EUROPEAN CENTRAL BANK
EUROSYSTEM

Working Paper Series

Nina Furbach

Non-homothetic housing demand and geographic worker sorting

No 3018

Abstract

Housing expenditure shares decline with income. A household's income determines its sensitivity to housing costs and drives its location decision. Has spatial skill sorting increased because low income individuals are avoiding increasingly expensive regions? I augment a standard quantitative spatial model with flexible non-homothetic preferences to estimate the effect of the national increase in the relative supply of high skilled workers that has put upward pressure on housing costs in skill-intensive cities. My model explains 10% of the increase in average house prices in Germany from 2007 to 2017 and 11% of the regional differences in house price increases. One third of the effects is due to an increase in spatial skill sorting driven by differences in housing expenditure shares. The observed degree of skill sorting was not significantly different from the optimal allocation in 2007 while skill sorting was larger than optimal in 2017.

JEL Classification: H21, H23, R12, R21

Keywords: Housing demand, non-homotheticiy, geographic worker sorting, Germany, quantitative spatial models

Non-technical summary

Western economies are experiencing a housing crisis unlike anything we have seen for decades. About half of Americans (49%) say the availability of affordable housing in their local community is a major problem (Schaeffer, 2022). At the same time, real house prices in the US have risen by 35% from 2010 to 2020. House price increases are sizable also in other Western economies: In Germany, house prices in 2020 were 47% higher than in 2010 (OECD, 2022). These developments are more and more seen as a driver of increasing economic inequality. The growing cost burden is not equally distributed across households since poorer households spend a significantly larger share of their income on housing. However, we know very little about how hard the lack of affordable housing hits individuals across the income distribution, nor do we know much about how it affects location decisions.

Individuals with lower incomes tend to allocate a larger portion of their budgets to housing, which might lead them to settle in lower-cost cities. Higher income individuals are less sensitive to housing costs and often choose to live in higher-cost cities. The higher the share of individuals with large incomes, the higher will be housing demand and house prices in large cities, and the more pronounced geographic sorting becomes. This paper studies whether the increase in the national supply of high skilled individuals can explain the trends in location choices observed over past decades: Skilled individuals have clustered in high-cost cities while unskilled workers avoid increasingly expensive cities. I ask whether individuals with lower skills (and therefore lower incomes) tend to avoid big cities to a larger extent than workers with higher skills since they suffer more from rising housing costs. A large degree of sorting, in turn, amplifies house price differences between large cities and rural areas. I further analyse how place-based policies optimally respond to the increase in spatial skill sorting.

I start by documenting that low income households spend a significantly larger share of their expenditures on housing. To do so, I run household-level regressions using large-scale German survey data from 2010 to 2014. Controlling for regional house prices, year fixed effects and a large set of household characteristics, I find that a 100% increase in total household expenditure causes a 30% decrease in the housing expenditure share.

To analyze how differences in housing expenditure shares affect location choices, I use an exogenous shock to housing costs: The national rise in the relative supply of workers with a university degree in Germany has increased the demand for housing. It thereby put upward pressure on house prices that was more pronounced in skill-intensive regions.

With the help of a spatial general equilibrium model, I analyze whether high school graduates avoided increasingly expensive regions to a larger extent than college graduates. The model is calibrated to be consistent with the empirically documented reduced-form estimates on the expenditure elasticity of the housing expenditure share. I estimate the model on the level of 141 labour market regions in Germany. My findings suggest that the increase in the national supply of high skilled workers from 2007 to 2017 led to an increase in spatial skill sorting. It accounts for roughly one tenth of the national increase in house prices and one tenth of the regional dispersion in house price increases, while one third of the effects is due to the heterogeneity in housing expenditure shares.

The results of this paper, which demonstrate that poorer individuals spend a significantly larger share of their income on housing, suggest that policies aimed at increasing the supply of affordable housing could substantially improve the well-being of low income individuals. My model provides evidence that low income individuals are avoiding increasingly expensive cities because they are hit harder by increases in housing costs. I find that place-based policies that aim at lowering the degree of geographic sorting have small welfare effects.

1 Introduction

Western economies are experiencing a housing crisis unlike anything we have seen for decades. About half of Americans (49%) say the availability of affordable housing in their local community is a major problem (Schaeffer, 2022). At the same time, real house prices in the US have risen by 35% from 2010 to 2020. House price increases are sizable also in other Western economies: In Germany, house prices in 2020 were 47% higher than in 2010 (OECD, 2022). These developments are more and more seen as a driver of increasing economic inequality. The growing cost burden is not equally distributed across households since poorer households spend a significantly larger share of their income on housing. However, we know very little about how hard the lack of affordable housing hits individuals across the income distribution, nor do we know much about how it affects location decisions.

Individuals with lower incomes tend to allocate a larger portion of their budgets to housing, which might lead them to settle in lower-cost cities. Higher income individuals are less sensitive to housing costs and often choose to live in higher-cost cities. The higher the share of individuals with large incomes, the higher will be housing demand and house prices in large cities, and the more pronounced geographic sorting becomes. This paper studies whether the increase in the national supply of high skilled individuals can explain the trends in location choices observed over past decades: Skilled individuals have clustered in high-cost cities while unskilled workers avoid increasingly expensive cities. I ask whether individuals with lower skills (and therefore lower incomes) tend to avoid big cities to a larger extent than workers with higher skills since they suffer more from rising housing costs. A large degree of sorting, in turn, amplifies house price differences between large cities and rural areas. I further analyse how place-based policies optimally respond to the increase in spatial skill sorting.

I start by estimating the degree of non-homotheticity using large-scale survey data of German households. My results establish that non-homotheticity in housing demand is both econometrically and economically significant. I set up a spatial general equilibrium model of non-homothetic housing demand to estimate the effect of the national rise in the skill share. Calibrating the model with the estimated preference parameters, I find that the rising skill share explains 3% of the increase in spatial sorting by skill, 10% of the national increase in house prices and 11% of the regional differences in house price increases from 2007 to 2017 in Germany. With homothetic preferences, the national shock does not change skill sorting which implies that it can explain only 6% of the national house price increase and 7% of the regional dispersion in house price increases. The national increase in the supply of high skilled workers has decreased welfare of low skilled workers by 0.9% and welfare of high skilled workers by 0.8%.

I next ask how a social planner should optimally respond to the observed changes in skill sorting. I analyze the optimal allocation in 2007 and 2017 using a utilitarian welfare function. I choose to study the allocation that equally benefits all worker types. The social planner maximizes welfare taking into account redistribution and efficiency considerations. To do so, she chooses regional type-specific taxes and transfers that set incentives for individuals to move across space. I find that the observed degree of skill sorting was not significantly different from the optimal allocation in 2007, while skill sorting was larger than optimal in 2017. From a social planner perspective, it would be optimal to set incentives for all workers to move toward rural areas, but to a larger extent for high skilled workers. Moving from the observed to the optimal allocation implies welfare gains of 0.4% in 2007 and 0.5% in 2017.

I arrive at these conclusions by studying sorting in a spatial general equilibrium model with heterogeneous workers that have non-homothetic preferences. I include two worker types: workers with and without a university degree. In my model, heterogeneous individuals with Generalized Elasticity of Substitution (GES) preferences trade off wages, housing costs and regional amenities when making their location decision. Locations differ exogenously in terms of housing supply, group-specific productivity and group-specific amenities. I further include heterogeneous preference shocks for locations that act as a form of migration costs. Identical firms combine labor from different worker groups to produce a final good that is traded between regions at zero cost.

The quantification follows the basic steps known from literature on quantitative spatial models (see Redding and Rossi-Hansberg, 2017 for an overview). First, I use observed data and the structure of the model to calibrate the key structural parameters. I estimate non-homothetic preferences over housing and non-housing consumption utilizing large-scale consumption microdata from the German Socio-Economic Panel (GSOEP). I start by linearizing the relationship between the housing expenditure share and total expenditure derived from the model. From the reduced-form estimation of this first-order approximation, I obtain parameters that are directly interpretable as elasticities and therefore comparable to estimates from the literature. Guided by the structure of the model, I control for local house prices since households' sorting decisions introduce a positive correlation between prices and incomes at the regional level (see Albouy et al., 2016 and Finlay and Williams, forthcoming). I find that a 100% increase in total expenditure causes a 30% decrease in the housing expenditure share. My estimates are well in line with those found in comparable studies (see Finlay and Williams, forthcoming for an overview). For the calibration of the model parameters, I estimate the non-linear relation between the housing expenditure share, total expenditure and house prices derived from the model directly by Generalized Method of Moments (GMM). I reject two alternative preferences used in the literature: Cobb-Douglas and a unit housing requirement.

In the second step of the quantification, I use observed data, the structure of the model, and the structural parameters to invert the structural productivity, housing and amenity fundamentals. For the model inversion, I leverage on matched employer-employee data from German social security records. In particular, for every year, I observe the local labor market in which individuals work (Kosfeld and Werner, 2012), the nominal wage and a range of individual-level characteristics. I use this information to construct a regional wage measure for every skill type that is purged from differences in observable worker characteristics between regions. Aggregation of the micro data yields total employment and average wage by region and worker group for 2007 and 2017. To these data, I merge a regional property price index, which is generated from property micro data from the largest German listing website (Ahlfeldt et al., 2022).

The model is quantified to match the observed data on house prices, skill-specific wages and skill-specific employment on the regional level. It is further calibrated to be consistent with the empirically documented estimates on the non-homotheticity of preferences. I use the model to quantify the importance of accounting for non-homothetic preferences when analyzing geographic worker sorting. To do so, I estimate the effect of the rise in housing congestion resulting from the national increase in the relative supply of high skilled workers. The size of the shock amounts to an increase in the national share of high skilled workers from 16% in 2007 to 22% in 2017. In the model with non-homothetic preferences, this shock leads to intensified geographic worker sorting since low skilled workers are hit harder by increases in housing costs that are more pronounced in skill-intensive regions. The increase in sorting, in turn, amplifies house price increases in large cities as compared to rural areas. The model allows for exogenous changes in productivity, housing and amenity fundamentals. Feeding the national shock of an increase in the skill share into the model, the model accounts for 10% of the national increase in house prices and 11% of the regional dispersion in house price increases. Roughly one third of the effects is due to the non-homotheticity of preferences. Non-homothetic preferences can explain 3% of the observed change in skill sorting. I find that the shock has decreased welfare of low skilled workers by 0.9% and welfare of high skilled workers by 0.8%.

The experiment explores the importance of allowing for non-homothetic preferences when analyzing sorting in spatial equilibrium models. I next ask: What is the optimal degree of skill sorting? There are two reasons why the social planner allocation differs from the observed equilibrium. First, because wages are on average lower, marginal utilities of tradable good consumption are larger in rural areas. The social planner thus increases welfare by redistributing from urban to rural regions which sets incentives for workers to move across space. Secondly, since congestion forces are not taken into account by workers when choosing their place of residence, there is space for welfare improvement by setting incentives for individuals to move toward rural areas. Non-homothetic preferences affect the optimal allocation in several ways. Since lower skilled workers have a larger housing expenditure share than with homothetic preferences, they demand more housing. This implies that they generate stronger congestion forces than with homothetic preferences, while at the same time being more sensitive to housing congestion. Workers with higher skills, on the other hand, generate lower congestion forces than with homothetic preferences, while being less sensitive to housing congestion. Furthermore, by changing the marginal utililities of tradable good consumption, non-homothetic preferences affect the optimal degree of redistribution between regions.

I use the model to compute the optimal allocation which provides insights into the welfare implications of spatial skill sorting. I solve the problem of a social planner who maximizes a utilitarian welfare function taking as given workers' location choices as well as resource constraints on housing and tradables. To do so, she chooses transfers between locations and worker types which I characterize. Welfare weights are calibrated such that both worker types experience the same welfare gain.

While the observed degree of skill sorting was not significantly different from the optimal allocation in 2007, skill sorting was larger than optimal in 2017. Since high skilled workers, by consuming more housing than low skilled workers, generate larger congestion forces, it is optimal to reallocate them to a larger extent toward regions with less congested housing markets. Furthermore, since the urban wage premium is larger for high skilled workers, spatial differences in marginal utilities of tradable good consumption are larger for high skilled than for low skilled workers. It is therefore optimal to redistribute consumption goods between regions to a larger extent for high skilled than for low skilled individuals. The smaller degree of agglomeration and skill sorting in the optimal allocation imply that house prices are less dispersed as reflected in a larger increase in house prices in rural as compared to urban areas. My findings indicate that moving from the observed to the optimal allocation implies welfare gains of 0.4% in 2007 and 0.5% in 2017. I further find that the modeling of preferences matters: the social planner implements significantly larger transfers in 2017 when assuming GES preferences as compared to Cobb Douglas preferences.

This paper is related to several strands of literature. One strand aims at explaining the diverging location choices between skilled and unskilled households. While some studies have stressed the role of endogenous amenities (Diamond, 2016) or the role of technology in generating skill-biased wage growth in certain locations (Giannone, forthcoming; Eckert et al., 2020; Rubinton, forthcoming), few studies show that non-homothetic housing demand significantly affects spatial sorting.

Ganong and Shoag (2017) connect changes in regional housing supply regulations to slowing regional income convergence. Finlay and Williams (forthcoming) find that skill-biased technological change has intensified skill sorting since it made lower skilled workers relatively more sensitive to housing costs.[1] My work is closely related to Gyourko et al. (2013) who analyse the effects of changes in the relative size of different skill groups. Based on empirical tests of a number of equilibrium relationships, they argue that the increase in spatial skill sorting can be explained by an increasing number of high skilled households nationally combined with an inelastic supply of land in superstar cities. I set up and estimate a quantitative spatial model that allows to quantify the effects of the national increase in the number of high skilled individuals. I find that it led to an increase in spatial sorting even when assuming uniform housing supply elasticities. My model further enables me to quantify the substantial contribution of the growing skill share to the increase in house prices, and assess its implications for welfare inequality.

In the second part of my analysis, I take one step further and ask how non-homothetic preferences shape the optimal allocation and the taxes and transfers that could implement it. I thereby complement a large literature on the extent of spatial misallocation and the role that transfer and taxation policies play (see Albouy, 2009, Ossa, forthcoming, Fajgelbaum et al., 2018 and Colas and Hutchinson, 2021). Rather than evaluating exogenous policies, I derive the optimal allocation in a quantitative spatial model with local congestion forces. Since my

[1] The result of Finlay and Williams (forthcoming) are in line with evidence from Couture et al. (forthcoming) on within-city sorting.

model is flexible enough to capture any degree of non-homotheticity, my results generalize those of Fajgelbaum and Gaubert (2020) who assume Cobb-Douglas preferences. Focusing on the effects of skill-specific productivity and amenity spillovers, Fajgelbaum and Gaubert (2020) find that the US economy would benefit from a larger skill sorting. According to the results of Rossi-Hansberg et al. (2021) on the other hand, it would be optimal to take advantage of scarce cognitive non-routine workers by clustering them in small cognitive hubs to maximize positive production externalities. My results are in line with Fajgelbaum and Gaubert (2020) and propose that spatial skill sorting is larger than optimal.

