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## Too levered for Pigou: carbon pricing, financial constraints, and leverage regulation

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**Abstract**

We analyze jointly optimal carbon pricing and leverage regulation in a model with financial constraints and endogenous climate-related transition and physical risks. The socially optimal emissions tax is below the Pigouvian benchmark (equal to the direct social cost of emissions) when emissions taxes amplify financial constraints, or above this benchmark if physical climate risks have a substantial impact on collateral values. Additionally introducing leverage regulation can be welfare-improving only if tax rebates are not fully pledgeable. A cap-and-trade system or abatement subsidies may dominate carbon taxes because they can be designed to have a less adverse effect on financial constraints.

**Keywords:** Pigouvian tax, carbon pricing, financial constraints, climate risk, financial regulation

**JEL classifications:** D62, G28, G32, G38, H23

# Non-technical Summary

Tackling climate change requires large-scale emissions reductions and investments in clean technologies, which have to be incentivized by environmental policies such as carbon taxes. The associated costs of transitioning to a low-carbon economy may imply significant losses for polluting firms and financial institutions exposed to "brown" assets. At the same time, physical damages caused by more frequent extreme weather events may hit asset values. This paper studies how these transition and physical climate risks interact with financial frictions and derives implications for optimal environmental policy and financial regulation.

We develop a tractable model in which financially constrained borrowers with polluting assets can decrease emissions through costly abatement investments or by liquidating assets, but liquidations are inefficient due to liquidation losses. Absent financial constraints, a carbon tax equal to the social cost of emissions (referred to as a *Pigouvian* tax in reference to the pioneering work by Pigou, 1932) can incentivize borrowers to invest in abatement and thereby implement the first-best allocation. This is no longer the case when borrowers are financially constrained. In this case, the optimal carbon tax differs from the benchmark Pigouvian solution because a regulator needs to trade off the intended emissions reduction against the side-effect the policy has on borrowers' financial constraints. Intuitively, an environmental regulator needs to take into account that increasing carbon taxes constitutes a materialization of climate transition risk that can trigger inefficient asset liquidations by constrained borrowers. As a result of these effects, the optimal carbon tax may be set below the benchmark Pigouvian rate.

A key insight from our analysis is that physical climate risks can reverse the relationship between emissions taxes and financial constraints. If physical climate risks have a substantial effect on collateral values, borrowers may benefit from an increase in pledgeable income when the aggregate level of emissions is brought down by a higher carbon tax. This collateral externality of emissions is not internalized by borrowers, and therefore optimal carbon taxes may alternatively be *above* the Pigouvian benchmark rate if the collateral effects of physical climate risk dominate the effects of transition risk.

Given that financial constraints can hinder the efficient implementation of optimal environmental policy, we also ask whether complementing emissions taxes with ex-ante leverage regulation can help. Doing so, we derive an important necessary condition: for

any additional policy tool to improve welfare, it needs to be the case that carbon taxes have a *direct effect* on borrower's financial constraints. This is only the case if taxes are not fully rebated to borrowers (for example, through a "carbon dividend"), or if such tax rebates are not fully pledgeable to outside investors. Analogously, in a cap-and-trade system purchases of pollution permits only have a direct effect on financial constraints if pollution permits are not 100% allocated for free to borrowers. This is the case, for example, in the EU Emissions Trading System where the number of freely allocated permits has been significantly reduced over recent years.

If tax rebates cannot be fully pledged to outside investors, then there is scope to complement carbon taxes with other policies. We show that a leverage mandate that requires borrowers to contribute either a minimum or a maximum level of own capital can be socially beneficial in this case. Whether a minimum (floor) or a maximum (ceiling) on capital is needed depends on the effect that relaxing financial constraints has on total emissions, which depends on whether looser constraints mostly induce borrowers to avoid liquidating polluting assets, or increase abatement. As an alternative policy, subsidies on firm's abatement activities can (somewhat trivially) improve on the solution achieved with carbon taxes if the subsidy redistributes resources from unconstrained economic agents to constrained borrowers. The model also highlights an important role the financial sector can play in enabling more efficient environmental policy in equilibrium, namely by offering hedging contracts that reduce financial frictions in high-social-cost-of-carbon states of the world, which in turn enables more efficient carbon pricing in equilibrium.

In summary, this paper sheds light on the relationship between environmental policy, financial regulation, and the green transition. It highlights the importance of considering financial frictions when designing environmental policies and demonstrates that optimal policies depend on the relative importance of climate-related transition and physical risks in a given economy. Furthermore, the paper establishes a key necessary condition for the effectiveness of supplementing environmental regulations with other policies when borrowers may face financial constraints.

# 1 Introduction

Tackling climate change requires large-scale emissions reductions and investments in clean technologies. Absent other frictions, such investments can be incentivized through emissions taxes set at a rate equal to the social cost of emissions, also known as Pigouvian taxes in reference to the pioneering work by Pigou (1932). However, during the transition to a low-carbon economy firms and financial institutions may suffer significant losses due to stranded assets that become technologically obsolete. At the same time, physical damages caused by more frequent extreme weather events may hit asset values. Such losses can aggravate financing frictions, limit the ability of firms to make the necessary investments in green technologies, and constrain regulators in designing environmental policies (see Hoffmann et al., 2017; Oehmke and Opp, 2022b; Biais and Landier, 2022). Accordingly, the risks posed by climate change have moved up the agenda of investors and policy makers.[1]

We contribute to the debate by providing an analytical evaluation of jointly optimal carbon pricing and financial regulation in a setting with financial constraints and endogenous climate-related transition and physical risks. Our analysis shows that the relative strength of these two climate-related risks crucially affects the way in which emissions taxes interact with financial constraints. We draw implications for optimal environmental policy and derive necessary conditions under which it can be welfare-improving to complement emissions taxes with ex-ante leverage regulation. The model also underlines the role of the financial sector in hedging climate-related risks, which may enable more efficient environmental policy in equilibrium.

In the model there are three dates and two types of agents: borrowers and deep-pocketed, risk-neutral lenders. Borrowers have an initial endowment and access to an investment project. At the initial date, they finance the project with a mix of inside equity and debt. Equity financing is costly because borrowers have a quasi-linear utility function and a limited initial endowment. The borrower's project generates a pecuniary

[1]For example, the European Central Bank and the Bank of England now include climate risks in their stress tests (see Alogoskoufis et al., 2021; Brunnermeier and Landau, 2022), and institutional investors view climate change as an important source of risk that they seek to mitigate (Krueger et al., 2020).

return as well as carbon emissions at the final date. The social cost of emissions is not known ex-ante, reflecting the uncertainty evident in the wide range of estimates of the social cost of carbon (e.g., see Nordhaus, 2018). At the interim date, all agents learn whether the economy is in a good state with a low social cost of emissions, or a bad state with a high cost of emissions. After learning the social cost, emissions can be reduced through costly abatement activities undertaken by borrowers at the interim date. At the same time, borrowers need to roll-over debt raised in the initial period, but new debt issuance is limited by a financial constraint because the project's returns are not fully pledgeable to outside investors. Cash-constrained borrowers can liquidate part of the initial investment at the interim date to generate resources and at the same time reduce emissions, yet liquidations are inefficient due to liquidation losses.

Borrowers are exposed to two different types of climate-related risks. First, we consider an environmental regulator imposing state-contingent emissions taxes to incentivize costly abatement activities, which represent the costs of transitioning to a low-carbon economy (often referred to as "transition risk" in the literature).[2] Second, we assume that the return of the project may decrease in the level of aggregate emissions to capture a borrower's exposure to financial losses due to environmental damages caused by a warming climate (often termed as "physical risk").[3] Both climate-related risks are endogenous in the model: transition risk is a consequence of emissions taxes optimally set by an environmental regulator, and financial losses due to physical climate risks depend on aggregate emissions that are a function of abatement activities and investment decisions by borrowers. This allows us to explore the differences in how these two types of climate-related risks interact with financial frictions and affect optimal environmental and financial policies in equilibrium.

[2] Consistent with transition risks being priced in financial markets, recent evidence documents that firm-level carbon emissions are priced in corporate bonds (see Seltzer et al., 2020), stocks (see Bolton and Kacperczyk, 2021), and options (see Ilhan et al., 2021), and that the risk of stranded fossil fuel assets is priced in bank loans (see Delis et al., 2019).

[3] Several contributions document the relevance of physical risk for asset prices and firm financing. For example, Giglio et al. (2021) find that the value of real estate in flood zones responds more to changes in climate attention, and Issler et al. (2020) document an increase in delinquencies and foreclosures after wildfires in California. Evidence in Ginglinger and Moreau (2019) indicates that physical climate risks affect a firm's capital structure. For a review discussing climate risks, see Giglio et al. (2021).

As a benchmark, we show that a state-contingent emissions tax equal to the social cost of emissions (i.e., a Pigouvian tax) implements the first-best allocation if financial constraints are slack in all states. In the first-best allocation, there are no liquidations and the optimal abatement scale trades off the social benefit of lower emissions against abatement costs. However, in equilibrium the financial constraint may bind (particularly in the bad state where a high social cost of emissions necessitates high emissions taxes and abatement investments). In this case, Pigouvian taxes cannot implement the first best, and optimal emissions taxes generally differ from the Pigouvian benchmark. The reason is that a constrained borrower has a limited ability to finance abatement and therefore needs to inefficiently liquidate some of the project at the interim date. Consequently, the socially optimal emissions tax needs to trade off the benefit of lower emissions against the cost of triggering inefficient liquidations. This implies an optimal emissions tax below the Pigouvian benchmark because borrowers are "too levered for Pigou".[4]

A key insight from our analysis is that physical climate risks can reverse the relationship between emissions taxes and financial constraints. If physical climate risk has a substantial effect on collateral values, borrowers may benefit from an increase in pledgeable income when the aggregate level of emissions is brought down by a higher emissions tax.[5] Because of this collateral externality the optimal emissions tax may be *above* the Pigouvian benchmark rate if the effects of physical climate risk dominate the effects of transition risk. More broadly, we show that financial constraints call for a generalized Pigouvian tax that takes climate-induced collateral externalities into account.

To evaluate whether it may be welfare-improving to combine emissions taxes with other policy tools, we analyze under what conditions the allocation implemented with emissions taxes is constrained efficient (i.e., equivalent to an allocation chosen by a planner maximizing social welfare subject to the same constraints as private agents). In a first

[4]The mechanism behind this result is consistent with recent evidence documenting that financial constraints affect firm abatement activities and emissions, see Xu and Kim (2022) and Bartram et al. (2021).

[5]This effect is similar to collateral externalities in models with pecuniary externalities, where borrowers do not internalize the effect of their choices on the financial constraints of other agents through prices (for a detailed discussion, see Dávila and Korinek, 2018). In our setting, the collateral externality operates through the physical costs of environmental damages caused by higher emissions, which reduce a borrowers' pledgeable income.

step, we consider a benchmark where emissions taxes are fully rebated to borrowers, and tax rebates are fully pledgeable to outside investors, so that emissions taxes have no *direct* effect on financial constraints. In this case, the competitive equilibrium with optimally set emissions taxes is constrained efficient. This implies that, while financial constraints generally imply optimal emissions taxes different from a Pigouvian benchmark, there is no scope to improve welfare using additional policy instruments when tax rebates are fully pledgeable.

By contrast, when tax rebates are partially non-pledgeable, the allocation is not constrained efficient, and combining emissions taxes with other policy tools can be welfare-improving. Given the central role of financial constraints, we consider a leverage mandate that allows the regulator to fix the initial level of equity of borrowers at a given level. Such a policy can be implemented through direct leverage mandates or, alternatively, through taxes and subsidies on initial leverage. To understand the role of leverage regulation in the model, note that, (i) when emissions taxes have a direct effect on financial constraints there remains a wedge between the social and the private cost of emissions even when emissions taxes are set optimally; and (ii) a borrower's initial leverage affects emissions because they affect financial constraints and therefore liquidations and abatement activities at the interim date. Together, these two points imply that borrowers make socially inefficient leverage choices, and consequently there is a role for leverage regulation to improve welfare.

The model focuses on an environment in which the presence of financial constraints alone does not motivate financial regulation. This is important because it allows us to establish the conditions under which the environmental externality provides a rationale for leverage regulation. We thus contribute to the debate on whether financial regulatory frameworks should consider climate-related risks beyond the prudential motive behind current regulatory frameworks (such as moral hazard problems due to government guarantees or pecuniary externalities, see, for example, Dewatripont and Tirole, 1994; Hellmann et al., 2000; Lorenzoni, 2008; Martinez-Miera and Repullo, 2010; Bahaj and Malherbe, 2020). While we are agnostic about whether borrowers in the model are non-financial firms or financial institutions, interpreting borrowers as firms may be more appealing given firms are the final holders of polluting assets and pay emissions taxes. Under this interpretation, the model prescribes that leverage regulation (or taxes and subsidies) should be directly applied to firms. Alternatively, we show in the appendix that the model is equivalent to one in which borrowers are banks that make loans to firms, which use these loans to finance investment, abatement costs, and emissions taxes (this equivalence holds if there is no friction between banks and firms, and banks capture all surplus). Under this interpretation, leverage regulation can be applied directly to banks within the current Basel regulatory framework.

