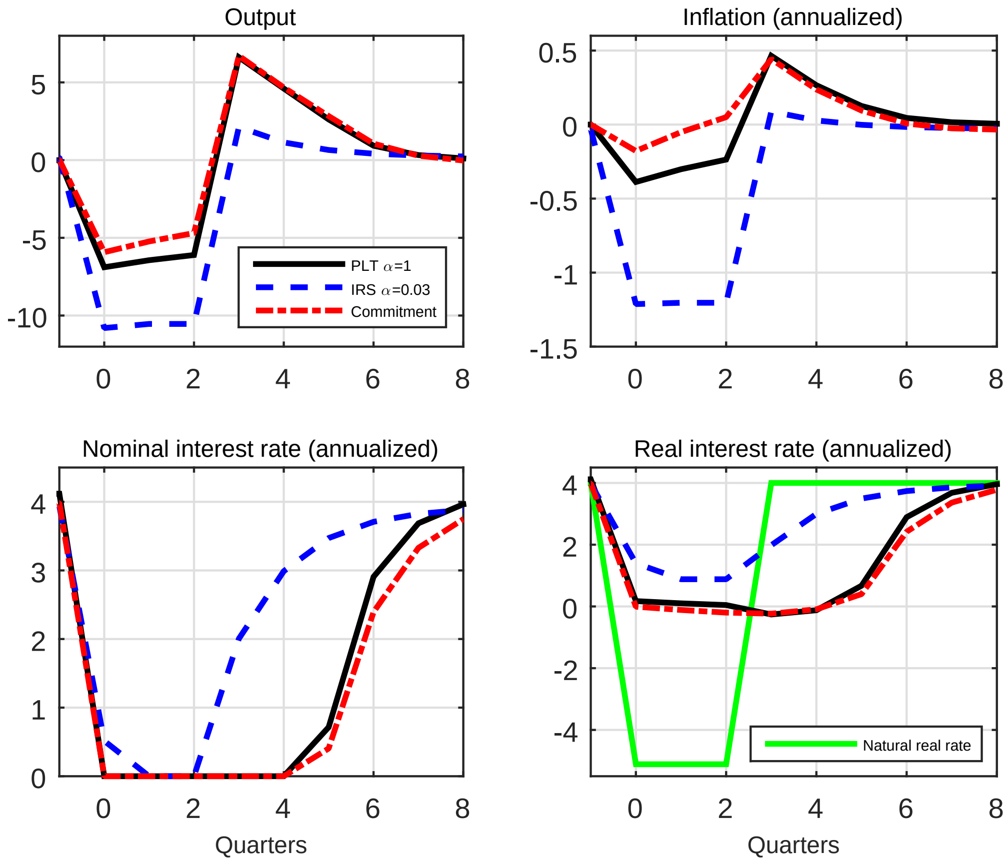
A grid search over $\alpha \in [0,1]$ reveals that society's welfare is maximized under strict price-level targeting, $\alpha = 1$, as shown in the sixth row of Table 2. According to the table, the welfare loss under price-level targeting is comparable to that under optimal commitment policy, and is substantially smaller than the loss observed under the interest-rate smoothing regime.

Figure 5 compares the evolution of the economy in the same liquidity trap scenario as in Section 3.2 under strict price-level targeting (solid black lines) with those under the optimized interest-rate smoothing regime (dashed blue lines) and the optimal commitment policy (dash-dotted red lines). The dynamics of the four variables shown in the figure under price-level targeting are closer to those under the optimal commitment policy than to those under the interest-rate smoothing policy, in line with the analysis by Eggertsson and Woodford (2003).

There are two advantages of price-level targeting over interest-rate smoothing. The first advantage is that, under price-level targeting, the temporary monetary stimulus upon exiting a liquidity trap is a function of the cumulated amount of inflation shortfalls during the trap, as under the shadow interest-rate smoothing regime. The larger the decline in the natural real rate, the larger the anticipated overshooting of inflation needed to meet the price-level target. As a result, the policy rate is kept lower for longer under strict price-level targeting than under interest-rate smoothing, as shown in the bottom-left panel. The more accommodative policy leads to a larger overshooting of inflation and output, which mitigates their initial declines at the ZLB, as shown in the two top panels.

