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The macroeconomics of liquidity in financial intermediation

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Abstract

In financial crises, the premium on liquid assets such as US Treasuries increases alongside credit spreads. This paper explains the link between the liquidity premium and spreads. We present a theory of endogenous bank fragility arising from a coordination friction among bank creditors. The theory's implications reduce to a single constraint on banks, which is embedded in a quantitative macroeconomic model to investigate the transmission of shocks to spreads and economic activity. Shocks that reduce bank net worth exacerbate the coordination friction. In response, banks lend less and demand more liquid assets. This drives up both credit spreads and the liquidity premium. By mitigating the coordination friction, expansions of public liquidity reduce spreads and boost the economy. Empirically, we identify high-frequency exogenous variation in liquidity by exploiting the time lag between auction and issuance of US Treasuries. We find a causal effect on spreads in line with the calibrated model.

Keywords: bank runs, bank-lending channel, liquid assets.

JEL Codes: E41, E44, E51, G01, G21.

Non-technical summary

In times of financial stress, banks find it hard to fund themselves and credit becomes more expensive. This paper documents that financial turmoil is also associated with an increase in the premium on liquid assets, such as US Treasury Bills. Why does the value of liquid assets go up when credit is tight? The answer to this question has implications for the effectiveness of expansions in liquid-asset supply, a frequently-deployed policy response to financial stress. To answer it, we develop a novel financial friction explaining this empirical relationship. Then, we embed it in a standard macroeconomic model to study implications for the economy and policy.

Maturity transformation, a key function of financial intermediation, results in a mismatch on banks' balance sheets. This creates the conditions for coordination failures in the form of "runs" by panicked creditors. The potential for runs leads investors to price run risk in the debt of intermediaries. Because runs are self-fulfilling phenomena, it is difficult to pin down the risk of runs. To do this, we depart from the assumption of common knowledge across bank creditors, a common approach to limit their ability to coordinate on arbitrary behaviour. With this, we find that the intensity of the friction depends on bank balance-sheet fundamentals. In particular, banks can mitigate run risk by holding more liquid assets or by having more net worth (i.e., equity). Thus, if a shock reduces their net worth, banks demand more liquid assets to keep their run risk in check. This friction generates a countercyclical liquidity premium.

With the coordination friction embedded in a standard real business cycle model, we can study its role quantitatively in the transmission of macroeconomic shocks and policy. The friction amplifies shocks: a bad shock that reduces banks' net worth increases banks' funding costs on account of heightened run risk. Higher bank funding costs reduce the supply of credit and thereby drive down investment. Moreover, the friction propagates shocks through time by making it harder for banks to make profits and thus accumulate net worth.

The model's liquid assets, defined as assets that keep their value in case of a systemic financial crisis, are the monetary and fiscal liabilities of the government. An increase in the supply of liquid assets crowds in private investment by expanding credit supply. This is because banks hold the additional liquid assets, and the resulting reduction in run risk is reflected in better funding terms for banks.

The real effects of liquidity supply imply that it can be used as a stabilizing tool in the face of shocks. If the government responds to disruptions to financial intermediation by accommodating the increased demand for liquid assets, it can dampen the amplification of shocks.

Finally, we test the model empirically. The key implication of the model is that a high liquidity premium pushes up bank-funding costs. We run the analysis at daily frequency and use the quantity of US Treasuries outstanding as an instrument for the liquidity premium. The instrument is predetermined at daily frequency on account of the time lag between auction and issuance of US Treasuries and therefore it is valid. With this econometric strategy, we find a significant positive effect of the liquidity premium on bank-funding costs which is quantitatively in line with the calibrated model.

1 Introduction

Disruptions to financial intermediation make credit more expensive and thereby harm the economy. This pattern motivated the introduction of a specific banking friction in macroeconomic models. In their seminal contribution, Gertler and Kiyotaki (2010) introduce a problem of moral hazard between banks and their creditors. Consequently, banks' ability to fund themselves is limited by the value of their equity. The resulting leverage constraint leads to a powerful impact of bank net worth on macroeconomic outcomes via credit spreads. This explains the general observation of plummeting bank values, higher bank-funding costs and increased credit spreads in financial crises.[1] However, this approach to banking is silent on why we observe soaring demands for liquidity and hence liquidity premiums in times of financial stress.

We observe a heightened liquidity premium, defined as the difference between the 3-month general-collateral repo rate and the 3-month treasury-bill rate, during banking crises.[2] Figure 1 shows this for the global financial crisis.[3] More systematically, this paper documents a positive relationship over time between the liquidity premium and banks' funding costs, as measured by the difference between the 3-month LIBOR and the 3-month repo rate. Figure 2 shows the positive correlation between these two variables.[4]

Since policymakers often react to banking crises through expansions of liquidity, it is crucial to understand the causes of the empirical relationship between the liquidity premium and funding costs.[5] Existing research (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016) has shown the liquidity premium responds to government policies. The facts documented in this paper suggest that tight bank funding drives demand for liquid assets. This is consistent with a view that scarce liquidity impairs bank lending in times of stress, suggesting a channel through which a greater supply of public liquidity can benefit the economy.

Motivated by this, we do two things in the paper. First, we develop a novel financial friction based on coordination failure among bank creditors. Liquid-asset holdings and bank net worth both mitigate the coordination friction and are substitutes. Hence, when net worth

[1] Figure 9 and Figure 10 in appendix A shows the average dynamics of these variables in banking crises as identified by Baron et al. (2021).

[2] This definition of liquidity premium is standard in the literature (Nagel, 2016). More discussion on this point is provided in section 7.

[3] Figure 11 in appendix A zooms in on the recent period (2019–2023).

[4] Figure 12 shows that the positive correlation holds both in expansions and recessions.

[5] There is a debate in the literature on the real effects of liquidity policies and the channels through which they operate (Kuttner, 2018).

Figure 1: Global financial crisis.

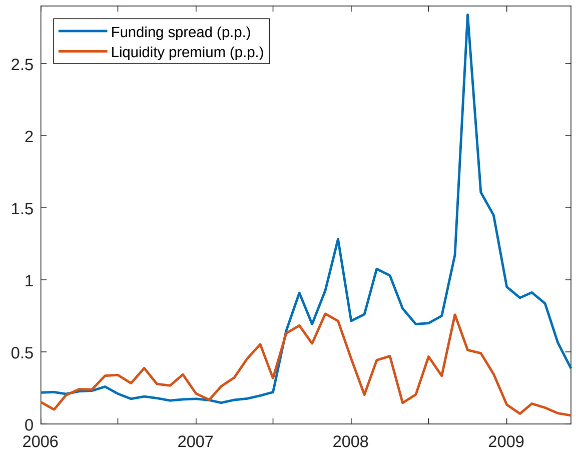
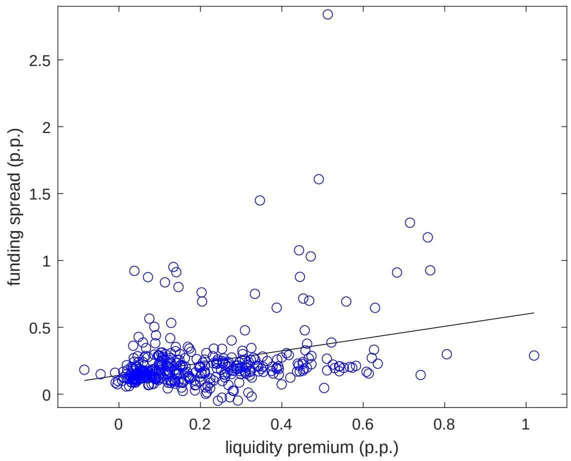


Figure 2: May 1991 – June 2023.



Note 1: Funding spread is 3-month (3M) LIBOR minus 3M general-collateral (GC) repo rate. Liquidity premium is 3M GC repo rate minus 3M T-bill rate.
Note 2: US monthly data. Data sources in appendix B.

is scarce, as in a financial crisis, banks demand more liquidity. This explains a high liquidity premium. It also implies policy can stabilize the economy by appropriately supplying liquid assets.

Second, we test whether the data supports the model's mechanism. In particular, the model implies that an increase in the liquidity premium pushes up the bank-funding spread. This is because it induces banks to economize on holding liquid assets. To identify exogenous variation in the liquidity premium, we use the quantity of outstanding US Treasuries as an instrumental variable. The instrument is strongly relevant and predetermined at daily frequency given the lag of a few days between the auction and issuance of Treasury securities. We find a significant positive effect.

Maturity transformation, a key function of financial intermediation, results in a mismatch on the balance sheets of banks.[6] This creates the conditions for coordination failures in the market for deposits (Diamond and Dybvig, 1983).[7] Such coordination failures take the form of "runs" by panicked creditors and played a central role in the global financial crisis in 2007, the crisis of US money-market funds in 2020 and the 2023 regional banking crisis (Shin, 2009; Bernanke, 2010; Li et al., 2020; Choi et al., 2023).

[6]For simplicity, we use "banks" as a general label for financial intermediaries and "deposits" for their short-term debt. The analysis applies more broadly to financial intermediaries with a maturity mismatch on their balance sheet.

[7]Perfect deposit insurance rules out coordination failures in these models. However, in the period 1984–2023Q3 deposits made up 73% of banks' liabilities and only 62% of deposits were insured on average. These values are respectively 79% and 57% in 2023Q3 (data source: FDIC QBP).

This paper models the deposits market as a coordination game. Strategic complementarities imply that under perfect information there are multiple equilibria. However, a deviation from common knowledge across depositors, which we introduce following the large literature on global games (Morris and Shin, 2003), leads to a unique equilibrium. Intuitively, without common knowledge it is impossible for depositors to coordinate on arbitrary equilibria. In the resulting unique equilibrium, depositors demand a level of compensation that is commensurate to a bank's fragility, defined as the minimum share of depositors that must not run for the bank to survive. If the bank offers an insufficient deposit rate, then depositors run even though the bank is solvent. Intuitively, banks must compensate depositors for run risk. However, as long as the bank offers a sufficiently high deposit rate, no run takes place because no depositor has an incentive to start the run that they fear.

Bank fragility, the heart of the coordination friction, is endogenous. It is a function of the bank's balance-sheet fundamentals. In particular, more levered banks and banks with fewer liquid assets as a share of total assets are more fragile. Therefore, they face higher funding costs. In other words, the coordination friction results in a mapping from higher capital and liquidity ratios into a lower funding spread. The capital and liquidity ratios are bank choices. In equilibrium, these choices trade off the returns on illiquid assets against the increased funding costs due to more fragility.

With the coordination friction embedded in a standard real business cycle model, we can study its role quantitatively in the transmission of macroeconomic shocks. The banking friction can be calibrated using observations on the average size of the liquidity premium, the credit spread, and banks' return on equity. The parameters of the macroeconomic model are set following the literature.

The friction amplifies shocks that affect banks' net worth. By making it more costly for banks to fund themselves, a reduction in net worth weakens the supply of credit and reduces the economy's output. The friction amplifies the effect on output of capital-destruction shocks, commonly studied in the literature on financial crises, by about one third on impact. At longer horizons, the amplification is greater. This persistence comes from banks' funding costs rising alongside credit spreads, implying banks' net worth is rebuilt very slowly in contrast to models with a leverage constraint. Furthermore, the increase in fragility due to scarcer net worth gives banks an incentive to demand more liquid assets. This generates a countercyclical liquidity premium.

Monetary and fiscal liabilities of the government are the natural source of liquidity supply. Banks create liquid assets for other sectors of the economy but they cannot produce assets that maintain their value in case of a systemic run.[8] Therefore, the relevant supply of liquid assets is a policy variable. In the model, an increase in the supply of liquid assets is expansionary. The liquid assets are absorbed by banks' balance sheets and reduce their fragility. With lower fragility, banks have access to funding on better terms and thus find it optimal to lend more. In other words, the supply of liquidity crowds in private investment. In the calibrated model, a shock that reduces the liquidity premium by 15 basis points leads to an expansion of credit supply reducing credit spreads by 24 basis points. This generates a 2-percent increase in investment on impact, with GDP also going up by a quarter of one percent. Moreover, the supply of liquidity can be used as a stabilizing policy tool in the face of shocks. If the government responds to disruptions to financial intermediation by accommodating the increased demand for liquid assets, it can dampen the amplification of shocks.

We test the key implication of the model: an increase in the liquidity premium causes an increase in the funding spread.[9] The econometric challenge is to find exogenous variation in the liquidity premium. Our strategy is to run the analysis at daily frequency and use the quantity of outstanding US Treasuries as an instrument. The instrument is strongly relevant to the liquidity premium. As for its validity, the quantity of treasuries is predetermined at daily frequency because there is a lag of a few days from auction, where it is determined, to issuance. Moreover, we include as controls 80 lags of financial and economic variables available at daily frequency, such as the dollar exchange rate and the liquidity premium itself. This cleans the autocorrelation out of the error term and ensures there is no endogeneity of the instrument driven by confounding variables or reverse causality. After all, if the error term only contains a non-autocorrelated daily shock, it cannot drive a variable determined on a previous day.

The empirical result is a robustly-significant positive effect of the liquidity premium on the funding spread. A 1-basis-point increase in the liquidity premium causes the funding spread to increase by about 1 basis point. This is in line with the size of the corresponding effect in the calibrated model. As a robustness check, we effectively split the sample between expansions and recessions. We find no evidence of a different size of the effect according to the state of the

[8] This is related to the seminal finding in Holmström and Tirole (1998) of a role for public liquidity supply in the presence of aggregate risk.

[9] We measure the funding spread as the difference between the 3-month LIBOR and the 3-month GC repo rate. More discussion on the measurement is provided in section 7.

economy.

Literature review. An extensive literature builds macroeconomic models around a leverage constraint on banks (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Boissay et al., 2016; Phelan, 2016; Karadi and Nakov, 2021; Van der Ghote, 2021; Fernández-Villaverde et al., 2023).[10] This friction, based on moral hazard, does not naturally give a role to banks' liquid-asset holdings unlike this paper's friction based on coordination failure. Moreover, models with the moral-hazard friction generate limited shock propagation because adverse shocks to bank net worth push up bank profitability by increasing credit spreads with little response of funding costs. Also, they struggle to match the observed procyclicality of banks' book leverage (Nuño and Thomas, 2017). The coordination friction is an improvement on the latter two counts, too. In this paper's model, shock propagation is strong because the positive effect of higher credit spreads on bank profitability after adverse shocks is largely offset by increased funding costs. And we find that leverage is procyclical because fragility is countercyclical.

In this paper, banks demand liquid assets to mitigate the risk of coordination failures among their creditors. This is a novel source of demand for liquid assets in the macroeconomic literature.[11] The existing literature posits an exogenous risk that bank creditors withdraw their funds. Banks demand liquid assets as a precaution to limit the amount they must borrow from the central bank at a punitive interest rate (Poole, 1968; Arce et al., 2020; Bianchi and Bigio, 2022) or the amount of assets they must sell at fire-sale prices (Drechsler et al., 2018; d'Avernas and Vandeweyer, forthcoming; Li, forthcoming) if hit by an adverse liquidity shock. In our model, the risk of deposit-holder withdrawals is a fully endogenous function of bank fundamentals.[12]

Studies evaluating quantitative-easing programmes, recent examples of policies that increased the supply of liquid assets, find reductions in interest-rate spreads in line with our model (Gagnon et al., 2011; Krishnamurthy and Vissing-Jorgensen, 2011). More recently,

[10]The first paper to derive a leverage constraint on banks from a moral-hazard problem is Holmström and Tirole (1997). Brunnermeier and Pedersen (2008) derive a leverage constraint based on value at risk.

[11]A strand of the banking literature formalizes this in static partial-equilibrium models (Rochet and Vives, 2004; Ahnert, 2016).

[12]A reduced-form approach to the demand for liquid assets is common in studies of the effects of liquidity supply (Krishnamurthy and Vissing-Jorgensen, 2012; Benigno and Benigno, 2022; Angeletos et al., 2023). Such approach may miss important characteristics of demand for liquid assets such as the substitutability of liquidity and bank capital, which is a feature of our model and which DeYoung et al. (2018) finds empirically.

Acharya and Rajan (2022) and Diamond et al. (2023) have sounded a cautionary note on the effects of liquid-asset supply in the context of QE. The former stresses that some of the benefit to bank fragility of additional liquidity supply is undone by banks taking on extra leverage. This result conforms to this paper's mechanism. The latter contribution finds empirically that liquid-asset holdings increase banks' marginal cost of lending. The authors suggest the reason for this may be limited balance-sheet space due to regulation. While the effect of regulation is beyond the scope of our paper, the driving force behind our paper's results, i.e. the positive effect of liquid-asset holdings by banks on the demand for their debt, is not considered in Diamond et al. (2023).

Banks' vulnerability to runs has been first formalized in Diamond and Dybvig (1983). That paper illustrates the possibility of runs, but it does not speak to their determinants because it has multiple equilibria. A literature in macroeconomics has adopted the multiple-equilibrium approach to study the effects of bank runs (Gertler and Kiyotaki, 2015; Gertler et al., 2016, 2020; Amador and Bianchi, forthcoming). A limitation of this approach is the need to assume an arbitrary relationship of the probability of runs with fundamentals. Because of this limitation, the role of liquidity in the determination of run risk does not emerge.