In additional analyses we consider alternative policy tools. In a frictionless world, emissions taxes are equivalent to a cap-and-trade system with tradable pollution permits, such as the EU Emission Trading System (EU ETS) (see Montgomery, 1972). We show that in the presence of financial constraints this equivalence only holds if the pledgeability of tax rebates is equal to the fraction of freely allocated permits. This implies that freely allocating pollution permits can eliminate the direct effect of carbon pricing on financial constraints and implement a constrained-efficient allocation. This is an important policy insight given real-world cap-and-trade systems (including the EU ETS) typically do not allocate 100% of permits for free.

Perhaps trivially, the most effective policy tools create financial slack by transferring resources from unconstrained investors to constrained borrowers. Such transfers can implement the first-best allocation and can either be implemented directly, or indirectly through abatement subsidies financed with taxes paid by unconstrained agents. We also show that hedging can have a positive effect on equilibrium environmental policy. When borrowers can hedge climate-related risks, financial constraints are less binding in the bad state and may even become slack, which may enable the environmental regulator to implement the first-best allocation using Pigouvian taxes. This highlights an important role the financial sector can play in the transition to a low-carbon economy, distinct from socially responsible investing that aims to reduce emissions by taking environmental and social factors into account in investment decisions (e.g., see Pástor et al., 2021; Oehmke and Opp, 2022b; Goldstein et al., 2022; Gupta et al., 2022).

This paper relates to several recent contributions that study environmental externalities and green investment under financial and other economic frictions (Tirole, 2010; Biais and Landier, 2022). Recent contributions by Hoffmann et al. (2017), Oehmke and Opp (2022b), and Heider and Inderst (2022) also find that, in the presence of financial constraints, Pigouvian taxes cannot implement a first-best allocation, and optimal emissions taxes generally differ from the standard Pigouvian solution.[6] Relative to these papers, our contribution is that we analyze jointly optimal carbon pricing and leverage regulation, and that our model features endogenous climate transition and physical risks. This allows us to derive novel insights on how these two climate-related risks differ in their impact on environmental and financial policies.[7] Another related contribution is Oehmke and Opp (2022a), who analyze capital requirements as a tool to incentivize bank lending to green firms when emissions taxes are not available. Dávila and Walther (2022) more generally study optimal regulation when policy instruments are imperfect, with an application to risk-weighted capital requirements that take environmental externalities into account. In contrast, we take optimally-set emissions taxes as a starting point, and ask under what conditions it may be beneficial to complement emissions taxes with leverage regulation in a setting in which there is no motive for financial regulation absent environmental externalities. Another related strand of literature uses DSGE models with financial frictions to simulate the effect and optimal design of macroprudential and monetary policies in the presence of environmental externalities (Carattini et al., 2021; Dafermos et al., 2018; Diluiso et al., 2020; Ferrari and Landi, 2021). We contribute by providing analytical results that allow to pinpoint the friction motivating financial regulation in this context.

[6]The literature also shows that a Pigouvian solution may be sub-optimal in the presence of heterogeneity or interaction between several externality-generating activities (Diamond, 1973; Rothschild and Scheuer, 2014). Moreover, a wedge between the optimal tax rate and the marginal social cost emerges when the planner seeks to regulate an externality in the presence of other distortionary taxes (Sandmo, 1975; Lee and Misiolek, 1986; Bovenberg and Goulder, 1997; Bovenberg and De Mooij, 1997; Barrage, 2020) or when consumers have self-control problems (Haavio and Kotakorpi, 2011). In these cases, as in our setting, the indirect effects of the policy motivate the deviation from the Pigouvian solution.

[7]Hoffmann et al. (2017) also consider credit subsidies that support abatement investment. These policy instruments are different from the ex-ante leverage regulation we consider but are similar to the abatement subsidy explored in Section 4.5. Both credit and abatement subsidies transfer resources to constrained agents while motivating green investment.

Section 2 describes the model setup and derives the first best benchmark. Section 3 solves the competitive equilibrium, and Section 4 analyzes optimal financial and environmental regulation. Section 5 concludes.

## 2 Model Setup

There are three dates, $t = 0, 1, 2$, a unit mass of investors, and a unit mass of borrowers. At $t = 1$ all agents learn whether the economy is in a good state ($s = G$) with a low social cost of emissions, or in a bad state ($s = B$) with a high social cost of emissions. The state of the world is drawn from a binomial distribution with the probability of the bad state given by $q$.

**Preferences and Endowments.** Investors are risk-neutral and deep-pocketed in that they have a large endowment $A_t^i$ at $t = 0$ and $t = 1$. Borrowers have a limited endowment $A_0^b$ only at $t = 0$ and quasi-linear utility over consumption. There is no discounting and all agents suffer disutility from aggregate carbon emissions $E_s^a$ at $t = 2$:

$$U^i = c_0^i + c_{1s}^i + c_{2s}^i - \gamma_s^u E_s^a,$$

$$U^b = u(c_0) + c_{1s} + c_{2s} - \gamma_s^u E_s^a,$$

where $\gamma_s^u$ is a parameter governing the cost of emissions in agent's utility, which depends on the state of the world $s \in \{G, B\}$. In the bad state $\gamma_s^u$ takes a high value $\gamma_B^u > \gamma_G^u$. In the good state, we normalize $\gamma_G^u = 0$.

The quasi-linear utility function introduces a meaningful trade-off for borrowers in how much own funds they contribute. To ensure an interior solution we assume that $u(c_0)$ satisfies the Inada conditions, i.e., that $u(c_0)$ is strictly increasing and strictly concave, and that in the limit $u'(0) = \infty$ and $u'(\infty) = 0$. Agents are atomistic, so that they do not internalize the effect of their decisions on aggregate carbon emissions $E_s^a$.

**Technology.** At $t = 0$ borrowers can invest in a productive technology with a fixed scale at an investment cost $I_0$. At $t = 1$ borrowers can liquidate some of the initial investment and adjust the investment scale to $I_{1s} \leq I_0$. The project generates a return of $R(I_{1s}, E_s^a, \gamma_s^p) = \rho I_{1s} - \gamma_s^p E_s^a$ at $t = 2$, and liquidations generate a payoff $\mu(I_0 - I_{1s})$ at $t = 1$, with $\mu < 1$.

The parameter $\gamma_s^p$ captures the project's exposure to physical climate risk from environmental damages. Just as the utility cost of emissions, the exposure to physical risk depends on the realized state of the world $s$, with $\gamma_B^p \geq 0$ and $\gamma_G^p = 0$. Thus, the total social cost of emissions consists of a direct utility cost as well as losses in asset values from environmental damages, $\gamma_s = 2\gamma_s^u + \gamma_s^p$. The social cost of emissions is uncertain from an ex-ante perspective, capturing the uncertainty evident in the wide range of estimates of the social cost of carbon (for example, see Nordhaus, 2018).[8]

The project emits carbon emissions $E(X_s, I_{1s})$ at $t = 2$, which aggregate to $E_s^a$ and may be subject to emissions taxes $\tau_s$.[9] Emissions can be reduced by abatement investments, denoted by $X_s$, at a cost $C(X_s, I_{1s})$ paid at $t = 1$. We offer two possible interpretations of this setup. Borrowers may represent non-financial firms that directly invest in a polluting asset, such as manufacturing firms investing in polluting plants. Alternatively, we show in Appendix B.2 that, under certain conditions, the setup is equivalent to one in which borrowers are financial institutions that lend to firms with polluting assets. In the latter case, borrowers pay for emissions taxes and abatement costs indirectly through the profitability of their loan portfolios.

We make the following functional form assumptions.

**Assumption 1.** *$E(X, I_1)$ and $C(X, I_1)$ satisfy*

1. $\frac{\partial E(X,I_1)}{\partial X} \leq 0, \quad \frac{\partial E(X,I_1)}{\partial I_1} \geq 0, \quad \frac{\partial C(X,I_1)}{\partial X} \geq 0, \quad \frac{\partial C(X,I_1)}{\partial I_1} \geq 0,$

2. $E(X \to \infty, I_0) = E(X, 0) = 0, \quad E(0, I_0) = \bar{E}, \ C(0, I_1) = C(X, 0) = 0,$

[8]While uncertainty is not a necessary model ingredient for our baseline results, it allows us to study the role that financial markets can play in facilitating the use of more efficient environmental policy by enabling hedging of climate risks (see Section 4.6). The framing also permits us to study how future environmental policy may affect long-run investments and result in stranded assets.

[9]Formally, using $b$ to index individual borrowers, $E_s^a = \int_0^1 E(X_s^b, I_{1s}^b)db$. To simplify notation, throughout the paper we do not use superscripts to index borrowers.

*3.* $\frac{\partial^2 E(X,I_1)}{\partial X^2} = 0$, $\frac{\partial^2 C(X,I_1)}{\partial X^2} > 0$.

Assumption 1.1 ensures that abatement investments are costly but reduce emissions, and that a higher final investment scale is associated with higher emissions and abatement costs. Assumption 1.2 defines boundaries such that costs and emissions are non-negative, and there is an upper bound $\bar{E}$ on emissions. Assumption 1.3 implies that emissions are linear in abatement, which simplifies the exposition, but that the cost of abatement is strictly convex, so that the borrower's optimal abatement choice has an interior solution.

**Environmental Regulation.** After production takes place, an environmental regulator can observe emissions and impose a state-contingent emissions tax $\tau_s$ per unit of emissions.[10] Emissions taxes are rebated lump-sum to borrowers, $T_s = \tau_s E^a_s$. Section 4 derives socially optimal emissions taxes and discusses efficiency. Subsections 4.4 and 4.5 also consider other environmental policies in the form of a pollution permit market and an abatement subsidy. Given the role of financial constraints in the model, in Subsection 4.3 we study whether there is scope for financial regulation to complement environmental policy.

**Financing.** Borrowers need to finance the upfront investment $I_0$ at $t = 0$ and abatement $X_s$ at $t = 1$. At $t = 0$ they can contribute their own funds as inside equity financing $e \leq A^b_0$. Additionally, borrowers can raise debt financing $d_0$ and $d_{1s}$ from investors at $t = 0, 1$. In Section 4.6 we also allow hedging and derive interesting additional insights on how it can affect equilibrium environmental policy.

Borrowing is limited by a moral hazard problem. We assume that borrowers can abscond with any resources except a fraction $\theta \in [0, 1]$ of asset returns, and a fraction $\psi \in [0, 1]$ of tax rebates at $t = 2$. Thus, there is a wedge between the project's return and pledgeable income, with pledgeable project returns given by $\tilde{R}(I_{1s}, E^a_s, \gamma^p_s) = \theta R(I_{1s}, E^a_s, \gamma^p_s)$ (as in Rampini and Viswanathan, 2013, among others). We introduce a separate pledgeability parameter for tax rebates to be able to perform key comparative

[10] We only consider a linear tax because there is no heterogeneity among borrowers, and therefore a non-linear tax cannot improve upon a linear tax. See Hoffmann et al. (2017) for a model with heterogeneity, in which a non-linear tax can be a superior policy instrument because it transfers less resources from more to less constrained firms.

statics exercises. For example, when $\psi = 1$ tax rebates are fully pledgeable and emissions taxes have no *direct* effect on financial constraints, while the opposite holds when $\psi < 1$.

At the interim date the liquidation proceeds $\mu(I_0 - I_{1s})$ can be seized by investors who provided $t = 0$ financing (that is, liquidation proceeds are pledgeable). Investors can demand liquidation if they choose not to roll over their debt and are not fully repaid at $t = 1$.

**Variable Definitions.** For the further analysis it will be useful to introduce the following variable definitions and assumptions:

**Definition 1.** *The project's private net marginal return* $r(\tau, X, I_1)$ *and pledgeable net marginal return* $\tilde{r}(\tau, X, I_1)$ *are respectively defined as*

$$r(\tau, X, I_1) = \rho - \mu - \frac{\partial C(X, I_1)}{\partial I_1} - \tau \frac{\partial E(X, I_1)}{\partial I_1},$$
$$\tilde{r}(\tau, X, I_1) = \theta\rho - \mu - \frac{\partial C(X, I_1)}{\partial I_1} - \tau \frac{\partial E(X, I_1)}{\partial I_1}.$$

**Assumption 2.** *Project returns* $\rho$ *are sufficiently large and pledgeability* $\theta$ *sufficiently small such that, given a threshold* $\bar{\tau} \geq \gamma_B$*,*

*1.* $r(\tau, X, I_1) > 0, \ \forall X, I_1, \tau \leq \bar{\tau}$

*2.* $\tilde{r}(0, X, I_1) < 0, \ \forall X, I_1$.