The second advantage is that, away from the ZLB constraint, price-level targeting is able to

Figure 5: Liquidity trap scenario: Comparison with price-level targeting



Note: In the considered liquidity trap scenario the economy is initially in the risky steady state. In period 0, the natural real rate falls into negative territory and stays at the new level for 3 quarters before jumping back to its steady state level.

fully stabilize inflation and the output gap in our baseline model with only demand shocks. In contrast, as previously discussed in Section 3.3, a gradualist central bank is slow in adjusting the policy rate in response to expansionary shocks or small contractionary shocks, allowing inflation and the output gap to deviate from their long-run targets.

5.2 A model with demand and supply shocks

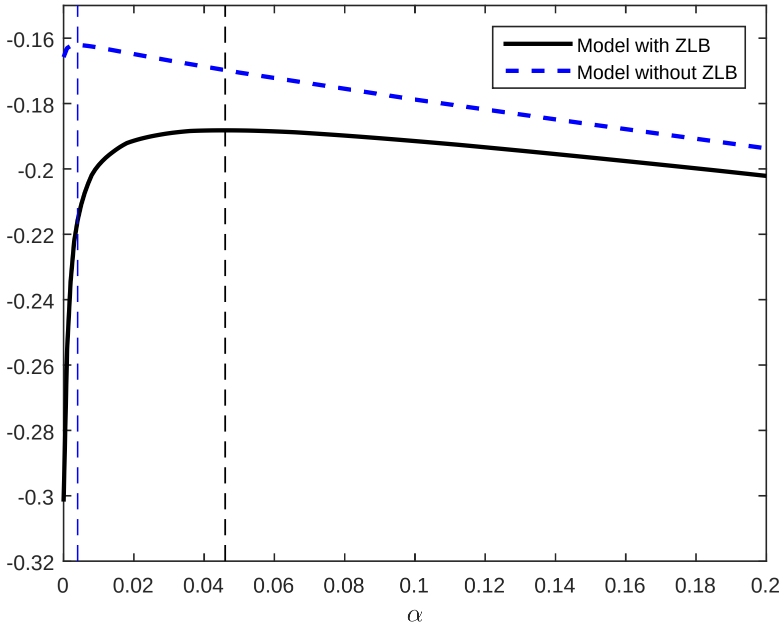
In our baseline model, the only exogenous shock is a demand shock. We now extend the analysis to an economy that is subject to both demand and supply shocks. Specifically, the New Keynesian Phillips curve is augmented with a price mark-up shock

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} + u_t \tag{22}$$

where u_t is $i.i.d.$ $N(0, \sigma_u^2)$. The remainder of the model structure stays the same as in Section 2. We set $\sigma_u = 0.17$ as estimated by Ireland (2011) for the U.S. economy.

Figure 6 plots the social welfare measure as defined in equation (10) for alternative values of $\alpha \in [0, 0.2]$.[14] The black solid line indicates welfare outcomes when accounting for the ZLB whereas the blue dashed line indicates welfare when ignoring the ZLB.

Figure 6: Welfare effects of interest-rate smoothing: Model with demand and supply shocks



Note: The figure shows how welfare as defined in (10) varies with the relative weight α on the interest-rate smoothing objective. The vertical black dashed line indicates the optimal weight on the smoothing objective in the model with ZLB and the vertical blue dashed line indicated the optimal weight in the model without the ZLB.