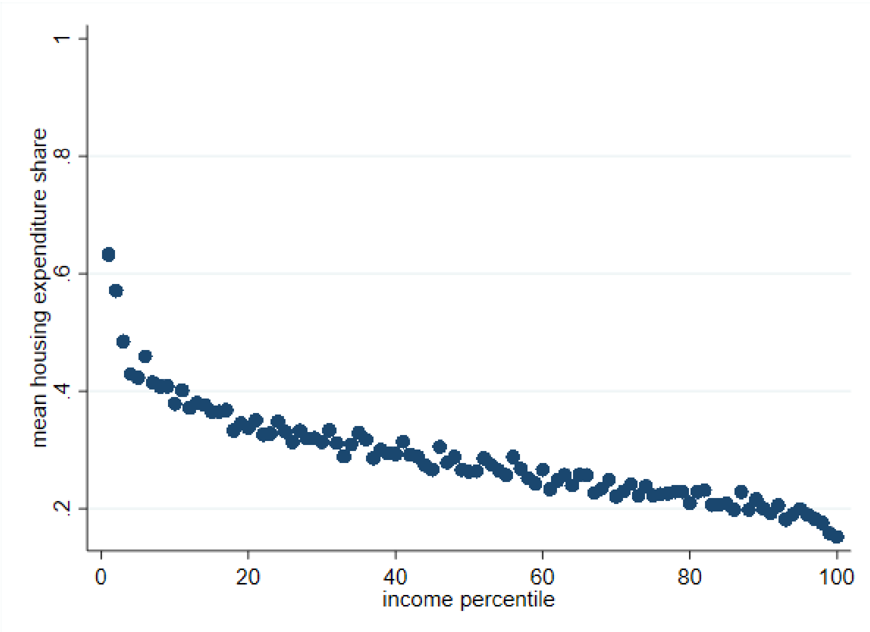
This paper is further related to literature estimating non-homotheticities in housing demand. At the level of cities, a common assumption is that preferences are Cobb-Douglas and therefore homothetic (see for example Eeckhout et al., 2014; Diamond, 2016; Fajgelbaum and Gaubert, 2020; Rossi-Hansberg et al., 2021). This assumption is often justified by the fact that housing expenditure shares vary little across cities with very different income levels (Davis and Ortalo-Magné, 2011). My results are in line with Finlay and Williams (forthcoming) and Albouy et al. (2016) who offer an alternative explanation for the similarity of housing expenditure shares across cities: offsetting price and income effects. While Albouy et al. (2016) rely on city-level variation in incomes, prices, and rental expenditure, I follow Finlay and Williams (forthcoming) and use consumption microdata. I find demand elasticities in Germany comparable to those estimated for other regions (see Finlay and Williams (forthcoming) for an overview), and quantify the role of these elasticities for the optimal degree of sorting and the response of house prices and welfare to changes in the supply of high-skilled workers.

The remainder of the paper is structured as follows. Section 2 presents stylized evidence that informs my modeling choices. Section 3 introduces a model with heterogenous workers that have non-homothetic preferences and Section 4 calibrates a quantitative version of this model. Section 5 uses the calibrated model to quantify the role of non-homothetic preferences when the national supply of different worker types changes. It further analyzes optimal region-specific taxes and transfers. Section 6 concludes.

2 Stylized Facts

To motivate the relevance of non-homothetic housing demand in the context of spatial skill sorting, I present some stylized facts on the spatial economy using data I describe in Section 4.1. I start by plotting the housing expenditure share for each percentile of the income distribution in Figure 1. It can be seen that housing expenditure shares are far from constant: Moving from the 10th percentile of the income distribution to the 90th percentile implies a decrease in the housing expenditure share from 38% to 21%.[2] The result is robust to controlling for household size. In Section 4.2, I causally estimate the degree of non-homotheticity and find that indeed housing expenditure shares decline substantially with income. When plotting the estimated preferences, they appear to fit the data better than preferences often assumed in the literature, which is confirmed by formal tests that reject the null hypothesis of constant housing expenditure shares.

Figure 1: Housing expenditure shares and income



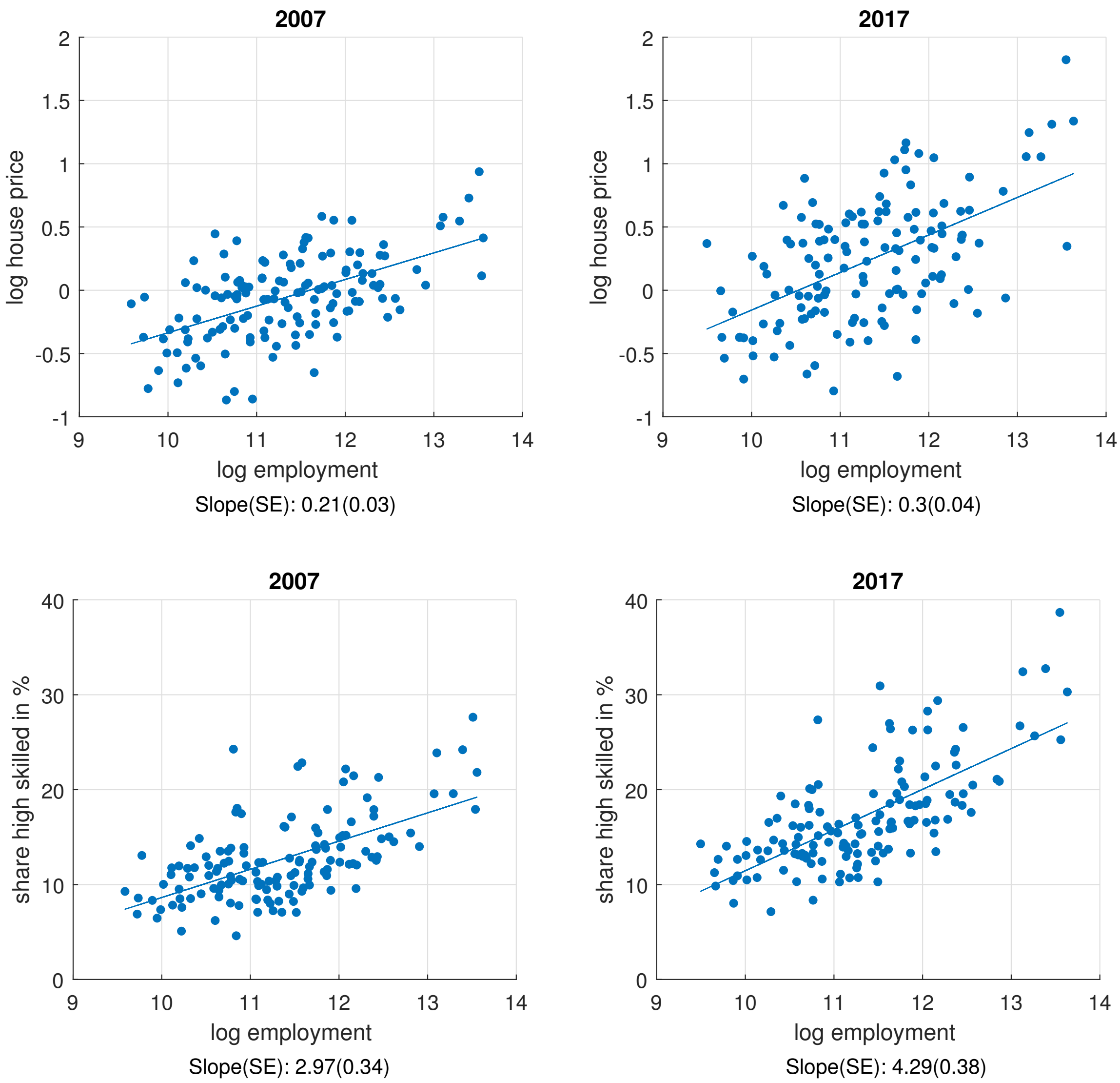
Source: GSOEP, own calculations. Note: Housing share is defined as housing expenditure (including heating and electricity) divided by total net income. The plot is based on household data from 2017. Number of observations: 6648

Non-homothetic preferences could potentially drive the patterns observed in the data and illustrated in Figure 2. The upper plots show that from 2007 to 2017, house prices have increased significantly more in large labor markets. I find that the elasticity of house prices with respect to employment has increased by almost 43%. At the same time, skill sorting has intensified since high skilled workers are increasingly attracted by dense regions, even to a larger extent than low skilled workers. The semi-elasticity of the share of high skilled workers with respect to city size

[2] A comparison to data 30 years ago reveals that the decrease in housing affordability is mainly a problem for low income households: The housing expenditure share has increased significantly more for low income than for high income households (see Figure A1).

has increased by 44%.

Figure 2: Geographic sorting and house prices



Source: SIAB, Ahlfeldt et al. (2022), own calculations. Note: Unit of observation is 141 labor market areas as defined by Kosfeld and Werner (2012).

In the following, I study a mechanism that links the three stylized facts. It can explain the simultaneous increase in regional house price differences and spatial skill sorting with the help of non-homothetic preferences: An exogenous national increase in the relative supply of high skilled workers has increased the demand for housing. The shock has put upward pressure on house prices that was more pronounced in skill-intensive regions. Due to lower housing expenditure shares, workers with higher incomes suffered less from increasing housing costs in these regions.

Non-homothetic preferences in combination with the national increase in high skilled workers can therefore explain why skilled households have clustered in dense labor markets while unskilled workers increasingly avoided these regions. The increase in sorting, in turn, amplified house price increases in large cities as compared to rural areas.

3 Model

3.1 Competitive allocation

In this section, I develop a spatial general equilibrium model with heterogeneous workers that have non-homothetic preferences. I consider an economy that is populated by $L = \sum_k L_k$ workers who I categorize into groups indexed by k. Heterogeneous workers choose a region i taking as given the decision of all other individuals. Local labor markets vary exogenously in their productivity, amenities, and housing supply. I further include worker-specific preference shocks for locations that act as a form of migration costs. Conditional on their labor market, individuals maximize utility over consumption of housing and tradable goods. I incorporate regional congestion forces by assuming an inelastic supply of housing. Homogeneous firms employ different worker types to produce goods that are traded at zero cost. I discuss potential mechanisms within the model framework that could rationalize the stylized facts presented in Section 2.

3.1.1 Workers

Preferences of a worker n belonging to group k and working in region i are defined over freely-tradable homogeneous goods c_{ik}, housing h_{ik}, regional amenities E_{ik} and the idiosyncratic amenity shock ϵ_{in}. I assume GES preferences that take the following form

$$u_{in} = \left(\gamma (c_{ik})^{1-\frac{1}{\rho}} + (1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}}\right)^{\frac{\rho}{\rho-1}} E_{ik}\epsilon_{in} \tag{1}$$

with $\gamma > 0$, $0 < \rho < 1$ and $\eta > \rho$. ρ is the elasticity of substitution and η captures the degree of non-homotheticity. GES preferences nest the specifications commonly used in the spatial literature. If $\rho \to 1$ and $\eta = 1$, I obtain Cobb-Douglas preferences where γ is the expenditure share on tradables.[3] In the case of $\eta \to \infty$, I get a unit housing requirement which is a very extreme form of non-homotheticity often assumed in the literature. It implies that individuals always purchase one unit of housing, so demand is perfectly price and income inelastic. GES

[3]Note that if $\eta = 1$ only, I get constant elasticity of substitution (CES) preferences.

preferences, in contrast, can accommodate any degree of non-homotheticity. In the empirically relevant case, $\eta > 1$ and expenditure shares on housing decrease with income.

Conditional on working in region i, a type-k worker solves the following problem:

$$\begin{aligned} v_{ik} &= \max_{c_{ik},h_{ik}} \left(\gamma(c_{ik})^{1-\frac{1}{\rho}} + (1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}}\right)^{\frac{\rho}{\rho-1}} E_{ik} \\ s.t. & \\ c_{ik} + p_i h_{ik} &= w_{ik} + \Pi_{ik} + t_{ik} \end{aligned} \tag{2}$$

where p_i is the price of housing, w_{ik} is the wage and t_{ik} is the net government transfer to a type-k worker in region i. The tradable good is chosen to be the numéraire. Π_{ik} is the return on a regional portfolio of housing that equals individual housing expenditure:

$$\Pi_{ik} = p_i h_{ik}. \tag{3}$$

I assume that ϵ_{nk} is drawn from a type-1 extreme value distribution with shape parameter ψ that reflects the extent of preference heterogeneity across regions. If the variation in regional amenity draws is large, workers show little sensitivity to differences in wages, house prices and amenities, which implies a low geographic mobility. The distributional assumption on region-specific amenity draws implies closed-form expressions for the number of workers in each region

$$L_{ik} = \frac{v_{ik}^{\frac{1}{\psi}}}{\sum_i v_{ik}^{\frac{1}{\psi}}} L_k \tag{4}$$

where $L_k = \sum_i L_{ik}$ is the total number of type-k workers.

3.1.2 Firms

Identical firms combine labor from different worker groups to produce the freely-traded final good. I assume a linear production function with group- and location-specific productivity shifters A_{ik}. Firm-level production functions translate directly to city-level production since firms face constant returns to scale and share an identical production technology. The regional-level production function is given by

$$Y_i = \sum_k A_{ik} L_{ik}. \tag{5}$$

Firms pay wages that are equal to the marginal product of labor

$$w_{ik} = A_{ik}. \tag{6}$$

The regional supply of housing H_i is determined by an exogenous part T_i that captures the availability of land and an endogenous part that depends on total city population $L_i = \sum_k L_{ik}$:

$$H_i = T_i L_i^{\gamma_H} \tag{7}$$

where γ_H is the housing supply elasticity.

3.1.3 Equilibrium

In equilibrium, the market for tradable goods clears:

$$\sum_i Y_i \quad = \quad \sum_i \sum_k L_{ik} c_{ik} \tag{8}$$

which follows from the household budget constraint in equation (2) combined with a balanced government budget $\sum_i \sum_k L_{ik} t_{ik} = 0$. Housing market clearing requires

$$H_i \quad = \quad \sum_k L_{ik} h_{ik} \tag{9}$$

where housing demand is given by the combined first-order conditions of the household maximization problem

$$h_{ik} \quad = \quad (\frac{1}{p_i} \frac{1-\gamma}{\gamma} \frac{\rho - \eta}{\rho - 1})^{\frac{\rho}{\eta}} (c_{ik})^{\frac{1}{\eta}}. \tag{10}$$

Labor markets clear when equation (4) and equation (6) hold.