The first condition ensures that continuing the investment project has positive NPV at $t = 1$ as long as emissions taxes do not exceed some threshold $\bar{\tau}$. Throughout the paper we focus on the interesting case $\tau_B \leq \bar{\tau}$, such that it is efficient to continue rather than liquidate the project even in the bad state with a high social cost of carbon. The second condition ensures that liquidation proceeds $\mu$ exceed the loss in pledgeable income due to a reduced investment scale. This implies that, while inefficient, liquidations relax financial constraints.

## 2.1 First-Best Benchmark

**Proposition 1.** *In the first-best allocation $I_{1s} = I_0$, and optimal $t = 0$ consumption by borrowers, $c_0$, and optimal abatement, $X_s$, are defined by the following conditions:*

$$u'(c_0) = 1,$$
$$\gamma_s \frac{\partial E(X, I_{1s})}{\partial X_s} = -\frac{\partial C(X_s, I_{1s})}{\partial X_s}.$$

*Proof.* See Appendix A.1 □

In the first-best allocation, the optimal abatement equates the marginal gain from lower emissions to the marginal cost of abatement. The borrower's consumption is at a level that ensures the marginal utility is equalized across agents and time. Crucially, there are no liquidations because liquidations are inefficient by Assumption 2. The next section shows that this may be different in the competitive equilibrium, where financially constrained borrowers may need to liquidate some of their initial investment.

# 3 Competitive Equilibrium

This section solves the problem of borrowers and defines a competitive equilibrium given a state-contingent emissions tax $\tau_s$ but without financial regulation. We analyze optimal emissions taxes and compare the allocation to an equilibrium with financial regulation and other policy tools in the next section.

## 3.1 Borrower Problem

The borrower's expected utility is given by

$$\mathbb{E}[U^b] = u(c_0) + \sum_{k \in \{G,B\}} Pr[s = k] \left( c_{1k} + c_{2k} - \gamma_k^u E_k^a \right).$$

Borrowers maximize their expected utility subject to the following constraints:

$$c_0 = A_0^b - e \geq 0, \tag{1}$$

$$c_{1s} = (I_0 - I_{1s})\mu + d_{1s} - (I_0 - e) - C(X_s, I_{1s}) \geq 0, \tag{2}$$

$$c_{2s} = R(I_{1s}, E_s^a, \gamma_s^p) - \tau_s E(X_s, I_{1s}) - d_{1s} + T_s \geq 0, \tag{3}$$

$$d_{1s} \leq \tilde{R}(I_{1s}, E_s^a, \gamma_s^p) - \tau_s E(X_s, I_{1s}) + \psi T_s, \tag{4}$$

$$I_{1s} \in [0, I_0]. \tag{5}$$

Equations (1), (2) and (3) are non-negativity constraints on consumption at $t = 0, 1,$ and 2, respectively. Eq. (4) is a financial constraint that ensures $t = 1$ borrowing does not exceed pledgeable income, which implies borrowers have no incentive to abscond at $t = 2$.[11]

Using the budget constraints to eliminate $c_0$, $c_{1s}$, $c_{2s}$, $d_0$, and $d_{1s}$, the borrower's problem can be formulated as a Lagrange function of $e, X_s, I_{1s}$ as well as Lagrange multipliers $\lambda_s$ for the $t = 1$ financial constraint in state $s$, and $\kappa$'s serving as multipliers for lower and upper bounds on variables. The Lagrangian is formally stated in Eq. (18) in Appendix A.2.1.

## 3.2 Borrower Decisions at $t = 1$

At $t = 1$ borrowers observe the realization of the aggregate state $s$, and then choose $X_s$ and $I_{1s}$. In principle, borrowers could also default on $t = 0$ debt, yet the following lemma shows that this is never optimal:

**Lemma 1.** *Borrowers prefer to roll-over $t = 0$ debt by raising $d_1 \geq d_0$, rather than defaulting on $t = 0$ debt.*

*Proof.* In Appendix A.2.2 □

The intuition is that investors can recoup $t = 0$ debt by forcing liquidation of the project, so that borrowers are better off rolling over the debt to avoid forced liquidations.

[11] Eq. (4) is equivalent to an incentive-compatibility condition $c_{2s} \geq (1 - \theta)R(I_{1s}, E_s^a, \gamma_s^p) + (1 - \psi)T_s$.

$X_s$ and $I_{1s}$ are chosen according to the following first order conditions:

$$(1+\lambda_s)\left(\tau_s \frac{\partial E(X_s, I_{1s})}{\partial X_s} + \frac{\partial C(X_s, I_{1s})}{\partial X_s}\right) = 0, \tag{6}$$

$$r(\tau_s, X_s, I_{1s}) + \lambda_s \tilde{r}(\tau_s, X_s, I_{1s}) - \overline{\kappa}_{Is} + \underline{\kappa}_{Is} = 0. \tag{7}$$

In Eq. (6) borrowers choose abatement trading off the tax bill associated with carbon emissions against the cost of abatement. Eq. (7) is the first order condition with respect to $I_{1s}$, which uses Definition 1 of private net marginal return and pledgeable net marginal return, $r(\cdot)$ and $\tilde{r}(\cdot)$. Together with the following condition,

$$\lambda_s[\tilde{R}(I_{1s}, E_s^a, \gamma_s^p) - \tau_s E(X_s, I_{1s}) + \psi T_s + e - I_0 + \mu(I_0 - I_{1s}) - C(X_s, I_{1s})] = 0, \tag{8}$$

which combines the complementary slackness conditions of the financial constraint (4) and non-negativity constraint of $c_{1s}$ (2), these conditions define the optimal state-contingent $t = 1$ allocations $I_{1s}, X_s$, and $\lambda_s$ for a given $\tau_s$ and $e$ (the optimality condition for equity is derived below).

**Lemma 2.** *Borrowers do not liquidate any investment if the financial constraint (4) is slack. That is, if $\lambda_s = 0$, then $I_{1s} = I_0$. In contrast, if $\lambda_s > 0$, then borrowers liquidate some investment so that $I_{1s} < I_0$.*

*Proof.* In Appendix A.2.3 □

Lemma 2 follows from Assumption 2, which implies that the net marginal return is positive and therefore it is optimal to continue the project without any liquidations, i.e., the optimum is a corner solution with $I_{1s} = I_0$ and $\overline{\kappa}_{Is} = r(\tau_s, X_s, I_{1s}) > 0$. By contrast, if the financial constraint is binding, $\lambda_s > 0$, the pledgeable income under the full investment scale is insufficient to support the required borrowing. Since liquidations relax financial constraints (by Assumption 2.2), in this case borrowers reduce the investment scale at $t = 1$ by choosing $I_{1s} < I_0$.

### 3.3 Borrower Decisions at $t = 0$

At $t = 0$ borrowers decide on their capital structure by choosing the optimal equity $e$ (debt financing follows as the residual $d_0 = I_0 - e$). The first order condition of the borrower's problem w.r.t. $e$ is given by

$$u'(A_0^b - e) = 1 + \kappa_e + (1-q)\lambda_G + q\lambda_B. \tag{9}$$

Condition (9) shows that borrowers contribute equity trading off the marginal utility cost of lower $t = 0$ consumption on the left-hand side against the marginal utility of $t = 1$ consumption plus the expected shadow cost of the financial constraint on the right-hand side. The first order conditions and complementary slackness condition together define the competitive equilibrium:

**Definition 2.** *Given a state-contingent emissions tax $\tau_s$, the competitive equilibrium is the set of allocations $I_{1s}^*(\tau_s), X_s^*(\tau_s), \lambda_s^*(\tau_s), e^*(\tau_G, \tau_B)$, defined by Equations (6), (7), (8), and (9). Aggregate emissions are given by $E_s^a(\tau_s) = E(X_s^*, I_{1s}^*)$. The allocations $c_0^*(\tau_G, \tau_B)$, $c_{1s}^*(\tau_s)$, $c_{2s}^*(\tau_s)$, and $d_0^*(\tau_G, \tau_B)$ follow as residuals from Eqs. (1), (2), (3), and $d_0 = I_0 - e$.*

For brevity we sometimes omit the dependence of equilibrium allocations on $\tau_s$. For instance, we refer to $X_s^*(\tau_s)$ as $X_s^*$, or to $e^*(\tau_G, \tau_B)$ as $e^*$.

### 3.4 Pigouvian Benchmark

**Proposition 2.** *If $\lambda_s^*(\gamma_s) = 0$, $\forall s \in \{G, B\}$, then the competitive equilibrium with $\tau_s = \gamma_s$ is equivalent to the first-best allocation.*

*Proof.* With $\lambda_s^*(\gamma_s) = 0$, $\forall s \in \{G, B\}$, it follows from Lemma 2 that $I_{1s}^* = I_0$. This investment level, as well as the FOCs of borrowers w.r.t. $X_s$ and $e$ in Eqs. (6) and (9), are then equivalent to those in the first best given in Proposition 1. □

Proposition 2 establishes an important benchmark result. If the financial constraint is slack in all states, then by Lemma 2 borrowers can avoid inefficient liquidations, and the optimal Pigouvian emissions tax can implement the first-best allocation. Accordingly, throughout we refer to a tax $\tau_s = \gamma_s \; \forall s \in \{B, G\}$ as the *Pigouvian benchmark*. In the next section we depart from this benchmark and analyze optimal emissions taxes when the financial constraint binds.

# 4 Carbon Pricing and Financial Regulation

To analyze optimal emissions taxes in the presence of financial constraints, we consider the problem of an environmental regulator who sets a state-contingent emissions tax $\tau_s^*$ after observing the social cost of emissions at $t = 1$. We then show under what conditions the resulting equilibrium allocation is constrained efficient, and ask whether there is a case to combine emissions taxes with leverage regulation.

## 4.1 Socially Optimal Emissions Tax

To derive the optimal $\tau_s$, we solve the problem of a regulator choosing the optimal tax at $t = 1$ so as to maximize social welfare. This problem can be written as the following Lagrangian with $\kappa_{\tau s}$ the multiplier on the non-negativity constraint on $\tau_s$:

$$\max_{\tau_G, \tau_B} W = A_0^i + A_1^i + u(A_0^b - e^*) + e^* - I_0 + \kappa_e(e^* - I_0 + \mu I_0)$$
$$+ \sum_{k \in \{B,G\}} Pr[s = k] \left\{ R(I_{1k}^*, E_k^a, \gamma_k^p) + \mu(I_0 - I_{1k}^*) - 2\gamma_k^u E(X_k^*, I_{1k}^*) - C(X_k^*, I_{1k}^*) + \kappa_{\tau k}\tau_k \right\}. \tag{10}$$

The regulator's first order condition with respect to $\tau_s$ can be written as:

$$r(\gamma_s, X_s^*, I_{1s}^*) \frac{\partial I_{1s}^*}{\partial \tau_s} - (\gamma_s - \tau_s) \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} \frac{\partial X_s^*}{\partial \tau_s} + \kappa_{\tau s} = 0. \tag{11}$$

In this condition, the final investment scale $I_{1s}^*$ and abatement $X_s^*$ are optimal choices by private agents that respond to changes in emissions taxes. In setting the emissions tax, the regulator takes into account the effect of the tax on these equilibrium allocations.

### 4.1.1 The Effect of Taxes on Equilibrium Allocations

Higher emissions taxes increase the cost of polluting, which incentivizes borrowers to invest more in abatement. But higher emissions taxes also affect the tightness of financial constraints, which may induce borrowers to abate less. Through this indirect effect, emissions taxes can have a perverse effect and decrease abatement due to tightening financial constraints. To focus on the interesting case in which emissions taxes are a useful tool to incentivize abatement to begin with, we introduce parameter assumptions that ensure the direct effect of emissions taxes on abatement dominates.

**Assumption 3.** *Model parameters are such that* $\frac{\partial X_s^*}{\partial \tau_s} > 0 \ \forall \tau_s$, *as characterized in Appendix A.3.1.*

The following Lemma additionally clarifies how liquidations and therefore the equilibrium investment scale $I_{1s}^*$ responds to emissions taxes.