In the presence of supply shocks, the optimal degree of interest-rate smoothing is no longer zero even when one ignores the ZLB constraint. This result arises because the optimal time-inconsistent response to a cost-push shock creates endogenous persistence in the inflation rate and the output gap. If the economy is buffeted by a transitory inflationary cost-push shock, the optimal commitment policy is to raise the policy rate above steady state for more than one period in order to undershoot the inflation target in the second period. Such a response improves the trade-off between inflation and output gap stabilization in the period when the shock hits the economy through the expectations channel (see, for instance, Gali (2008)). Putting a small positive weight on the interest-rate smoothing objective allows a discretionary central bank to mimic the gradual response of the optimal commitment policy to price mark-up shocks.

As in our baseline model that is exposed to demand shocks only, the presence of the ZLB increases the optimal degree of interest-rate smoothing. In the model with the ZLB, the optimal weight is $\alpha = 0.046$, as indicated by the vertical dotted line, versus $\alpha = 0.004$ in the model without the ZLB. Reflecting the additional benefit of interest-rate smoothing arising from the presence

[14]For each candidate we conduct 2000 simulations, each consisting of 1050 periods where the first 50 periods are discarded as burn-in periods.

of cost-push shocks in the model with the ZLB, this optimal weight is larger than that in the model with demand shocks only, which is 0.03 as shown in Figure 1. As before, the welfare gains from interest-rate smoothing are quantitatively important. At the optimal weight $\alpha = 0.046$, the welfare costs are more than one-third smaller than under the standard discretionary monetary policy regime.

6 Conclusion

Our analysis provides a novel rationale for policy rate gradualism. In a liquidity trap, a gradualist central bank keeps the policy rate low for longer than is warranted by the dynamics of output and inflation alone, mimicking a key feature of the optimal commitment policy. This low-for-longer policy creates a transitory boom in future inflation and output, which dampens the declines of inflation and real activity during the liquidity trap via expectations.

A discretionary central bank that is only concerned with output and inflation stabilization will find itself unable to credibly commit to keep the policy rate low, for it has an incentive to renege on its past promise and increase the policy rate once the liquidity-trap conditions recede. However, modifying the objective function of a discretionary central bank to include an interest-rate smoothing objective allows society to make low-for-longer policies credible. An optimally chosen weight on the interest-rate smoothing objective relative to the central bank's objectives for inflation and output stabilization leads to a significant improvement in society's welfare even though society itself is not intrinsically concerned with the stabilization of changes in the policy rate.

References

Berriel, T., and A. Mendes (2015): "Central Bank Balance Sheet, Liquidity Trap, and Quantitative Easing," Working Paper.

Bhattarai, S., G. B. Eggertsson, and B. Gafarov (2015): "Time Consistency and the Duration of Government Debt: A Signalling Theory of Quantitative Easing," NBER Working Papers 21336, National Bureau of Economic Research, Inc.

Billi, R. M. (2016): "A note on nominal GDP targeting and the zero lower bound," *Macroeconomic Dynamics*, pp. 1–20.

Burgert, M., and S. Schmidt (2014): "Dealing with a Liquidity Trap when Government Debt Matters: Optimal Time-Consistent Monetary and Fiscal Policy," *Journal of Economic Dynamics and Control*, 47, 282 – 299.

Calvo, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.

Coibion, O., and Y. Gorodnichenko (2012): "Why Are Target Interest Rate Changes so Persistent?," *American Economic Journal: Macroeconomics*, 4(4), 126–162.

Cukierman, A. (1991): "Why Does the Fed Smooth Interest Rates," in *Monetary Policy on the 75th Anniversary of the Federal Reserve System*, ed. by M. Belongia, pp. 111–147. Kluwer Academic Publishers, Boston.

Eggertsson, G. B. (2006): "The Deflation Bias and Committing to Being Irresponsible," *Journal of Money, Credit and Banking*, 38(2), 283–321.

Eggertsson, G. B., and M. Woodford (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 34(1), 139–235.

Federal Open Market Committee (2015): "Press release, Washington, DC, December 16," available at https://www.federalreserve.gov/newsevents/press/monetary/20151216a.htm.

Gali, J. (2008): *Monetary Policy, Inflation, and the Business Cycle*. Princeton: Princeton University Press.