Leveraging theoretical results from Carlsson and van Damme (1993), Goldstein and Pauzner (2005) show that a small departure from perfect information produces a unique equilibrium in a bank-run game. This is an attractive feature because the evidence points to a strong relationship between poor bank fundamentals and banking crises (Gorton, 1988; Baron et al., 2021). A large literature in banking uses variations of such second-generation bank-run models to study optimal policy (Vives, 2014; Kashyap et al., 2024; Ikeda, 2024). Our paper is the first to integrate a second-generation bank-run model in a macroeconomic framework.[13]

Outline of the paper. The coordination game among depositors is laid out in section 2. This results in a constraint on bank behaviour, which is integrated in a standard macroeconomic model in the following two sections. In section 5, we discuss properties of banks' demand for liquid assets. The model is calibrated and quantitative experiments are carried out in section 6. In section 7, the empirical results of the study are reported. The appendices contain: (A) figures, (B) details about data sources, (C) proofs, (D) steady-state results, and (E) the full model

[13] A small strand of the banking literature has studied the relationship of bank runs with selected macroeconomic variables (Ennis and Keister, 2003; Martin et al., 2014; Porcellacchia, 2020; Mattana and Panetti, 2021; Leonello et al., 2022).

solution.

2 Coordination game

This section sets up the coordination game played by bank depositors. It solves for the unique equilibrium, which implies a relationship between banks' balance sheets and the interest rates required to induce households to hold deposits. Since banks anticipate the outcome of the coordination game, in the remainder of the paper this relationship constrains the choices made by banks.

The economy contains a unit continuum of banks (more generally, financial intermediaries) indexed by $b \in [0,1]$. Deposits at bank b are demand deposits paying interest rate j_b if held to the next time period, but with an option to withdraw on demand. While referred to as 'demand deposits', this bank debt can be interpreted more broadly as short-term unsecured borrowing in money markets that is frequently rolled over.

A coordination game among depositors is played in each discrete time period, but time subscripts are omitted in this section given the essentially static nature of the game. Just before the coordination game begins, all deposits $D_b \geq 0$ at bank b are held equally by a unit continuum of households indexed by $h \in [0,1]$. Expected payoffs in the next time period are discounted at rate ρ by all households.[14]

Bank fragility. Before households decide whether to hold deposits in the coordination game, banks make portfolio and leverage decisions. Bank b chooses how much to invest in illiquid and liquid assets A_b and M_b respectively, where the notion of liquidity is defined below. Taking as given net worth (equity) N_b, these choices result in deposit creation up to a level of deposits D_b consistent with the balance-sheet identity $A_b + M_b = D_b + N_b$. These deposits are in the hands of households at the point where the coordination game among depositors is played.

If a positive fraction $1 - H_b$ of households chooses not to hold deposits D_b at bank b, the bank must make a total payment $(1 - H_b)D_b$ to these households by disposing of some assets. The full value M_b of the liquid assets acquired earlier can be obtained at this point, but disposal of illiquid assets A_b during the coordination game only recovers a fraction λ of their value at acquisition. If the proceeds of these asset liquidations are insufficient to cover the withdrawals,

[14] Since there is a continuum of banks, depositor behaviour can be analysed as if households were risk neutral and ρ taken as given. In the full model, the common discount rate ρ is an endogenous variable.

then the bank fails. The condition for failure is given by

$$(1 - H_b)D_b > \lambda A_b + M_b. \tag{1}$$

The parameter $\lambda \in [0,1]$ measures the liquidity of assets A_b relative to the benchmark of the perfectly liquid asset M_b. Rearranging the condition above and using the balance-sheet identity, bank b does not fail if $H_b \geq F_b$, where fragility F_b is given by

$$F_b = \frac{(1-\lambda)A_b - N_b}{A_b + M_b - N_b}. \tag{2}$$

If net worth is positive, fragility is a number between 0 and $1 - \lambda$, and greater net worth lowers fragility. Increased holdings of liquid assets M_b reduce a bank's fragility where it is initially positive, while holding more illiquid assets A_b raises fragility when it is below $1 - \lambda$ initially. A noteworthy feature of fragility is that it can be expressed in terms of familiar liquidity and capitalization ratios, respectively

$$m_b = \frac{M_b}{A_b + M_b} \quad \text{and} \quad n_b = \frac{N_b}{A_b + M_b}, \tag{3}$$

as

$$F_b = \frac{(1-\lambda)(1-m_b) - n_b}{1 - n_b}. \tag{4}$$

Hence, the bank's scale plays no role in determining its fragility.

Notice that the illiquidity of assets A_b is key to the existence of a coordination problem. If the full value of any assets can always be realized (i.e., $\lambda = 1$), then banks with positive net worth can never be fragile. It is also important that banks' portfolio choice is made before people decide whether to hold deposits: once illiquid assets are funded by deposit creation, there is strategic complementarity in depositors' holding decisions. This timing assumption could capture the fact that banks create deposits when they make a loan and then someone in the economy must be willing to hold the deposits if the bank is to avoid having to dispose of assets. More generally, it could be interpreted as a mismatch between the timing of capital investment, which is typically long-term, and banks' more short-term funding sources.

Structure of the game. Independently for each bank b, households make a simultaneous binary choice whether to hold deposits D_b until the next period.[15] This choice is captured by the indicator function H_{bh}, which equals 1 if household h holds and 0 if it chooses to withdraw. Withdrawing households receive funds in the same time period.[16]

Holding bank deposits exposes households to credit risk because banks can fail. If this happens, those holding deposits recover the principal after incurring a cost θ per unit of deposits. The parameter $\theta > 0$ represents losses associated with the bankruptcy process, and these costs are paid by depositors at the beginning of the next time period.[17]

In this economy, banks fail because of 'runs' — too many depositors deciding to withdraw. The share of households who hold bank b's deposits is $H_b = \int_0^1 H_{bh}\,\mathrm{d}h$, and there is some endogenous minimum fraction F_b who must hold for the bank not to fail. Thus, the indicator function Φ_b for the failure of bank b is

$$\Phi_b(F_b, H_b) = \begin{cases} 0 & \text{if } H_b \geq F_b, \\ 1 & \text{otherwise.} \end{cases} \tag{5}$$

The variable F_b is bank b's fragility. This is the measure of the bank's fundamentals in the coordination game, and it depends on the liquidity of the bank's portfolio of assets and its leverage according to equation (2) as seen above. $F_b = 1$ means that bank b needs each and every household to trust it and hold its deposits in order to survive. On the other hand, an intermediary with $F_b = 0$ is not fragile at all — it will not fail even if all households refuse to hold its deposits.

Conditional on knowing a bank's fragility and the share of households holding its deposits, the net payoff per unit of deposits from holding versus withdrawing is

$$\Gamma(F_b, H_b) = \frac{(j_b - \rho)(1 - \Phi_b) - \theta \Phi_b}{1 + \rho}, \tag{6}$$

with Φ_b given by equation (5). Households want to hold deposits at bank b if $\Gamma(F_b, H_b) \geq 0$ given

[15] To simplify the game, holding is a binary choice, but households would not gain from being able to make partial withdrawals here.

[16] To keep the analysis tractable, households would need to wait until the next time period to deposit these funds at another bank.

[17] This timing assumption is not essential, but is chosen for consistency with the full macroeconomic model presented later.

their discount rate ρ and the interest rate j_b offered by the bank.[18] Given $j_b \geq \rho$ as required for the net payoff to be non-negative, $\Gamma(F_b, H_b)$ is weakly decreasing in fragility F_b, representing a deterioration in the bank's fundamentals, and weakly increasing in the fraction H_b holding deposits, indicating the presence of strategic complementarity in the coordination game. With complete information, there would be multiple Nash equilibria whenever fragility is positive and $j_b \geq \rho$: an equilibrium where everyone holds with $H_{bh} = 1$, and a 'bank-run' equilibrium with $H_{bh} = 0$.

Incomplete information. In this paper, non-fundamental bank runs are ruled out by a small deviation from complete information. Households cannot observe bank b's fragility F_b. Instead, each receives an independent signal centred around the true fragility $\hat{F}_{bh} \sim \mathrm{U}[F_b - \omega, F_b + \omega]$ for some $\omega > 0$.

This information structure is a key ingredient of a global game. Even if the noise is vanishingly small and thus households are virtually certain about bank fundamentals, uncertainty about the information held by other households makes it hard to coordinate behaviour. Coordination on other publicly available information is ruled out here by assuming all depositors hold a uniform prior on F_b before observing their signals, and that it is common knowledge everyone holds this prior.[19] Formally, in the bank-b coordination game, all households' prior information is $\mathcal{I}_b = \{F_b \sim \mathrm{U}_{\mathbb{R}}, D_b, j_b\}$.[20] Household h updates this prior using signal $\hat{F}_{bh}$ to form expectations $\mathbb{E}_{bh}[\cdot] = \mathbb{E}\left[\cdot \,\middle|\, \hat{F}_{bh}, \mathcal{I}_b\right]$.

Equilibrium strategies. As is well-established in the literature, this small deviation from complete information in combination with well-behaved strategic complementarities in the net-payoff function rules out sunspot equilibria in which households coordinate regardless of the fundamentals. The game is left with a unique Bayesian Nash equilibrium that depends on bank fundamentals.

Households follow strategies where conditional on their signals and prior information, deposits are held if and only if $\mathbb{E}_{bh}[\Gamma(F_b, H_b)] \geq 0$. If $j_b < \rho$ then the net payoff is negative irrespective of H_b, and hence withdrawing is a strictly dominant strategy for all signals $\hat{F}_{bh}$. In

[18] If indifferent, households are assumed to hold deposits to break ties.

[19] Relaxing this assumption to consider information provided by publicly observed endogenous variables would make the analysis much more complex.

[20] For simplicity, we specify an improper prior. The results are unchanged for a proper uniform prior with positive density at least on the set $[-\omega, 1 + \omega)$.

the interesting case where $j_b \geq \rho$, standard results from the literature on global games show that the unique strategy profile surviving rounds of iterated deletion of dominated strategies approaches a threshold rule.

Lemma 1. *In the unique equilibrium strategy, household h holds bank b's deposits if and only if $\hat{F}_{bh} \leq F_b^*$ with*

$$F_b^* = \frac{j_b - \rho}{j_b - \rho + \theta} + \frac{j_b - \rho - \theta}{j_b - \rho + \theta}\omega \tag{7}$$

and $j_b \geq \rho$.

Proof. Please refer to appendix C. □

Given $j_b \geq \rho$, each household h compares its signal $\hat{F}_{bh}$ to a common threshold $F_b^* \in [-\omega, 1+\omega)$ and holds deposits if the signal indicates fragility is low enough. As the deposit interest-rate spread over ρ becomes larger, households use a higher threshold in (7) and thus accept greater levels of fragility while choosing to hold deposits.

To understand the threshold F_b^* intuitively, consider vanishing noise $\omega \to 0$. In this case, each household is almost perfectly informed about bank fragility F_b from looking at its own signal. But while observing a signal $\hat{F}_{bh}$ exactly at the common threshold F_b^* means a household is confident that true fragility is very close to $\hat{F}_{bh}$ and others will receive signals close to its own, the household has no information about whether others' signals are above or below F_b^*. If everyone is using the same threshold strategy, the household's rational beliefs are that the fraction H_b holding deposits is uniformly distributed over $[0,1]$. Knowing that F_b is approximately $\hat{F}_{bh}$, the probability of bank failure is $\hat{E}_{bh}(\Phi_b) = \hat{F}_{bh}$ for $\hat{F}_{bh} \in [0,1]$ using (5). The expected net payoff (6) from holding deposits is $\left[(j_b - \rho)(1 - \hat{F}_{bh}) - \theta \hat{F}_{bh}\right] / (1+\rho)$, and the signal for which this is zero is exactly the threshold F_b^* from (7).

It is possible to aggregate up the equilibrium behaviour of individual households and obtain the share of a bank's deposits that are held given the bank's fragility.

Lemma 2. *Consider a bank b with fragility F_b and F_b^* given by (7). The share of households holding deposits in equilibrium is given by*

$$H_b = \begin{cases} 1 & \text{if } F_b \leq F_b^* - \omega, \\ \frac{1}{2} + \frac{F_b^* - F_b}{2\omega} & \text{if } F_b \in \left(F_b^* - \omega, F_b^* + \omega\right], \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

if $j_b \geq \rho$. If $j_b < \rho$, then $H_b = 0$.

Proof. Please refer to appendix C. □

The law of large numbers ensures that there is no uncertainty about the share of households holding the deposits. Liquidity and leverage, which determine fragility F_b, and the interest rate j_b, which determines households' tolerance of fragility F_b^*, deterministically pin down the share of households holding bank-b deposits in equilibrium.

In general, partial runs with $H_b \in (0,1)$ are possible. To keep the model tractable, we work under conditions that rule them out. These conditions are (1) small enough noise $\omega \to 0$ and (2) a degree of randomness in bank fragility. Vanishing noise implies that partial runs can only take place for $F_b = F_b^*$ and the randomness of bank fragility ensures that the probability of the realization of precisely this value is zero. The introduction of randomness in bank fragility is in the spirit of a trembling-hand equilibrium refinement. Accordingly, we study the case with the variance of fragility approaching zero.

Proposition 1. *Consider $\omega \to 0$ and fragility $\tilde{F}_b \sim U[F_b - \chi, F_b + \chi]$ with $\chi \to 0$ and $\omega/\chi \to 0$. Then, we have that:*

1. *Partial runs with $H_b \in (0,1)$ have probability zero for any F_b, F_b^* and j_b.*
2. *$H_b = 1$ with probability one if and only if $F_b \leq F_b^*$ and $j_b \geq \rho$.*

Proof. Please refer to appendix C. □

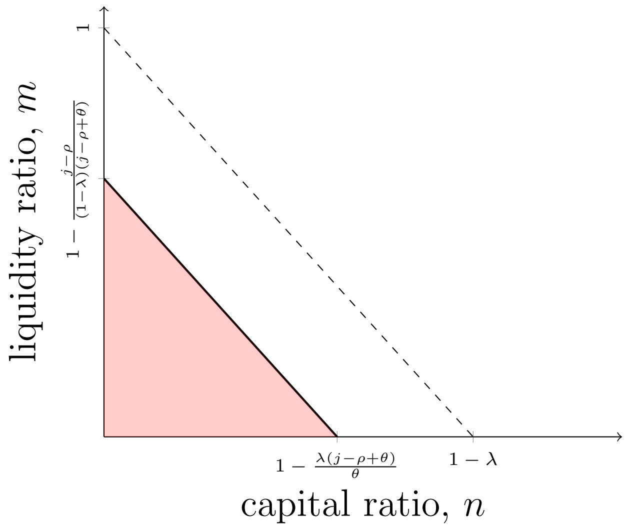
The proposition specifies the combinations of bank fragility and interest rate on deposits that lead to either all households holding the deposits or a complete run that makes the bank fail.

Bank funding costs. Substituting the equilibrium run threshold (7) into the conditions to avoid a bank run derived in Proposition 1 yields a link between the required interest rate on deposits and a bank's fragility:

$$j_b - \rho \geq \max\left\{\frac{F_b}{1-F_b}, 0\right\}\theta. \tag{9}$$

The expression suggests that strategic behaviour of depositors can be thought of as demanding a premium $j_b - \rho$ for run risk, with fragility F_b appearing as the probability of a run. Because ω is small, the risk premium is not due to uncertainty about a bank's fundamentals. It is due to

Figure 3: Fundamentals and bank runs.



The dashed and solid lines represent equation (11) respectively for a zero spread and an arbitrary strictly positive spread. Given the strictly positive spread, a run takes place in the red region.

uncertainty about what other depositors think about the bank's fundamentals, coupled with strategic complementarity in deposit-holding decisions. However, there is a limitation to this intuition: the probability of a run is actually zero in equilibrium as long as a sufficient premium on bank deposits is paid.

Substituting the determinants of bank fragility, given by equation (2), into condition (9), yields a mapping from the bank's balance sheet to the deposit rate required to avoid a run:

$$j_b - \rho \geq \max\left\{\frac{(1-\lambda)A_b - N_b}{\lambda A_b + M_b}, 0\right\}\theta. \tag{10}$$

In the full model, this equation plays the role of a constraint on banks, because banks have positive net worth in equilibrium and therefore want to make choices that avoid a bank run.[21]

The substitution of familiar financial ratios (3) in the no-run condition gives further intuition into the result. The mapping from financial ratios to the deposit rate required to avoid bank runs is

$$j_b - \rho \geq \max\left\{\frac{(1-\lambda)(1-m_b) - n_b}{\lambda + (1-\lambda)m_b}, 0\right\}\theta. \tag{11}$$

A graphical representation is provided in Figure 3. In the figure, the dashed line depicts the

[21] A bank with negative fragility can violate this constraint by setting $j_b < \rho$ and not fail. However, it is not profitable to do so. Instead of creating deposits and generating a run on them, the bank is at least equally well off by not issuing the deposits in the first place.

combinations of capital ratio and liquidity ratio that rule out bank failure with zero spread on deposits. The region of bank fundamentals that lead to bank failure, coloured in red, is always within the dashed line for a positive spread on deposits. All else equal, a higher interest on deposits makes the failure region smaller. The key implication of this result is that there is a three-way substitutability from the bank's perspective between equity, liquidity and interest on deposits. For instance, a bank can lever up while keeping its interest-rate expenses in check by boosting its liquid-asset holdings.