**Lemma 3.** *If the financial constraint is slack,* $\lambda_s^*(\tau_s) = 0$, *then* $\frac{\partial I_{1s}^*}{\partial \tau_s} = 0$ *and* $\frac{\partial X_s^*}{\partial \tau_s} > 0$. *Under Assumption 3, if the financial constraint binds,* $\lambda_s^*(\tau_s) > 0$, *then* $\frac{\partial X_s^*}{\partial \tau_s} > 0$ *and there exists a threshold characterized by* $\hat{\gamma}_s^p(\tau_s) = \frac{\psi}{\theta}\tau_s + \frac{(1-\psi)}{\theta} E(X_s^*, I_{1s}^*) \frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial (X_s^*)^2} / \left(\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\right)^2$, *such that*

- $\frac{\partial I_{1s}^*}{\partial \tau_s} < 0$ *if* $\gamma_s^p < \hat{\gamma}^p(\tau_s)$,
- $\frac{\partial I_{1s}^*}{\partial \tau_s} = 0$ *if* $\gamma_s^p = \hat{\gamma}^p(\tau_s)$,
- $\frac{\partial I_{1s}^*}{\partial \tau_s} > 0$ *if* $\gamma_s^p > \hat{\gamma}^p(\tau_s)$.

*Proof.* See Appendix A.3.1 □

Only if the financial constraint binds, $\lambda_s^*(\tau_s) > 0$, borrowers need to liquidate investments to be able to roll-over their debt. Interestingly, higher emissions taxes can result in more or less liquidations, depending on how strongly asset values are affected by physical climate risk, as captured by $\gamma_s^p$. The overall effect of emissions taxes on the final investment scale follows from totally differentiating (8) with respect to $\tau_s$:

$$\frac{\partial I_{1s}^*}{\partial \tau_s} = \frac{\overbrace{(1-\psi)E(X_s^*, I_{1s}^*)}^{\text{Direct effect}} + \overbrace{(\theta\gamma_s^p - \psi\tau_s)\frac{\partial E_s^a}{\partial X_s^*}\frac{\partial X_s^*}{\partial \tau}}^{\text{Collateral externality}}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)} \tag{12}$$

This equation highlights that emissions taxes affect the final investment scale via two channels that operate through financial constraints. First, changes in the tax directly affect the size of the tax bill and the tax rebate. Since only a fraction $\psi$ of the tax rebate is pledgeable this *direct effect* of the emissions tax on the tightness of the financial constraint is proportional to $(1-\psi)E(X_s^*, I_{1s}^*)$.

Second, changes in abatement also affect the aggregate level of emissions, which impact borrowers' pledgeable income via two *collateral externalities.* Physical climate risk represents a negative collateral externality because higher aggregate emissions result in larger physical damages to borrowers' assets, decreasing pledgeable income by $\theta\gamma_s^p$. Thus in the presence of physical climate risk higher emissions taxes partly relax financial constraints. At the same time, there is a positive collateral externality because tax rebates are a function of aggregate emissions. This implies that lower aggregate emissions reduce the tax rebate, decreasing pledgeable income by $\psi\tau_s$.

Overall, the effect of emissions taxes on financial constraints and liquidations depends on the relative strength of the direct effect of taxes on pledgeable income, and the indirect effects due to collateral externalities.[12] When borrowers' exposure to physical climate risk is low such that $\gamma_s^p < \hat{\gamma}^p$, the direct effect and tax rebate externality dominate, so that higher emissions taxes imply tighter constraints and more liquidations. If borrowers' exposure to physical climate risk is high such that $\gamma_s^p > \hat{\gamma}^p$, the equilibrium effect of emissions taxes that lowers the physical risk dominates, so that higher emissions taxes

[12] Note that, because higher taxes induce an endogenous change in abatement by borrowers, they also affect abatement costs. On one hand, higher abatement increases abatement costs, tightening financial constraints. On the other hand, higher abatement reduces emissions and thereby the tax bill, easing financial constraints. Therefore, an additional term that shows up in the numerator of Eq. (12) is $-\left(\frac{\partial C(X_s^*, I_{1s}^*)}{\partial X_s^*} + \tau\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\right)\frac{\partial X_s^*}{\partial \tau}$. However, by the borrower's optimal abatement choice in Eq. (6), this term is equal to zero, so that this channel has no marginal effect on financial constraints and drops out from Eq. (12).

relax financial constraints and result in fewer liquidations.

#### 4.1.2 Optimal Emissions Tax

Because emissions taxes interact with financial constraints, the regulator considers not only the direct effect of taxes on emissions, but also their side effect on asset liquidations.

**Proposition 3.** *The optimal emissions tax $\tau_s^*$ solves (11). If $\lambda_s^*(\gamma_s) = 0$ or $\gamma_s = 0$, then $\tau_s^* = \gamma_s$. If $\lambda_s^*(\gamma_s) > 0$ and $\gamma_s > 0$, then the optimal emissions tax depends on the strength of physical risk $\gamma_s^p$, and on the pledgeability of tax rebates $\psi$ and cash flows $\theta$. If $\psi \geq \theta$, the optimal emissions tax is always below the direct social cost of emissions, $\tau_s^* < \gamma_s$. If $\psi < \theta$, then*

- $\tau_s^* < \gamma_s$ *if* $\gamma_s^p < \hat{\gamma}^p(\tau_s^*)$,
- $\tau_s^* = \gamma_s$ *if* $\gamma_s^p = \hat{\gamma}^p(\tau_s^*)$,
- $\tau_s^* > \gamma_s$ *if* $\gamma_s^p > \hat{\gamma}^p(\tau_s^*)$,

*Proof.* See Appendix A.3.2 □

With binding financial constraints, $\lambda_s^*(\gamma_s) > 0$, the optimal emissions tax generally differs from the Pigouvian benchmark equal to the direct social cost of emissions $\gamma_s$, because the regulator needs to account for the effect of the policy on liquidations. To disentangle the results in Proposition 3, we discuss three polar cases: (i) tax rebates are not pledgeable and physical climate risk has no effect on collateral values ($\psi = \gamma_s^p = 0$); (ii) tax rebates are not pledgeable but physical climate risk has an effect on collateral values ($\psi = 0, \gamma_s^p > 0$); and (iii) tax rebates are pledgeable and physical climate risk has an effect on collateral values ($\psi > 0, \gamma_s^p > 0$).

**(i) No physical risk ($\psi = \gamma_s^p = 0$).** With non-pledgeable tax rebates and absent physical climate risk effects, there is no collateral externality and emissions taxes affect financial constraints only through their *direct effect* on pledgeable income. In this case, higher taxes trigger inefficient liquidations (see Lemma 3). Internalizing this undesired side effect, an environmental regulator sets an emissions tax below the direct social cost of emissions, $\tau_s^* < \gamma_s$. Intuitively, regulators set a lower carbon tax because they understand that higher taxes constitute a realization of climate transition risk for financially constrained borrowers. Put differently, optimal emissions taxes are below the Pigouvian benchmark because borrowers are "too levered for Pigou".

**(ii) Physical risk ($\psi = 0, \gamma_s^p > 0$).** Physical climate risk implies that emissions taxes affect borrower's financial constraints not only through their *direct effect*, but also through a *collateral externality.* The relative importance of this effect depends on how strongly collateral values are exposed to physical climate risk, as measured by $\gamma_s^p$. If $\gamma_s^p < \hat{\gamma}_s^p$, the direct effect dominates and the trade-off resembles the one in case (i) above. This case applies when climate transition risks dominate physical climate risk effects, for example in economies with large polluting industries. By contrast, if the effect of physical climate risk on collateral values is sufficiently high such that $\gamma_s^p > \hat{\gamma}_s^p$, then higher emissions taxes ease financial constraints (see Lemma 3). As a result, the trade-offs faced by an environmental regulator change fundamentally, implying optimal emissions taxes above the direct social cost of emissions, $\tau_s^* > \gamma_s$. Such a case may apply to economies that are heavily exposed to the risk of weather disasters such as droughts or floodings that have a negative effect on asset values.

**(iii) Plegeability ($\psi > 0, \gamma_s^p > 0$).** With (partially) pledgeable tax rebates, the overall collateral externality effect of emissions taxes depends not only on the impact due to physical climate risk, but also due to changes in the size of tax rebates. The latter represents a positive collateral externality of emissions, thereby counteracting the negative collateral externality due to physical risk. Which of the two collateral externalities dominates depends on whether tax rebates or asset returns have a greater pledgeability. If $\psi \geq \theta$, tax rebates are more pledgeable than the firm's asset returns, and the positive collateral externality due to tax rebates dominates. In this case, optimal emissions taxes are unambiguously below the direct social cost of emissions, $\tau_s^* < \gamma_s$, irrespectively of the level of $\gamma_s^p$. By contrast, if $\psi < \theta$ the optimal emissions tax may be above the direct social cost of emissions if $\gamma_s^p$ is sufficiently large, as discussed under case (ii) above.

An interesting implication is that, in economies where firms' assets have a low pledgeability (such as knowledge-based economies with much intangible capital), optimal emissions taxes are lower because the effect of physical risk on collateral values is less relevant (small $\theta$). Similarly, emissions taxes may be optimally lower in economies where tax rebates are more pledgeable (large $\psi$; for example, due to stronger political institutions).

**Generalized Pigouvian Tax.** The results in Proposition 3 highlight that, in the presence of financial constraints, the total social cost of emissions includes not only the direct social cost of emissions $\gamma_s$, but also the indirect costs due to collateral externalities driven by physical climate risk, $\lambda_s \theta \gamma_s^p$, and the pledgeability of tax rebates, $\lambda_s \psi \tau_s$.[13] Therefore, another useful benchmark to compare the optimal emissions tax to is a *generalized* Pigouvian tax, defined as the emissions tax that equalizes the private cost of emissions $\tau_s$ to the *total* social cost of emissions $\gamma_s + \lambda_s \theta \gamma_s^p + \lambda_s \psi \tau_s$.

**Proposition 4.** *Let the generalized Pigouvian tax be defined as*

$$\tau_s^{GP} = \frac{\gamma_s + \lambda_s^* \theta \gamma_s^p}{1 + \psi \lambda_s^*}.$$

*With $\lambda_s^* > 0$ and $\gamma_s > 0$, the optimal emissions tax is $\tau_s^* = \tau_s^{GP}$ if $\psi = 1$, and $\tau_s^* < \tau_s^{GP}$ if $\psi < 1$. With $\lambda_s^* = 0$ or $\gamma_s = 0$, the optimal emissions tax is $\tau_s^* = \tau_s^{GP} = \gamma_s$.*

*Proof.* In Appendix A.3.3 □

While the optimal emissions tax may be above a standard Pigouvian benchmark equal to the direct social cost of emissions $\gamma_s$ (see Proposition 3), Proposition 4 shows that, if tax rebates are not fully pledgeable, the optimal emissions tax is always below a generalized Pigouvian benchmark that accounts for collateral externalities. This highlights that, even with $\tau_s^* > \gamma_s$, the adverse direct effect of emissions taxes on financial constraints can limit

[13] Collateral externalities can also emerge in models with pecuniary externalities, where borrowers do not internalize how their choices affect the financial constraint of other agents through their impact on prices (for a detailed discussion, see Dávila and Korinek, 2018). As in these settings, here borrowers choose a socially sub-optimal leverage because they do not internalize their impact on financial constraints. Unlike in the pecuniary externality literature, in our setting the collateral externality arises due to the effect of aggregate emissions on borrowers' pledgeable income.

the regulator in setting a tax that accounts for all direct and indirect social costs of emissions. The next subsection shows this has implications for the efficiency of the allocation.

## 4.2 Efficiency

To evaluate efficiency, we compare the allocation that can be implemented with the optimal emissions tax $\tau_s^*$ to the constrained-efficient allocation in which a social planner can choose $X_s$, $I_{1s}$ and $e$ directly, subject to the same resource and financial constraints as private agents. This constrained-efficient allocation is formally defined and characterized in Appendix A.4.1.

**Proposition 5.** *If $\psi = 1$, then the competitive equilibrium with a socially optimal emissions tax equal to the generalized Pigouvian tax $\tau_s^{GP} = \frac{\gamma_s + \lambda_s^* \theta \gamma_s^p}{1+\lambda_s^*}$ is constrained efficient. If $\psi < 1$ and the financial constraint binds in some state, $\lambda_s^* > 0$, then the competitive equilibrium with a socially optimal emissions tax $\tau_s^*$ is not constrained efficient.*

*Proof.* In Appendix A.4.1 □

We show in Appendix A.4.1 that the constrained-efficient level of abatement solves

$$-(\gamma_s + \lambda_s \theta \gamma_s^p) \frac{\partial E(X_s, I_{1s})}{\partial X_s} = (1 + \lambda_s) \frac{\partial C(X_s, I_{1s})}{\partial X_s}. \tag{13}$$

When choosing the optimal level of abatement, a constrained social planner trades off the benefits associated with lower aggregate emissions on the left-hand side against the cost of abatement on the right-hand side of Eq. (13). The total marginal benefit of lowering emissions consists of the avoided direct social cost $\gamma_s$, plus the indirect social cost due to the collateral externality associated with physical climate risk $\lambda_s \theta \gamma_s^p$. On the right-hand side, the marginal abatement cost is scaled by the marginal utility of consumption plus the shadow cost of the financial constraint, $(1 + \lambda_s)$, because spending on abatement tightens borrowers' financial constraints.