Giannoni, M. P., and M. Woodford (2003): "How Forward-Looking is Optimal Monetary Policy?," *Journal of Money, Credit and Banking*, 35(6), 1425–1469.

Givens, G. E. (2012): "Estimating Central Bank Preferences under Commitment and Discretion," *Journal of Money, Credit and Banking*, 44(6), 1033–1061.

Gust, C. J., J. D. Lopez-Salido, and M. E. Smith (2012): "The empirical implications of the interest-rate lower bound," Discussion paper.

Hills, T. S., and T. Nakata (2014): "Fiscal Multipliers at the Zero Lower Bound: The Role of Policy Inertia," Finance and Economics Discussion Series 2014-107, Board of Governors of the Federal Reserve System (U.S.).

Hills, T. S., T. Nakata, and S. Schmidt (2016): "The Risky Steady State and the Interest Rate Lower Bound," Finance and Economics Discussion Series 2016-9, Board of Governors of the Federal Reserve System (U.S.).

Ireland, P. N. (2011): "A New Keynesian Perspective on the Great Recession," *Journal of Money, Credit and Banking*, 43(1), 31–54.

Jeanne, O., and L. E. O. Svensson (2007): "Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank," *American Economic Review*, 97(1), 474–490.

Levin, A., V. Wieland, and J. C. Williams (2003): "The Performance of Forecast-Based Monetary Policy Rules Under Model Uncertainty," *American Economic Review*, 93(3), 622–645.

Nakata, T., and S. Schmidt (2014): "Conservatism and Liquidity Traps," Finance and Economics Discussion Series 2014-105, Board of Governors of the Federal Reserve System (U.S.).

Orphanides, A., and J. C. Williams (2002): "Robust Monetary Policy Rules with Unknown Natural Rates," *Brookings Papers on Economic Activity*, 33(2), 63–146.

——— (2007): "Robust Monetary Policy with Imperfect Knowledge," *Journal of Monetary Economics*, 54(5), 1406–1435.

Persson, T., and G. Tabellini (1993): "Designing Institutions for Monetary Stability," *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 53–84.

——— (1999): "Chapter 22 Political Economics and Macroeconomic Policy," vol. 1, Part C of *Handbook of Macroeconomics*, pp. 1397 – 1482. Elsevier.

Reifschneider, D., and J. C. Williams (2000): "Three Lessons for Monetary Policy in a Low-Inflation Era," *Journal of Money, Credit and Banking*, 32(4), 936–966.

Rogoff, K. (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," *The Quarterly Journal of Economics*, 100(4), 1169–89.

Sack, B. (1998): "Uncertainty, Learning, and Gradual Monetary Policy," Discussion paper, Board of Governors of the Federal Reserve System (U.S.).

Sack, B., and V. Wieland (2000): "Interest-Rate Smoothing and Optimal Monetary Policy: A Review of Recent Empirical Evidence," *Journal of Economics and Business*, 52(1-2), 205–228.

Stein, J. C., and A. Sunderam (2015): "Gradualism in Monetary Policy: A Time-Consistency Problem," Working Paper.

Svensson, L. E. O. (1997): "Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts," *American Economic Review*, 87(1), 98–114.

Walsh, C. (2003): "Speed Limit Policies: The Output Gap and Optimal Monetary Policy," *American Economic Review*, 93(1), 265–278.

Walsh, C. E. (1995): "Optimal Contracts for Central Bankers," *American Economic Review*, 85(1), 150–67.

Woodford, M. (2003a): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

——— (2003b): "Optimal Interest-Rate Smoothing," *Review of Economic Studies*, 70, 861–886.