The coordination game creates a motive for banks to hold liquid assets, keep a buffer of net worth, and pay extra interest on deposits. These three courses of action are substitutes and costly for banks. In section 4, we study how banks choose among these options on the basis of economic conditions. Analysing the implications for macroeconomic outcomes first requires integrating the coordination game with a full macroeconomic model.

3 Banks in a macroeconomic model

This section embeds the coordination friction faced by holders of banks' deposits into macroeconomic model. The core of the economy is a real business cycle model as in Kydland and Prescott (1982).

Timeline. Each discrete time period $t = 0, 1, 2, \ldots$ is divided into two stages. At the first stage, competitive markets for goods, labour, capital, liquid assets, and illiquid bonds are open. Aggregate shocks are realized, households choose labour supply and holdings of non-bank assets, and firms produce final goods and incomes are distributed. The government chooses the supply of liquid assets and adjusts fiscal policy. At this stage, banks choose deposit creation and deposit interest rates, and select a portfolio of liquid and illiquid assets to hold. At the second stage, depositors play the coordination game described in section 2. Signals about bank fragility are received, and households choose whether to hold deposits at each bank. Banks may fail at this stage, and if so, assets are immediately liquidated and bankruptcy costs are paid at the beginning of the next period. Finally, on the basis of what happens at both stages of period t, households' consumption is determined.

Physical capital as the illiquid asset. The illiquid asset held by banks is physical capital. A surviving bank b holding illiquid assets $A_{b,t-1}$ at the period $t-1$ has a stock of physical capital

$K_{bt} = \xi_t A_{b,t-1}$ to rent out to firms for use in production of final goods. The variable ξ_t is an exogenous capital-quality shock common to all banks. Physical capital depreciates at rate δ during each time period.

At the first stage of period t, final goods can be transformed into new capital through investment $I_{bt} = A_{bt} - (1-\delta)K_{bt}$ financed by banks (or existing capital transformed back into final goods if investment is negative). Only goods transformed into capital by this stage can be stored and carried into period $t+1$.

Capital is illiquid at the second stage of period t in the sense that investment is partially irreversible at that point. Only a fraction λ of a bank's physical capital can be immediately converted back into consumption goods without causing the bank to fail. More than this amount can be recovered, but at the cost of bank failure, with the wiping out of bank equity acting as an adjustment cost.[22] Those holding deposits at the point of bank failure must also incur a cost θ to recover each unit of deposits through the bankruptcy process described in section 2.

Other frictions. For the model of banks introduced earlier to be relevant for macroeconomics, three other frictions are needed. First, households cannot directly hold physical capital (banks' illiquid asset), so financial intermediation is necessary for capital accumulation and production. Second, bank debt takes the form of the short-term demand deposits described earlier, so there is a mismatch between the liquidity of bank liabilities and assets. Third, banks face limits on accumulating net worth, and thus their asset holdings cannot be entirely financed through equity.

While the model does not speak to why such frictions are present, these are all standard assumptions in the existing macro-banking literature. The first could be justified if holding illiquid assets requires expertise possessed only by bankers, or diversification through the scale at which bankers operate. The second might come from some short-term liquidity needs of households that preclude them locking up wealth in a long-term asset.

The third is often formally built into macro-banking models through exogenous exit of banks or bankers. Here, a simpler foundation for the assumption is a problem of separation of ownership and control of banks. Suppose bank employees are able to divert bank profits to their bonus pools if these funds are not swiftly returned to shareholders. Formally, suppose a constant fraction $\gamma/(1+\gamma)$ of pre-dividend net worth is vulnerable to diversion as bonuses W_{bt},

[22] Banks will operate with positive net worth.

where γ is a positive parameter. Even if bank shareholders would otherwise prefer earnings to be retained, they need to pay out at least the funds vulnerable to diversion. This motivates a minimum dividend condition

$$\Pi_{bt} \geq \gamma N_{bt}, \tag{12}$$

where Π_{bt} is the dividend paid by bank b at the beginning of period t, and N_{bt} denotes net worth after distribution of dividends.

3.1 Production

Homogeneous final goods for consumption or investment are produced by a continuum of perfectly competitive firms $f \in [0,1]$. These firms hire homogeneous labour L_{ft} at wage w_t and rent physical capital K_{ft} from banks at price x_t in competitive markets. Firms face a constant-returns-to-scale Cobb-Douglas production function

$$Y_{ft} = Z_t K_{ft}^{\alpha} L_{ft}^{1-\alpha}, \tag{13}$$

where Z_t is the exogenous level of total factor productivity and α is the capital elasticity of output ($0 < \alpha < 1$). All prices and wages are fully flexible, and the price of goods is normalized to one so that all variables are in real terms.

Firms maximize profits $\Pi_{ft} = Y_{ft} - x_t K_{ft} - w_t L_{ft}$ that are immediately paid out as dividends. Profit maximization implies capital is used up to the point where the marginal product of capital equals the rental rate x_t, and labour is hired up to where the marginal product of labour equals the wage w_t:

$$\alpha Z_t \left(\frac{L_{ft}}{K_{ft}}\right)^{1-\alpha} = x_t, \quad \text{and} \quad (1-\alpha) Z_t \left(\frac{K_{ft}}{L_{ft}}\right)^{\alpha} = w_t. \tag{14}$$

With constant returns to scale, profits are equal to zero ($\Pi_{ft} = 0$) in equilibrium. The ex-post return received by owners of physical capital between $t-1$ and t is

$$R_t = \xi_t (1 - \delta + x_t) - 1.$$

3.2 Households

At the beginning of period t, household $h \in [0,1]$ has expected lifetime utility

$$\mathbb{E}_t\left[\sum_{\ell=0}^{\infty}\beta^{\ell}\left\{\frac{C_{h,t+\ell}^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}-\chi\frac{L_{h,t+\ell}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\right\}\right], \tag{15}$$

where C_{ht} is consumption and L_{ht} is labour supply, β is the subjective discount factor, χ is a parameter representing the disutility of labour, and σ and ψ are preference parameters that will be the elasticity of intertemporal substitution and Frisch elasticity of labour supply, respectively. All households have the same preferences, and start from equal wealth in an initial period 0. The only heterogeneity among households is in the signals they receive at the banking stage of each time period.

The information set for the conditional expectation $\mathbb{E}_t[\cdot]$ in (15) contains commonly known aggregate shocks, prices, and macroeconomic variables from date t and earlier. As discussed in section 2, at the banking stage, while households know the size of deposits D_{bt}, the interest rate j_{bt} offered, and receive arbitrarily precise signals $\hat{F}_{bht}$ about the fragility F_{bt} of each bank, they hold uninformative priors over F_{bt} before seeing their signals, and these priors are common knowledge.[23]

Competitive-markets stage. Based on past decisions and outcomes, household h begins period t with deposits including accrued interest $(1+j_{b,t-1})(1-\Phi_{b,t-1})H_{bh,t-1}D_{b,t-1}$ that were held at surviving banks b. The household pays a cost $\theta\Phi_{b,t-1}H_{bh,t-1}D_{b,t-1}$ in the recovery of funds on deposit at a failing bank b in period $t-1$.

Household h chooses labour supply L_{ht} and receives wage income $w_t L_{ht}$, and everyone pays a common lump-sum net tax T_t. Each household also receives a dividend Π_t from owning an equal share of a non-tradable investment fund comprising all banks and non-financial firms in the economy.[24] Household h may also choose to borrow between periods t and $t+1$ an amount B_{ht} (or if negative, hold savings outside banks) in the form of a risk-free but illiquid bond with

[23] Formally, banking-stage decisions are made using information set $\mathcal{I}_{bt} = \{F_{bt} \sim \mathrm{U}_{\mathbb{R}}, D_{bt}, j_{bt}\}$ for bank b updated with household h's signal $\hat{F}_{bht}$. Past idiosyncratic signals do not convey any additional information beyond the commonly known aggregate variables in the information set for $\mathbb{E}_t[\cdot]$, so it is not necessary to indicate a household-specific information set for the conditional expectation in (15).

[24] Consistent with the informational assumptions, individual bank dividends Π_{bt} are not observed. In equilibrium, there are no gains from trading shares in the investment fund among households.

interest rate ρ_t.[25] Any past borrowing $(1+\rho_{t-1})B_{h,t-1}$ must be repaid, and a no-Ponzi condition must be respected.[26]

The flow budget constraint for gross non-financial wealth U_{ht} (not net of debt, and excluding savings outside banks) carried into the banking stage of time period t is

$$U_{ht} = \int_0^1 \left[(1+j_{b,t-1})(1-\Phi_{b,t-1}) - \theta\Phi_{b,t-1}\right] H_{bh,t-1} D_{b,t-1}\,\mathrm{d}b + w_t L_{ht} + \Pi_t - T_t + W_{ht} + \\ + B_{ht} - (1+\rho_{t-1})B_{h,t-1} + (1+i_{t-1})M_{h,t-1} - M_{ht}. \quad (16)$$

Households directly holding physical capital is ruled out by assumption, so capital is excluded from (16). Some additional terms are present. Bonuses W_{ht} are obtained through diversion of bank net worth, though these will be zero because (12) holds in equilibrium.[27] The budget constraint also allows for the possibility that households might hold non-bank liquid assets $M_{ht} \geq 0$ paying risk-free interest rate i_t. However, as long as $i_t \leq \rho_t$, households maximize utility by choosing $M_{ht} = 0$, which will hold in equilibrium.[28]

If μ_{ht} denotes the current-value Lagrangian multiplier on the date-t flow budget constraint (16) of household h, the choices of labour supply L_{ht} and net bond issuance B_{ht} that maximize expected utility (15) satisfy $L_{ht}^{1/\psi} = w_t\mu_{ht}$ and $\mu_{ht} = \beta(1+\rho_t)\mathbb{E}_t\mu_{h,t+1}$. The transversality condition is $\lim_{\ell\to\infty}\beta^\ell\mu_{h,t+\ell}(U_{h,t+\ell} - B_{h,t+\ell} + M_{h,t+\ell}) \leq 0$.

Banking stage. By this stage, banks have already chosen interest rates j_{bt}, made deposit creation D_{bt} decisions, and purchased physical capital and liquid assets when date-t competitive markets were open. It has been implicitly assumed that deposits are accepted by firms and households as a means of payment and circulate at the competitive-markets stage.[29] Since non-financial firms are entirely static, paying out all sales revenue immediately as factor payments, any deposits must be in the hands of households once the competitive-markets stage is over. As households are ex ante identical, any newly created deposits are assumed to flow to all

[25] Illiquid in that no value from this asset can be realized until the next competitive-markets stage.

[26] The no-Ponzi condition is $\lim_{\ell\to\infty}\frac{\mathbb{E}_t}{1+\rho_t}\cdots\frac{\mathbb{E}_{t+\ell-1}}{1+\rho_{t+\ell-1}}B_{h,t+\ell} = 0$.

[27] Each household receives an equal share W_{ht} of the bonus pool $\frac{1}{1+\gamma}\int_0^1 \max\{0, \gamma N_{bt} - \Pi_{bt}\}\,\mathrm{d}b$.

[28] This can be interpreted as households choosing to deposit in banks any outside money obtained from fiscal transfers, and selling any liquid financial assets to banks.

[29] The medium-of-exchange role of deposits is not explicitly modelled here. Deposits are accepted in exchange for goods if agents believe no bank failures will occur, as is true in equilibrium.

households equally, which means that $D_{bht} = D_{bt}$ for all households h at each bank b.[30]

Households begin the banking stage with some amount of gross non-financial wealth U_{ht}. They simultaneously make their deposit holding decisions $H_{bht} \in \{0, 1\}$, which collectively determine any bank failures $\Phi_{bt} \in \{0, 1\}$. Based on these decisions and the outcomes for banks, household h's consumption is U_{ht} minus deposits held at surviving banks. Those choosing not to hold bank b's deposits can consume an extra amount D_{bt}. Those holding deposits at failing banks can recover them through the bankruptcy process by paying a per-unit cost θ at the beginning of the next period. Denoting consumption for a general outcome of the coordination game by $\tilde{C}_{ht}$:

$$\tilde{C}_{ht} = U_{ht} - \int_0^1 (1 - \Phi_{bt}) H_{bht} D_{bt} \, \mathrm{d}b - \left(\tilde{T}_t - T_t\right), \tag{17}$$

where the final term $\tilde{T}_t - T_t$ allows for the government to adjust lump-sum taxes on households from T_t to $\tilde{T}_t$ if bank failures occur. Equation (17) implicitly assumes households spend funds withdrawn from banks when they choose not to hold deposits. Since the market for illiquid bonds is closed at this stage and banks have already chosen D_{bt}, the only means of saving is by holding liquid non-bank assets $\tilde{M}_{ht} \geq 0$ paying interest i_t. If such assets exchange one-for-one with goods at all stages of period t — which is what makes them liquid — it is optimal to choose $\tilde{M}_{ht} = 0$ if $i_t \leq \rho_t$.[31]

Given (15) and (17), the Lagrangian multiplier on the flow budget constraint (16) equals the expected marginal utility of consumption, $\mu_{ht} = \mathbb{E}_t\left[\tilde{C}_{ht}^{-1/\sigma}\right]$. From (16) and (17), the expected utility benefit from choosing $H_{bht} = 1$ instead of $H_{bht} = 0$ is proportional to $(1 - \mathbb{E}_{bht}[\Phi_{bt}])(1 + j_{bt})\beta\mathbb{E}_t\mu_{h,t+1} + \mathbb{E}_{bht}[\Phi_{bt}](\mu_{h,t} - \theta\beta\mathbb{E}_t\mu_{h,t+1}) - \mu_{ht}$, where $\mathbb{E}_{bht}[\cdot] = \mathbb{E}\left[\cdot|\mathcal{I}_{bt}, \hat{F}_{bht}\right]$ denotes expectations conditional on households' information sets at the banking stage. This expression comes from H_{bht} and Φ_{bt} for one individual bank of a continuum not affecting the marginal utility of consumption μ_{ht}, so behaviour in the coordination game can be studied as if households are risk neutral. Since $\beta\mathbb{E}_t\mu_{h,t+1}/\mu_{ht} = 1/(1 + \rho_t)$ for all h, the net payoff per unit of deposits held can be calculated using (6) with a common discount rate ρ_t for all households, as was supposed in section 2.

Household behaviour when the no-run condition holds. In equilibrium, all banks choose to satisfy the no-run condition (10). Hence, $H_{bht} = 1$ for all $h \in [0, 1]$ and $\Phi_{bt} = 0$ for all $b \in [0, 1]$.

[30] In equilibrium, all households begin each time period with the same deposits at a given bank.

[31] In general, $\tilde{M}_{ht}$ also appears on the left-hand side of equation (17), but it is zero in equilibrium.

Moreover, (17) implies $C_{ht} = U_{ht} - \int_0^1 D_{bt}\mathrm{d}b$, so $C_{ht}^{-1/\sigma}$ is known exactly at the competitive-markets stage. With no heterogeneity in wealth or preferences, all households have the same marginal utility of consumption $\mu_{ht} = C_t^{-1/\sigma}$ and make the identical consumption C_t and labour supply L_t choices satisfying:

$$C_t^{-\frac{1}{\sigma}} = \beta(1+\rho_t)\mathbb{E}_t\left[C_{t+1}^{-\frac{1}{\sigma}}\right], \quad \text{and} \quad L_t^{\frac{1}{\psi}} = w_t C_t^{-\frac{1}{\sigma}}. \tag{18}$$

At the macroeconomic level, there is effectively a representative household.

3.3 Government

Supply of liquid assets. The government issues liabilities M_t that are liquid assets in that they can be exchanged one-for-one with consumption goods at any stage of time period t. These liabilities are broadly interpretable as government bonds, reserves, or outside money more generally, though the model has a single type of liquid government liability for simplicity. This offers a risk-free return i_t between periods t and $t+1$.

Policy instruments and flow budget constraint. The government can also choose to purchase illiquid bonds B_t (or if negative, issue illiquid bonds).[32] The other instrument of government policy is a net lump-sum tax T_t levied on all households. Changes in fiscal and monetary policy are represented through different combinations of M_t, B_t, and T_t. Consolidating across all branches of government, the flow budget constraint is

$$T_t = (1+i_{t-1})M_{t-1} - (1+\rho_{t-1})B_{t-1} - M_t + B_t. \tag{19}$$

Liquidity of government bonds. The government does not have access to any special production technology allowing it directly to create an asset that is a liquid store of value. There is no reason why the value of a unit of M_t has to be the same at the first and second stages of period t, for example, if liquid assets held by banks were distributed to satisfy withdrawals. To ensure the real value of a unit of M_t is always 1, the government must be willing to adjust the supply of liquid assets to accommodate demand $\tilde{M}_t$ at the banking stage. Since the market for illiquid

[32] Purchases of $B_t > 0$ financed by issuing M_t can be interpreted as a form of unconventional monetary policy. However, the government never buys physical capital here, so it does not ever directly take on the financial intermediation role performed by banks or other financial intermediaries.

bonds is closed at that point, the equivalent of budget constraint (19) requires that taxes adjust to $\tilde{T}_t = T_t - (\tilde{M}_t - M_t)$.