In contrast to a social planner, the environmental regulator cannot choose abatement directly, but instead uses emissions taxes as a policy instrument to incentivize abatement.

If tax rebates are fully pledgeable, the regulator can implement the abatement level defined by Eq. (13) without introducing additional distortions to the final investment scale by setting the emissions tax equal to the generalized Pigouvian tax $\tau_s^{GP}$. However, if tax rebates are not fully pledgeable, $\psi < 1$, taxes have a direct adverse effect on financial constraints because $\tau_s E(X_s, I_{1s}) - \psi T_s = (1-\psi)\tau_s E(X_s, I_{1s}) > 0$, and the regulator needs to set an emissions tax below $\tau_s^{GP}$ (see Proposition 4). As a result, emissions taxes can only implement the constrained-efficient allocation if tax rebates are fully pledgeable.

This result implies that when $\psi < 1$ there may be scope to improve welfare by complementing emissions taxes with other policies. Since borrowers' initial leverage directly affects the tightness of the collateral constraint, ex-ante leverage regulation is a natural candidate policy we consider in the next subsection. Another way to improve social welfare could be to use an alternative environmental policy instrument with no direct adverse effect on financial constraints. Section 4.4 explores a cap-and-trade system with tradable pollution permits and Section 4.5 abatement subsidies instead of emissions taxes.

## 4.3 Leverage Regulation

This section introduces leverage regulation complementing emissions taxes when tax rebates are not fully pledgeable ($\psi < 1$). We analyze a leverage mandate that fixes the borrower's equity at a level $\bar{e}$, which can be implemented through a direct mandate, or through taxes and subsidies (see Appendix B.1). To streamline the discussion, we focus on the case in which the model parameters are such that in the competitive equilibrium the financial constraint binds when $s = B$ and is slack when $s = G$.

### 4.3.1 The Effect of a Leverage Mandate on Equilibrium Allocations

To understand the trade-offs faced by the regulator when choosing the leverage mandate, we first study the effect of leverage on the equilibrium final investment scale $I_{1s}^*$ and abatement $X_s^*$.

**Lemma 4.** *If $\lambda_s^* = 0$, then borrower equity does not affect the final investment scale and abatement, $\frac{\partial X_s^*}{\partial \bar{e}} = \frac{\partial I_{1s}^*}{\partial \bar{e}} = 0$. If $\lambda_s^* > 0$, then higher borrower equity increases the final* *investment scale,* $\frac{\partial I_{1s}^*}{\partial \bar{e}} > 0$*. The equilibrium abatement*

- *increases in borrower equity,* $\frac{\partial X_s^*}{\partial \bar{e}} > 0$*, if* $\tau_s^* \frac{\partial^2 E(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} + \frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} < 0$
- *decreases in borrower equity,* $\frac{\partial X_s^*}{\partial \bar{e}} < 0$*, if* $\tau_s^* \frac{\partial^2 E(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} + \frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} > 0$

*Proof.* In Appendix A.4.2 □

Equity affects the optimal choices of borrowers at $t = 1$ only if they face a binding financial constraint. Generally, a greater equity buffer relaxes financial constraints. This enables borrowers to liquidate less, so that $\frac{\partial I_{1s}^*}{\partial \bar{e}} > 0$ if $\lambda_s^* > 0$. This change in the investment scale has an indirect effect on the optimal abatement, as both the marginal cost and the marginal benefit of abatement (in terms of avoided tax expenditures) depend on $I_{1s}^*$. If abatement is more efficient at a higher investment scale, i.e. when the technologies are such that $\tau_s^* \frac{\partial^2 E(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} + \frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} < 0$, then more equity results in a higher equilibrium level of abatement, $\frac{\partial X_s^*}{\partial \bar{e}} > 0$. The opposite holds if abatement is less efficient at a higher investment scale. Combining these effects, the total effect of equity on emissions can be represented as

$$\frac{dE(X_s^*, I_{1s}^*)}{d\bar{e}} = \underbrace{\left( \frac{\partial E(X_s^*, I_{1s}^*)}{\partial I_{1s}^*} - \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} \frac{\tau_s \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} + \frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*}}{\frac{\partial^2 C(X_s^*, I_{1s}^*)}{(\partial X_s^*)^2}} \right)}_{= \frac{dE(X_s^*, I_{1s}^*)}{dI_{1s}^*}} \frac{\partial I_{1s}^*}{\partial \bar{e}}.$$

As equity increases the final investment scale whenever financial constraints bind, the effect of equity on emissions depends on $\frac{dE(X_s^*, I_{1s}^*)}{dI_{1s}^*}$. The first term in $\frac{dE(X_s^*, I_{1s}^*)}{dI_{1s}^*}$ captures the direct effect of a greater investment scale on emissions. The second term captures the endogenous response of abatement, through which emissions may decline in equity.

### 4.3.2 Optimal Leverage Regulation

We now consider the problem of a regulator who sets an equity mandate $\bar{e}$ at $t = 0$ and state-contingent emissions taxes $\tau_s$ at $t = 1$, so as to maximize welfare. That is, we re-consider the optimization problem (10) but allow the regulator to also set $e = \bar{e}$ at

$t = 0$. The regulator's first order condition w.r.t. $\bar{e}$ is given by

$$
\begin{aligned}
&u'(A_0^b - \bar{e}) - 1 - \kappa_e = \\
&\sum_{k \in \{B,G\}} Pr[s=k] \left[ r(\tau_k, X_k, I_{1k}) \frac{\partial I_{1k}^*}{\partial \bar{e}} - (\gamma_k - \tau_k) \frac{dE(X_s^*, I_{1s}^*)}{d\bar{e}} \right].
\end{aligned} \tag{14}
$$

In setting the optimal equity mandate, the regulator considers the effect of leverage on borrower profits and emissions. Since equity increases the final investment scale when the financial constraint binds, it results in a higher profit earned by borrowers. The regulator internalizes this effect, similarly to private agents. This is captured by $r(\tau_s, X_s, I_{1s}) \frac{\partial I_{1s}^*}{\partial \bar{e}}$ in the regulator's FOC. The regulator also accounts for the effect of leverage on emissions and the marginal social cost that these generate in excess of what is already accounted for by the borrower, captured by $(\gamma_s - \tau_s) \frac{dE(X_s^*, I_{1s}^*)}{d\bar{e}}$. Comparing Eq. (14) with the corresponding borrower's FOC in Eq. (9) yields the optimal equity mandate.

**Proposition 6.** *If in the competitive equilibrium the borrower's financial constraint is slack when $s = G$ and binding when $s = B$, then the optimal equity mandate coincides with the borrower's choice of equity if and only if*

$$
\frac{dE(X_s^*, I_{1s}^*)}{dI_{1s}^*} \underbrace{\left[\gamma_B - \tau_B^* + \lambda_B \left(\theta \gamma_B^p - \psi \tau_B^*\right)\right]}_{\text{T-SCC wedge}} = 0. \tag{15}
$$

*If $\psi < 1$ the T-SCC wedge is positive and the optimal equity mandate $\bar{e}^*$ is*

- $\bar{e}^* > e^*(\tau_G^*, \tau_B^*)$ *if* $\frac{dE(X_B^*, I_{1B}^*)}{dI_{1B}^*} < 0$,
- $\bar{e}^* = e^*(\tau_G^*, \tau_B^*)$ *if* $\frac{dE(X_B^*, I_{1B}^*)}{dI_{1B}^*} = 0$,
- $\bar{e}^* < e^*(\tau_G^*, \tau_B^*)$ *if* $\frac{dE(X_B^*, I_{1B}^*)}{dI_{1B}^*} > 0$.

*Proof.* See Appendix A.4.3 □

What motivates leverage regulation is the difference in the marginal social and private costs of changes in emissions induced by higher levels of equity. The left-hand side of Eq. (15) captures this intuition, consisting of $\frac{dE(X_s^*, I_{1s}^*)}{dI_{1s}^*}$ and the expression in square brackets labeled T-SCC wedge, where T-SCC stands for total social cost of carbon. The T-SCC wedge is the difference between the total social cost and private cost of emissions and consists of two components. First, $\gamma_B - \tau_B$ is the wedge between the direct social cost of emissions $\gamma_B$ and the private cost of emissions $\tau_B$. Second, $\lambda_B\left(\theta\gamma_B^p - \psi\tau_B\right)$ is the effect of emissions on pledgeable income caused by the collateral externalities due to physical climate risk and tax rebates.

The optimal equity mandate can be above or below the level in the competitive equilibrium, depending on the effect of borrower equity on emissions. From Proposition 4, the optimal emissions tax is below $\tau_B^{GP}$ if $\psi < 1$, which implies a positive T-SCC wedge. This positive T-SCC wedge results in a socially inefficient leverage choice by borrowers and motivates an equity mandate. If higher equity primarily results in more abatement rather than lower liquidations, such that $\frac{dE(X_s^*, I_{1s}^*)}{dI_{1s}^*} < 0$, then the regulator opts for an equity level that is above the privately optimal level of equity, $\bar{e}^* > e^*$. By contrast, if $\frac{dE(X_s^*, I_{1s}^*)}{dI_{1s}^*} > 0$, then higher equity implies higher emissions, and the optimal equity mandate is below a borrower's optimal choice of equity in the competitive equilibrium, $\bar{e}^* < e^*$.

#### 4.3.3 A Motive to Include Climate Externalities in Financial Regulation

The finding in Proposition 6 that leverage regulation can improve welfare may not seem surprising given the large body of literature that shows how financial constraints can motivate financial regulation (for an overview, see Dewatripont and Tirole, 1994). Yet the following corollary shows that the financial constraint in itself does not motivate leverage regulation in our model:

**Corollary 1.** *If $\gamma_s^u = \gamma_s^p = 0$, then $\bar{e}^* = e^*$ regardless of whether $\lambda_B^* = 0$ or not.*

*Proof.* Follows from the result in Proposition 3 that $\tau_s = 0$ if $\gamma_s^u = \gamma_s^p = 0$, which implies a zero T-SSC wedge as defined in Proposition 6. □

In the absence of environmental externalities there is no benefit to introducing leverage regulation – irrespective of whether the financial constraint binds or not. This is important because it implies that financial constraints alone are not enough to motivate leverage regulation in our model. Instead, the motive for implementing an equity mandate $\bar{e}$ comes from the interaction between environmental externalities and financial frictions because binding financial constraints imply that the optimal emissions tax is below the total social cost of emissions. The results in Proposition 6 thus contribute to the debate on whether environmental externalities should be included in the mandate of financial regulatory frameworks (also see Oehmke and Opp, 2022a).

### 4.4 Cap and Trade

An alternative policy that a regulator could use is a cap-and-trade system with a limited quantity $Q_s$ of tradeable pollution permits (similar to the EU ETS). For each unit of emissions borrowers need to surrender a permit to the regulator. Remaining permits can be sold at the market price $p_s$. Absent other frictions, such pollution permit markets are equivalent to emissions taxes (see Montgomery, 1972). In what follows we show under what conditions the pollution permit market is equivalent to emissions taxes when the financial constraint binds and explore whether a pollution permit trading system can achieve higher welfare than emissions taxes.

A key feature of a pollution permit trading system is the mode through which polluters acquire the permits. We assume that a share $\phi$ of all permits $Q_s$ is freely allocated to borrowers ex-ante and that the remaining $(1-\phi)Q_s$ permits need to be purchased by the borrower at the market price $p_s$. Note that with freely allocated permits borrowers have the same incentives to invest in abatement because of the opportunity cost of selling unused permits. For now, the regulator takes the freely allocated share $\phi$ as given. Later we discuss the welfare-maximizing level of $\phi$. The budget constraints of the borrower under the pollution trading scheme are:

$$c_{1s} = \mu(I_0 - I_{1s}) + d_{1s} + e - I_0 - C(X_s, I_{1s}) + p_s(\phi Q_s - E(X_s, I_{1s})) \geq 0, \tag{2'}$$

$$c_2 = R(I_{1s}, E_s^a, \gamma_s^p) - d_{1s} \geq 0, \tag{3'}$$

$$d_{1s} \leq \tilde{R}(I_{1s}, E_s^a, \gamma_s^p). \tag{4'}$$

The first order conditions of the borrower stated in Appendix A.4.4 are equivalent to those in the baseline problem, with $p_s$ taking the place of $\tau_s$. The borrower's FOC with respect to abatement given by (6') in Appendix A.4.4 determines the relationship between the privately optimal level of abatement and $p_s$, and mirrors (6) of the original problem. This condition, together with the market clearing for permits, $Q_s = E_s^a$, jointly determine a mapping from $p_s$ to $E_s^a$. Thus, the regulator can implement a desired market price of permits by altering the total amount of permits. Consequently, we can express the regulator's problem as maximizing social welfare by choosing $p_s$ in each state $s = \{B, G\}$. Appendix A.4.4 reports the first order condition of the regulator. As in the baseline setting, the regulator internalizes the effect of the policy on borrowers' profits and emissions. Comparing the FOC under the cap-and-trade system with the one in the original problem yields the following result.