Appendix

A Policy Regimes

A.1 Interest-Rate Smoothing

The Lagrange problem of the central bank with an interest-rate smoothing objective at period t is given by

$$
\begin{aligned}
V_t^{CB}(r_t, i_{t-1}) = \max_{\pi_t, y_t, i_t} \Big[& -\frac{1}{2}\left[(1-\alpha)\left(\pi_t^2 + \lambda y_t^2\right) + \alpha(i_t - i_{t-1})^2\right] + \beta E_t V_{t+1}^{CB}(r_{t+1}, i_t) \\
& + \phi_t^{PC}(\pi_t - \beta E_t \pi_{t+1}(r_{t+1}, i_t) - \kappa y_t) \\
& + \phi_t^{IS}(y_t - E_t y_{t+1}(r_{t+1}, i_t) + \sigma(i_t - E_t \pi_{t+1}(r_{t+1}, i_t) - r_t)) \\
& + \phi_t^{ZLB} i_t \Big]
\end{aligned} \tag{A.1}
$$

where the central banker takes the value and policy functions next period as given. The FONC are

$$
(1-\alpha)\pi_t - \phi_t^{PC} = 0 \tag{A.2}
$$

$$
(1-\alpha)\lambda y_t + \kappa \phi_t^{PC} - \phi_t^{IS} = 0 \tag{A.3}
$$

$$
\begin{aligned}
& \alpha(i_t - i_{t-1}) - \beta \frac{\partial E_t V_{t+1}^{CB}(r_{t+1}, i_t)}{\partial i_t} + \beta \frac{\partial E_t \pi(r_{t+1}, i_t)}{\partial i_t} \phi_t^{PC} \\
& + \left(\frac{\partial E_t y(r_{t+1}, i_t)}{\partial i_t} + \sigma \frac{\partial E_t \pi(r_{t+1}, i_t)}{\partial i_t} - \sigma \right) \phi_t^{IS} - \phi_t^{ZLB} = 0
\end{aligned} \tag{A.4}
$$

as well as the complementary slackness conditions and the NKPC and IS equation. Combining the first two conditions, we get

$$
(1-\alpha)(\lambda y_t + \kappa \pi_t) = \phi_t^{IS} \tag{A.5}
$$

Furthermore, note that

$$
\frac{\partial V_t^{CB}(r_t, i_{t-1})}{\partial i_{t-1}} = \alpha(i_t - i_{t-1}) \tag{A.6}
$$

We can then consolidate the third optimality condition to obtain an interest-rate target criterion

$$
\begin{aligned}
0 = & \alpha(1+\beta) i_t - \alpha i_{t-1} - \beta \alpha E_t i(r_{t+1}, i_t) \\
& + \beta(1-\alpha) \frac{\partial E_t \pi(r_{t+1}, i_t)}{\partial i_t} \pi_t - (1-\alpha) \left(\frac{\partial E_t y(r_{t+1}, i_t)}{\partial i_t} + \sigma \frac{\partial E_t \pi(r_{t+1}, i_t)}{\partial i_t} \right) (\lambda y_t + \kappa \pi_t) \\
& - (1-\alpha)\sigma(\lambda y_t + \kappa \pi_t) - \phi_t^{ZLB}
\end{aligned} \tag{A.7}
$$

A.2 Asymmetric Interest-Rate Smoothing

The value of the central bank with an asymmetric interest-rate smoothing (AIRS) objective is given by

$$V_t^{CB,AIRS} = u^{CB,AIRS}(\pi_t, y_t, \Delta i_t) + \beta E_t V_{t+1}^{CB,AIRS} \tag{A.8}$$

where the central bank's contemporaneous objective function, $u^{CB,AIRS}(\cdot,\cdot,\cdot)$, is given by

$$u^{CB,AIRS}(\pi_t, y_t, \Delta i_t) = -\frac{1}{2}\left[(1-\alpha)\left(\pi_t^2 + \lambda y_t^2\right) + \alpha\left(\Delta i_t^2 + \max(\Delta i_t, 0)\right)\right] \tag{A.9}$$

Each period t, the central bank with an asymmetric interest-rate smoothing objective chooses the inflation rate, the output gap, and the nominal interest rate in order to maximize the value today subject to the behavioral constraints of the private sector, with the policy functions at time $t+1$ taken as given

$$V_t^{CB,AIRS}(r_t, i_{t-1}) = \max_{\pi_t, y_t, i_t} \quad u^{CB,AIRS}(\pi_t, y_t, \Delta i_t) + \beta E_t V_{t+1}^{CB,AIRS}(r_{t+1}, i_t) \tag{A.10}$$