Thus, for given parameters γ, ρ, η, ψ and γ_H, location-specific fundamentals A_{ik}, T_i, E_{ik} and taxes t_{ik}, an equilibrium is a vector of L_{ik}, w_{ik}, c_{ik}, h_{ik}, Π_{ik}, p_i, H_i and Y_i satisfying equations (2) to (10).[4]

[4]When estimating the model, I impose $t_{ik} = 0 \; \forall \; i, k$ both in the observed and counterfactual scenario. In this case, equation (8) follows from the household budget constraint in equation (2) and is therefore redundant.

3.1.4 Potential mechanisms

In a quantitative spatial model as laid out in this section, a number of different shocks could rationalize the patterns observed in the data and presented in Section 2. With homothetic preferences, a simultaneous increase in both spatial skill sorting and regional house price differences could result from shocks to A_{ik} or E_{ik}, i.e. from region-specific shocks to productivity or amenity fundamentals that differ across skill types. One example would be skill-biased technological change with differential effects across regions depending on their industry composition. If high skilled workers become more productive mainly in dense regions, house prices increase in these regions since more high skilled workers move there. An alternative explanation would be regional shocks that are symmetric across skill types in combination with asymmetric spillovers. Consider the case in which all workers become more productive in denser regions. Such productivity shock would increase the spatial concentration of population. If high skilled workers benefit more from knowledge spillovers in dense cities, we would observe an increase in both house prices and geographic skill sorting. A large literature combines both asymmetric spillovers with asymmetric region-specific shocks to fundamentals such that spillovers amplify the effects of shocks to regional fundamentals (see for example Diamond, 2016 and Giannone, forthcoming). Another explanation is that agglomeration spillovers themselves have changed over time. Baum-Snow and Pavan (2013) argue that agglomeration forces of high skilled workers have become stronger relative to those of low skilled workers which led to an increase in spatial skill sorting.

With non-homothetic preferences, a number of additional shocks could rationalize the increase in both regional house price differences and spatial skill sorting. Even in the absence of spillovers, productivity and amenity shocks that are common across regions might lead to a change in spatial skill sorting. Finlay and Williams (forthcoming) model skill-biased technological change as a national shock to productivity fundamentals of high skilled relative to low skilled workers. Assuming non-homothetic preferences, they find that skill-biased technological change can explain 23% of the increase in skill sorting in the US since 1980. Gyourko et al. (2013) argue that the simultaneous increase in geographic skill sorting and spatial house price differences can be explained by an inelastic supply of land in superstar cities combined with an increasing number of high income households nationally. In the following, I show that the national increase in the relative supply of high skilled individuals leads to an increase in spatial sorting even with uniform housing supply elasticities γ_h.

3.2 The planner's problem

In this section, I characterize the optimal allocation and the taxes and transfers that implement it. There are two reasons why the social planner allocation differs from the observed equilibrium. First, the decentralized world is inefficient due to congestion forces on regional housing markets. The fact that housing supply is inelastic ($\gamma_h < \infty$) implies an externality: Workers do not generate the same degree of congestion in all regions which is not taken into account when choosing a place of residence. Thus, there is space for welfare improvement by reallocating workers across space (see Fajgelbaum and Gaubert, 2020). Second, due to differences in wages across space, marginal utilities of tradable good consumption are not constant across regions. The social planner increases welfare by redistributing toward regions with low marginal utilities of consumption.

My aim is to contrast the decentralized allocation with the solution to the planner's problem. I solve the problem of a social planner who takes as given that workers can freely move across labor markets. Under this assumption, expected utility of a type-k worker is given by

$$u_k^{exp} \equiv \psi \log(\sum_i e^{\frac{\frac{\rho}{\rho-1} \log\left(\gamma(c_{ik})^{1-\frac{1}{\rho}} + (1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}}\right) + \log E_{ik}}{\psi}}). \tag{11}$$

Then, if ω_k denotes the welfare weight for skill type k, I can postulate the generalized social welfare function

$$\mathcal{W} = \sum_k \omega_k \psi \log(\sum_i e^{\frac{\frac{\rho}{\rho-1} \log\left(\gamma(c_{ik})^{1-\frac{1}{\rho}} + (1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}}\right) + \log E_{ik}}{\psi}}). \tag{12}$$

The planner maximizes the expression in equation (12) subject to workers' location choices (equation (4)), the resource constraint on housing (equation (9)), as well as the resource constraint on tradables (equation (8)). I turn next to characterizing the solution to this planning problem. Details on the calculation of the social planner solution can be found in Appendix B.

Competitive equilibria according to the definition in Section 3.1.3 may not correspond to a point on the Pareto frontier due to spatial inefficiencies: Workers do not internalize the impact that their location choice has on other workers in the form of housing congestion. The social planner takes the social costs of additional workers in different regions into account when setting incentives for workers to move between labor markets. In the optimal allocation, the social marginal cost of an additional type-k worker in region i has to equal its social marginal value. More formally, I can express optimal expenditures as

$$\mu^Y c_{ik} + \mu_i^h h_{ik} = w_{ik} + \widetilde{\Pi}_{ik} + \lambda_{ik} \tag{13}$$

where μ^Y, μ_i^h and λ_{ik} are Lagrange multipliers on the government budget constraint, the resource constraint on housing and the mobility constraint. $\widetilde{\Pi}_{ik}$ denotes the social marginal value generated in the housing sector:

$$\widetilde{\Pi}_{ik} = \mu_i^h \frac{\partial H_i}{\partial L_{ik}}. \tag{14}$$

Thus, the social planner implements transfers according to

$$t_{ik} = \Phi_{ik} + \lambda_{ik} \tag{15}$$

where $\Phi_{ik} = \widetilde{\Pi}_{ik} - \Pi_{ik}$ is the wedge between the private and the social marginal value of an extra type-k worker in region i.

The proposition generalizes a key insight in Fajgelbaum and Gaubert (2020) to an economy with non-homothetic preferences and imperfect worker mobility between regions. As pointed out by Fajgelbaum and Gaubert (2020), the optimal transfers t_{ik} take care of inefficiencies due to spillovers as well as distributional concerns. In the absence of spillovers, I would still have $t_{ik} = \lambda_{ik}$, so that transfers would redistribute according to differences in the marginal utility of consumption across individuals, as implied by the second welfare theorem. In Fajgelbaum et al. (2018), workers are perfectly mobile and hold Cobb-Douglas preferences. In this case, λ_{ik} does not depend on the region. The burden of dealing with spatial inefficiencies falls on the other component of the optimal transfer scheme, corresponding to the first term in equation (15). Since my model does not incorporate positive externalities and incomes are on average lower in rural areas, which implies higher marginal utilities of tradable goods consumption, the social planner will transfer resources from urban to rural areas, and thereby incentivise individuals to move away from expensive large cities.

Both determinants of the optimal transfer scheme are affected by the non-homotheticity of preferences. In terms of efficiency considerations, incomes are on average higher in urban areas, which implies that individuals have lower housing expenditure shares and suffer less from the lack of affordable housing. Due to the weaker congestion forces, the social planner will redistribute less workers towards rural areas. The redistribution of workers is also less efficient when housing expenditure shares decline with income. Implementing negative transfers in urban expensive cities leads to a decline in the disposable income of individuals in these regions, which implies an increase in their housing expenditure share and an increase in their housing demand. Non-homothetic preferences therefore mitigate the decline in housing congestion achieved from setting negative transfers in expensive regions that incentivise individuals to move towards less expensive areas.

Non-homothetic preferences also affect the optimal degree of skill sorting. Since lower skilled workers have a larger housing expenditure share than with homothetic preferences, they demand more housing. This implies that they generate stronger congestion forces than with homothetic preferences, while at the same time being more sensitive to housing congestion. Workers with higher skills, on the other hand, generate lower congestion forces than with homothetic preferences, while being less sensitive to housing congestion. Furthermore, by changing the marginal utililities of tradable good consumption, non-homothetic preferences affect optimal transfers which determine the optimal degree of redistribution between regions for both skills. With non-homoethetic preferences affecting the optimal transfer scheme in several different ways and many of those working into different directions, it is ex-ante not clear how the optimal allocation with a generalized utility function compares to the results obtained when assuming Cobb Douglas preferences.

Note that with GES preferences, the housing expenditure share depends not only on individuals' income but also on regional house prices. In Section 5.3, I show that accounting for a positive elasticity of the housing expenditure share with respect to house prices does matter: the results when assuming CES preferences are significantly different from those obtained when assuming Cobb Douglas preferences.

4 Quantification

I calibrate the model to German labor market regions in 2007 and 2017. The quantification of the model consists of two steps that follow the literature on quantitative spatial models (see Redding and Rossi-Hansberg, 2017 for an overview). First, I obtain values of the structural parameters. I estimate the preference parameters γ, ρ and η using variables observed in the data and the structure of the model. The housing supply elasticity γ_H and the migration elasticity ψ

are taken from the literature. Second, I use data from 2007 and 2017, the calibrated parameter values, and the structure of the model to invert the structural fundamentals A_{ik}, T_i and E_{ik} separately for 2007 and 2017.

4.1 Data

I estimate the model on the level of 141 German labor market regions as defined by Kosfeld and Werner (2012) based on commuting data. The areas are constructed by combining one or more districts with the aim of creating self-contained labor markets. The boundaries of local labor markets are defined such that commuting flows between labor market regions are minimized. I drop all regions in which the number of observations for any worker group is smaller than 20. I end up with a sample of 138 labor markets.

I obtain information on regional employment and wages for different worker groups from the microdata on individual employment histories from the Sample of Integrated Labor Market Biographies (SIAB) provided by the Institute for Employment Research (Antoni et al., 2019). The SIAB is a 2% representative sample of administrative data on all workers who are subject to social security contributions excluding self-employed and civil servants. I restrict the sample to full-time workers between 20 and 64 and use the consumer price index from Statistisches Bundesamt (2019) to calculate real wages. In the SIAB data, I only observe wages up to the social security contribution ceiling. To impute top-coded wages for the roughly 5% of observations above the social security contribution ceiling, I use the approach from Dauth and Eppelsheimer (2021).

I split the sample into 2 groups: Workers with and workers without a university degree.[5] I aggregate wages to the labor market level by running the following regression for every worker group k and for the years 2007 and 2017 separately:

$$\ln w_n^{raw} = \alpha_k + \beta_k X_n + d_{ik} + \epsilon_n \tag{16}$$

where X_n is a set of observable worker characteristics, d_{ik} is a group-region dummy, and ϵ_n is an error term.[6] Given the Mincerian regressions and assuming ϵ_n is normally ditributed, I can

[5]Individuals are assigned the highest qualification level that they achieve throughout their working life.

[6]The controls include sex, a dummy that indicates whether a person is German, detailed level of educational attainment, duration of past unemployment periods, and duration of past unemployment periods squared.

rescale average wages according to

$$w_{ik} = \exp\left(\alpha_k + \frac{1}{L_k}(\sum_{n\in k} X_n)\beta_k + d_{ik} + \frac{\sigma^2}{2}\right) \tag{17}$$

which represents the average wage of a type-k worker in region i while assuming that workers have otherwise identical characteristics between regions.

I use a house price index from Ahlfeldt et al. (2022) who utilize data from the FDZ (Forschungsdatenzentrum) Ruhr on real estate offers published on the largest German listing website ImmobilienScout24 with a self-reported market share of about 50% (see Klick and Schaffner, 2019). By combining a hedonic regression approach with recent extensions that treat spatial units as the nucleus of a spatial price gradient, Ahlfeldt et al. (2022) generate an index that controls for property characteristics and distance from the center of the labor market region.

Table 1: Summary Statistics

	2007		2017	
	mean	sd	mean	sd
Wage low skill	83.17	9.25	94.91	10.09
Wage high skill	148.17	22.42	157.64	21.03
Total employment (in thd)	126.91	142.75	126.13	146.04
Share high skill (in %)	12.62	4.36	17.10	5.57
House purchase price	1	0.34	1.43	0.80

Note: The table shows descriptive statistics for cross-sectional data on the level of 138 labor markets. Wages are gross daily wages, house prices are relative to the national mean in 2007.

To calibrate the preference parameters, I use consumption microdata from the GSOEP which is a yearly survey with information on income, expenditure and education of individuals in approximately 11000 private households.[7] I aggregate the data set to the household-level and merge it with the house price index on the district-level from Ahlfeldt et al. (2022).

[7] I exclude households where the household head is non-employed, doing an apprenticeship, is younger than 18 or older than 64 years, has refugee status or is seeking asylum, as well as all households with owner-occupied housing.

4.2 Structural parameters

Preference parameters ρ and η

To calibrate the preference parameters of the model, I utilize the household first order condition as defined in equation (10). Defining total expenditure $x_{ik} = c_{ik} + p_i h_{ik}$, multiplying with $\dfrac{p_i}{x_{ik}}$ and substituting for c_{ik} yields

$$s_{ik} = \left(\frac{1-\gamma}{\gamma}\frac{\rho-\eta}{\rho-1}\right)^{\frac{\rho}{\eta}} p_i^{1-\frac{\rho}{\eta}} (x_{ik})^{-(1-\frac{1}{\eta})}(1-s_{ik})^{\frac{1}{\eta}} \tag{18}$$

where $s_{ik} = \dfrac{p_i h_{ik}}{x_{ik}}$ denotes the housing expenditure share. To estimate this equation, I use the variation across households h and years t. I interpret $\alpha \equiv (\frac{1-\gamma}{\gamma}\frac{\rho-\eta}{\rho-1})^{\frac{\rho}{\eta}}$ as an idiosyncratic shock to a household's taste for housing, so that equation (18) becomes

$$s_{ht} = \alpha_{ht} p_{it}^{1-\frac{\rho}{\eta}} (x_{ht})^{-(1-\frac{1}{\eta})}(1-s_{ht})^{\frac{1}{\eta}}. \tag{19}$$

I follow Finlay and Williams (forthcoming) and log-linearize equation (19) around the mean housing expenditure share $\bar{s}$ to obtain

$$\widehat{s}_{ht} = \frac{\eta(1-\bar{s})}{\eta(1-\bar{s})+\bar{s}}\left(\widehat{\alpha}_{ht} + (1-\frac{\rho}{\eta})\widehat{p}_{it} - (1-\frac{1}{\eta})\widehat{x}_{ht}\right) \tag{20}$$

where $\widehat{y}$ denotes the log deviation of a variable y from its mean. Defining $\beta_{ht} \equiv \frac{\eta(1-\bar{s})}{\eta(1-\bar{s})+\bar{s}}\widehat{\alpha}_{ht}$, $\theta \equiv \frac{\eta(1-\bar{s})}{\eta(1-\bar{s})+\bar{s}}(1-\frac{\rho}{\eta})$ and $\zeta \equiv -\frac{\eta(1-\bar{s})}{\eta(1-\bar{s})+\bar{s}}(1-\frac{1}{\eta})$, equation (20) simplifies to

$$\widehat{s}_{ht} = \beta_{ht} + \theta\widehat{p}_{it} + \zeta\widehat{x}_{ht}. \tag{21}$$

Under the null of homothetic preferences, $\theta = \zeta = 0$. I bring equation (21) to the data by modeling the demand shifter β_{ht} as a function of observables, year fixed effects, and an additive error. Formally, I get

$$\widehat{s}_{ht} = \beta_t + \delta X_{ht} + \theta\widehat{p}_{it} + \zeta\widehat{y}_{ht} + \epsilon_{ht} \tag{22}$$

where X_{ht} is a vector of demographic characteristics which includes household size, the number of earners in the household as well as the gender and age of the household head. I observe total expenditure x_{ht}, the housing expenditure share s_{ht}, and prices p_{it}. The error term ϵ_{ht} represents measurement error in expenditure plus random shocks to housing demand which both are assumed to be uncorrelated with expenditure and prices conditional on the controls. In my preferred specification, I estimate the nonlinear equation (19) directly by GMM.