**Proposition 7.** *The allocation implemented with a pollution permit market in which the quantity of permits is chosen to implement a permit price $p_s = \tau_s$ and a fraction $\phi$ of permits are allocated freely, is equivalent to the allocation implemented with an emissions tax $\tau_s$ if the fraction of freely allocated permits is equal to the fraction of tax rebates that can be pledged, $\phi = \psi$.*

*Proof.* See Appendix A.4.4 □

In both the baseline setting and the current extension the regulator's policy amounts to choosing the private marginal cost of emissions represented either by the tax rate $\tau_s$ or the price of permits $p_s$. The direct effect of the policies on the financial constraints depend, respectively, on $\psi$, the pledgeability of the tax rebates, and $\phi$, the share of freely allocated permits. Pollution permits have a direct effect on the financial constraint if the borrower needs to purchase some of them ex-ante (i.e. if $1 - \phi > 0$). This corresponds to the direct effect of the tax bill on pledgeable income under emissions taxes. The price of permits also affects the tightness of the financial constraint through the collateral externalities, which mirror those discussed in Section 4.1.2.

So far we assumed that the regulator takes the share of freely allocated permits as given. However, the advantage of using a cap-and-trade system instead of emissions taxes is that the regulator can choose $\phi$ optimally. The equivalence result in Proposition 7 implies that a version of Proposition 5 in which $\tau_s = p_s$ and $\psi = \phi$ holds in the current setting, giving rise to the following corollary.

**Corollary 2.** *The regulator can implement a constrained-efficient allocation by setting* $\phi = 1$ *and issuing a quantity of permits that implements a permit price* $p_s^* = \frac{\gamma_s + \lambda_s^* \theta \gamma_s^p}{1+\lambda_s^*}$.

The regulator can avoid the problem of the carbon price's direct effect on borrowers' financial constraints by allocating all permits for free and setting $\phi = 1$. In this case, the shadow cost of permits induces borrowers to engage in a constrained-efficient level of abatement. As in the baseline with $\psi = 1$, the optimal policy is below the Pigouvian benchmark $p_s^* < \gamma_s$ whenever the financial constraint binds (see Proposition 3).

An important policy implication is that a pollution permit market with free allowances may be a superior policy instrument when financial constraints are a first-order concern, and that such a pollution permit market can render financial regulation unnecessary. Yet, in practice emissions permit markets often do not allocate permits for free. For example, the EU ETS (the largest emissions permit market in the world), only grants free allowances equal to a fraction of total emissions, and is gradually reducing the amount of free allowances over time.[14]

We acknowledge that there may be considerations outside our model that motivate these real-life policy choices.[15] For example, it may be difficult for regulators to correctly allocate free permits if polluters were privately informed about heterogeneous abatement costs, potentially triggering undesirable distributional consequences. Similarly, determining the amount of freely allocated emissions by past emissions (a policy referred to as "grandfathering"), may weaken incentives to reduce emissions as firms may want to avoid a reduction in the amount of freely allocated permits in the future (see Clò, 2010). Modeling these frictions is beyond the scope of our model. In as far as they constrain the regulator's ability to allocate all permits for free, the results in Propositions 6 and 7

[14]For example, the manufacturing industry received 80% of its allowances for free in 2013. This proportion has been decreased down to 30% in 2020, see European Commission website.

[15]The European Commission states that it reduces the amount of free allowances "to reflect more accurately the technological progress and to incentivize further deployment of innovative low-carbon technologies", see European Commission website.

suggest potential benefits from complementing permit markets with leverage regulation in this case.

## 4.5 Abatement Subsidy

Another alternative policy is a subsidy to abatement investments instead of a tax on emissions. To analyze such a policy in the context of our baseline model, suppose that $\tau_s = 0$ and consider instead a subsidy $\sigma_s$ on abatement financed by lump-sum taxes. For now, suppose these lump-sum taxes are fully financed by borrowers and that $-\sigma_s X_s = T_s$. Borrowers have to raise financing at the beginning of $t = 1$ to pay the lump-sum taxes and invest in abatement, then receive the subsidy $\sigma_s$ per unit of abatement. To map the subsidy to the baseline model, we assume that a fraction $\psi$ of the subsidy is pledgeable to outside investors, and borrowers can absond with $1-\psi$. The first order condition with respect to $X_s$ in Eq. (6) becomes

$$(1+\lambda_s)\left(\sigma_s - \frac{\partial C(X_s, I_{1s})}{\partial X_s}\right) = 0.$$

This equation is equivalent to the original first order condition (6) when setting $\sigma_s = \tau_s \frac{\partial E(X_s, I_{1s})}{\partial X_s}$. Whether the subsidy can implement the constrained-efficient allocation depends on $\psi$, as can be seen from the complementary slackness condition (8), which now becomes

$$\lambda_s \left[\tilde{R}(I_{1s}, E_s^a, \gamma_2^p) + \psi\sigma_s X_s - T_s + e - I_0 + \mu(I_0 - I_{1s}) - C(X_s, I_{1s})\right] = 0.$$

In equilibrium, this condition maps to Eq. (8) with $-(1-\psi)T_s$ replaced by $-T_s + \psi\sigma_s X$. This implies that the results from the baseline model apply. Notably, Proposition 5 still holds, so that the allocation is constrained efficient only if the subsidy is fully pledgeable, i.e., if $\psi = 1$.

**Transfers.** A subsidy may dominate emissions taxes if it is financed through taxes raised from unconstrained investors. In this case, the subsidy constitutes a net transfer

$\mathcal{T}_s = \sigma X_s$ from unconstrained to constrained agents, and it can implement the first-best allocation if the transfer is sufficiently large to ensure the financial constraint is slack in all states.

More generally, consider the baseline model with a generic transfer $\mathcal{T}_s$ to borrowers paid at $t = 1$, financed by lump-sum taxes from investors. With this transfer the complementary slackness condition (8) becomes

$$\lambda_s \left[ \tilde{R}(I_{1s}, E_s^a, \gamma_2^p) - \tau E(X_s, I_{1s}) + \psi T_s + \mathcal{T}_s + e - I_0 + \mu(I_0 - I_{1s}) - C(X_s, I_{1s}) \right] = 0.$$

Clearly, if $\mathcal{T}_s$ is sufficiently large, then the financial constraint becomes slack. As shown in Proposition 2, this implies that an emissions tax equal to the Pigouvian benchmark can implement the first best. Perhaps trivially, complementing Pigouvian emissions taxes with transfers from unconstrained investors to constrained borrowers can circumvent the financial constraint.

## 4.6 Hedging

In the baseline model, borrowers can take out non-state-contingent debt and cannot hedge. This extension allows fairly-priced hedging contracts that pay $h_B$ in the bad state and $h_G$ in the good state. Such contracts can also be implemented through state-contingent financing, for example, bonds that write off some of the principal when the social cost of emissions or taxes are high. Fair pricing of the hedging contract requires that

$$(1 - q)h_G + qh_B = 0. \tag{16}$$

Using this expression, the problem of borrowers can be expressed in terms of choosing the optimal $h_G$, while $h_B$ follows as $h_B = -\frac{(1-q)h_G}{q}$. The borrower's problem with hedging is formally shown in Appendix A.4.5. The first order conditions are the same as in the baseline model, except for the new first order condition w.r.t. $h_G$, which states that borrowers equalize the shadow cost of the financial constraints across states:

$$\lambda_G = \lambda_B. \tag{17}$$

This implies that borrowers optimally shift resources from the good, low SCC state to the bad, high SCC state. If this allows borrowers to ensure that financial constraints are slack in both states ($\lambda_G = \lambda_B = 0$), then a Pigouvian emissions tax $\tau_s = \gamma_s, \forall s \in \{B, G\}$ can implement the first best allocation (see Proposition 2). By allowing firms to hedge climate-related transition risk, the financial sector can enable efficient emissions taxation in equilibrium. This result highlights that hedging of climate-related risks may be an important role the financial sector can play in supporting the transition to a low-carbon economy, distinct from socially responsible investing that aims to direct firm policies by taking into account environmental and social factors in investment decisions (e.g., see Pástor et al., 2021; Oehmke and Opp, 2022b; Goldstein et al., 2022; Gupta et al., 2022).

If under optimal hedging $\lambda_G = \lambda_B > 0$, then emissions taxes are different from the Pigouvian benchmark, see Proposition 3. Appendix A.4.5 shows that in this case the efficiency results in Proposition 5 apply, so that emissions taxes alone can implement a constrained-efficient allocation only if tax rebates are fully pledgeable.

# 5 Conclusion

This paper provides an analytical framework to shed light on how to design and combine carbon pricing with other regulatory tools when firms are subject to financial constraints and to endogenous climate-related transition and physical risks. We find that emissions taxes alone can only implement a constrained-efficient allocation if tax rebates are fully pledgeable. Otherwise, welfare can be improved by complementing emissions taxes with leverage regulation, or by replacing emissions taxes with a cap-and-trade system with ex-ante freely allocated pollution permits.

Another important insight is that the way in which financial constraints interact with emissions taxes critically depends on the relative strength of climate-related transition and physical risks on pledgeable income. Higher emissions taxes tighten financial constraints if borrowers are exposed to climate transition risk, but they can ease financial constraints if borrowers' assets are exposed to physical climate risk, because lower emissions have a positive effect on their collateral value. Optimal emissions taxes need to account for climate-induced collateral externalities, and thus may be either above or below a Pigouvian benchmark rate equal to the direct social cost of emissions.

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# A Appendix

## A.1 First Best (Proposition 1)

*Proof.* The first best allocation corresponds to the abatement, investment and consumption levels that maximize social welfare defined by the sum of agent's utilities

$$\begin{aligned}\max_{I_{1s},X_s,c_0,c_0^i,c_{ts},c_{ts}^i} \quad & W = u(c_0) + c_0^i \\ & + (1-q)\left[c_{1G} + c_{1G}^i + c_{2G} + c_{2G}^i - \gamma_G E(X_G, I_{1G})\right] \\ & + q\left[c_{1B} + c_{1B}^i + c_{2B} + c_{2B}^i - \gamma_B E(X_B, I_{1B})\right],\end{aligned}$$

subject to $I_{1s} \leq I_0$, $c_0 \geq 0$, $c_0^i \geq 0$, $c_{ts} \geq 0$,$c_{ts}^i \geq 0$ and the aggregate resource constraints

$$\begin{aligned}c_0 + c_0^i &= A_0^b + A_0^i - I_0, \\ c_{1s} + c_{1s}^i + C(X_s, I_{1s}) &= A_1^i + \mu(I_0 - I_{1s}), \\ c_{2s} + c_{2s}^i &= \rho I_{1s} - \gamma_s E(X_s, I_{1s}),\end{aligned}$$

for all $s \in \{G, B\}$. Eliminating $c_0^i, c_{1s}, c_{1s}^i, c_{2s} + c_{2s}^i$ the problem can be formulated as:

$$\begin{aligned}\max_{I_{1s},X_s,c_0} \quad & W = u(c_0) + A_0^b + A_0^i - I_0 - c_0 \\ & + A_1^i + \mu(I_0 - (1-q)I_{1G} - qI_{1B}) - (1-q)C(X_G, I_{1G}) - qC(X_B, I_{1B}) \\ & + (1-q)\rho I_{1G} + q\rho I_{1B} - \gamma_B E(X_B, I_{1B}) \\ & + (1-q)\bar{\kappa}_{I_1 G}(I_0 - I_{1G}) + q\bar{\kappa}_{I_1 B}(I_0 - I_{1B}),\end{aligned}$$

with $\bar{\kappa}_{I_1s}$ the Lagrange multiplier on the constraint that $I_{1s} \leq I_0$. The first order conditions w.r.t. $c_0, I_{1s}$ and $X_s$ are given by, respectively,

$$\begin{aligned} u'(c_0) &= 1, \\ \rho - \mu - \gamma_s \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}} - \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}} - \bar{\kappa}_{I_1s} &= 0, \\ \gamma_s \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}} + \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}} &= 0. \end{aligned}$$

By Assumption 2 liquidations are inefficient, which implies $\bar{\kappa}_{I_1s} > 0$ and $I_{1s} = I_0$. □

## A.2 Competitive Equilibrium

### A.2.1 Borrower's Lagrangian

The non-negativity constraint for $c_0$ is always satisfied since we assume that $u'(0) = \infty$. Moreover, due to the financial constraint (4) $c_{2s}$ is always positive, so that (3) never binds.