A Markov-Perfect equilibrium with an asymmetric interest-rate smoothing objective is defined as a set of time-invariant value and policy functions $\{V^{CB,AIRS}(\cdot),\ \pi(\cdot),\ y(\cdot),\ i(\cdot)\}$ that solves the problem of the central bank above, together with the value function $V(\cdot)$ that is consistent with $\pi(\cdot)$ and $y(\cdot)$.

A.3 Shadow Interest-Rate Smoothing

The value of the central bank with a shadow interest-rate smoothing (SIRS) objective is given by

$$V_t^{CB,SIRS} = u^{CB,SIRS}(\pi_t, y_t, i_t, i_{t-1}^*) + \beta E_t V_{t+1}^{CB,SIRS} \tag{A.11}$$

where the central bank's contemporaneous objective function, $u^{CB,SIRS}(\cdot,\cdot,\cdot,\cdot)$, is given by

$$u^{CB,SIRS}(\pi_t, y_t, i_t, i_{t-1}^*) = -\frac{1}{2}\left[(1-\alpha)\left(\pi_t^2 + \lambda y_t^2\right) + \alpha(i_t - i_{t-1}^*)^2\right] \tag{A.12}$$

Each period t, the central bank with a shadow interest-rate smoothing objective first chooses the shadow nominal interest rate in order to maximize the value today subject to the behavioral constraints of the private sector, with the value and policy functions at time $t+1$—$V_{t+1}^{CB,SIRS}(\cdot,\cdot)$, $y_{t+1}(\cdot,\cdot)$, $\pi_{t+1}(\cdot,\cdot)$—taken as given:

$$i_t^* = \text{argmax}_x \qquad u^{CB,SIRS}(\pi(x), y(x), x, i_{t-1}^*) + \beta E_t V_{t+1}^{CB,SIRS}(r_{t+1}, x) \tag{A.13}$$

with

$$\begin{aligned} y(x) =& E_t y_{t+1}(r_{t+1}, x) - \sigma(x - E_t \pi_{t+1}(r_{t+1}, x) - r_t) \\ \pi(x) =& \kappa y(x) + \beta E_t \pi_{t+1}(r_{t+1}, x) \end{aligned} \tag{A.14}$$

The actual policy rate i_t is given by

$$i_t = \max(i_t^*, 0) \tag{A.15}$$

That is, the actual policy rate today is zero when $i_t^* < 0$, and it is equal to i_t^* when $i_t^* \geq 0$.

The central bank's value today is given by

$$V_t^{CB,SIRS}(r_t, i_{t-1}^*) = u^{CB,SIRS}(\pi_t, y_t, i_t, i_{t-1}^*) + \beta E_t V_{t+1}^{CB,SIRS}(r_{t+1}, i_t^*) \tag{A.16}$$

where inflation and the output gap are given by

$$\begin{aligned} y_t =& E_t y_{t+1}(r_{t+1}, i_t^*) - \sigma(i_t - E_t \pi_{t+1}(r_{t+1}, i_t^*) - r_t) \\ \pi_t =& \kappa y_t + \beta E_t \pi_{t+1}(r_{t+1}, i_t^*) \\ i_t \geq& 0 \end{aligned} \tag{A.17}$$

A Markov-Perfect equilibrium with an shadow interest-rate smoothing objective is defined as a set of time-invariant value and policy functions $\{V^{CB,SIRS}(\cdot),\ \pi(\cdot),\ y(\cdot),\ i^*(\cdot),\ i(\cdot)\}$ that solves the problem of the central bank above, together with the value function $V(\cdot)$ that is consistent with $\pi(\cdot)$ and $y(\cdot)$.