Since expenditure data is only available from 2010 to 2014, I restrict my sample to these years. I drop households in the top and bottom 1% of the income and expenditure distribution each year to guard against serious misreporting errors. I further restrict the sample to renters. Since homeownership rates are increasing with income and homeowners spend on average less on housing than renters, I expect my estimates to be a lower bound of non-homotheticity. Finlay and Williams (forthcoming) use data on housing expenditures of homeowners and find similar results to those for renters.

Table 2: Preference Estimates

Dependent variable: Log housing expenditure share

	(1) OLS	(2) OLS IV	(3) OLS	(4) OLS IV	(5) GMM	(6) GMM IV
Log expenditure	-0.477***	-0.212***	-0.529***	-0.270***		
	(0.013)	(0.021)	(0.012)	(0.021)		
Log price			0.311***	0.233***		
			(0.017)	(0.017)		
eta			2.576***	1.520***	1.746***	1.807***
			(0.079)	(0.057)	(0.025)	(0.028)
rho			1.649***	1.072***	0.643***	0.598***
			(0.066)	(0.044)	(0.034)	(0.036)
Demographic controls	✓	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓	✓
R2	.331	.331	.055	.055		
adj. R2	.331	.33	.054	.054		
First stage F-statistic		122970		118590		
N	8232	8232	8232	8232	8232	8232
No. of clusters	4659	4318	4659	4659	4318	4318

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: Standard errors in parentheses, clustered at the household level. Renters only. Instrument is log household income. Demographic controls include household size, number of earners as well as gender and age of the household head.

The estimation results are shown in Table 2. To deal with measurement error in expenditure, I follow Finlay and Williams (forthcoming) and use income as an instrument for expenditure. I find an expenditure elasticity of $\zeta = -0.27$ which is in line with estimates found in the literature that range from -0.88 to -0.01 (see the literature review in Finlay and Williams, forthcoming). Controlling for house prices on the district level increases the absolute size of the expenditure elasticity. Since higher income households sort into expensive regions, the estimated elasticity will be biased toward zero when not controlling for house prices. Offsetting price and income effects are in line with the findings in Albouy et al. (2016) and Finlay and Williams (forthcoming).

Finally, column (6) shows my preferred specification where I estimate the nonlinear equation (19) directly by GMM and instrument for expenditure. I estimate $\rho = 0.60$ and $\eta = 1.81$. Finlay and Williams (forthcoming) find estimates of the price elasticity $\theta = 0.39$ and the expenditure elasticity $\zeta = -0.25$. Assuming a mean housing expenditure share of $\bar{s} = 0.29$ as observed in the data, these estimates imply $\rho = 0.78$ and $\eta = 1.44$. In Appendix A.2, I show that my results are close to Finlay and Williams (forthcoming) when estimating preference parameters from NHCES preferences using GMM.

I follow Finlay and Williams (forthcoming) and compare the preferences estimated in Table 2 to two benchmarks from the literature: Cobb-Douglas preferences and a unit housing requirement. The GES preferences estimated above nest both of these special cases. The null hypothesis of Cobb-Douglas preferences, corresponding to $\rho \to 1$ and $\eta = 1$, can be rejected at the 1% level. A unit housing requirement corresponds to $\eta \to \infty$. Column (6) allows me to reject the null hypothesis that $\eta = 1.87$ or any number above at the 1% level.

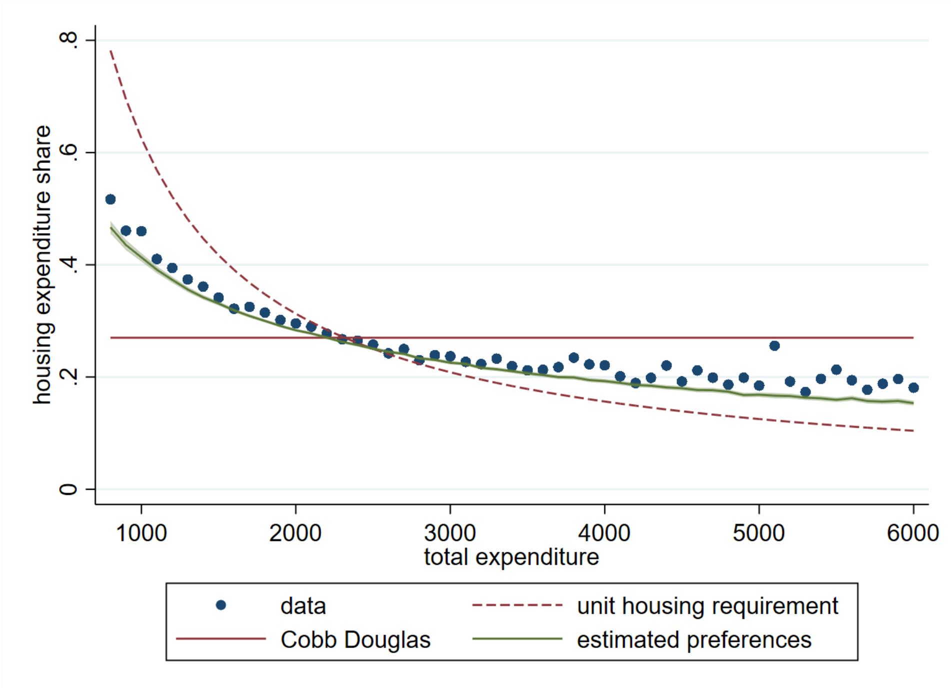
Although my GES specification is more general than these special cases, it still imposes a particular functional form on the relationship between total expenditure and the housing expenditure share. I assess the validity of this assumption by constructing a binned scatterplot of expenditure against housing shares as shown in Figure 3. My estimated preferences appear to fit the data well. For comparison, I also include housing expenditure shares as implied by Cobb-Douglas and a unit housing requirement. Neither alternative comes close to matching the data.

Housing congestion γ_H

To calibrate the elasticity of housing supply to population, I combine equation (9) and (10) and solve for p_i

$$p_i = \frac{1-\gamma}{\gamma}\frac{\rho-\eta}{\rho-1}\Big(\frac{\sum_k L_{ik} c_{ik}^{\frac{1}{\eta}}}{H_i}\Big)^{\frac{\eta}{\rho}}. \tag{23}$$

Figure 3: Preference calibration



Note: 'Estimated Preferences' plots equation (20) at the parameter values obtained in Table 2, column (4). The shaded area represents a 95% confidence interval. 'Cobb-Douglas' plots the preferences with $\rho \to 1$ and $\eta = 1$ and 'Unit Housing Requirement' with $\eta \to \infty$. The scale parameter γ is chosen to match the expenditure share as in the data. 'Data' plots the average housing share in 100 evenly sized bins defined by total expenditure.

The elasticity of house prices with respect to population is

$$\mathcal{E}_k \equiv \frac{\partial p_i}{\partial L_{ik}} \frac{L_{ik}}{p_i} = \frac{\eta}{\rho} \Big(\frac{L_{ik} c_{ik}^{\frac{1}{\eta}}}{\sum_k L_{ik} c_{ik}^{\frac{1}{\eta}}} - \gamma_H \frac{L_{ik}}{L_i} \Big). \tag{24}$$

Summing over k yields

$$\sum_k \mathcal{E}_k = \frac{\eta}{\rho}(1 - \gamma_H). \tag{25}$$

Assuming equal elasticities for all worker types

$$\mathcal{E} = \frac{1}{K} \frac{\eta}{\rho}(1 - \gamma_H). \tag{26}$$

where K denotes the number of worker groups. I take the parameter $\mathcal{E} = 0.208$ from Combes et al. (2019). This implies $\gamma_H = 1 - K\frac{\rho}{\eta}\mathcal{E} = 0.861$.

Scale parameter γ

The scale parameter γ is not identified separately from the scale of prices and consumption, so

I normalize it to match the aggregate housing share. Plugging $x_{ik} = \frac{c_{ik}}{1-s_{ik}}$ into equation (18), I get

$$\frac{s_{ik}}{1-s_{ik}} = \left(\frac{1-\gamma}{\gamma}\frac{\rho-\eta}{\rho-1}\right)^{\frac{\rho}{\eta}} \frac{p_i^{1-\frac{\rho}{\eta}}}{(c_{ik})^{1-\frac{1}{\eta}}} \tag{27}$$

which I numerically solve for s_{ik} and γ using the additional constraint that the mean housing expenditure share matches the observed data ($\frac{1}{L}\sum_i \sum_k L_{ik}s_{ik} = 0.29$).

Table 3: Calibrated Parameters

Parameter	Value	Source
Preferences		
ρ	0.60	Estimated
η	1.81	Estimated
ψ	0.5	Gaubert et al. (forthcoming)
Congestion forces		
γ_H	0.86	Combes et al. (2019), own calculation

4.3 Structural fundamentals

I obtain the location-specific productivity, housing supply and amenity shifters A_{ik}, T_i and E_{ik} by inverting the model so that it exactly matches the observed data on p_i, w_{ik}, t_{ik} and L_{ik} for all regions i and skill types k. Abstracting from income taxes, social security contributions and transfers, I set $t_{ik} = 0$.[8] From equation (6), I calculate productivity fundamentals

$$A_{ik} = w_{ik}. \tag{28}$$

Plugging housing supply (equation (10)) and housing demand (equation (7)) in the housing market clearing condition (equation (9)), I get an expression for the housing supply shifter that

[8] There are no location-specific income taxes in Germany. Note that linear taxes do not change the results in the case of Cobb-Douglas preferences. I abstract from non-linearities due to GES preferences and from non-linearities in income taxes, social security contributions and transfers.

depends solely on variables that I observe in the data

$$T_i = L_i^{-\gamma_H}\Big(\frac{1}{p_i}\frac{\rho-\eta}{\rho-1}\frac{1-\gamma}{\gamma}\Big)^{\frac{\rho}{\eta}}\sum_k L_{ik}(w_{ik}+t_{ik})^{\frac{1}{\eta}}. \tag{29}$$

Finally, I combine the mobility constraint (equation (4)) with the budget constraint (equation 2) to get an equation that I can numerically solve for amenities E_{ik}

$$L_{ik} = \frac{\Big((\gamma(w_{ik}+t_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}})^{\frac{\rho}{\rho-1}}E_{ik}\Big)^{\frac{1}{\psi}}}{\sum_i\Big((\gamma(w_{ik}+t_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}})^{\frac{\rho}{\rho-1}}E_{ik}\Big)^{\frac{1}{\psi}}}L_k$$

with housing demand from equation (10):

$$h_{ik} = (\frac{1}{p_i}\frac{1-\gamma}{\gamma}\frac{\rho-\eta}{\rho-1})^{\frac{\rho}{\eta}}(w_{ik}+t_{ik})^{\frac{1}{\eta}}. \tag{30}$$

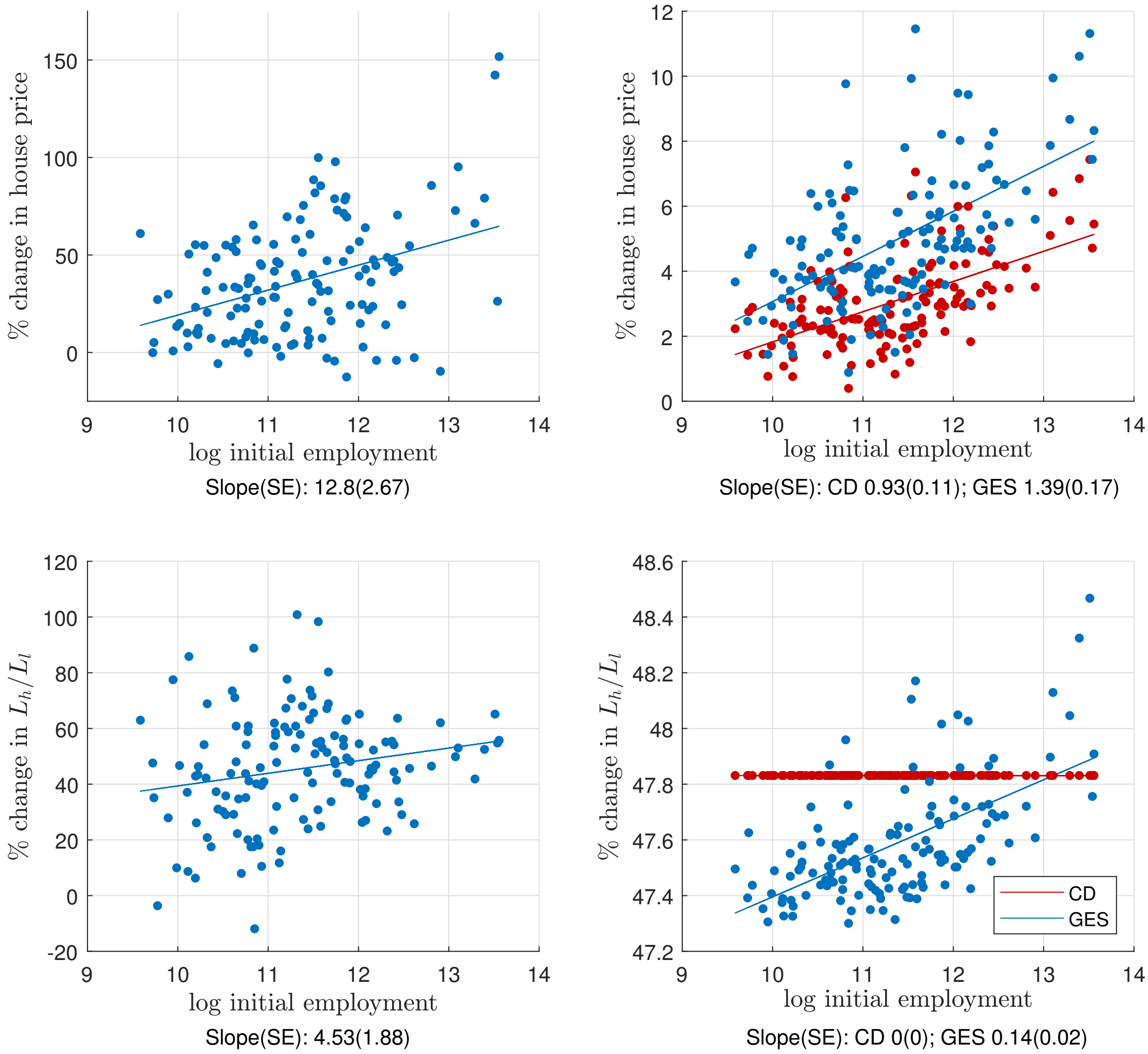
5 Results

5.1 Changes in house prices and geographic sorting

I use the calibrated model to quantify the importance of accounting for non-homothetic preferences when analyzing geographic worker sorting. To do so, I estimate the effect of the rise in housing congestion resulting from the national growth in the relative supply of high skilled workers. The shock amounts to an increase in the share of high skilled workers from 16% in 2007 to 22% in 2017. I plot a decomposition of the increase in the regional dispersion of house prices and the increase in skill sorting from 2007 to 2017. I compare the decomposition results obtained from a model with non-homothetic preferences with those from a model with constant housing expenditure shares. When assuming Cobb-Douglas preferences, I calibrate the scale paraemter γ to match the mean housing expenditure share observed in the data. However, by calibrating $\rho \to 1$ and $\eta = 1$, I do not match any other moments obtained from the GSOEP consumption microdata which I match in the case of GES preferences (see Section 4.2).