The problem of borrowers can be stated as the following Lagrangian:

$$\begin{aligned} &\max_{X_s, I_{1s}, d_{1s}, e} \mathcal{L} = u(A_0^b - e) \\ &+ \sum_{k \in \{G,B\}} Pr[s = k] \left[ \mu(I_0 - I_{1k}) + e - I_0 - C(X_k, I_{1k}) + R(I_{1k}, E_k^a, \gamma_s^p) - \tau_k E(X_k, I_{1k}) + T_k \right] \\ &+ \sum_{k \in \{G,B\}} Pr[s = k] \left\{ \lambda_k \left[ \tilde{R}(I_{1k}, E_k^a, \gamma_s^p) - \tau_k E(X_k, I_{1k}) + \psi T_k - d_{1k} \right] + \underline{\kappa}_{Ik} I_{1k} + \overline{\kappa}_{Ik} [I_0 - I_{1k}] \right\} \\ &+ \sum_{k \in \{G,B\}} Pr[s = k] \kappa_{c_{1k}} \left[ d_{1k} + \mu(I_0 - I_{1k}) + e - I_0 - C(X_k, I_{1k}) \right], \end{aligned} \tag{18}$$

where $\lambda_s$ is the Lagrange multiplier for the financial constraint and $\kappa$'s are the multipliers for lower and upper bounds on variables. The first order condition w.r.t. $d_{1s}$ implies that the multiplier on the non-negativity constraint for $c_{1s}$ is equal to the multiplier on the financial constraint. If the financial constraint binds, borrowers are at a corner solution and do not consume at $t = 1$, so that $c_{1s} = 0$ and $\lambda_s = \kappa_{c_{1s}} > 0$. The FOC's given in Section 3 follow.

### A.2.2 Proof of Lemma 1

Consider two cases: (i) $d_0 \geq \mu I_0$ and (ii) $d_0 < \mu I_0$.

(i) If $d_0 \geq \mu I_0$, then defaulting on $t = 0$ debt implies that investors force liquidation of the entire project, i.e. $I_1 = 0$. This implies a residual payoff to the borrower of 0 plus tax rebates the borrower can abscond with $(1 - \psi)T_s$. Not defaulting, the borrower can do at least as well because the borrower may not have to liquidate the entire project, so that $I_1 \geq 0$. Consequently, the borrower can earn $\tilde{R}(I^*_{1s}, E^a_s, \gamma^p_s)$ plus $(1 - \psi)T_s$ and is therefore weakly better off not defaulting.

(ii) If $d_0 < \mu I_0$, defaulting on $t = 0$ debt implies that investors force liquidation s.t. $d_0 = \mu(I_0 - I_1)$. The borrower can then decide to continue the project, abate and potentially raise new debt $d_1$, subject to the constraint that liquidations are at least s.t. $d_0 = \mu(I_0 - I_1)$. But the borrower can already achieve this by not defaulting and instead rolling over $d_0$. Therefore, defaulting introduces an additional constraint on how much the borrower at a minimum needs to liquidate. Again, the borrower is weakly better off not defaulting to avoid this constraint.

### A.2.3 Proof of Lemma 2

*Proof.* Equation (7) evaluated at $\lambda_s = 0$ is $r(\tau_s, X_s, I_{1s}) - \overline{\kappa}_{Is} + \underline{\kappa}_{Is} = 0$. By Assumption 2.1 $r(\tau_s, X_s, I_{1s}) > 0$, which implies that the solution requires $\overline{\kappa}_{Is} > 0$ (i.e., $I_0 = I^*_{1s}$).

The complementary slackness condition (8) can be reformulated as

$$\lambda_s S(\tau_s, X_s, I_1, e, \gamma^p_s) = 0. \tag{8'}$$

Assumption 2.2 implies that liquidating investments eases financial constraints. Thus, if the financial constraint is slack at full investment scale, $S(\tau_s, X_s, I_0, e, \gamma^p_s) \geq 0$, it is slack for any $I_{1s}$. If the reverse holds, $S(\tau_s, X_s, I_0, e, \gamma^p_s) < 0$, such that the pledgeable resources are insufficient to cover the expenses at $t = 1$ in the absence of liquidations, then the financial constraints binds, $\lambda_s > 0$. In this case the complementary slackness condition (8') requires that borrowers liquidate the investment up to the point where

$S(\tau_s, X_s, I_{1s}^*, e, \gamma_s^p) = 0$. Thus, if $\lambda_s > 0$ it must be that $I_{1s}^* < I_0$ and $\overline{\kappa}_{Is} = 0$.

□

## A.3 Optimal Policy

### A.3.1 Proof of Lemma 3 and statement of Assumption 3

Recall that $X_s^*(\tau_s)$ is pinned down by:

$$\tau_s \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} = -\frac{\partial C(X_s^*, I_{1s}^*)}{\partial X_s^*}, \tag{6}$$

Totally differentiating (6) with respect to $\tau_s$ allows us to find $\frac{\partial X_s^*}{\partial \tau_s}$:

$$\frac{\partial X_s^*}{\partial \tau_s} = \frac{\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*} - \frac{\partial^2 N(X_s^*, I_{1s}^*, \tau_s)}{\partial X_s^* \partial I_{1s}^*} \frac{\partial I_{1s}^*}{\partial \tau_s}}{-\frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial (X_s^*)^2}} \tag{19}$$

where $N(X_s, I_{1s}, \tau_s) = -\tau_s E(X_s, I_{1s}) - C(X_s, I_{1s})$ and we use that $\frac{\partial^2 E(X, I_1)}{(\partial X)^2} = 0$. If the financial constraint is slack, $\lambda_s^*(\tau_s, \bar{e}) > 0$, then $I_{1s}^* = I_0$, so $\frac{\partial I_{1s}^*}{\partial \tau_s} = 0$ and $\frac{\partial X_s^*}{\partial \tau_s} > 0$. If the financial constraint is binding, $\lambda_s^*(\tau_s, \bar{e}) > 0$, then the interior solution of $I_{1s}^*(\tau_s)$ follows from:

$$\tilde{R}(I_{1s}^*, E_s^a, \gamma_s^p) + \mu(\bar{I}_0 - I_{1s}^*) + \bar{e} - I_0 - C(X_s^*, I_{1s}^*) - \tau E(X_s^*, I_{1s}^*) + \psi T_s = 0 \tag{8}$$

Totally differentiating (8) with respect to $\tau_s$ allows us to find $\frac{\partial I_{1s}^*}{\partial \tau_s}$:

$$\frac{\partial I_{1s}^*}{\partial \tau_s} = \frac{(1-\psi)E(X_s^*, I_{1s}^*) - (\psi\tau_s - \theta\gamma_s^p)\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial \tau}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)} \tag{20}$$

To further simplify, we use a shorthand notation: $E(X_s^*, I_{1s}^*) = E$, $E_x' = \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}$, $N_{wv}'' = \frac{\partial^2 N(X_s^*, I_{1s}^*, \tau_s)}{\partial W_s \partial V_s}$ and $\tilde{r}(\tau_s) = \tilde{r}(\tau_s, X_s^*, I_{1s}^*)$. Moreover, we use (19) and (20) to get:

$$\frac{\partial I_{1s}^*}{\partial \tau_s} = \frac{(1-\psi)EC_{x^2}'' + (\psi\tau_s - \theta\gamma_s^p)(E_x')^2}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p)C_{x^2}'' + (\psi\tau_s - \theta\gamma_s^p)E_x' N_{xI}''} \tag{21}$$

$$\frac{\partial X_s^*}{\partial \tau_s} = \frac{(1-\psi)EN_{xI}'' - \tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p)E_x'}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p)C_{x^2}'' + (\psi\tau_s - \theta\gamma_s^p)E_x' N_{xI}''} \tag{22}$$

**Assumption 3** requires that the model parameters are such that $\frac{\partial X_s^*}{\partial \tau_s} > 0$. This is the case when the numerator and the denominator of (22) have the same sign.

Notice that the denominator of (22) is negative for $\psi = 0$ and $\gamma_s^p = 0$. More generally, this expression is negative if and only if $\tilde{r}(\tau_s - \tau_s\psi + \theta\gamma_s^p)C''_{x^2} < -(\psi\tau_s - \theta\gamma_s^p)N''_{xI}E'_x$.

The numerator of (22) is negative if $\tilde{r}(\theta\gamma_s^p)E'_x > (1-\psi)(\tau_s E'_I E'_x + EN''_{xI})$. This is true whenever $\psi = 1$. Since the RHS of the inequality is monotone in $\psi$, the numerator of (22) is negative across the full range of $\psi$ if $\tilde{r}(\theta\gamma^p)E'_x > \tau_s E'_I E'_x + EN''_{xI}$

Thus, Assumption 3 can be restated as:

- $\tilde{r}(\theta\gamma_s^p, X_s^*, I_{1s}^*)\frac{\partial E(X_s^*,I_{1s}^*)}{\partial X_s^*} > E(X_s^*, I_{1s}^*)\frac{\partial^2 N(X_s^*,I_{1s}^*,\tau_s)}{\partial X_s^* \partial I_{1s}^*} + \tau_s \frac{\partial E(X_s^*,I_{1s}^*)}{\partial X_s^*}\frac{\partial E(X_s^*,I_{1s}^*)}{\partial I_{1s}^*}$

  $\forall X_s^*(\tau_s), I_{1s}^*(\tau_s), \tau_s < \bar{\tau}$

- $\tilde{r}(\tau_s - \tau_s\psi + \theta\gamma_s^p, X_s^*, I_{1s}^*)\frac{\partial^2 C(X_s^*,I_{1s}^*)}{\partial (X^*)^2} < -(\psi\tau_s - \theta\gamma_s^p)\frac{\partial^2 N(X_s^*,I_{1s}^*,\tau_s)}{\partial X_s^* \partial I_{1s}^*}\frac{\partial E(X_s^*,I_{1s}^*)}{\partial X_s^*}$

  $\forall X_s^*(\tau_s), I_{1s}^*(\tau_s), \tau_s < \bar{\tau}$

**Lemma 3** follows from observing that the numerator of equation (21) which defines $\frac{\partial I_{1s}^*}{\partial \tau_s}$ is negative if $\gamma_s^p > \frac{\psi}{\theta}\tau_s + \frac{(1-\psi)EC''_{x^2}}{\theta(E'_x)^2} = \hat{\gamma}^p(\tau_s)$ and positive if $\gamma_s^p < \hat{\gamma}^p(\tau_s)$. The denominator of (21) is the same as that of $\frac{\partial X^*}{\partial \tau_s}$, i.e. negative under Assumption 3.

### A.3.2 Proof of Proposition 3

The first order condition of the regulator with respect to $\tau$ is given by:

$$\left(\rho - \mu - \gamma_s \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}} - \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}}\right)\frac{\partial I_{1s}^*}{\partial \tau_s} - \left(\gamma_s \frac{\partial E(X_s, I_{1s})}{\partial X_s} + \frac{\partial C(X_s, I_{1s})}{\partial X_s}\right)\frac{\partial X_s^*}{\partial \tau_s} + \kappa_\tau = 0$$

Using (6) and the definition of $r(\tau, X, I_1)$ the above simplifies to (11).

Since $\frac{\partial X_s^*}{\partial \tau_s} > 0$ and $r(\tau_s, X_s, I_{1s}) > 0$ the optimal tax:

- is lower than the direct social cost of carbon $\tau_s < \gamma_s$ if $\frac{\partial I_{1s}^*}{\partial \tau_s} < 0$ and $\gamma_s > 0$
- is equal to the direct social cost of carbon $\tau_s = \gamma_s$ if $\frac{\partial I_{1s}^*}{\partial \tau_s} = 0$ or if $\frac{\partial I_{1s}^*}{\partial \tau_s} < 0$ and $\gamma_s = 0$
- is higher than the direct social cost of carbon $\tau_s > \gamma_s$ if $\frac{\partial I_{1s}^*}{\partial \tau_s} > 0$

Using Lemma 3 to determine the sign of $\frac{\partial I_{1s}^*}{\partial \tau_s}$ yields the result in Proposition 3.