A.4 Price-Level Targeting

The value of the central bank with a price-level targeting (PLT) objective is given by

$$V_t^{CB,PLT} = u^{CB,PLT}(\pi_t, y_t, p_t) + \beta E_t V_{t+1}^{CB,PLT} \tag{A.18}$$

where $p_t \equiv p_{t-1} + \pi_t$ denotes the price level in period t and the central bank's contemporaneous objective function, $u^{CB,PLT}(\cdot,\cdot,\cdot)$, is given by

$$u^{CB,PLT}(\pi, y, p) = -\frac{1}{2}\left[(1-\alpha)\left(\pi^2 + \lambda y^2\right) + \alpha p^2\right] \tag{A.19}$$

The last term, αp^2, captures the price-level targeting objective, and the parameter $\alpha \in [0,1]$ determines how the smoothing objective weighs against the inflation and output gap objective in the central bank's objective function.

Each period t, the central bank with a price-level targeting objective chooses the inflation rate, the output gap, and the nominal interest rate in order to maximize its objective function subject to the behavioral constraints of the private sector, with the policy functions at time $t+1$ taken as given

$$V_t^{CB,PLT}(r_t, p_{t-1}) = \max_{\pi_t, y_t, i_t} \quad u^{CB,PLT}(\pi_t, y_t, p_t) + \beta E_t V_{t+1}^{CB,PLT}(r_{t+1}, p_t) \tag{A.20}$$

A Markov-Perfect equilibrium with a price-level targeting objective is defined as a set of time-invariant value and policy functions $\{V^{CB,PLT}(\cdot),\ \pi(\cdot),\ y(\cdot),\ p(\cdot),\ i(\cdot)\}$ that solves the problem of the central bank above, together with the value function $V(\cdot)$ that is consistent with $\pi(\cdot)$ and $y(\cdot)$.

The Lagrangean problem of the central bank with a price-level targeting objective is given by

$$
\begin{aligned}
V_t^{CB,PLT}(r_t, p_{t-1}) = \max_{p_t, y_t, i_t} \Bigg[& -\frac{1}{2}\left[(1-\chi)\left((p_t - p_{t-1})^2 + \lambda y_t^2\right) + \chi p_t^2\right] + \beta E_t V_{t+1}^{CB,PLT}(r_{t+1}, p_t) \\
& + \phi_t^{PC}\left(p_t - \frac{1}{1+\beta}p_{t-1} - \frac{\beta}{1+\beta}E_t p(r_{t+1}, p_t) - \frac{\kappa}{1+\beta}y_t\right) \\
& + \phi_t^{IS}(y_t - E_t y(r_{t+1}, p_t) + \sigma(i_t - E_t p(r_{t+1}, p_t) + p_t - r_t)) = 0 \\
& + \phi_t^{ZLB} i_t \Bigg] \qquad \text{(A.21)}
\end{aligned}
$$

Note, that the central banker takes the decision rules of the private sector and of future central bankers as given. The FONC are

$$
\begin{aligned}
& (1-\chi)(p_t - p_{t-1}) + \chi p_t - \beta \frac{\partial E_t V_{t+1}^{CB}(r_{t+1}, p_t)}{\partial p_t} + \left(\frac{\beta}{1+\beta}\frac{\partial E_t p(r_{t+1}, p_t)}{\partial p_t} - 1\right)\phi_t^{PC} \\
& + \left(\frac{\partial E_t y(r_{t+1}, p_t)}{\partial p_t} + \sigma \frac{\partial E_t p(r_{t+1}, p_t)}{\partial p_t} - \sigma\right)\phi_t^{IS} = 0 \qquad \text{(A.22)} \\
& \lambda(1-\chi)y_t + \frac{\kappa}{1+\beta}\phi_t^{PC} - \phi_t^{IS} \qquad \text{(A.23)} \\
& \sigma\phi_t^{IS} + \phi_t^{ZLB} = 0 \qquad \text{(A.24)}
\end{aligned}
$$

as well as the complementary slackness conditions and the NKPC and IS equation. Note that

$$
\frac{\partial V_t^{CB}(r_t, p_{t-1})}{\partial p_{t-1}} = (1-\chi)(p_t - p_{t-1}) - \frac{1}{1+\beta}\phi_t^{PC} \qquad \text{(A.25)}
$$