The left panels of Figure 4 plot changes as observed in the data, while the right panels isolate changes resulting from the national increase in the relative supply of high skilled workers. I plot data in a counterfactual scenario in which the national skill share increased as observed in the data while fundamentals remain at their 2007 level. The upper panels in Figure 4 illustrate that the increase in housing congestion is more pronounced in large regions. I find that the national change in the relative supply of high skilled workers can explain 11% of the regional dispersion in house price increases. With homothetic Cobb-Douglas preferences, a change in the national skill share can explain only 7% of the regional dispersion in house price increases.

Figure 4: Decomposition of changes in sorting and house prices



Note: Unit of observation is 141 labor market areas as defined by Kosfeld and Werner (2012). The left panels show changes in house prices and sorting as observed in the data from 2007 to 2017. The right panels show changes in house prices and sorting in a counterfactual scenario in which the national skill share increases as observed in the data while fundamentals remain at their 2007 level.

Why do house price differences increase more with GES preferences? With non-homothetic preferences, geographic worker sorting intensifies since low skilled workers are hit harder by increases in housing costs that are more pronounced in skill-intensive regions. The increase in skill sorting, in turn, amplifies differences in house price increases. The slope parameters in the lower panels quantify the change in skill sorting. The parameter in the data is 4.53, while in the counterfactual scenario, I find a slope parameter of 0.14. These values indicate that the national change in the relative supply of high skilled workers can explain 3% of the observed change in skill sorting. With homothetic preferences, there is no change in worker sorting. When calculating the average national increase in house prices in the data and in the counterfactual, I find that roughly 10% of the national increase in housing costs can be explained by the model, while roughly one third of this increase comes from the assumption of non-homothetic preferences. In Section 5.3, I show that the results are robust to the calibration of the non-homotheticity parameter η.

Next, I calculate welfare changes implied by the national increase in the relative supply of high skilled workers. I measure the percentage change of pre-shock tradable good consumption that would make the worker as bad off as after the shock. I equalize expected utility as defined in equation (11) before and after the shock

$$u_k^{exp}\left((1+\Delta_k)c_{ik}^{2007}, h_{ik}^{2007}, E_{ik}^{2007}\right) = u_k^{exp}\left(c_{ik}^{sh}, h_{ik}^{sh}, E_{ik}^{sh}\right) \tag{31}$$

where $c_{ik}^{2007}, h_{ik}^{2007}, E_{ik}^{2007}$ are consumption of tradables, housing and amenities in the observed equilibrium. $c_{ik}^{sh}, h_{ik}^{sh}, E_{ik}^{sh}$ are values in the counterfactual scenario in which the national share of high skilled workers changes as in the data, while fundamentals remain as in 2007. Solving numerically for Δ_k, I find that the increase in the national share of high skilled workers has decreased expected utility of high skilled workers by 0.8% and expected utility of low skilled workers by 0.9%. Since low skilled workers spend a larger share of their income on housing, they were hit harder by increases in housing congestion.

5.2 The size of inefficiencies

After having explored the importance of allowing for non-homothetic preferences when analyzing sorting in spatial equilibrium models, I next estimate the optimal degree of sorting. To do so, I solve the problem of a social planner who uses transfers between locations and worker types which change the spatial distribution of economic activity. By changing the location incentives of workers, they affect spatial sorting and the spatial concentration of population. These reallocations in turn impact house prices, which feed back to location choices. In the following, I describe the spatial equilibrium resulting from this process.

I start by calibrating the welfare weights ω_k such that both worker types experience the same welfare gain as compared to the observed allocation. I measure welfare gains as the percentage change in tradable good consumption that would make individuals as well off as in the optimal allocation. Similarly to the measurement in equation (31), I obtain the welfare change Δ_k from numerically solving

$$u_k^{exp}\left((1+\Delta_k)c_{ik}^{obs}, h_{ik}^{obs}, E_{ik}^{obs}\right) = u_k^{exp}\left(c_{ik}^{opti}, h_{ik}^{opti}, E_{ik}^{opti}\right) \tag{32}$$

where $c_{ik}^{obs}, h_{ik}^{obs}$ and E_{ik}^{obs} are consumption and amenities in the observed allocation, while $c_{ik}^{opti}, h_{ik}^{opti}$ and E_{ik}^{opti} are values in the optimal allocation.[9]

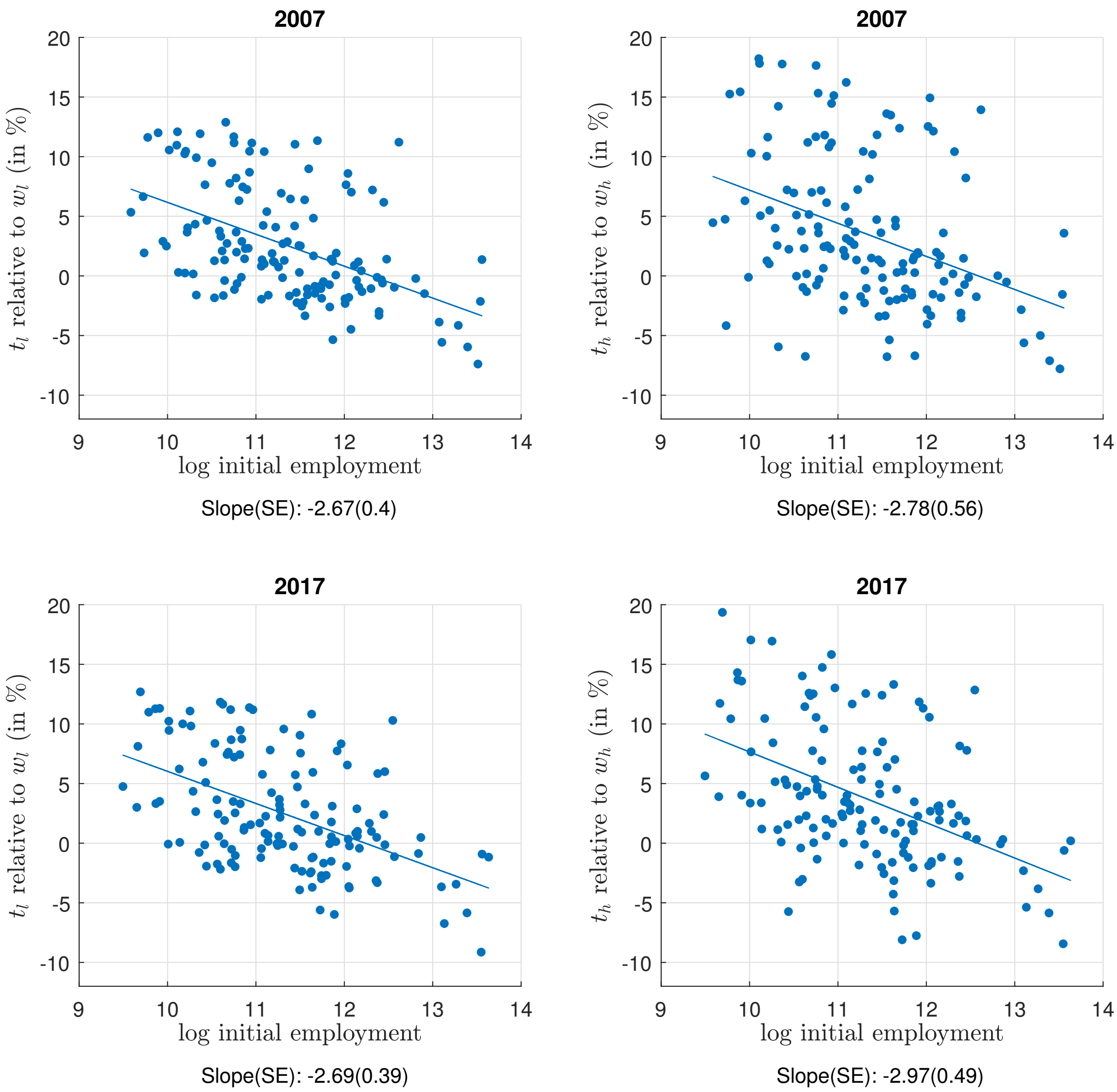
Figure 5 illustrates the transfer scheme that implements the optimal allocation. Both in 2007 and 2017, it is optimal to set incentives for high and low skilled workers to move toward less populated areas. As formally shown in Section 3.2, the social planner takes into account redistribution and efficiency considerations when choosing a regional skill-specific transfer scheme. In terms of efficiency, it is optimal to set larger transfers in rural areas because I include only negative externalities in the form of housing congestion. In terms of redistribution, optimal transfers are larger in rural areas since wages are on average lower than in urban areas which implies larger marginal utilities of tradable good consumption. It is therefore intuitive that the spatial concentration of population is smaller in the optimal allocation as compared to the observed allocation.

As illustrated in the upper panels of Figure 6, it is further optimal to decrease skill sorting by moving a larger share of high skilled workers toward less populated regions. Since high skilled workers, by consuming more housing than low skilled workers, generate larger congestion forces, it is optimal to reallocate them to a larger extent toward regions with less congested housing markets. Furthermore, since the urban wage premium is larger for high skilled workers, spatial differences in marginal utilities of tradable good consumption are larger for high skilled than for low skilled workers.[10] The smaller degree of agglomeration and skill sorting in the optimal allocation imply that house prices are substantially less dispersed as reflected in a larger increase

[9]Note that calibrating welfare weights such that welfare gains are equal for both worker types implies welfare weights to be differentially calibrated for 2007 and 2017. However, when estimating the social planner solution, I find negligible changes in welfare weights: Calibrating the welfare weights to sum up to 1, I find the social planner to choose a weight for high skilled workers of 0.627 in 2007 and of 0.639 in 2017. The results are robust to calibrating the welfare weights in 2017 to the weights found for the calibration in 2007.

[10]The urban wage premium is plotted in Figure A.3.

Figure 5: Optimal transfers

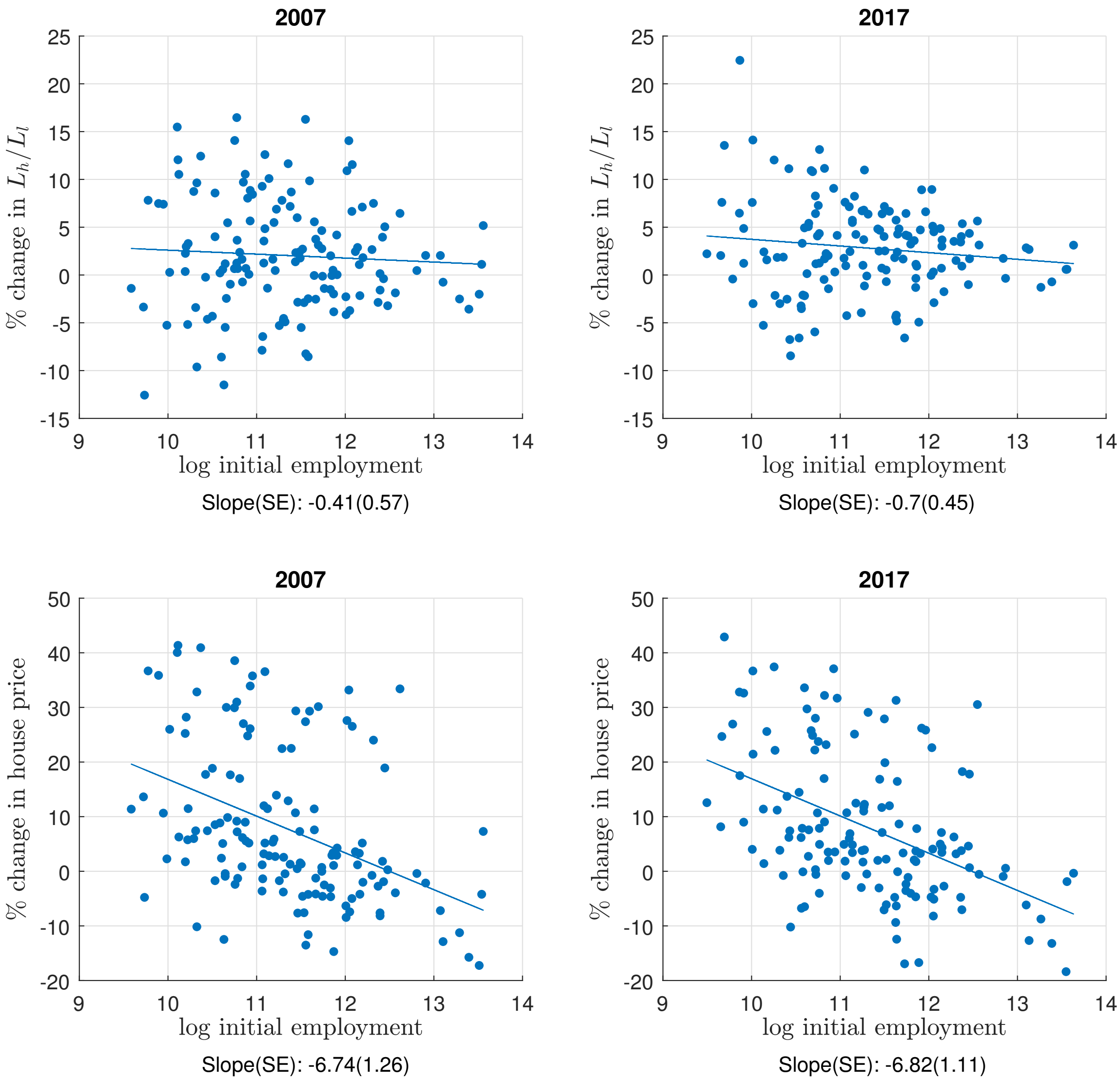


Note: Unit of observation is 141 labor market areas as defined by Kosfeld and Werner (2012). The plots show optimal transfers for low and high skilled workers relative to their wage. The planner's weights are chosen such that both types of workers experience the same welfare gains.

in house prices in rural as compared to urban areas (see the lower panels of Figure 6). Doubling region size is associated with a roughly seven percentage point lower change in house prices.