3.4 Banks

Ownership. Each bank $b \in [0,1]$ is owned by an investment fund that is itself owned equally by all households. Dividends $\mathit{\Pi}_{bt}$ from banks are aggregated and passed on to households as $\mathit{\Pi}_t$ along with any dividends $\mathit{\Pi}_{ft}$ from non-financial firms:

$$\mathit{\Pi}_t = \int_0^1 \mathit{\Pi}_{bt}\,\mathrm{d}b + \int_0^1 \mathit{\Pi}_{ft}\,\mathrm{d}f. \tag{20}$$

To focus on bank failures owing to illiquidity rather than insolvency, assume that any bank b that has survived up to the beginning of period t ($\mathit{\Phi}_{b,t-1} = 0$) but which has negative net worth and is unable to distribute positive dividends (owing to the realization of asset returns) must be recapitalized by the investment fund. A resource cost $\varkappa$ is incurred per unit of recapitalization funds provided. The funds are raised by requiring other banks owned by the fund to pay higher dividends.[33] As seen later, in the presence of a recapitalization cost $\varkappa > 0$, banks acting in the interests of their owners avoid insolvency with probability one.

Objective and constraints. Banks act in the interests of their owners, and as there is effectively a representative household in equilibrium, the objective function of bank b is $\mathit{\Pi}_{bt} + V_{bt}$, where V_{bt} is the present value of future dividends (the ex-dividend value of bank b) obtained using households' common stochastic discount factor P_t:

$$V_{bt} = \mathbb{E}_t\left[\sum_{\ell=1}^{\infty}\left\{\prod_{l=1}^{\ell} P_{t+l}\right\}\mathit{\Pi}_{b,t+\ell}\right], \quad \text{where } P_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}. \tag{21}$$

At each date t, each bank makes an interest rate j_{bt} and deposit creation decision that results in a stock of deposits D_{bt}. In addition, the bank chooses the amount of physical capital A_{bt} and liquid assets M_{bt} to hold, and the dividend $\mathit{\Pi}_{bt}$ to distribute. Each bank is competitive in goods and asset markets and hence takes prices (x_t, R_t, i_t), and the stochastic discount factor P_t as given. There is no competitive market for deposits, so banks can choose the quantity supplied D_{bt} and

[33] A failure of the whole banking system where recapitalization through this means is impossible will not occur on the equilibrium path.

the interest rate j_{bt}, but they know that households will decide whether to hold or not during the coordination game.

Constraints. If bank b avoids failure prior to date t, its net worth (bank equity or capital) N_{bt} after paying dividend Π_{bt} depends as follows on past decisions and the realized return on physical capital R_t:

$$N_{bt} = (1+R_t)A_{b,t-1} + (1+i_{t-1})M_{b,t-1} - (1+j_{b,t-1})D_{b,t-1} - \Pi_{bt}. \tag{22}$$

This assumes no diversion of funds to employee bonuses, which requires the minimum-dividend condition (12) to hold. Given net worth N_{bt}, the balance-sheet identity is

$$A_{bt} + M_{bt} = D_{bt} + N_{bt}. \tag{23}$$

Since failure wipes out a bank's ability to pay dividends, banks want to ensure the no-run condition (10) derived in section 2 holds.[34] The present value of dividends (21) is maximized subject to (12), (22), (23), and (10). Since (10) specifies a minimum threshold for j_{bt} and net worth $N_{b,t+1}$ is decreasing in j_{bt}, the no-run condition must bind:

$$j_{bt} = \rho_t + \max\left\{\frac{1}{\lambda + \frac{\lambda N_{bt} + (1-\lambda)M_{bt}}{D_{bt}}} - 1, 0\right\}\theta. \tag{24}$$

3.5 Market clearing

Equilibrium in competitive factor markets requires

$$\int_0^1 K_{ft}\,\mathrm{d}f = \int_0^1 K_{bt}\,\mathrm{d}b = K_t, \quad \text{and} \quad \int_0^1 L_{ft}\,\mathrm{d}f = \int_0^1 L_{ht}\,\mathrm{d}h = L_t, \tag{25}$$

[34]For completeness, the actions taken by bank b if a positive fraction of households choose not to hold its deposits ($H_{bt} < 1$) are described. These 'withdrawals' mean the balance sheet $A_{bt} + M_{bt} = D_{bt} + N_{bt}$ chosen at the competitive-markets stage must change. The bank first disposes of liquid assets, reducing them to $\tilde{M}_{bt} = \max\{0, M_{bt} - (1-H_{bt})D_{bt}\}$. If this is insufficient, physical capital is liquidated down to $\tilde{A}_{bt} = A_{bt} + (M_{bt} - \tilde{M}_{bt}) - (D_{bt} - \tilde{D}_{bt})$, where $\tilde{D}_{bt}$ denotes the amount of deposits still held at the end of the banking stage. Liquidating physical capital necessitates bank failure ($\Phi_{bt} = 1$) if $\tilde{A}_{bt} < \lambda A_{bt}$. Final deposits held are $\tilde{D}_{bt} = (1-\Phi_{bt})H_{bt}D_{bt}$, and investment funded by the bank is $\tilde{I}_{bt} = \tilde{A}_{bt} - (1-\delta)K_{bt}$. Failing banks incur an adjustment cost that wipes out net worth $\tilde{N}_{bt} = N_{bt}$ at the beginning of the next period: formally, the future capital stock is $\tilde{K}_{b,t+1} = \xi_{t+1}(1-\Phi_{bt})\tilde{A}_{bt}$. Any remaining depositors of failing banks also incur recovery costs in the bankruptcy process, and these resource costs are counted as part of investment. Formally, for b with $\Phi_{bt} = 1$, depositors' costs appear as $\tilde{I}_{b,t+1} = \theta\Phi_{bt}H_{bt}D_{bt}$.

and equilibrium in the markets for liquid assets and illiquid bonds requires

$$\int_0^1 M_{bt}\,\mathrm{d}b = M_t, \quad \text{and} \quad B_t = \int_0^1 B_{ht}\,\mathrm{d}h. \tag{26}$$

4 Analysis of bank behaviour

This section analyses banks' optimal choices of holdings of liquid assets, creation of deposits, and distribution of dividends subject to the banking friction developed in section 2. The full dynamic optimization problem can be solved as a series of static problems in liquidity and leverage choices taking as given the path of net worth, and then finally considering dividend policy to characterize the evolution of net worth.

Liquidity demand. The expected discounted value of $N_{b,t+1} + \Pi_{b,t+1}$ conditional on information from date t is denoted by

$$\Omega_{bt} = (1 + r_t)N_{bt} + (r_t - j_{bt})D_{bt} - (r_t - i_t)M_{bt}, \tag{27}$$

where r_t denotes a risk-adjusted expected value of R_{t+1} that is defined below. In maximizing Ω_{bt} with net worth N_{bt} and market prices r_t, i_t, and ρ_t given, there are three choice variables j_{bt}, D_{bt}, and M_{bt} and one binding no-run condition (24). This constraint gives j_{bt} as a function of D_{bt} and M_{bt}, and the first-order condition for maximizing Ω_{bt} with respect to M_{bt} is $r_t - i_t = -D_{bt}\partial j_{bt}/\partial M_{bt}$, where (24) implies

$$-D_{bt}\frac{\partial j_{bt}}{\partial M_{bt}} = (1-\lambda)\theta\left[\lambda + \frac{(1-\lambda)M_{bt} + \lambda N_{bt}}{D_{bt}}\right]^{-2} = \frac{(1-\lambda)\theta}{(1-F_{bt})^2}. \tag{28}$$

An increase in M_{bt} given D_{bt} and N_{bt} means a switch from illiquid to liquid assets, which has cost equal to the difference in the expected risk-adjusted returns $r_t - i_t$. The marginal benefit is the reduction in funding costs $-D_{bt}\partial j_{bt}/\partial M_{bt}$ as bank fragility falls, and the first-order condition equates the marginal cost and benefit of a balance sheet of a given size having more liquid assets.[35] As banks all face the same cost $r_t - i_t$ and the marginal benefit of a unit of

[35] Note that the second-order condition is satisfied because $-D_{bt}\partial j_{bt}/\partial M_{bt}$ is decreasing in M_{bt} as seen from (28). Since $M_{bt} \geq 0$, it is necessary to check for corner solutions. However, cases where there is a corner solution for some banks but not others can be ruled out, and a positive aggregate supply of liquidity means that there cannot be a corner equilibrium for all banks.

liquidity depends only on a bank's fragility F_{bt} and parameters, the first-order condition implies that banks trade liquidity in money markets to the point where fragility is equalized across banks. With $F_{bt} = F_t$ for all b, the level of systemic bank fragility F_t is derived from (2) and the balance-sheet identity (23):

$$F_t = 1 - \lambda - \frac{(1-\lambda)M_t + \lambda N_t}{D_t}, \tag{29}$$

where N_t, M_t, and D_t are the aggregate amounts of equity, liquid assets, and deposits in the banking system. An immediate consequence is that all banks obtain the same funding cost $j_t = j_{bt}$ on their deposits. Using (9), this interest rate satisfies $j_t - \rho_t + \theta = \theta/(1 - F_t)$, and by combining with (28):

$$r_t - i_t = \frac{(1-\lambda)}{\theta}(j_t - \rho_t + \theta)^2. \tag{30}$$

Intuitively, holding an additional unit of liquid assets improves a bank's ability to pay depositors on demand by $1 - \lambda$. With no change in the size of the deposit base, this reduces the bank's total funding cost by $(j_t - \rho_t + \theta)^2/\theta$.

Deposit creation. The objective function Ω_{bt} in (27) can be written as follows:

$$\Omega_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right)N_{bt} + \left[(r_t - j_t) - \left(\frac{r_t - i_t}{1-\lambda}\right)\left(\frac{(1-\lambda)M_{bt} + \lambda N_{bt}}{D_{bt}}\right)\right]D_{bt}.$$

Given that banks optimize over liquidity M_{bt} to equalize fragility, equations (28), (29) and (30) imply that the objective function becomes

$$\Omega_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right)N_{bt} + \left(\frac{r_t - \lambda i_t}{1-\lambda} - j_t - \frac{\theta}{1-F_t}\right)D_{bt}.$$

Substituting the no-run constraint $\theta/(1 - F_t) = j_t - \rho_t + \theta$:

$$\Omega_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right)N_{bt} + \left[\frac{r_t - \lambda i_t}{1-\lambda} - j_t - (j_t - \rho_t + \theta)\right]D_{bt}. \tag{31}$$

Intuitively, creating an extra deposit can earn the bank an expected return r_t if invested in physical capital. If liquid assets were reduced by $\lambda/(1-\lambda)$ to acquire additional physical capital then the bank would obtain a net payoff $(r_t - \lambda i_t)/(1-\lambda) - j_t$ if the deposit rate remained at j_t. However, increasing physical capital by $1/(1-\lambda)$ and reducing liquid assets by $\lambda/(1-\lambda)$ leaves the amount of funds the bank is able to repay depositors on demand unchanged. The cost of holding sufficient extra liquid assets to keep fragility constant is equal to $j_t - \rho_t + \theta$ per unit of deposits, so this term must be included as an additional cost.

With the demand for liquid assets optimally chosen to equalize fragility and leave it as effectively constant at the level of an individual bank, the objective function (31) is linear in deposits D_{bt}. If the coefficient is positive, there is no limit to banks' desire to create, while if negative, no deposit creation occurs. Equilibrium with a positive but finite supply of deposits requires the coefficient on D_{bt} is zero:

$$\frac{r_t - \lambda i_t}{1-\lambda} - j_t = j_t - \rho_t + \theta. \tag{32}$$

This means that the static objective function (27) is ultimately linear in only net worth:

$$\Omega_{bt} = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right) N_{bt}. \tag{33}$$

Note that exact distribution of deposits D_{bt} across banks is not uniquely determined, only the aggregate amount of deposits D_t consistent with (29).

Spreads and the liquidity premium. Putting together the liquidity demand (30) and deposit creation (32) optimality conditions implies a relationship between bank funding costs $j_t - \rho_t$ and the liquidity premium $\rho_t - i_t$:

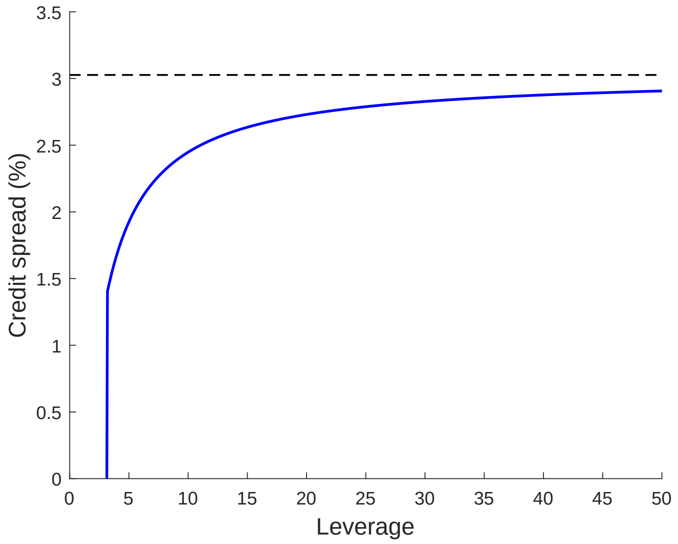
$$j_t - \rho_t = \sqrt{\theta}\sqrt{\rho_t - i_t}. \tag{34}$$

The funding cost $j_t - \rho_t$ is the geometric average of the deposit default cost parameter θ and the liquidity premium $\rho_t - i_t$. Similarly, the resulting credit spread is a multiple of a generalized mean of θ and the liquidity premium $\rho_t - i_t$:

$$r_t - i_t = 4(1-\lambda)\left(\frac{1}{2}\sqrt{\theta} + \frac{1}{2}\sqrt{\rho_t - i_t}\right)^2. \tag{35}$$

The equilibrium link among spreads is driven by adjustments to bank balance sheets. A higher credit spread makes banks choose to increase their leverage, $1/n_t$, and thus increase their fragility according to equation (24). This is represented in Figure 4 for an inelastic supply of liquid assets. The increase in fragility drives up banks' funding spreads and the resulting

Figure 4: Credit supply



Note 1: The credit spread is $r_t - i_t$ and leverage is $1/n_t$.
Note 2: Annualized calibrated parameter values from Table 2 and the implied steady-state M are used.
Note 3: The dashed line is the spread at which credit supply is infinite.

increase in the demand for liquid assets drives up the liquidity premium.

Figure 4 can be interpreted as the model's credit-supply schedule. For low levels of the credit spread, the supply of credit (i.e., investment in physical capital by banks) is inelastic at the level that ensures banks are not fragile and they pay the risk-free rate on their deposits. Above a given credit spread, banks have the incentive to lever up and become fragile. In this region, the supply of credit is elastic. Increases in the supply of liquid assets expand banks' credit supply in the fragile region. They are irrelevant when banks are not fragile.

Dividend policy and net worth. The remaining aspect of bank behaviour to analyse is its dividend policy, which affects the evolution of bank net worth. Suppose first that insolvency never occurs with positive probability, so dividends Π_{bt} are never negative. The minimum dividend condition is $\Pi_{bt} \geq \gamma N_{bt}$, and let ζ_{bt} denote the Lagrangian multiplier on this constraint in the problem of maximizing the present-discounted value of dividends (21). The multipliers satisfy $\zeta_{bt} \geq 0$ and the Kuhn-Tucker condition $\zeta_{bt}(\Pi_{bt} - \gamma N_{bt}) = 0$ to allow for the constraint $\Pi_{bt} \geq \gamma N_{bt}$ being slack. In terms of the multiplier ζ_{bt}, the static objective Ω_{bt} from (27) is defined by

$$\Omega_{bt} = \frac{\mathbb{E}_t\left[P_{t+1}(1+\zeta_{b,t+1})(\Pi_{b,t+1} + N_{b,t+1})\right]}{\mathbb{E}_t\left[P_{t+1}(1+\zeta_{b,t+1})\right]}. \tag{36}$$

Conjecturing that the Lagrangian multiplier ζ_{bt} is the same for all banks, $\zeta_t = \zeta_{bt}$, the risk-adjusted expected return r_t on illiquid assets in the earlier analysis of liquidity demand and deposit creation is

$$r_t = \frac{\mathbb{E}_t[P_{t+1}(1+\zeta_{t+1})R_{t+1}]}{\mathbb{E}_t[P_{t+1}(1+\zeta_{t+1})]}.$$

With the same r_t for all banks, the static objective function Ω_{bt} is linear in net worth N_{bt}, as seen in (33). The first-order conditions for the optimal choice of dividends Π_{bt} and hence net worth N_{bt} from date $t+1$ onwards are:

$$1 + (1+\gamma)\zeta_t = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right)\mathbb{E}_t\left[P_{t+1}(1+\zeta_{t+1})\right].$$

This first-order condition is independent of any bank-specific variables, confirming that ζ_t is independent of b. Imposing the first-order conditions, the present-discount value of future dividends must be linear in net worth N_{bt}:

$$V_{bt} = v_t N_{bt}, \quad \text{where } v_t = \left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right)\mathbb{E}_t\left[P_{t+1}(1+\zeta_{t+1})\right].$$

The coefficient v_t is the market-to-book ratio: the market value of a bank's future dividends divided by its ex-dividend net worth. This ratio is common to all banks. Note that $v_t = 1+(1+\gamma)\zeta_t$ for all t from $t+1$ onwards, so the market-to-book ratio satisfies

$$v_t = \frac{\gamma}{1+\gamma}\frac{1+\frac{r_t-\lambda i_t}{1-\lambda}}{1+\rho_t} + \frac{1}{1+\gamma}\left(1 + \frac{r_t - \lambda i_t}{1-\lambda}\right)\mathbb{E}_t\left[P_{t+1}v_{t+1}\right]. \tag{37}$$

The minimum dividend constraint must bind if $v_t > 1$.