Substituting this expression as well as the last two optimality conditions into the first, we get

$$
\begin{aligned}
& (1 + \beta(1-\chi))\, p_t + \left(\frac{\beta}{1+\beta}\frac{\partial E_t p(r_{t+1}, p_t)}{\partial p_t} - 1\right)\phi_t^{PC} - \frac{1}{\sigma}\left(\frac{\partial E_t y(r_{t+1}, p_t)}{\partial p_t} + \sigma\frac{\partial E_t p(r_{t+1}, p_t)}{\partial p_t} - \sigma\right)\phi_t^{ZLB} \\
& = (1-\chi)p_{t-1} + \beta(1-\chi)E_t p(r_{t+1}, p_t) - \frac{\beta}{1+\beta}E_t\phi^{PC}(r_{t+1}, p_t) \qquad \text{(A.26)}
\end{aligned}
$$

B Numerical algorithm

We use the policy function iteration algorithm described below in Subsection B.1 to solve the model for the various monetary policy regimes. The only exception is the AIRS regime for which we solve the model using a value function iteration algorithm described in Subsection B.2.[15]

[15] For some values of the relative weight on the interest-rate smoothing term in the central bank's objective function we were not able to solve the model with AIRS regime using the algorithm described in Subsection B.1, which is why we used the value function iteration algorithm described in Subsection B.2.

B.1 Policy function iteration

We approximate the policy functions for the inflation rate, output and the policy rate with a finite elements method using collocation. For the basis functions we use cubic splines. The algorithm proceeds in the following steps (here exemplified for the interest-rate smoothing regime):

1. Construct the collocation nodes. The nodes are chosen such that they coincide with the spline breakpoints. Use a Gaussian quadrature scheme to discretize the normally distributed innovation to the natural real rate shock.

2. Start with a guess for the basis coefficients.

3. Use the current guess for the basis coefficients to calculate the partial derivatives of expected inflation and expected output with respect to the current policy rate at the collocation nodes.

4. Approximate the expectation functions for inflation, output and the policy rate, using the same quadrature scheme as in step 3.

5. Solve the system of equilibrium conditions for inflation, output and the policy rate at the collocation nodes, assuming that the zero lower bound is not binding. For those nodes where the zero bound constraint is violated solve the system of equilibrium conditions associated with a binding zero bound.

6. Update the guess for the basis coefficients. If the new guess is sufficiently close to the old one, proceed with step 7. Otherwise, go back to step 4.

7. Check whether the new set of partial derivatives based on the updated basis coefficients is sufficiently close to the previous ones. If this is the case, you are done. Otherwise, go back to step 3.

B.2 Value function iteration

We approximate the central bank's value function and the policy functions with a finite elements method using collocation. Cubic splines are used for the basis functions approximating the value function and linear splines are used for the basis functions approximating the policy functions. The algorithm proceeds in the following steps:

1. Construct the collocation nodes. The nodes are chosen such that they coincide with the spline breakpoints. Construct a choice vector for the policy rate. Use a Gaussian quadrature scheme to discretize the normally distributed innovation to the natural real rate shock.

2. Start with a guess for the basis coefficients.

3. Approximate the expectation functions for inflation, output and the central bank's value function for each element of the choice vector using the current guess for the basis coefficients.

4. Solve the New Keynesian Phillips curve and the consumption Euler equation for inflation and output taking as given the policy rate and expectations of inflation and output in the next period. Calculate the difference between the policy rate (choice variable) and lagged policy rate (state variable).

5. Determine the element of the choice vector that maximizes the central bank's value function at each collocation node.

6. Update the guess for the basis coefficients. If the new guess is sufficiently close to the old one, you are done. Otherwise, go back to step 3.

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