How should transfers adjust to changes from 2007 to 2017? Figure 5 illustrates that in 2017, optimal policies imply a greater degree of redistribution between regions compared to 2007. A larger dispersion in transfers, in turn, implies larger incentives for individuals to move across space. The upper panels in Figure 6 show that it is optimal to decrease sorting by 70% more than in 2007: The slope parameter decreases from -0.41 in 2007 to -0.7 in 2017. In Figure A.3,

Figure 6: Optimal sorting and house prices

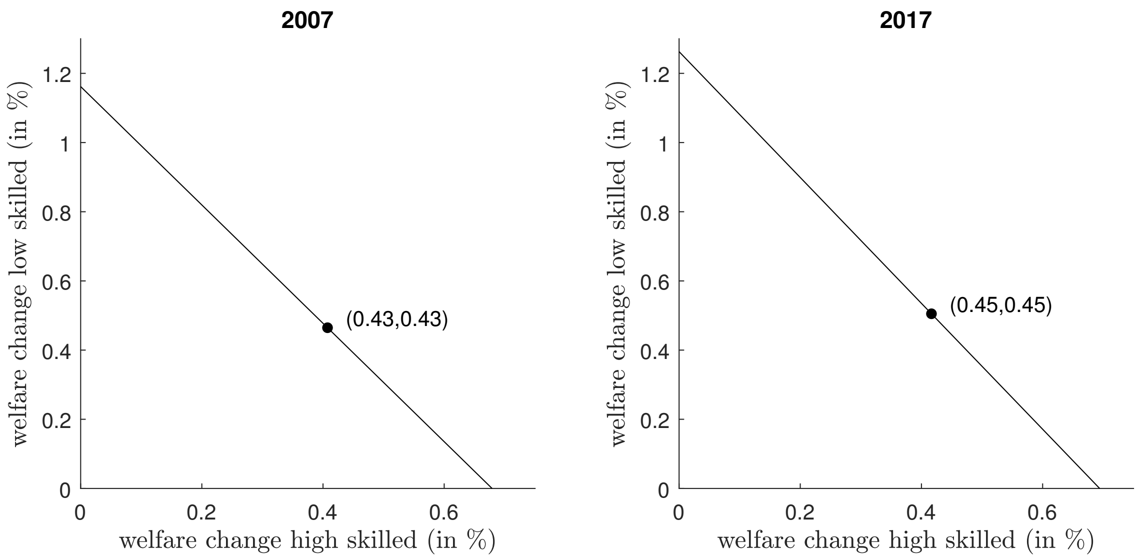


Note: Unit of observation is 141 labor market areas as defined by Kosfeld and Werner (2012). The plots show the change in skill sorting and house prices between the observed equilibrium and the optimal allocation. The planner's weights are chosen such that both types of workers experience the same welfare gains.

it can be seen that the urban wage premium has decreased for both worker types, which implies that the stronger decrease in sorting in 2017 is driven by efficiency considerations rather than differences in marginal utilities of consumption. Since sorting has increased from 2007 to 2017, congestion externalities generated by high skilled individuals have increased to a larger extent. It is therefore optimal from a social planner perspective to decrease sorting more than in 2007. As a consequence, moving from the observed to the optimal allocation implies a larger decrease in house price differences between rural and urban areas than in 2007, as shown in the lower panels of Figure 6.

Next, I allow for differential welfare gains for workers with and without a university degree moving from the observed to the optimal allocation. Figure 7 plots the utility frontier obtained from solving for the optimal allocation on a grid of welfare weights ω_k. Welfare gains are measured as given in Equation (32). I choose the grid of welfare weights such that welfare gains for both worker types are positive. I find that for any combination of welfare gains, the benefits from moving from the observed to the optimal allocation are larger in 2017 than in 2007. When both worker types benefit equally, welfare gains amount to 0.43% of tradable good consumption in 2007 and 0.45% in 2017. Since sorting and the spatial dispersion in house prices have increased from 2007 to 2017, it is optimal to redistribute more in 2017, which implies larger welfare gains.

Figure 7: Utility frontier between high and low skilled workers



Note: The plots show the change in welfare between the observed equilibrium and the optimal allocation. Welfare changes are measured as the percentage change in tradable good consumption in the observed allocation that would make the individual as well off as moving to the optimal allocation.

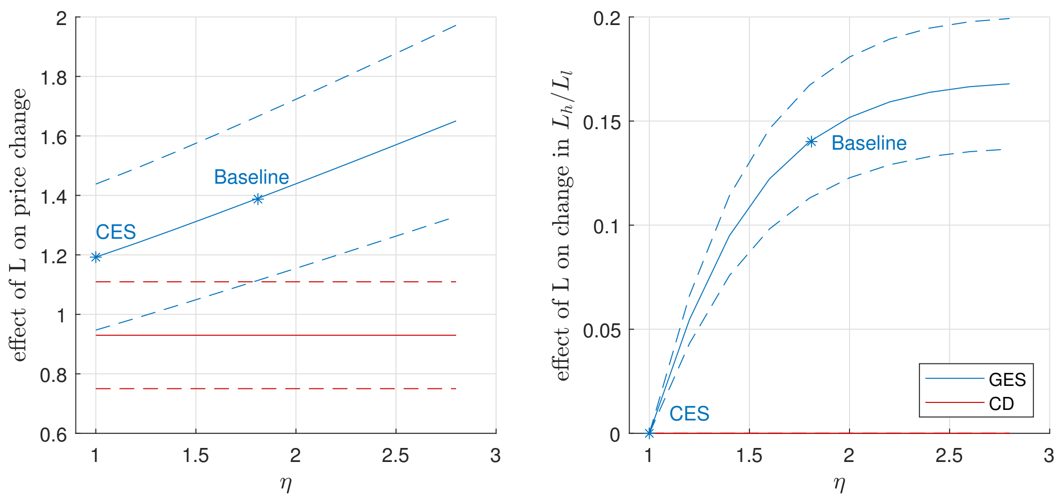
Both in 2007 and 2017, the maximum possible welfare gain of low skilled workers is substantially larger than that of high skilled workers. While low skilled workers can gain more than 1.2% from moving from the observed to the optimal allocation in 2017, the welfare gain for high skilled workers does not exceed 0.7%. Since low skilled workers spend a larger share of their income on housing, they benefit more from reduced housing congestion.

5.3 Robustness to the calibration of the preference parameters

In the baseline specification, I have focused on renting households which likely provide a lower bound of non-homotheticity. To test the robustness of the results to the calibration of the non-homotheticity preference parameter, I solve the model on a grid of η from one, which corresponds to CES preferences, to 2.8, which is the average that existing studies find for homeowners.[11]

Figure 8 shows for different calibrations of the key preference parameter the slope coefficients of the fitted lines in Figure 4, which capture how changes in house prices and sorting vary across regions of different size. The results are robust even when calibrating the model with a high non-homotheticity parameter corresponding to the value for homeowners. With $\eta = 2.8$, the national increase in the supply of high skilled workers explains roughly 13% of the regional dispersion in house price increases, while it still explains roughly 10% of the national increase in house prices. In this specification, it accounts for 4% of the increase in spatial skill sorting. Note that the plotted calibrations are not necessarily empirically relevant, since I keep the price elasticity ρ constant, which might not be independent of the degree of non-homotheticity.

Figure 8: The effect of region size on model-implied changes in sorting and house prices

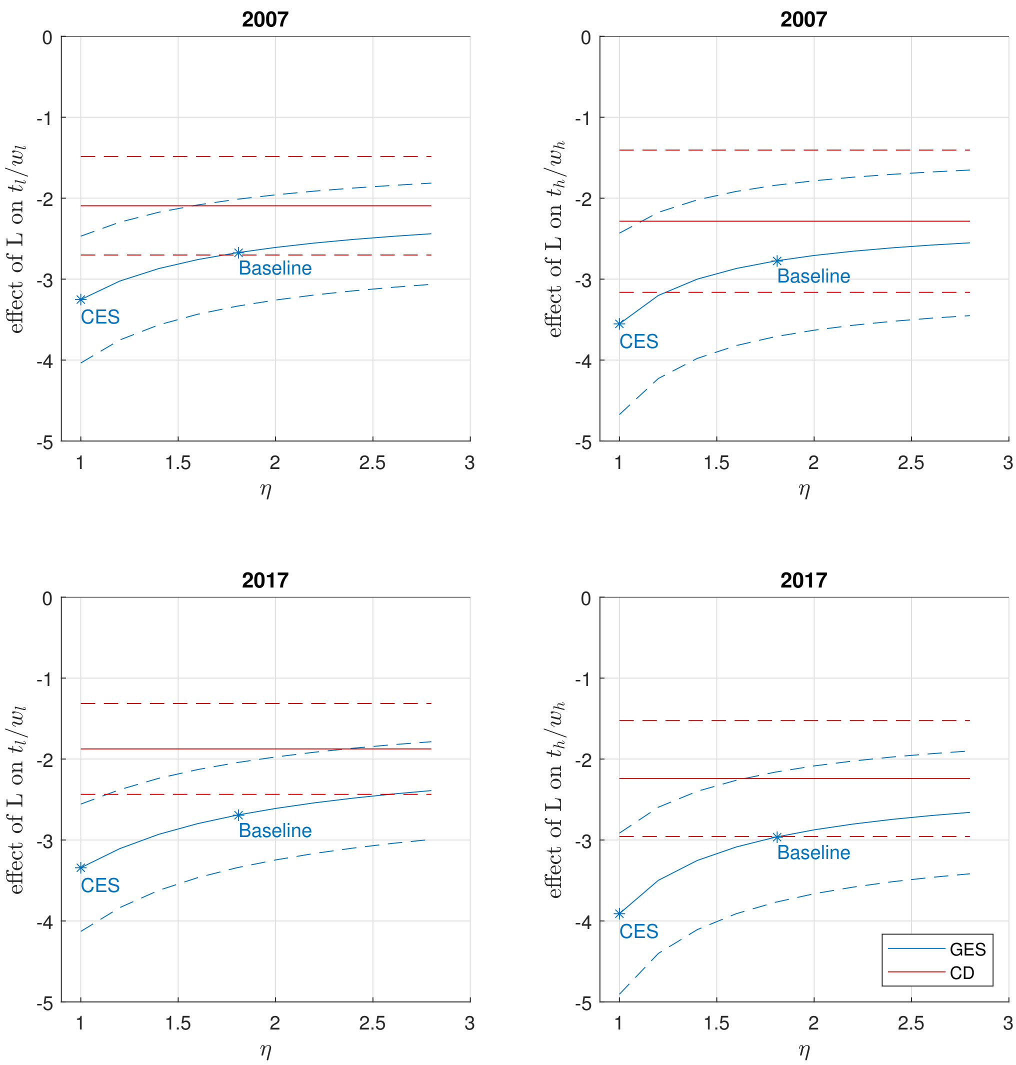


Note: Slope parameters from a linear regression of model-implied changes in house prices and geographic sorting on region size with different calibrations of the preference parameter η. The dashed lines show 90% confidence intervals. The model-implied changes refer to a counterfactual scenario in which the national skill share increases as observed in the data while fundamentals remain at their 2007 level. The slope parameters in the data are 12.8 for house prices and 4.5 for skill sorting.

[11] I take the average of the studies by Finlay and Williams, forthcoming, Ioannides and Zabel, 2008, Larsen, 2014 and Zabel, 2004).

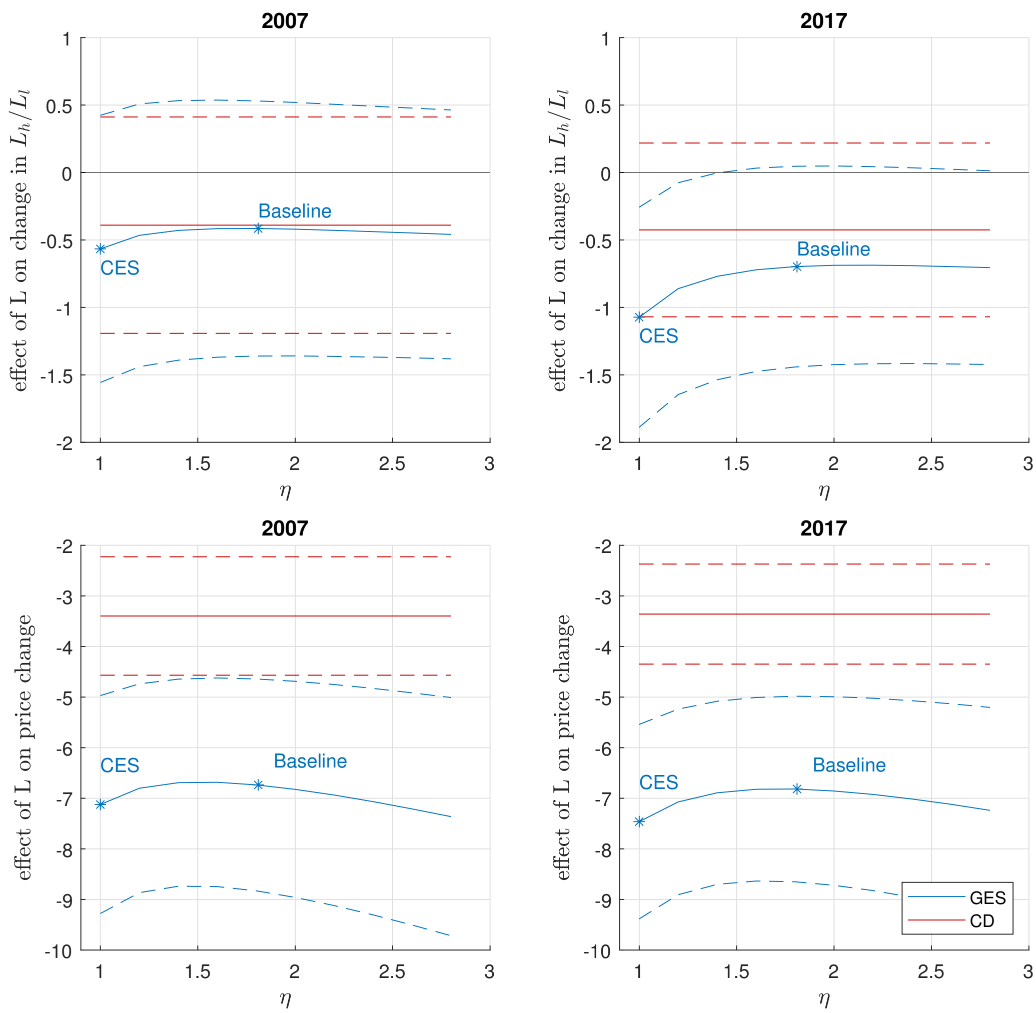
Next, I analyse the optimal allocation with different calibrations of the preference parameters. Figure 9 and Figure 10 show the results presented in Section 5.2 on a grid of the non-homotheticity parameter η, in particular the slope parameters of the fitted lines in Figure 5 and Figure 6. Optimal transfers as well as changes in geographic sorting and house prices are robust to higher calibrations of η. The results that I obtain with Cobb Douglas preferences are broadly in line with the findings of Fajgelbaum and Gaubert (2020) for the US. I further find that the modeling of preferences matters: the social planner implements significantly larger transfers in 2017 when assuming GES preferences as compared to Cobb Douglas preferences.