The ex-post return Q_{bt} on bank b's book equity is

$$Q_{b,t+1} = \frac{\Pi_{b,t+1} + (N_{b,t+1} - N_{bt})}{N_{bt}}, \quad \text{which is} \quad Q_{b,t+1} = R_{t+1}\frac{A_{bt}}{N_{bt}} + i_t\frac{M_{bt}}{N_{bt}} - j_t\frac{D_{bt}}{N_{bt}}. \tag{38}$$

It has been supposed that banks' actions do not lead to a realization of $Q_{b,t+1}$ where equity $N_{b,t+1}$ becomes negative. Recall that recapitalization by the investment fund costs $1+\varkappa$ units of net worth in other banks for each unit of capital injected into an insolvent bank. First note that since $V_{bt} = v_t N_{bt}$ with $v_t \geq 1$ for solvent banks, this recapitalization reduces the present value of dividends the investment fund is able to distribute to households. Second, deposit creation decisions at individual banks are not restricted by (32), and there are different combinations of leverage and liquid asset demand consistent with achieving a given level of bank fragility. It follows that individual banks can choose leverage to ensure that net worth remains positive with probability one without having to take actions that reduce the present value of dividends.

The risk-adjusted expected return on book equity is

$$q_{bt} = \frac{\mathbb{E}_t[P_{t+1}(1+\zeta_{t+1})Q_{b,t+1}]}{\mathbb{E}_t[P_{t+1}(1+\zeta_{t+1})]}, \text{ which is } \quad q_{bt} = \frac{r_t - \lambda i_t}{1-\lambda}, \tag{39}$$

using (33), (36), and (38). This expected return is the same for all banks, $q_t = q_{bt}$.

5 Substitutability of net worth and liquidity

Banks have a motive to hold liquidity in this paper's economy. Liquidity holdings reduce banks' fragility and thus mitigate the coordination friction that they face. This motive generates a demand for liquidity. Combining and re-arranging equations (10) and (34), we obtain banks' demand for liquid assets conditional on the liquidity premium, their holdings of physical capital and net worth as

$$M_t = \max\left\{\left[(1-\lambda)\sqrt{\frac{\theta}{\rho_t - i_t}} - \lambda\right]A_t - \sqrt{\frac{\theta}{\rho_t - i_t}}N_t, 0\right\}. \tag{40}$$

Conditional on bank leverage, an increase in the liquidity premium makes banks demand more liquid assets.

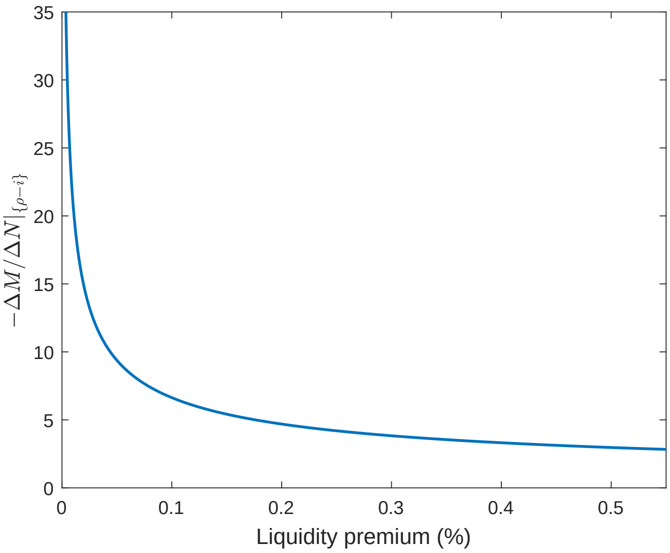
The equation implies that liquidity and net worth are substitutes for banks. Note that the RBC block of equations implies that the banking state variable N_t only affects macroeconomic variables via the credit spread and that equation (35) maps the equilibrium credit spread into the liquidity premium. With this, we can write that[36]

$$\left.\frac{\mathrm{d}M_t}{\mathrm{d}N_t}\right|_{\{\rho_s - i_s\}_{s=t}^{\infty}} = -\sqrt{\frac{\theta}{\rho_t - i_t}}. \tag{41}$$

Holding spreads constant, an increase in net worth reduces the demand for liquidity. In this sense, net worth and liquidity are substitutes from banks' viewpoint. The rate at which net worth and liquidity are traded off is decreasing in the liquidity premium as shown in Figure 5. To gain intuition in this, we can combine equations (24), (29) and (34) and note that the liquidity

[36] We also assume that $M_t > 0$ in order to ignore the non-negativity constraint. The condition is satisfied for low enough net worth and is satisfied in steady state.

Figure 5: Substitutability of net worth for liquidity



Note: Annualized calibrated parameter values from Table 2 are used.

premium determines bank fragility according to $F_t = (\rho_t - i_t)/(\theta + \rho_t - i_t)$. Hence, we can write

$$\left.\frac{\mathrm{d}M_t}{\mathrm{d}N_t}\right|_{\{\rho_s - i_s\}_{s=t}^{\infty}} = -\frac{1-F_t}{F_t}. \tag{42}$$

In an economy that is less fragile, a lost unit of net worth pushes banks to demand more units of liquid assets because each additional unit is less beneficial in bringing down the bank's fragility. Intuitively, the marginal effect of liquid-asset holdings on fragility is decreasing.

The rate of substitution of net worth for liquidity holding spreads constant, as given in equation (41), is interesting for its policy interpretation. It is the amount of additional liquidity the government must supply in reaction to a unit reduction in banks' net worth in order to insulate macroeconomic quantities. For an annualized liquidity premium of 28 points, to which we calibrate the model in section 6, the government can insulate the economy from the effects of a unit reduction in bank net worth by increasing the supply of liquid assets by about 4 units. A policy regime with a lower liquidity premium, and therefore less bank fragility, requires a stronger reaction by the government to rule out macroeconomic effects from a reduction in bank net worth. For example, in a policy regime with a 10-bp liquidity premium the government must react to a unit reduction in net worth with 6.7 extra units of liquid assets.

6 Quantitative analysis

This section quantifies the importance of bank fragility in the transmission of economic shocks. The model is simulated using a log linearization around its non-stochastic steady state. This steady state itself is analysed in appendix D, and the log-linearized equations are given in appendix E.

6.1 Calibration

The banking sector of the economy is described by the three parameters λ, θ, and γ. These are calibrated using information on the average values of the liquidity premium, credit spread, and return on bank equity. The parameter β is calibrated using the average level of interest rates. The approach is to choose parameters to match the model's implications for targeted variables in a non-stochastic steady state to the average values observed. The policy-determined supply of liquid assets in steady state consistent with liquidity premium can be inferred from the average capitalization ratio of banks. Finally, the other macroeconomic parameters α, δ, σ, and ψ are set to conventional values following the literature.

The model is calibrated to the US economy using data from 1991 up to the 2007–8 financial crisis. Data availability for banking variables determines the start of the sample in 1991Q3, and stopping in 2008Q4 accounts for the substantially different provision of liquidity after 2008 resulting from the many policy responses to the crisis.

The liquidity premium is defined with reference to the 3-month Treasury Bill as the most liquid asset. The average T-Bill yield over the period 1991Q3–2008Q4 is 3.7% in nominal terms. In the model, all interest rates are real interest rates, so the average 2.2% rate of inflation according to the personal consumption expenditure (PCE) over the same period is subtracted, leaving a real yield of 1.5%. The macroeconomic model is formulated in discrete time, and it is natural to align the length of one period with the 3-month maturity of the T-Bill. The steady-state quarterly real interest rate on the liquid asset is i, so $i = 1.5\%/4$, where a variable without a time subscript denotes its non-stochastic steady-state value. The liquidity premium as measured by the 3-month GC repo rate minus the T-Bill yield is 28 basis points on average, therefore $\rho = i + 0.28\%/4$.

The credit spread $r - i$ for illiquid bank assets is proxied by the yield on Moody's seasoned Baa-rated corporate bonds over 10-year Treasuries, which is 2.2% annual, hence $r = i + 2.2\%/4$.

The return on bank equity q is measured by the average ratio of cash dividends to equity for commercial banks covered by the FDIC, which is 8.4% at an annual rate, giving $q = 8.4\%/4$. In the model, the real return on bank equity coincides with the dividend-net worth ratio.[37]

Since $r = (1-\lambda)q + \lambda i$ from (39), the parameter λ measuring the liquidity of bank assets is calibrated as $\lambda = (q-r)/(q-i)$. As the formula shows, a low value of λ arises if r is large relative to i, because the illiquidity of assets makes it challenging for the bank to lend without increasing fragility. The calibration targets imply $\lambda = 0.681$.

The parameter θ measuring the costs of bank failure for depositors can be calibrated with information on ρ, i, and q. Using equations (35) and (39), $q - i = (\sqrt{\theta} + \sqrt{\rho - i})^2$, so θ can be set as $\theta = (\sqrt{q-i} - \sqrt{\rho - i})^2$. High values of θ arise when the return on bank equity q is far above the risk-free interest rate ρ because a more severe credit friction between banks and their depositors increases spreads. The value resulting from the calibration targets is $\theta = 4.4\%/4$.

In a steady state where the return on bank equity exceeds the risk-free rate, the return on equity q is equal to the minimum fraction γ of equity distributed as dividends. This immediately implies $\gamma = 8.4\%/4$. Households' Euler equation from (18) in steady state implies the discount factor β satisfies $\beta = 1/(1+\rho)$. With $\rho = 1.78\%/4$, the implied value of the discount factor is $\beta = 0.996$. In summary, the calibration makes use of the following equations linking the model parameters to the targets:

$$\lambda = \frac{r-i}{q-i}, \quad \theta = \left(\sqrt{q-i} - \sqrt{\rho - i}\right)^2, \quad \gamma = q, \quad \text{and} \quad \beta = \frac{1}{1+\rho}.$$

Information on all the calibration targets is collected in Table 1, and the implied banking parameters are shown in Table 2.

The observed liquidity premium as the price of liquidity effectively pins down, along with the other spreads, the quantity of liquidity supplied in the steady state by the government. Using equation (29) for bank fragility, the steady-state liquidity ratio is

$$m = 1 - \left(\frac{q-i}{r-i}\right)\left(n + (1-n)\sqrt{\frac{\rho - i}{q-i}}\right),$$

where n is the steady-state capital ratio of banks. Using data on total equity capital and total

[37] The nominal return on book equity for FDIC banks is 11.6%, implying an annual real return of 9.4%, which is close to the dividend-equity ratio.

Table 1: Targets used to calibrate the parameters of the model

Description	Notation	Value
Liquidity premium	$\rho - i$	0.28%/4
Credit spread	$r - i$	2.2%/4
Real return on bank equity	q	8.4%/4
Real Treasury Bill rate	i	1.5%/4
Bank capital ratio	n	8.8%

Table 2: Calibrated parameters of the model

Description	Notation	Value
Bank-asset liquidity relative to T-bills	λ	0.681
Loss given bank default	θ	4.4%/4
Minimum dividend distribution	γ	8.4%/4
Subjective discount factor	β	0.996
Elasticity of intertemporal substitution	σ	1
Frisch elasticity of labour supply	ψ	3
Capital elasticity of output	α	1/3
Depreciation	δ	7.5%/4
Steady-state liquidity ratio	m	0.148

assets from the Federal Deposit Insurance Corporation, the average bank capital ratio is 8.8% from 1991Q3 to 2008Q4. This and the other calibration targets implies $m = 0.148$.

The parameters describing the macroeconomic features of the model are set following the literature. The elasticity of intertemporal substitution σ is 1 and the Frisch elasticity of labour supply ψ is 3. The capital elasticity of output α is set to 1/3 to match the capital share of national income. The depreciation parameter δ is chosen to give a 7.5% annualized depreciation rate.

6.2 Results

Capital destruction shock. We simulate the model to show the effects of a one-off capital destruction shock. The impulse response functions are shown in Figure 6 alongside those for an RBC model with the same macroeconomic features but no banking sector. In the RBC model, households can directly invest in capital. To make the models comparable, the RBC model includes an exogenous but time-invariant spread between the return on capital and the marginal rate of substitution. Variables such as interest rates, spreads, and ratios are percentage point deviations from steady state (annualized for interest rates and spreads), with 1 meaning 1 percentage point. All other variables are percentage deviations from steady state, with 1 denoting 1%. To begin with, we assume policy is completely passive and the supply of liquid assets is not changed.

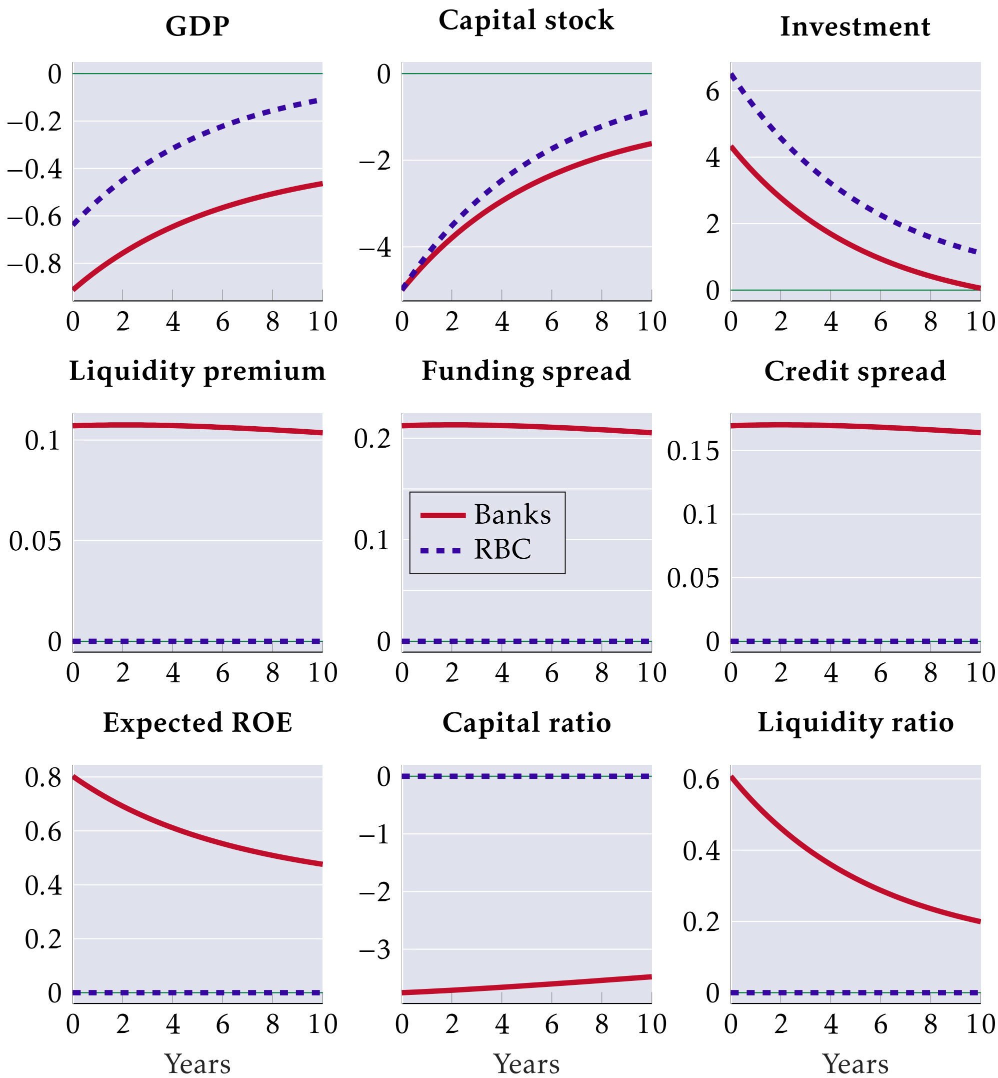
Consider first the effectively frictionless responses of variables in the RBC model. The shock directly reduces the capital stock by 5%, which brings down GDP. Investment rises to equate the marginal product of capital to the interest rate (in the RBC model, all interest rates move together one-for-one).

In the model with banks, the loss of some of the assets held by banks reduces their equity, which increases fragility and causes them to demand more liquid assets, pushing up the liquidity premium $\rho_t - i_t$ by 11 basis points. Banks must offer a higher interest rate on deposits to avoid runs, and the funding spread $j_t - \rho_t$ rises by 21 basis points. The increase in funding costs reduces lending, and the credit spread $r_i - i_t$ rises by 17 basis points. This results in less investment and a slower recovery of the capital stock compared to the RBC model. Consequently, GDP is lower and returns to its steady state at a slower rate.

Liquidity premium shock. The no-run constraint implies that the quantity of liquid assets held by banks has the effect of reducing fragility. We simulate the effects of an increase in liquidity by considering an exogenous shift in policy such that there is an unexpected 15 basis points decline in the liquidity premium with a half-life of 5 years. The impulse response functions are shown in Figure 7.

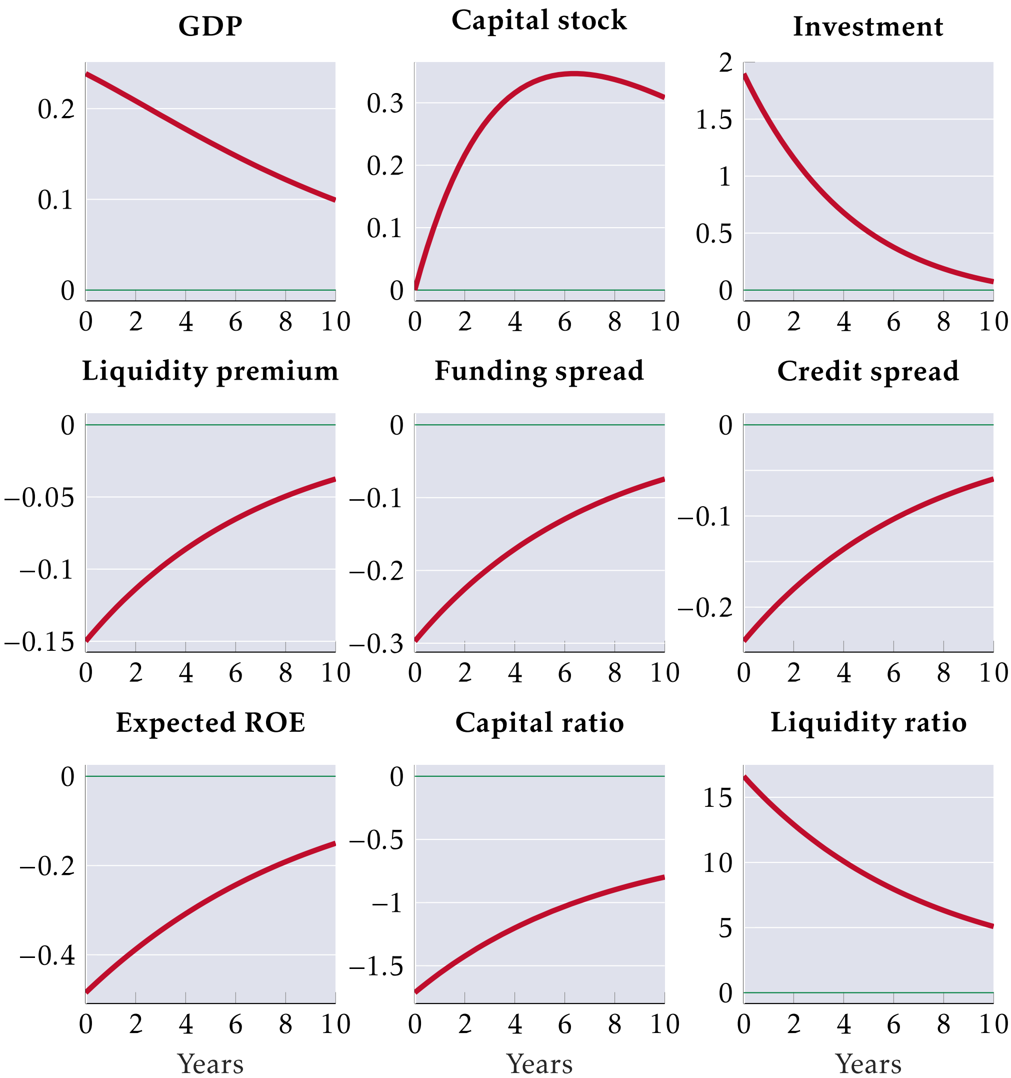
The reduction in fragility brought about by the shock allows banks to take on more leverage and pay a lower interest rate on their debt. The funding spread falls by 30 basis points, and the credit spread by 24 basis points. This leads to a rise in investment, which raises GDP.

Figure 6: Impulse response functions for a capital destruction shock



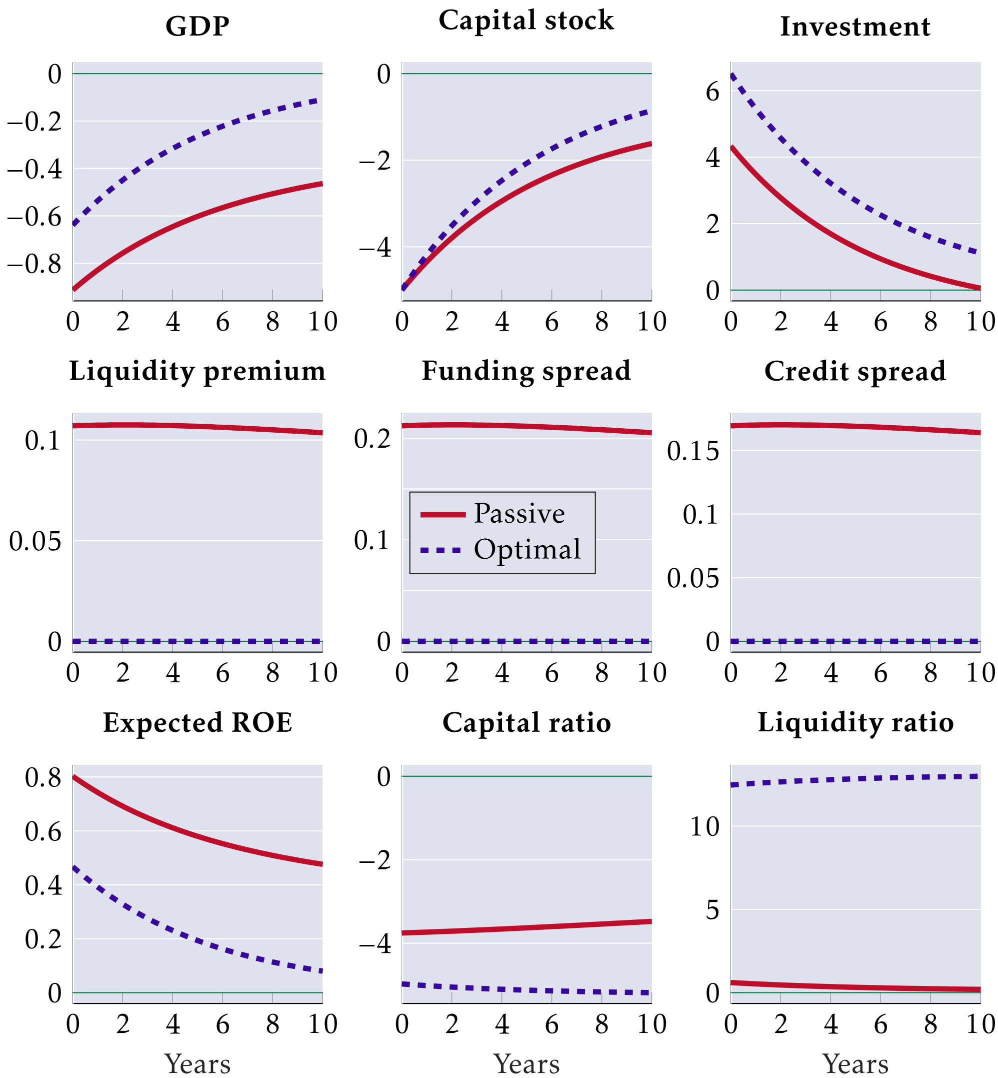
Stabilizing the liquidity premium. We can also study the supply of liquid assets as a systematic response to shocks. In this case, the optimal policy is to respond to shocks by supplying enough liquid assets to keep the liquidity premium constant at its initial steady-state value. Impulse responses to a one-off 5% capital destruction shock under optimal policy are represented by the blue dashed line in Figure 8. The red solid line represents the case of an inelastic supply of liquid assets. The optimal policy completely stabilizes the liquidity premium and hence the bank funding spread and the credit spread. To accomplish this, the quantity of liquid assets must increase significantly and persistently. The high persistence is necessary because in the

Figure 7: Impulse response functions for an expansion of liquid assets



absence of spreads bank equity does not fully recover. The greater supply of liquid assets leads to banks' liquidity ratio going up by 12 percentage points, which reduces bank fragility so much that the responses of macroeconomic variables are the same as those of the RBC model seen in Figure 6.

Figure 8: Capital destruction shock with and without the optimal policy response



7 Empirical Analysis

In this section, we empirically test the key prediction that distinguishes our model from other macroeconomic models with financial frictions. Our model predicts that liquidity is an important factor for banks' ability to fund lending. Specifically, it predicts that an increase in the liquidity premium increases banks' funding spread.

Specification. Equation (34) in the model describes the equilibrium relationship between liquidity premium and funding spread. If we linearize the equation and add an error term ϵ_t

that captures possible drivers of the funding spread not considered in the model, we can write

$$\mathrm{FS}_t = \alpha + \beta\,\mathrm{LP}_t + \epsilon_t. \tag{43}$$

We allow the error term to be autocorrelated with L lags of a data vector $\boldsymbol{y}_t$, to contain time fixed effects $\boldsymbol{d}_t$ and a linear trend.[38] Thus, we can re-write the empirical specification as

$$\mathrm{FS}_t = \alpha + \beta\,\mathrm{LP}_t + \sum_{l=1}^{L} \mathbf{y}_{t-l}^{\top}\zeta_{t-l} + \mathbf{d}_t^{\top}\boldsymbol{\eta} + \kappa t + \nu_t, \tag{44}$$

where ν_t is a stochastic innovation that is not autocorrelated but is potentially heteroskedastic. Vectors ζ_{t-l} and $\boldsymbol{\eta}$ and scalar κ contain parameters.

Data. We include in the data vector $\mathbf{y}_t$ eleven variables at daily frequency with the first observation on 3 January 2006 and the last on 30 June 2023.[39] (1) The funding spread measured as the difference between 3-month LIBOR and 3-month general-collateral (GC) repo rate. (2) The liquidity premium measured as the difference between the 3-month GC repo rate and the 3-month T-bill rate.[40] (3) The log-transformed quantity of outstanding treasuries. (4) The log-transformed balance on the Treasury General Account. 5) The spread between Moody's seasoned Baa corporate bond yield and the 10-year treasury rate. (6) The log-transformed value of the S&P 500 stockmarket index. (7) The log-transformed value of the S&P 500 financials stockmarket index. (8) The log-transformed VIX. (9) The level of the 3-month GC repo rate. (10) The level of the 10-year treasury rate. (11) The trade-weighted exchange rate of the US dollar. We set $L = 80$ to ensure we control for at least one quarter of data as lags. Our vector $\mathbf{d}_t$ includes time dummies for (1) weekday, (2) day of the month, (3) month, and (4) NBER recessions. The linear time trend does not allow for gaps in the observed dates.[41]

Identification. The econometric challenge is to find exogenous variation in the liquidity premium to estimate our coefficient of interest β. Because of omitted variables, measurement

[38] The data vector also contains the funding spread and liquidity premium.

[39] Before 2006 we do not have daily data for the dollar's trade-weighted exchange rate. The dataset's end date coincides with the final discontinuation date of LIBOR in the US. After merging the series, we are left with 4157 observations over the period. Data sources are reported in appendix B.

[40] Our adopted measure of the liquidity premium is standard in the literature (Nagel, 2016; Krishnamurthy and Li, 2023). The funding spread is the difference between the rate at which banks can borrow without collateral and the risk-free rate as measured by the GC repo rate.

[41] On average, our dataset contains 59 observations per quarter, nearly the universe of business days.

error and reverse causality, OLS estimates are unlikely to be consistent. For example, it is possible that unobserved shocks to uncertainty are driving both the funding spread and the liquidity premium. Or perhaps the GC repo rate is a noisy measure of the risk-free rate, and measurement error is driving a correlation between the measured liquidity premium and funding spread. It is also possible that shocks to the funding spread are driving demand for liquidity and thus the liquidity premium.[42]

Our identification strategy is to instrument the liquidity premium with the quantity of outstanding treasury debt. The quantity of treasuries is relevant to the liquidity premium as shown in a vast literature studying the convenience yield on treasuries (Krishnamurthy and Li, 2023). We confirm the instrument's relevance in the first-stage regression.

As for the instrument's validity, treasury debt is issued a few days after it is auctioned as can be seen in figure 14 in appendix A.[43] This institutional feature makes outstanding treasury debt predetermined at daily frequency. This rules out confounding variables in the error term v_t driving treasury debt and thus making it invalid. It also rules out reverse causality.

Another threat to the instrument's validity are alternative mechanisms through which the quantity of treasuries affects the funding spread for a given liquidity premium. We can assuage this concern by noting that an implication of outstanding treasuries being predetermined at daily frequency is that they are also perfectly anticipated. In other words, there is no new information revealed when treasuries are issued and mature. All the information, for instance regarding fiscal policy, is revealed at the latest during the auction. This rules out a direct information effect of the quantity of treasuries.

Finally, treasury debt is a highly persistent variable. To rule out a persistent omitted variable driving both treasury debt and the funding spread, it is important that the controls included in the regression succeed in removing the autocorrelation from the residual. Suppose we omitted lags of an element of the data vector $\boldsymbol{y}_t$ from the analysis. Then, the residual would contain the omitted lags as well as the stochastic innovation. If the omitted lags are also driving treasury debt because for instance they drive fiscal policy, then the instrument is no longer valid. As described above, we include as controls 80 lags of eleven variables available at daily frequency. As a result, the estimated residuals are not autocorrelated as can be seen in figure 13 in appendix A.

[42] The results from OLS, reported in table 5 in appendix A, supports the view that measurement error in the risk-free rate is an important driver of endogeneity.

[43] Even more days pass from announcement to auction.

Table 3: Regression table

	Funding spread
Liquidity premium	0.99**
	(0.45)
Lags	Y
Time dummies	Y
Linear trend	Y
R-squared	97%
Observations	4077
1^{st}-stage F statistic	15

Note 1: Outstanding treasuries as external instrument.
Note 2: Heteroskedasticity-consistent standard errors in parentheses.
Note 3: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

Key result. Table 3 contains the results of the benchmark IV regression. An exogenous one basis-point increase in the liquidity premium increases banks' funding spread by 1 basis point.[44] The effect is robustly significant with a p-value of 2.8%.[45]

The instrument is highly relevant as confirmed by the first-stage F statistic of 15. In the first-stage regression, we find that a one-percent increase in treasuries reduces the liquidity premium by 2.1 basis points (p-value is 0.3%). The direction is consistent with a movement along the downward-sloping demand for treasuries.

To check the robustness of the results, we look for evidence of state-dependence in the effect of the liquidity premium on the funding spread. We add as regressor an interaction term of the liquidity premium with the recession dummy to see to what extent the effect differs according to the state of the economy. As an additional instrument, we use the interaction of treasuries with the recession dummy. As reported in table 5 in appendix A, the effect of the liquidity premium on the funding spread in recessions is not significantly different from the same effect in expansions.

In table 5 in appendix A, we check alternative specifications and find that excluding the time dummies or the lag structure does not affect the results. Finally, to understand the import

[44] The size of the effect in the calibrated model is 2 basis points, which is in the 99% confidence interval of the estimate.

[45] We use heteroskedasticity-consistent standard errors although a Pagan-Hall general test overwhelmingly fails to reject homoskedasticity of the residuals (the test's p-value is 100%). With regular standard errors, the p-value is 0.4%.

of lag selection for the result, we run the benchmark regression with different values for the number of lags and report the results for the coefficient of interest in figure 15 in appendix A.

8 Conclusion

This paper has developed a novel financial friction based on coordination failure in the market for bank deposits. The friction implies that fragile banks borrow on worse terms. Liquid-asset holdings and net worth are substitutable factors that keep banks' fragility in check. Hence, when net worth is scarce, banks demand more liquid assets. Introducing this friction in a canonical macroeconomic model, we have found that the model matches the positive correlation of the liquidity premium with indicators of financial stress. This is a fact that current macroeconomic models with financial frictions do not speak to. Moreover, the friction gives a role for policy in adjusting the supply of liquid assets to stabilize the economy. Finally, we have tested empirically a key prediction of the model: a high liquidity premium leads to high funding costs for banks. Exploiting exogenous variation in the liquidity premium at daily frequency due to predetermined changes in the supply of treasuries, we find a robustly-significant positive effect. The corresponding effect in the calibrated model is within the 99% confidence interval of the empirical estimate.