Figure 9: Optimal transfers with different calibrations of the preference parameters



Note: Slope parameters from a linear regression of optimal transfers relative to wages (in %) on region size with different calibrations of the preference parameter η. The dashed lines show 90% confidence intervals. The planner's weights are chosen such that both types of workers experience the same welfare gains.

Figure 10: Optimal spatial equilibrium with different calibrations of the preference parameters



Note: Slope parameters from a linear regression of optimal changes in house prices and the share of high to low skilled workers (both in %) on region size with different calibrations of the preference parameter η. The dashed lines show 90% confidence intervals. The planner's weights are chosen such that both types of workers experience the same welfare gains.

Figure 9 shows that the social planner implements substantially larger transfers with CES preferences than with Cobb Douglas preferences. The reason is that relocating workers is more efficient with CES preferences. When the social planner sets incentives for workers to move towards rural areas, housing costs decline in urban regions. With CES preferences, a reduction in housing costs leads to a decline in the housing expenditure share, which in turn helps to bring down the high costs of housing in dense expensive regions. This mechanism is reflected in a signifiantly larger decline in the regional dispersion of house prices with CES preferences than with Cobb Douglas preferences (see Figure 10). The slope paramter with CES and GES preferences is more than twice as high as the coefficient obtained when calibrating the model with Cobb Douglas preferences.

The non-homotheticity of preferences, in contrast, makes the redistribution of workers less efficient. Implementing negative transfers in urban expensive cities leads to a decline in the disposable income of individuals in these regions, which implies an increase in their housing expenditure share and an increase in their housing demand. Non-homothetic preferences therefore mitigate the decline in housing congestion achieved from setting negative transfers in expensive regions that incentivise individuals to move towards less expensive areas. Note, however, that optimal transfers take care of inefficiencies due to spillovers as well as distributional concerns, as laid out in Section 3.2. Both determinants of the optimal transfer scheme are affected by the assumptions on the utility function.

6 Conclusion

With non-homothetic preferences, an increase in the national supply of high skilled individuals intensifies spatial sorting. The reason is that low skilled workers are hit harder by increases in housing costs that are more pronounced in skill-intensive regions. The increase in sorting, in turn, amplifies house price increases in large cities as compared to rural areas. I find that 10% of the increase in average house prices in Germany from 2007 to 2017 can be explained by the growth in the national share of high skilled workers. My model further explains 11% of the observed regional differences in house price increases. Roughly one third of these effects is due to the increase in spatial sorting.

The results suggest that low income individuals are hit harder by increases in housing costs which is why they are avoiding increasingly expensive cities. I find that place-based policies that aim at lowering the degree of geographic sorting have small welfare effects. However, geographic sorting might have implications for welfare inequality beyond of the effects estimated in this paper. Workers' location choices might determine, besides housing market developments, also their labour market conditions and access to education and health services, amplifying the increases in inequality observed in many Western economies over the past decades.

References

Ahlfeldt, Gabriel, Stephan Heblich, and Tobias Seidel, "Micro-geographic property price and rent indices," *Regional Science and Urban Economics*, 2022, p. 103836.

Albouy, David, "The Unequal Geographic Burden of Federal Taxation," *Journal of Political Economy*, 2009, *117* (4), 635–667.

_ , Gabriel Ehrlich, and Yingyi Liu, "Housing demand, cost-of-living inequality, and the affordability crisis," *National Bureau of Economic Research No. w22816*, 2016.

Antoni, Manfred, Philipp vom Berge, Tobias Graf, Stephan Grießemer, Steffen Kaimer, Markus Köhler, Claudia Lehnert, Martina Oertel, Alexandra Schmucker, Stefan Seth, and Christan Seysen, "Weakly anonymous Version of the Sample of Integrated Labour Market Biographies (SIAB) – Version 7517 v1," 2019.

Baum-Snow, Nathaniel and Ronni Pavan, "Inequality and City Size," *The Review of Economics and Statistics*, 2013, *95* (5), 1535–1548.

Colas, Mark and Kevin Hutchinson, "Heterogeneous Workers and Federal Income Taxes in a Spatial Equilibrium," *American Economic Journal: Economic Policy*, 2021, *13* (2), 100–134.

Combes, Pierre-Philippe, Gilles Duranton, and Laurent Gobillon, "The Costs of Agglomeration: House and Land Prices in French Cities," *Review of Economic Studies*, 2019, *86* (4), 1556–1589.

Comin, Diego, Danial Lashkari, and Martí Mestieri, "Structural Change With Long-Run Income and Price Effects," *Econometrica*, 2021, *89* (1), 311–374.

Couture, Victor, Cecile Gaubert, Jessie Handbury, and Erik Hurst, "Income growth and the distributional effects of urban spatial sorting," *Review of Economic Studies*, forthcoming.

Dauth, Wolfgang and Johann Eppelsheimer, "Preparing the sample of integrated labour market biographies (SIAB) for scientific analysis: a guide," *Journal for Labour Market Research*, 2021, *54* (1), 1–14.

Davis, Morris A. and François Ortalo-Magné, "Household expenditures, wages, rents," *Review of Economic Dynamics*, 2011, *14* (2), 248–261.

Diamond, Rebecca, "The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980-2000," *American Economic Review*, 2016, *106* (3), 479–524.

Eckert, Fabian, Sharat Ganapati, and Conor Walsh, "Skilled Scalable Services: The New Urban Bias in Economic Growth," *Available at SSRN 3736487*, 2020.

Eeckhout, Jan, Roberto Pinheiro, and Kurt Schmidheiny, "Spatial Sorting," *Journal of Political Economy*, 2014, *122* (3), 554–620.

Fajgelbaum, Pablo D. and Cecile Gaubert, "Optimal Spatial Policies, Geography, and Sorting," *The Quarterly Journal of Economics*, 2020, *135* (2), 959–1036.

_ , Eduardo Morales, Juan Carlos Suárez Serrato, and Owen Zidar, "State Taxes and Spatial Misallocation," *The Review of Economic Studies*, 2018, *86* (1), 333–376.

Finlay, John and Trevor Williams, "Sorting and the Skill Premium: The Role of Nonhomothetic Housing Demand," *Journal of International Economics*, forthcoming.

Ganong, Peter and Daniel Shoag, "Why has regional income convergence in the US declined?," *Journal of Urban Economics*, 2017, *102*, 76–90.

Gaubert, Cecile, Patrick M. Kline, and Danny Yagan, "Place-Based Redistribution," *American Economic Review*, forthcoming.

Giannone, Elisa, "Skilled-biased technical change and regional convergence," *Journal of Political Economy*, forthcoming.

Gyourko, Joseph, Christopher Mayer, and Todd Sinai, "Superstar Cities," *American Economic Journal: Economic Policy*, 2013, *5* (4), 167–99.

Ioannides, Yannis M and Jeffrey E Zabel, "Interactions, neighborhood selection and housing demand," *Journal of urban economics*, 2008, *63* (1), 229–252.

Klick, Larissa and Sandra Schaffner, "FDZ data description: Regional real estate price indices for Germany (RWI-GEO-REDX)," *RWI Projektberichte*, 2019.

Kosfeld, Reinhold and Alexander Werner, "Deutsche Arbeitsmarktregionen-Neuabgrenzung nach den Kreisgebietsreformen 2007-2011," *Raumforschung und Raumordnung*, 2012, *70* (1), 49–64.

Larsen, Erling Røed, "The Engel curve of owner-occupied housing consumption," *Journal of Applied Economics*, 2014, *17* (2), 325–352.

OECD, "Housing prices (indicator)," *Accessed on 08 December 2022*, 2022.

Ossa, Ralph, "A quantitative analysis of subsidy competition in the US," *Journal of Public Economics*, forthcoming.

Redding, Stephen J. and Esteban Rossi-Hansberg, "Quantitative Spatial Economics," *Annual Review of Economics*, 2017, *9* (1), 21–58.

Rossi-Hansberg, Esteban, Pierre-Daniel Sarte, and Felipe Schwartzman, "Local industrial policy and sectoral hubs," *AEA Papers and Proceedings*, 2021, *111*, 526–531.

Rubinton, Hannah, "The geography of business dynamism and skill biased technical change," *Review of Economic Studies*, forthcoming.

Schaeffer, Katherine, "Key facts about housing affordability in the U.S.," *Pew Research Center*, 2022.

Statistisches Bundesamt, "Preise - Verbraucherpreisindizes für Deutschland," 2019.

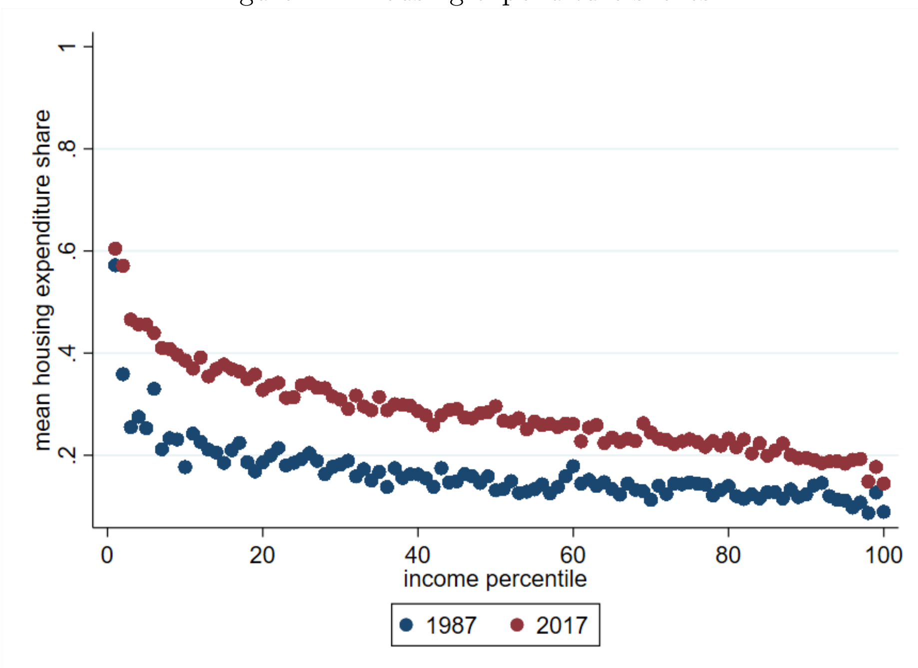
Zabel, Jeffrey E, "The demand for housing services," *Journal of Housing Economics*, 2004, *13* (1), 16–35.

A Quantification Appendix

A.1 Stylized Facts

To analyze how hard increases in housing costs have hit households across the income distribution, I plot housing expenditure shares in 1987 compared to housing expenditure shares in 2017. Figure A1 provides suggestive evidence that the decrease in housing affordability is mainly a problem for low income households: The housing expenditure share has increased significantly more for low income than for high income households.

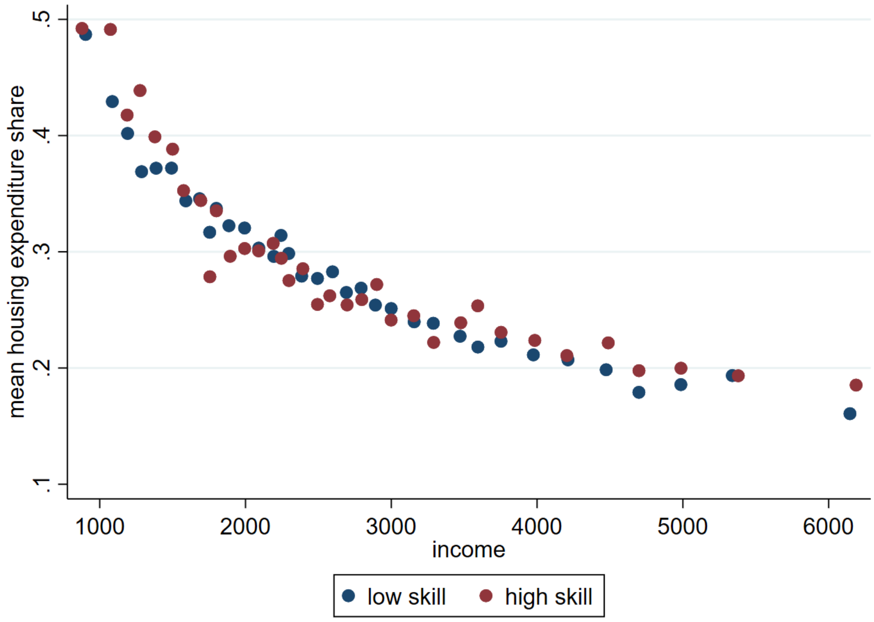
Figure A1: Housing expenditure shares



Source: GSOEP, own calculations. Note: West Germany only. Housing share is defined as housing expenditure (including heating and electricity) divided by total net income. Number of observations: 2725 in 1987, 5188 in 2017

Next, I investigate whether differences in housing expenditure shares are driven by education levels rather than income. Figure A2 plots housing expenditure shares for 50 evenly sized bins defined by total income separately for households with a household head not holding a university degree and households with a household head holding a university degree. It can be seen that for given income levels, the two types of households do not spend different shares of their expenditure on housing.