The paper provides a quantitative framework to understand and evaluate policies that change the quantity of liquid assets in the economy. A case in point is quantitative easing, as enacted in response to the financial disruptions of the global financial crisis. The current generation of macroeconomic models largely appraise such policy as a credit policy: QE is effective because the central bank makes loans that banks cannot make on account of a binding leverage constraint. In this paper's framework, the real effects of QE stem from the liability-side of the central-bank balance sheet regardless of its asset holdings. Lots of liquid reserves on banks' balance sheets make creditors willing to lend to banks at more favourable conditions. The two effects are not exclusive. Hence, there is scope for studying moral-hazard and coordination frictions together for a rounder account of central-bank balance-sheet policies. More generally, the interaction of liquid-asset supply with other policy levers warrants further investigation. For this, the introduction of additional frictions from the literature, such as distortionary taxes or nominal rigidities, will be necessary.

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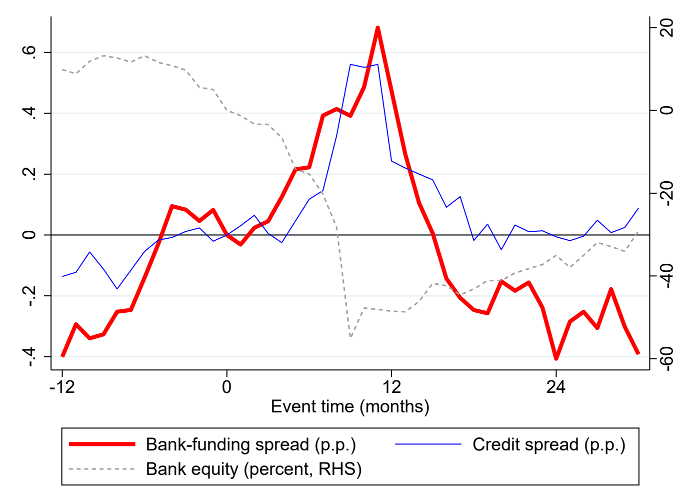
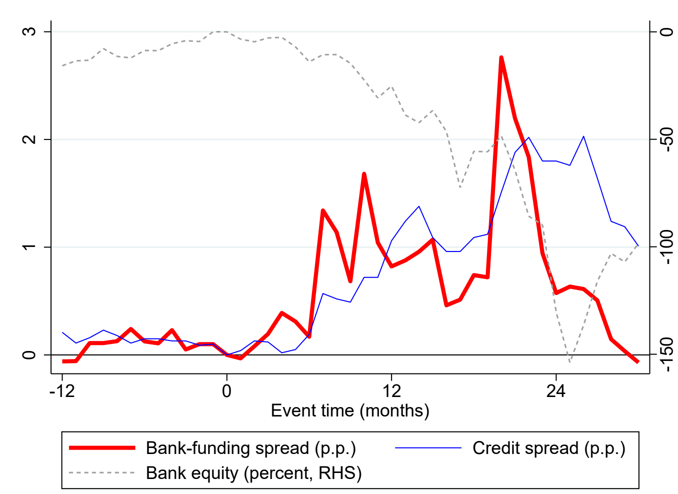
A Figures

Figure 9: Interest-rate spreads in banking crises.



Note 1: Because of data limitations, bank-funding spread is defined as the 3-month interbank rate minus 3-month government-bond rate (funding spread plus liquidity premium in the paper's definition). *Note 2:* The figure plots the average evolution of bank-funding spreads, credit spreads and bank equity (cumulated bank-share returns in log points) around banking-crisis years identified in Baron et al. (2021). The variables are normalized to 0 at event time 0, which is January of the banking-crisis year. The plots are averages over 66 episodes for which data is available. *Note 3:* The list of included banking crises is reported in table 4 in appendix A.

Figure 10: The dynamics of the global financial crisis in the US.



Plot of the evolution of the US bank-funding spread (the TED spread between 3-month LIBOR and 3-month T-bill rate), the US credit spread (Moody's Aaa corporate yield minus the 10-year treasury rate) and bank equity (cumulated bank-share returns in log points). Event time 0 is January 2007.

Table 4: List of banking crises underlying figure 9.

Country	Year
Australia	1989
Austria	2008, 2011
Belgium	2008, 2011
Czechia	1995
Denmark	1877, 1885, 1907, 1992, 2008, 2011
Finland	1990
France	1882, 1889, 1937, 2008
Germany	1891, 1901, 1930, 2008
Hong Kong	1998
Hungary	1995, 2008
Iceland	2008
Ireland	2007
Italy	1992, 2008, 2011, 2016
Japan	1990, 1997, 2001
Korea	1997
Luxembourg	2008
Malaysia	1985, 1997
Norway	1987, 2008
Philippines	1997
Portugal	2008, 2011
Russia	2008
Spain	2008, 2010
Sweden	1991, 2008
Switzerland	1990, 2008
Taiwan	1998
Thailand	1979, 1983, 1997
UK	1878, 1890, 1914, 1991, 2008
US	1890, 1893, 1907, 1930, 1984, 1990, 2007

Figure 11: Pandemic and tightening cycle.

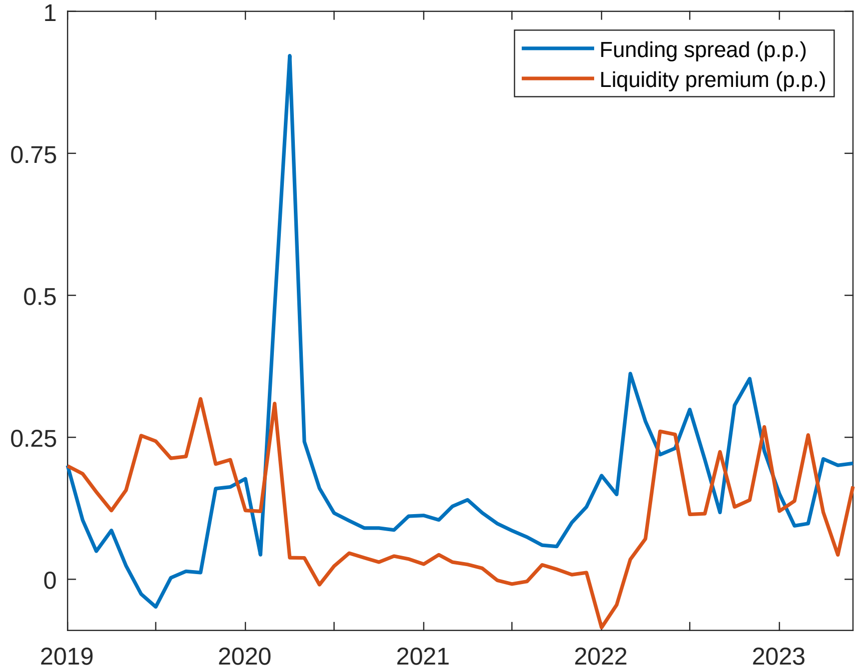


Figure 12: Expansions vs recessions.

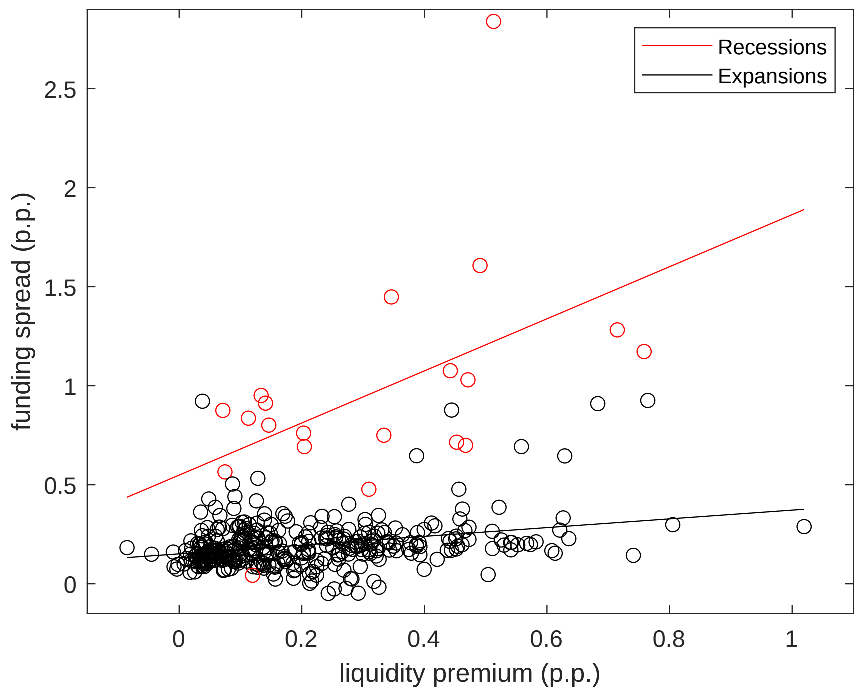
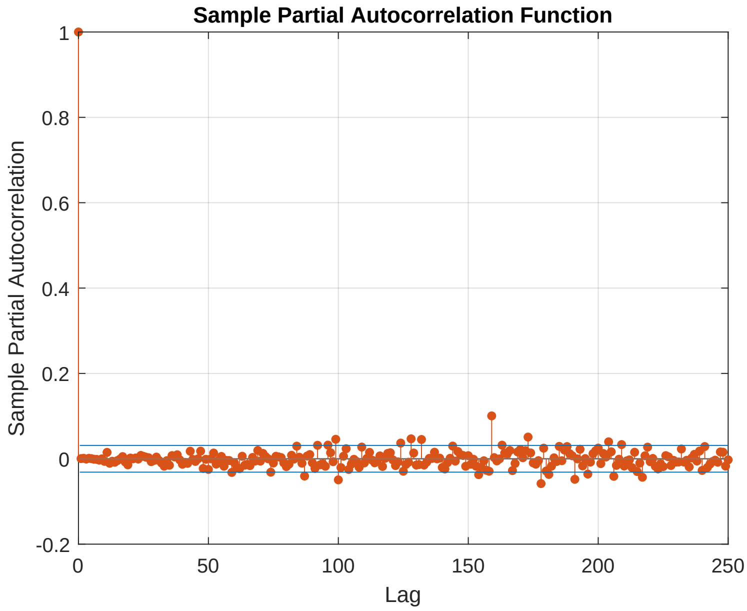


Table 5: OLS and alternative specifications

Funding spread	OLS	IV	IV	IV	IV	IV
Liquidity premium	-0.30***	1.4	1.0**	0.31***	1.28***	0.99**
	(0.06)	(1.0)	(0.48)	(0.04)	(0.06)	(0.45)
Liquidity premium × Recession		-0.54				
		(1.0)				
Lags	Y	Y	Y	N	N	Y
Time dummies	Y	Y	N	Y	N	Y
Linear trend	Y	Y	Y	Y	Y	Y
R-squared	99%	96%	96%	57%	17%	97%
Observations	4077	4077	4077	4157	4157	4077
1^{st}-stage F statistic		3.9	13	1560	1823	15

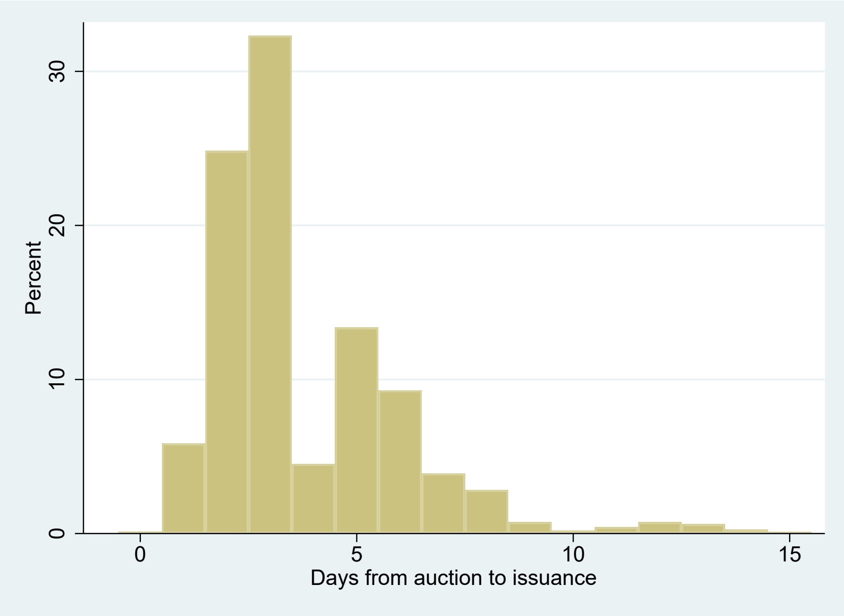
Note 1: IV estimation uses outstanding treasuries as external instrument.
Note 2: In regression with interaction term, estimation uses outstanding treasuries × recession as additional external instrument.
Note 3: Heteroskedasticity-consistent standard errors in parentheses.
Note 4: Funding spread = 3M LIBOR - 3M repo rate. Liquidity premium = 3M repo rate - 3M T-bill rate.

Figure 13: Partial autocorrelation function of funding-spread innovations.



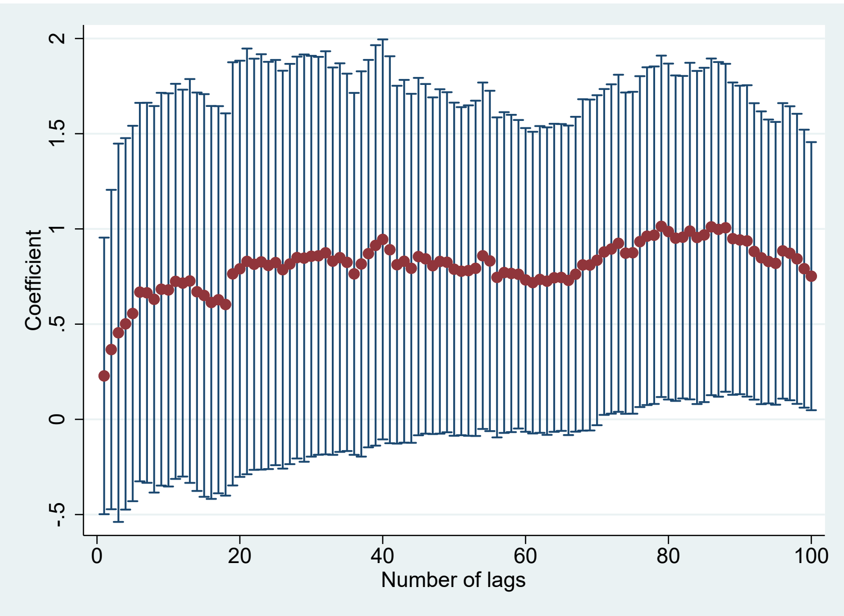
Note 1: The residuals are estimated in the benchmark IV regression.
Note 2: The blue lines are 95% confidence intervals for estimates of sample partial autocorrelation with a white-noise process.

Figure 14: Time from treasury auction to issuance



Note 1: Data source is TreasuryDirect.gov.
Note 2: There are 5320 observations of CUSIP-level treasury bills, notes and bonds issued between January 2006 and June 2023.
Note 3: The instances of auction and issuance on the same day are 7, corresponding to 0.1% of observations.

Figure 15: Robustness to lag selection.



Note: The blue bar represents the 95% confidence interval for the coefficient of interest β estimated with IV.

B Data sources

We obtain the 3-month GC repo rate (mid-price from ticker "USRGCGC ICUS Currncy") and the 3-month LIBOR from Bloomberg. Daily data on quantity of outstanding treasuries (series "Debt held by the public" in dataset "Debt to the Penny") and on the TGA closing balance (series "Treasury General Account (TGA) Closing Balance" in dataset "Daily Treasury Statement (DTS)") is available on the website Fiscaldata maintained by the US Treasury Department. From the website FRED maintained by the Federal Reserve Bank of St. Louis, we retrieve the 3-month T-bill rate (series "DTB3"), the spread between Moody's seasoned Baa corporate bond yield and the 10-year treasury rate (series "BAA10Y"), the 10-year treasury rate (series "DGS10"), the VIX (series "VIXCLS"), and the nominal broad US dollar index (series "DTWEXBGS"). The closing values of the S&P 500 stockmarket index and the S&P 500 financials stockmarket index are downloaded from the website Yahoo! Finance.

C Proofs

Proof of Lemma 1. A strategy in the coordination game is a correspondence that maps a household's signal $\tilde{F}_h$ into deposit-holding decision H_h.

Consider other households playing the same threshold strategy such that they hold a bank's deposits with $H_h = 1$ if they receive signal $\tilde{F}_h \leq k$ and do not hold the deposits otherwise. Given a household i's (improper) uniform prior and signal $\tilde{F}_i$ about the fundamental, its expected net payoff of holding deposits can be written as

$$\tilde{\pi}^*(\tilde{F}_i, k) = \int_{\tilde{F}_i - \omega}^{\tilde{F}_i + \omega} \frac{\tilde{\pi}(F, k)}{2\omega} \, \mathrm{d}F, \tag{45}$$

where $\tilde{\pi}(F, k)$ is the payoff of holding deposits for a given fundamental F.

Using (6), we can show that for $j < \rho \; \tilde{pi} < 0$. Hence, it is a dominant strategy for households not to hold deposits. Focusing on the interesting case $j \geq \rho$, we have that net payoff $\tilde{\pi}(F, k)$ is $-\theta < 0$ for $F > 1$, because in this case the bank fails even if everyone else's strategy implies holding for any signal. It is $j - \rho \geq 0$ for $F \leq 0$ because in this case the bank does not fail regardless of everyone else's strategy. As for the intermediate range, notice that by the law of large numbers the share of households holding is

$$H = \begin{cases} 1 & \text{if } F \leq k - \omega, \\ \frac{1}{2} + \frac{k-F}{2\omega} & \text{if } F \in (k-\omega, k+\omega] \\ 0 & \text{otherwise.} \end{cases} \tag{46}$$

This result implies that

$$\tilde{\pi}(F,k) = \begin{cases} -\theta & \text{if } F > 1, \\ -\theta & \text{if } F \in (0,1] \text{ and } F > \frac{k+\omega}{1+2\omega}, \\ j-\rho & \text{if } F \in (0,1] \text{ and } F \leq \frac{k+\omega}{1+2\omega}, \\ j-\rho & \text{otherwise.} \end{cases} \tag{47}$$

Using (45) and (47), we can verify that a dominant strategy sets $H_i = 0$ for $\tilde{F}_i > 1+\omega$. A dominant strategy also sets $H_i = 1$ for $\tilde{F}_i < -\omega$ if $j > \rho$. If $j = \rho$, then it is only weakly dominant to set $H_i = 1$ for $\tilde{F}_i < -\omega$. Nonetheless, the equilibrium strategy must imply this behaviour because of the tie-breaking assumption that households hold deposits if indifferent.

Now, we start to iteratively delete strictly dominated strategies. We start with a strategy of holding deposits if and only if $\tilde{F}_i \leq 1+\omega$. As already noticed, a strategy that implies holding for $\tilde{F}_h > 1+\omega$ is dominated. This restricts the strategies we consider for other households. Let us consider other households playing a threshold strategy with $k = 1+\omega$. Because of strategic complementarity, if a strictly better strategy for household i can be found under this conjecture about other households' behaviour, then household i's strategy under consideration is strictly dominated. The expected net payoff of holding for a household receiving signal $1+\omega$ given $k = 1+\omega$ is $\tilde{\pi}^*(\tilde{F}_i, k) = -\theta$. Hence, the strategy is dominated.

This logic can be extended by studying

$$\tilde{\pi}^*(z,z) = \int_{z-\omega}^{\frac{z+\omega}{1+2\omega}} \frac{j-\rho}{2\omega}\,\mathrm{d}F - \int_{\frac{z+\omega}{1+2\omega}}^{z+\omega} \frac{\theta}{2\omega}\,\mathrm{d}F = \frac{j-\rho+\omega(j-\rho-\theta)}{1+2\omega} - \frac{j-\rho+\theta}{1+2\omega} z. \tag{48}$$

The function is monotonically decreasing and crosses zero at

$$z^* = \frac{j-\rho}{j-\rho+\theta} + \frac{j-\rho-\theta}{j-\rho+\theta}\omega. \tag{49}$$

This allows us to delete as dominated all strategies such that a household holds deposits with $\tilde{F}_i > z^*$. We can apply an analogous analysis in reverse to delete as dominated all strategies that set $H_h = 0$ for $\tilde{F}_i < z^*$.

Deletion of strictly dominated strategies and the assumption that households hold a bank's deposits if indifferent give us the strategy played in the unique Bayesian Nash equilibrium of the coordination game as

$$H_h^* = \begin{cases} 1 & \text{if } \tilde{F}_h \geq \frac{j-\rho}{j-\rho+\theta} + \frac{j-\rho-\theta}{j-\rho+\theta}\omega, \\ 0 & \text{otherwise.} \end{cases} \tag{50}$$

□

Proof of Lemma 2. The lemma is proven by substituting the equilibrium run threshold (7) into equation (46), which gives the share of households holding deposits for a general run threshold k. The result for $j_b < \rho$ follows directly from Lemma 1. □

Proof of Proposition 1. We start with the proof of statement 1. First, consider the case with $(F_b - F_b^*)/\chi \in (-1+\omega/\chi, 1-\omega/\chi)$, a non-empty set since $\omega/\chi \to 0$. Using equation (8), we can compute $\Pr(H_b \in (0,1)) = \omega/\chi \to 0$. Second, consider the case with $(F_b - F_b^*)/\chi = -1+\omega/\chi$, which implies $(F_b - F_b^*)/\chi < 1-\omega/\chi$ since $\omega/\chi \to 0$. In this case too, we have that $\Pr(H_b \in (0,1)) = \omega/\chi \to 0$. Third, consider the case with $(F_b - F_b^*)/\chi \in (-1-\omega/\chi, -1+\omega/\chi)$. In this case, we can compute $\Pr(H_b \in (0,1)) = (1/2)\left[1 + (F_b - F_b^*)/\chi + \omega/\chi\right]$, which is smaller than $3\omega/2\chi \to 0$. For the last case with $(F_b - F_b^*)/\chi \leq -1-\omega/\chi$, it is trivially true that $\Pr(H_b \in (0,1)) = 0$. Finally, for $\chi \to 0$ we have that $F_b = \tilde{F}_b$. Hence, I do not have to refer to $\tilde{F}_b$ separately in the statement.

Second, we prove statement 2. We work with $j_b \geq \rho$. For $j_b < \rho$, there probability of $H_b = 1$ is trivially zero due to Lemma 2. Given statement 1 of the proposition, for $(F_b - F_b^*)/\chi \leq -1+\omega/\chi$ we have that $\Pr(H_b = 1) \to 1$ because clearly $H_b = 0$ is impossible under equation (8). In the alternative case $(F_b - F_b^*)/\chi > -1+\omega/\chi$, we have that

$$\Pr(H_b = 1) = \max\left\{0, \frac{1}{2}\left(1 - \frac{F_b - F_b^*}{\chi} - \frac{\omega}{\chi}\right)\right\} < 1. \tag{51}$$

Hence, we have that $H_b = 1$ has probability one if and only if $F_b \leq F_b^* - \chi(1-\omega/\chi)$ and $j_b \geq \rho$. Given $\chi \to 0$, this is equivalent to $\tilde{F}_b \leq F_b^*$ and $j_b \geq \rho$.

D Steady state

In this section, we analyse the long-run dynamics of the model by studying the steady state. The model's steady state is a constant sequence for prices and quantities that satisfies the model's equilibrium conditions. Along the steady state, the quantity of liquid assets M, i.e. the policy variable, is constant.

We look for a steady state with a strictly positive liquidity premium $\rho - i > 0$ and bank net worth $N > 0$. Combining equations (35) and (38), we obtain

$$q - \rho = \theta + 2\sqrt{\theta(\rho - i)} > 0. \tag{52}$$

Evaluating the formula for the banks' market-to-book ratio (37) in steady state, we obtain

$$v = \frac{\gamma(1+q)}{(1+\gamma)(1+\rho) - (1+q)} > 1, \tag{53}$$

which implies that the minimum dividend constraint is binding in steady state so that

$$\Pi = \gamma N. \tag{54}$$

Together with the law of motion for banks' net worth in (39), a binding minimum dividend constraint implies that in a steady state

$$q = \gamma \tag{55}$$

for $N > 0$.[46] First, we notice from (52) that a parametric restriction

$$\gamma > \rho + \theta \tag{56}$$

is necessary for (55) to be sustained with a strictly positive liquidity premium.[47] Under this

[46] The upper limit on M identified below rules out $N = 0$ and $q < \gamma$ in the steady state with strictly positive liquidity premium.

[47] If this is violated, then net worth grows up to the point where there is no fragility and the liquidity premium is zero.

restriction, we pin down the steady-state liquidity premium as

$$\rho - i = \frac{[\gamma - (\rho + \theta)]^2}{4\theta}. \tag{57}$$

This liquidity premium creates the right level of returns on bank net worth so that bank net worth is stable. Interestingly, it is independent of policy. We can determine the steady-state balance-sheet structure of banks with equations (23), (24) and (34) as

$$N = \left[1 - \lambda - \frac{\gamma - (\rho + \theta)}{2\theta}\left(\lambda + \frac{M}{K}\right)\right]K. \tag{58}$$

To have positive net worth in steady state, we need to restrict the equity friction with

$$\gamma \leq \rho + \theta\frac{2 - \lambda}{\lambda} \tag{59}$$

and policy with

$$M < \left[\frac{2\theta(1 - \lambda)}{\gamma - (\rho + \theta)} - \lambda\right]K. \tag{60}$$

An excessively strong equity friction makes it impossible to sustain positive net worth in steady state even with no liquidity. An excessively large supply of liquid assets rules out a fragile steady state with positive liquidity premium for any positive level of net worth.

The key finding that in the long run liquidity policy has no effect on the liquidity premium, and thus fragility, is due to the endogenous structure of banks' balance sheet. Increases in liquid-asset supply crowd out bank net worth in the long run to the point where fragility is unchanged.

As is standard in a real business cycle model, the steady-state risk-free rate is pinned down by the Euler equation in (18) as $\rho = (1-\beta)/\beta$ and the steady-state level of capital is the unique strictly-positive solution to the system of equations given by

$$(1-\alpha)K^{\alpha(1+\frac{1}{\sigma})} = L^{\alpha+\frac{1}{\psi}}\left(ZL^{1-\alpha} - \delta\right)^{-\frac{1}{\sigma}} \tag{61}$$

and

$$K = \left(\frac{\alpha}{r-\delta}\right)^{\frac{1}{1-\alpha}}L. \tag{62}$$

E Solving the full macroeconomic model

The full macroeconomic model is represented by the following set of equations:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{63}$$

$$C_t + I_t = Y_t \tag{64}$$

$$(1-\alpha)\frac{Y_t}{L_t} = w_t \tag{65}$$

$$C_t^{\frac{1}{\sigma}} L_t^{\frac{1}{\psi}} = w_t \tag{66}$$

$$K_t = X_t A_{t-1} \tag{67}$$

$$A_t = (1-\delta)K_t + I_t \tag{68}$$

$$P_t = \beta\left(\frac{C_t}{C_{t-1}}\right)^{-\frac{1}{\sigma}} \tag{69}$$

$$\frac{1}{1+\rho_t} = \mathbb{E}_t P_{t+1} \tag{70}$$

$$R_t = \left(\alpha\frac{Y_t}{K_t} + 1 - \delta\right)\frac{K_t}{A_{t-1}} - 1 \tag{71}$$

$$r_t = \frac{\mathbb{E}_t[P_{t+1}(1+\zeta_{t+1})R_{t+1}]}{\mathbb{E}_t P_{t+1}(1+\zeta_{t+1})} \tag{72}$$

$$A_t + M_t = D_t + N_t \tag{73}$$

$$N_t = \frac{1+Q_t}{1+\gamma}N_{t-1} \tag{74}$$

$$F_t = 1 - \lambda - \frac{\lambda N_t + (1-\lambda)M_t}{D_t} \tag{75}$$

$$Q_t = q_{t-1} + (R_t - r_{t-1})\frac{A_{t-1}}{N_{t-1}} \tag{76}$$

$$r_t = (1-\lambda)q_t + \lambda i_t \tag{77}$$

$$r_t - i_t = (1-\lambda)\left(\sqrt{\theta} + \sqrt{\rho_t - i_t}\right)^2 \tag{78}$$

$$j_t - \rho_t = \sqrt{\theta}\sqrt{\rho_t - i_t} \tag{79}$$

$$\rho_t - i_t = \theta\frac{F_t^2}{(1-F_t)^2} \tag{80}$$

$$V_t = \mathbb{E}_t[P_{t+1}(V_{t+1} + \Pi_{t+1})] \tag{81}$$

$$\Pi_t = \gamma N_t \tag{82}$$

The variables Z_t and X_t denote exogenous levels of TFP and capital quality. There is also an equation describing how the supply of liquidity M_t is determined, which could be exogenous or endogenous. In what follows, $\hat{}$ denotes the deviation of a variable from its steady-state value: a simple deviation for variables already measured as percentages, and log deviations for all other variables.

The production function (63) in log deviations is:

$$\hat{Y}_t = \hat{Z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t \tag{83}$$

Letting $\kappa = K/Y$ denote the steady-state capital-output ratio, since $I = \delta K$, the log linearization of the aggregate demand equation (64) is:

$$(1-\delta\kappa)\hat{C}_t + \delta\kappa\hat{I}_t = \hat{Y}_t \tag{84}$$

Labour demand (65) and supply (66) in log deviations are:

$$\hat{Y}_t - \hat{L}_t = \hat{w}_t \tag{85}$$

$$\frac{1}{\sigma}\hat{C}_t + \frac{1}{\psi}\hat{L}_t = \hat{w}_t \tag{86}$$

Since $X = 1$, the supply of capital to firms (67) and capital investment (68) have the following log-linear forms:

$$\hat{K}_t = \hat{X}_t + \hat{A}_{t-1} \tag{87}$$

$$\hat{A}_t = (1-\delta)\hat{K}_t + \delta\hat{I}_t \tag{88}$$

Using $P = \beta$ and $\rho = (1-\beta)/\beta$, the stochastic discount factor P_t from (69) and the implied risk-free rate (70) in terms of deviations from steady state:

$$\hat{P}_t = -\frac{1}{\sigma}(\hat{C}_t - \hat{C}_{t-1}) \tag{89}$$

$$\hat{\rho}_t = -(1+\rho)\mathbb{E}_t\hat{P}_{t+1} \tag{90}$$

Using $r = R = (\alpha/\kappa) - \delta$, the equations (71) and (72) for the ex-post and risk-adjusted expected returns on physical capital have the following approximate forms:

$$\hat{R}_t = (r+\delta)(\hat{Y}_t - \hat{K}_t) + (1+r)(\hat{K}_t - \hat{A}_{t-1}) \tag{91}$$

$$\hat{r}_t = \mathbb{E}_t \hat{R}_{t+1} \tag{92}$$

In terms of $n = N/(A+M)$ and $m = M/(A+M)$, the bank balance sheet (73) and accumulation of net worth (74) equations can be approximated as follows (noting $Q = \gamma$):

$$(1-m)\hat{A}_t + m\hat{M}_t = (1-n)\hat{D}_t + n\hat{N}_t \tag{93}$$

$$\hat{N}_t = \hat{N}_{t-1} + \frac{\hat{Q}_t}{1+\gamma} \tag{94}$$

The approximation of the equation for bank fragility (75) is:

$$\hat{F}_t = \frac{(\lambda n + (1-\lambda)m)(1-m)}{(1-n)^2}\hat{A}_t - \frac{(\lambda + (1-\lambda)m)n}{(1-n)^2}\hat{N}_t - \frac{((1-\lambda)(1-m)-n)m}{(1-n)^2}\hat{M}_t \tag{95}$$

Equations (76), (77) and (78) for the returns on bank assets and liabilities become:

$$\hat{Q}_t = \hat{q}_{t-1} + \frac{(1-m)}{n}(\hat{R}_t - \hat{r}_t) \tag{96}$$

$$\hat{r}_t = (1-\lambda)\hat{q}_t + \lambda\hat{i}_t \tag{97}$$

$$\hat{r}_t - \hat{i}_t = (1-\lambda)\left(1 + \frac{\sqrt{\theta}}{\sqrt{\rho - i}}\right)(\hat{\rho}_t - \hat{i}_t) \tag{98}$$

Equations (79) and (80) linking bank fragility, funding costs, and the liquidity premium have the following approximations:

$$\hat{j}_t - \hat{\rho}_t = \frac{1}{2}\frac{\sqrt{\theta}}{\sqrt{\rho - i}}(\hat{\rho}_t - \hat{i}_t) \tag{99}$$

$$\hat{\rho}_t - \hat{i}_t = 2\frac{\sqrt{\rho - i}}{\sqrt{\theta}}\left(\sqrt{\theta} + \sqrt{\rho - i}\right)^2 \hat{F}_t \tag{100}$$

The log linearization of the stock-market value equation (81) is:

$$\hat{V}_t = \frac{1}{1+\rho}\mathbb{E}_t\hat{V}_{t+1} + \frac{\rho}{1+\rho}\mathbb{E}_t\hat{N}_{t+1} - \frac{1}{1+\rho}\hat{\rho}_t \tag{101}$$

Together with these main equations, there are auxiliary equations defining other variables. The funding spread is $\hat{j}_t - \hat{\rho}_t$, the liquidity premium $\hat{\rho}_t - \hat{i}_t$, the credit spread $\hat{r}_t - \hat{i}_t$, the total size of banks' balance sheets $(1-m)\hat{A}_t + m\hat{M}_t$, the capitalization ratio $\hat{n}_t = n(1-n)(\hat{N}_t - \hat{D}_t)$, the liquidity ratio $\hat{m}_t = m(1-m)(\hat{M}_t - \hat{A}_t)$, and the market-to-book value ratio $\hat{v}_t = \hat{V}_t - \hat{N}_t$.

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