Figure A2: Housing expenditure shares by skill



Source: GSOEP, own calculations. Note: The plot shows mean housing expenditure shares for 50 evenly sized bins defined by total income. Housing share is defined as housing expenditure divided by total net income. The plot is based on household data from 2017. Skill refers to the skill of the household head. Number of observations: 3259 low skill, 1322 high skill

A.2 Calibration

I compare my non-homotheticity estimates to those assuming non-homothetic constant elasticity of substitution (NHCES) preferences as applied in Finlay and Williams (forthcoming)

$$(U_{ik})^{\frac{\sigma-1}{\sigma}} = \Omega^{\frac{1}{\sigma}} (h_{ik})^{\frac{\sigma-1}{\sigma}} (U_{ik})^{\frac{\epsilon}{\sigma}} + (c_{ik})^{\frac{\sigma-1}{\sigma}} \tag{33}$$

where $0 < \sigma < 1$, $\epsilon \geq \sigma - 1$, and $\Omega > 0$ are parameters.[12] Cobb-Douglas preferences are obtained by taking $\epsilon = 0$ and $\sigma \to 1$. The opposite case, a unit housing requirement, is obtained by taking $\epsilon = -1$ and $\sigma \to 0$. Instead of equation (18), I get

$$s_{ik} = \Omega p_i^{1-\sigma} (x_{ik})^{\epsilon} (1 - s_{ik})^{1+\frac{\epsilon}{1+\sigma}}. \tag{34}$$

[12] Note that the preferences in equation (33) are equivalent to those in Comin et al. (2021)

$$1 = \gamma^{\frac{1}{\sigma}} \left(c_{ik} (u_{ik})^{-1} \right)^{1-\frac{1}{\sigma}} + (1-\gamma)^{\frac{1}{\sigma}} \left(h_{ik} (u_{ik})^{-(1+\epsilon')} \right)^{1-\frac{1}{\sigma}}$$

with $u_{ik} = U_{ik} \gamma^{\frac{1}{\sigma-1}}$, $\epsilon' = \frac{\epsilon}{1-\sigma}$ and $\Omega = \frac{1-\gamma}{\gamma^{1+\epsilon'}}$.

I estimate equation (34) by GMM. The results as given in Table A1 are similar to those in Finlay and Williams (forthcoming) who find $\epsilon = -0.306$ and $\sigma = 0.522$.

Table A1: Preference Estimates for NHCES preferences

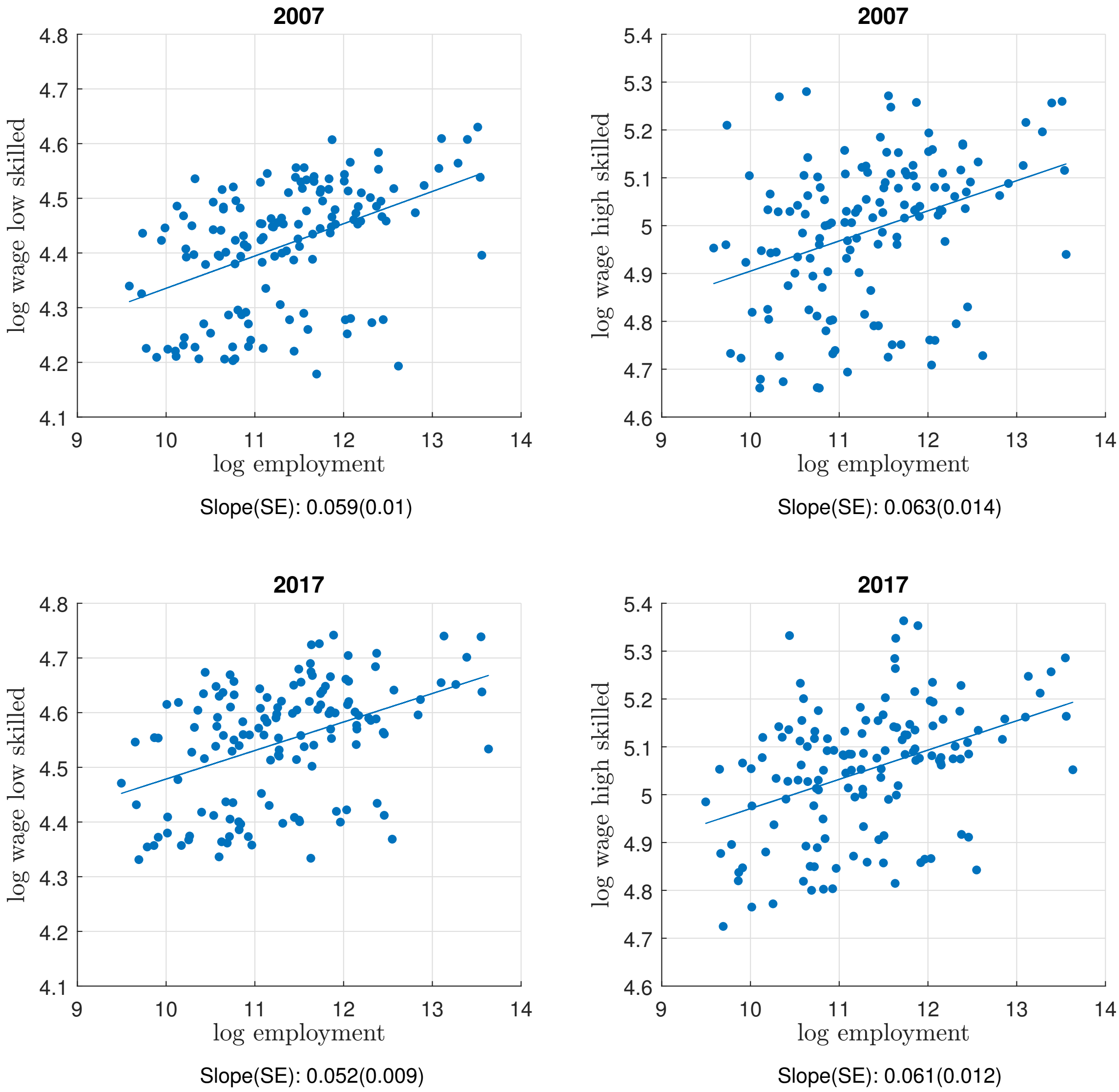
	(1) GMM	(2) GMM IV
ϵ	-0.299***	-0.301***
	(0.003)	(0.003)
σ	0.882***	0.886***
	(0.002)	(0.002)
Demographic controls	✓	✓
Year FE	✓	✓
N	8231	8231
No. of clusters	4333	4333

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: Standard errors in parentheses, clustered at the household level. Renters only. Instrument is log family income. Demographic controls include household size, number of earners as well as gender and age of the household head.

A.3 The urban wage premium

Figure A3: The urban wage premium by skill



Source: SIAB, own calculations. Note: Unit of observation is 141 labor market areas as defined by Kosfeld and Werner (2012).

B Model Appendix - Constrained efficient allocation

The program of the constrained social planner maximizing ex-ante utility with the constrained that she does not know the realizations of the idiosyncratic shocks and therefore needs to respect location choices can be written as

$$
\begin{aligned}
\mathcal{L} &= \sum_k \omega_k \psi \log\Big(\sum_i e^{\frac{\frac{\rho}{\rho-1}\log\left(\gamma(c_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}}\right)+\log E_{ik}}{\psi}}\Big) \\
&- \sum_k \sum_i \lambda_{ik}\Big(\frac{\left((\gamma(c_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}})^{\frac{\rho}{\rho-1}}E_{ik}\right)^{\frac{1}{\psi}}}{\sum_i \left((\gamma(c_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}})^{\frac{\rho}{\rho-1}}E_{ik}\right)^{\frac{1}{\psi}}}L_k - L_{ik}\Big) \\
&- \mu^Y \sum_i \sum_k (L_{ik}c_{ik} - L_{ik}A_{ik}) \\
&- \sum_i \mu_i^h \Big(\sum_k L_{ik}h_{ik} - H_i\Big)
\end{aligned}
$$

where I omit notation for the non-negativity constraints and solve for interior solutions. Fajgelbaum and Gaubert (2020) show that the social planner problem is concave when congestion forces are at least as large as agglomeration forces. The fact that my model features congestion forces only and that the generalization of non-homothetic preferences does not act as an agglomeration force ensure that there is a unique solution to the maximization problem. The first-order conditions are given by

$$
[h_i^k]\ \eta_i^h L_{ik}h_{ik} = \frac{\frac{\rho-\eta}{\rho-1}(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}}}{\gamma(c_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}}}\Big[\omega_k L_{ik} - \frac{\lambda_{ik}L_{ik}}{\psi} + \sum_j \lambda_{jk}\frac{L_{jk}L_{ik}}{\psi}\Big]
$$

$$
[c_{ik}]\ \eta^Y L_{ik}c_{ik} = \frac{\gamma(c_{ik})^{1-\frac{1}{\rho}}}{\gamma(c_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}}}\Big[\omega_k L_{ik} - \frac{\lambda_{ik}L_{ik}}{\psi} + \sum_j \lambda_{jk}\frac{L_{jk}L_{ik}}{\psi}\Big]
$$

$$
[L_{ik}]\ -\lambda_{ik} = \mu^Y(c_{ik}-A_{ik}) - \mu_i^h\Big(h_{ik} - \frac{\partial H_i}{\partial L_{ik}}\Big)
$$

I have 4xIxK+I+1 equations in $L_{ik}, c_{ik}, h_{ik}, \lambda_{ik}, \mu_i^h$ and μ^Y:

- Dividing $[h_{ik}]$ through $[c_{ik}]$ gives housing demand

$$h_{ik} = \Big(\frac{\mu^Y}{\mu_i^h}\frac{\rho-\eta}{\rho-1}\frac{1-\gamma}{\gamma}\Big)^{\frac{\rho}{\eta}}(c_{ik})^{\frac{1}{\eta}} \tag{35}$$

- Optimal consumption $[c_{ik}]$

$$\frac{\mu^Y}{\gamma}c_{ik}\Big(\gamma+(1-\gamma)\frac{(h_{ik})^{1-\frac{\eta}{\rho}}}{(c_{ik})^{1-\frac{1}{\rho}}}\Big) \quad = \quad \omega_k - \frac{\lambda_{ik}}{\psi} + \sum_j \lambda_{jk}\frac{L_{jk}}{\psi} \equiv x_{ik} \tag{36}$$

- Optimal labor allocation $[L_{ik}]$

$$-\lambda_{ik} \quad = \quad \mu^Y\Big(A_{ik}-c_{ik}\Big)+\mu_i^h\Big(\frac{\partial H_i}{\partial L_{ik}}-h_{ik}\Big) \tag{37}$$

- Mobility constraint

$$L_{ik} \quad = \quad \frac{\Big((\gamma(c_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}})^{\frac{\rho}{\rho-1}}E_{ik}\Big)^{\frac{1}{\psi}}}{\sum_i\Big((\gamma(c_{ik})^{1-\frac{1}{\rho}}+(1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}})^{\frac{\rho}{\rho-1}}E_{ik}\Big)^{\frac{1}{\psi}}}L_k \tag{38}$$

- Housing market clearing

$$\sum_k L_{ik}h_{ik} - H_i = 0 \tag{39}$$

- Balanced government budget

$$\sum_i\sum_k(L_{ik}c_{ik}-L_{ik}A_{ik}) = 0 \tag{40}$$

Decentralized vs planner

From $[L_{ik}]$, I get

$$\mu^Y c_{ik} + \mu_i^h h_{ik} \quad = \quad w_{ik} + \Pi_{ik} + t_{ik}$$

where I define

$$\begin{aligned} w_{ik} &\equiv \mu^Y A_{ik} && (41) \\ \Pi_{ik} &\equiv \mu_i^h \frac{(c_{ik})^{\frac{1}{\eta}}}{\sum_k L_{ik}(c_{ik})^{\frac{1}{\eta}}} H_i && (42) \end{aligned}$$

and

$$t_{ik} \equiv \lambda_{ik} + \mu_i^h \Big(\frac{\partial H_i}{\partial L_{ik}} - \Pi_{ik} \Big). \qquad (43)$$

Together with housing demand in equation (35), I have the first order conditions from the decentralized allocation with $\mu^Y = 1$ and $\mu_i^h = p_i$.

B.1 Computation

To solve the system of equations numerically, I can substitute in the Lagrange multipliers and housing to express the system of equations in terms of L_{ik} and c_{ik}. Combining housing demand in equation (10) with housing market clearing in equation (9), I get

$$h_{ik} = \frac{(c_{ik})^{\frac{1}{\eta}}}{\sum_k L_{ik}(c_{ik})^{\frac{1}{\eta}}} H_i.$$

Summing $[c_{ik}]$ over i and normalizing population of each type to 1 ($\sum_i L_{ik} = 1$) yields

$$\omega_k \quad = \quad \frac{\mu^Y}{\gamma} \sum_i \Big(\gamma L_{ik} c_{ik} + (1-\gamma) L_{ik} (h_{ik})^{1-\frac{\eta}{\rho}} (c_{ik})^{\frac{1}{\rho}} \Big)$$

which, after summing over k, can be rearranged to

$$\mu^Y \quad = \quad \frac{\gamma}{\sum_i \sum_k \Big(\gamma L_{ik} c_{ik} + (1-\gamma) L_{ik} (h_{ik})^{1-\frac{\eta}{\rho}} (c_{ik})^{\frac{1}{\rho}} \Big)}.$$

Summing housing demand over k and rearranging, I obtain

$$\mu_i^h = \mu^Y \frac{1-\gamma}{\gamma} \frac{\rho-\eta}{\rho-1} \Big(\frac{\sum_k c_{ik}^{\frac{1}{\eta}}}{\sum_k h_{ik}} \Big)^{\frac{\eta}{\rho}}.$$

The first-order condition with respect to $[L_i^k]$ gives an expression for λ_i^k:

$$-\lambda_{ik} \quad = \quad \eta^Y (A_{ik} - c_{ik}) + \mu_i^h \Big(\frac{\partial H_i}{\partial L_{ik}} - h_{ik} \Big).$$

Thus, after substituting in housing h_{ik} and the Lagrange multipliers μ^Y, μ_i^h and λ_{ik}, I have a system of 2xIxK equations in L_{ik} and c_{ik}:

$$\frac{\mu^Y}{\gamma} c_{ik} \Big(\gamma + (1-\gamma) \frac{(h_{ik})^{1-\frac{\eta}{\rho}}}{(c_{ik})^{1-\frac{1}{\rho}}} \Big) \quad = \quad \omega_k - \frac{\lambda_{ik}}{\psi} + \sum_j \lambda_{jk} \frac{L_{jk}}{\psi}$$

$$L_{ik} = \frac{\Big(\big(\gamma (c_{ik})^{1-\frac{1}{\rho}} + (1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}} \big)^{\frac{\rho}{\rho-1}} E_{ik} \Big)^{\frac{1}{\psi}}}{\sum_i \Big(\big(\gamma (c_{ik})^{1-\frac{1}{\rho}} + (1-\gamma)(h_{ik})^{1-\frac{\eta}{\rho}} \big)^{\frac{\rho}{\rho-1}} E_{ik} \Big)^{\frac{1}{\psi}}} L_k.$$

Acknowledgements

I thank Philip Jung, Lukas Buchheim, participants of several conferences and seminars and an anonymous referee for their helpful comments and suggestions. This paper uses confidential data from the Research Data Centre of the German Federal Employment Agency at the Institute for Employment Research. Details on possibilities of data processing can be found on the homepage of the Research Data Centre (fdz.iab.de).

Nina Furbach

Central Bank, Frankfurt am Main, Germany, email: nina.furbach@ecb.europa.eu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

PDF ISBN 978-92-899-7113-3 ISSN 1725-2806 doi:10.2866/0803901 QB-01-25-059-EN-N