**Optimal emissions tax**

The interior solution to the optimal emissions-taxation problem solves:

$$\frac{r(\gamma_s, X_s^*, I_{1s}^*)[(1-\psi)EC''_{x^2} + (\psi\tau_s - \theta\gamma_s^p)(E'_x)^2]}{(1-\psi)EN''_{xI} - \tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)E'_x} = (\gamma_s - \tau_s)E'_X \tag{23}$$

which can be rewritten as the following polynomial:

$$\begin{aligned}
&\tau^2(1-\psi)\left(\frac{E''_{xI}}{E'_x} - E'_I\right) + \\
&\tau\left[\rho(\theta-\psi) - w(1-\psi) - (\theta\gamma^p - \gamma)E'_I + \frac{1-\psi}{E'_x}E(C''_{xI} - \gamma E''_{xI})\right] + \\
&\theta\gamma^p r(\gamma) - \gamma\tilde{r}(\theta\gamma^p) - \frac{1-\psi}{(E'_x)^2}E(\gamma E'_x C''_{xI} - r(\gamma)C''^2_x) = 0
\end{aligned} \tag{24}$$

### A.3.3 Proof of Proposition 4

Focusing on the interior solution ($\kappa_{\tau k} = 0$) to Eq. (11) and using Eq. (12) yields:

$$\begin{aligned}
-\frac{r(\gamma_s, X_s^*, I_{1s}^*)}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)}(1-\psi)E(X_s^*, I_{1s}^*) = \\
\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial \tau_s}\left[\tau_s - \gamma_s + \frac{(\theta\gamma_s^p - \psi\tau_s)r(\gamma_s, X_s^*, I_{1s}^*)}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)}\right]
\end{aligned}$$

With some algebra this simplifies to:

$$r(\gamma_s, X_s^*, I_{1s}^*)(1-\psi)E(X_s^*, I_{1s}^*) = \frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial \tau_s}\left[\gamma_s - \tau_s + \lambda_s^*(\theta\gamma_s^p - \gamma\tau_s)\right]\tilde{r}(\tau_s, X_s^*, I_{1s}^*)$$

If $\psi = 1$, then the LHS of the above is equal to zero, so the tax must solve $\gamma_s - \tau_s + \lambda_s^*(\theta\gamma_s^p - \tau_s)$. If $\psi < 1$, then the LHS of the above is positive, so it must be that $\gamma_s - \tau_s + \lambda_s^*(\theta\gamma_s^p - \tau_s) > 0$.

If $\gamma_s = 0$ then $\tau_s = 0$ and $\kappa_{\tau k} > 0$ solve Eq. (11).

## A.4 Efficiency and Other Policies

### A.4.1 Proof of Proposition 5

As a first step, we define the constrained efficient allocation in which a social planner can choose $X_s, I_{1s}$ and $e$ directly without any policy instruments, but subject to the same constraints as private agents. The planner's problem can be written as a Lagrangian similar to the borrower's Lagrangian in Eq. (18):

$$\begin{aligned}
&\max_{X_s, I_{1s}, d_{1s}, e} \mathcal{L} = A_0^i + A_1^i + u(A_0^b - e) + e - I_0 \\
&+ \sum_{k\in\{B,G\}} Pr[s=k]\left\{R(I_{1k}, E_k^a, \gamma_k^p) + \mu(I_0 - I_{1k}) - 2\gamma_k^u E(X_k, I_{1k}) - C(X_k, I_{1k})\right\} \\
&+ \sum_{k\in\{B,G\}} Pr[s=k]\lambda_k^{SP}\left\{\tilde{R}(I_{1k}, E_k^a, \gamma_k^p) + \mu(I_0 - I_{1k}) - C(X_k, I_{1k}) + e - I_0\right\} \\
&+ \sum_{k\in\{B,G\}} Pr[s=k]\left[\underline{\kappa}_{Ik} I_{1k} + \overline{\kappa}_{Ik}(I_0 - I_{1k})\right].
\end{aligned} \tag{25}$$

The first order condition with respect to $X_s$, $I_{1s}$, and $e$ are given by, respectively,

$$-(\gamma_s + \lambda_s^{SP}\theta\gamma_s^p)\frac{\partial E(X_k, I_{1s})}{\partial X_s} - (1+\lambda_s^{SP})\frac{\partial C(X_s, I_{1s})}{\partial X_s} = 0 \tag{26}$$

$$r(\gamma_s, X_s, I_{1s}) + \lambda_s^{SP}\tilde{r}(\theta\gamma_s^p, X_s, I_{1s}) + \underline{\kappa}_{Ik} - \overline{\kappa}_{Ik} = 0 \tag{27}$$

$$-u'(A_0^b - e) + 1 + \kappa_e + \sum_{k\in\{B,G\}} Pr[s=k]\lambda_k = 0 \tag{28}$$

The complementary slackness condition in state $s$ is given by

$$\lambda_s^{SP}[\tilde{R}(I_{1s}, E_s^a, \gamma_s) - I_0 + \mu(I_0 - I_{1s}) + e - C(X_s, I_{1s})] = 0. \tag{29}$$

**Definition 3.** *The constrained efficient allocation is the set of allocations $I_{1s}^{SP}, X_s^{SP}, \lambda_s^{SP}, e^{SP}$, defined by Equations (26), (27), (29), and (28). Aggregate emissions are given by $E^a = E(X_s^{SP}, I_{1s}^{SP})$. The allocations $c_0^{SP}, c_{1s}^{SP}, c_{2s}^{SP}$, and $d_0^{SP}$ follow as residuals from Eqs. (1), (2), (3), and $d_0^{SP} = I_0 - e^{SP}$.*

The equilibrium is constrained efficient if and only if $X_s^*(\tau^*) = X_s^{SP}$, $I_{1s}^* = I_{1s}^{SP}$ and

$e^* = e^{SP}$. We first establish when $X_s^*(\tau^*) = X_s^{SP}$ and then move to the remaining conditions.

Using the private FOC's wrt. $X_s$ given by (6) to find the level of $\tau^{SP}$ that would implement the constrained efficient level of abatement $X_s^* = X_s^{SP}$ consistent with (26) we get: $\gamma_s + \lambda_s^{SP}\theta\gamma_s^p = (1+\lambda_s^{SP})\tau_s^{SP}$, where:

$$\lambda_s^{SP} = -\frac{r(\gamma_s, X_s^{SP}, I_{1s}^{SP}) + \underline{\kappa}_{Is} - \overline{\kappa}_{Is}}{\tilde{r}(\theta\gamma_s^p, X_s^{SP}, I_{1s}^{SP})}$$

Focusing on the case when $I_{1s}^{SP}$ is in the interior solution, the emissions tax that implements the constrained efficient allocation is

$$\tau_s^{SP} = \frac{\gamma_s \tilde{r}(\theta\gamma_s^p, X_s^{SP}, I_{1s}^{SP}) - \theta\gamma_s^p r(\gamma_s, X_s^{SP}, I_{1s}^{SP})}{\tilde{r}(\theta\gamma_s^p, X_s^{SP}, I_{1s}^{SP}) - r(\gamma_s, X_s^{SP}, I_{1s}^{SP})}. \tag{30}$$

To determine whether the equilibrium level of $X_s^*$ is constrained efficient, we plug in the tax that can implement the constrained efficient allocation $\tau_s^{SP}$ into the condition that defines the optimal tax set by the regulator (23).

$$\frac{r(\gamma_s, X_s^*, I_{1s}^*)[(1-\psi)Ec''_{x^2} + (\psi\tau_s^{SP} - \theta\gamma_s^p)(E'_x)^2]}{(1-\psi)EN''_{xI} - \tilde{r}(\tau_s^{SP}(1-\psi) + \theta\gamma_s^{;}, X_s^*, I_{1s}^*)E'_x} = (\gamma_s - \tau_s^{SP})E'_x \tag{31}$$

which can be rewritten as:

$$(1-\psi)\left[r(\gamma_s, X_s^*, I_{1s}^*)\left(Ec''_{x^2} - \tau_s^{SP}(E'_x)^2\right) - (\gamma_s - \tau_s^{SP})\left(E'_x EN''_{xI} + (E'_x)^2\tau^{SP}E'_I\right)\right] = 0 \tag{32}$$

The LHS of (32) is equal to zero whenever $\psi = 1$. In this case $\tau_s^{SP}$ corresponds to the tax implemented by the regulator. To show that when $\psi = 1$ also $I_{1s}^* = I_{1s}^{SP}$ notice that the complementary slackness condition (8) collapses to (29). Moreover, private and planner's FOC's with respect to $e$ are equal whenever

$$-\frac{r(\gamma_s) + \underline{\kappa}_{sI}^{SP} - \overline{\kappa}_{sI}^{SP}}{\tilde{r}(\theta\gamma_s^p)} = -\frac{r(\tau_s^{SP}) - \overline{\kappa}_{sI} + \underline{\kappa}_{sI}}{\tilde{r}(\tau_s^{SP})} \tag{33}$$

which is holds at $\tau_s^{SP}$ defined in (30). Thus, if $\psi = 1$ the competitive equilibrium is constrained efficient.

If $\psi < 1$ then the LHS of (32) is equal to zero only if:

$$\begin{aligned}
&(\tau_s)^2 E'_x[E'_x E'_I - E E''_{xI}]+ \\
&\tau_s E'_x[E(\gamma E''_{xI} - C''_{xI}) - r(0, X^*_s, I^*_{1s})E'_x]+ \\
&[r(\gamma, X^*_s, I^*_{1s})EC''_{x^2} + \gamma E'_x E C''_{xI}] = 0
\end{aligned} \tag{34}$$

Let $\tau_s = \tilde{\tau}_s^a$ and $\tau_s = \tilde{\tau}_s^b$ denote the solutions of (34). Given that LHS is quadratic in $\tau_s$, if the solution to (34) exists $\tilde{\tau}_s^a$ and $\tilde{\tau}_s^b$ are functions of $\frac{\partial^2 C(X,I_1)}{\partial X \partial I_1}$, $\frac{\partial^2 E(X,I_1)}{\partial X \partial I_1}$ and $\frac{\partial^2 C(X,I_1)}{(\partial X)^2}$. Notice that the tax rate that is needed to implement the constrained efficient level of abatement, $\tau^{SP}$, given in (30) does not depend on these cross- and second-order-derivatives. Thus, condition (34), which ensures that $X^*_s = X^{SP}_s$ is generally not satisfied except in a knife's edge case in which the values of these derivatives are coincidentally such that $\tilde{\tau}_s^a = \tau^{SP}$. This implies that the allocation implemented by the tax optimally set by the regulator is constrained inefficient when $\psi < 1$.

### A.4.2 Proof of Lemma 4

Totally differentiating (6) with respect to $\bar{e}$ allows us to find $\frac{\partial X^*}{\partial \bar{e}}$ are:

$$\left[\frac{\partial^2 C(X^*_s, I^*_{1s})}{\partial (X^*_s)^2}\right] \frac{\partial X^*_s}{\partial \bar{e}} + \left[\tau_s \frac{\partial^2 E(X^*_s, I^*_{1s})}{\partial X^*_s \partial I^*_{1s}} + \frac{\partial^2 C(X^*_s, I^*_{1s})}{\partial X^*_s \partial I^*_{1s}}\right] \frac{\partial I^*_{1s}}{\partial \bar{e}} = 0$$

Which can be simplified using $N(X_s, I_{1s}, \tau_s) = -\tau_s E(X_s, I_{1s}) - C(X_s, I_{1s})$ to yield:

$$\frac{\partial X^*_s}{\partial \bar{e}} = \frac{\frac{\partial^2 N(X^*_s, I^*_{1s}, \tau_s)}{\partial X^*_s \partial I^*_{1s}}}{\frac{\partial^2 C(X^*_s, I^*_{1s})}{\partial (X^*_s)^2}} \frac{\partial I^*_{1s}}{\partial \bar{e}} \tag{35}$$

If the financial constraint is slack, $\lambda^*_s(\tau_s) > 0$, then $I^*_{1s} = I_0$, so $\frac{\partial I^*_{1s}}{\partial \bar{e}} = 0$ and $\frac{\partial X^*_s}{\partial \bar{e}} = 0$ . If the financial constraint is binding, $\lambda^*_s(\tau_s) > 0$, then the interior solution of $I^*_{1s}(\tau_s)$ is pinned down by (8). By totally differentiating (8) with respect to $\bar{e}$ we get:

$$\frac{\partial I_{1s}^*}{\partial \bar{e}} = \frac{-1 - (\psi\tau_s - \theta\gamma_s^p)\frac{\partial E(X_s^*, I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial \tau_s}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p, X_s^*, I_{1s}^*)} \tag{36}$$

Combining (35) and (36) and using the shorthand notation introduced in Appendix A.3.1, yields:

$$\frac{\partial I_{1s}^*}{\partial \bar{e}} = \frac{-C''_{x^2}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p)C''_{x^2} + (\psi\tau_s - \theta\gamma_s^p)E'_x N''_{xI}} \tag{37}$$

$$\frac{\partial X_s^*}{\partial \bar{e}} = \frac{-N''_{xI}}{\tilde{r}(\tau_s(1-\psi) + \theta\gamma_s^p)C''_{x^2} + (\psi\tau_s - \theta\gamma_s^p)E'_x N''_{xI}} \tag{38}$$

The denominator of (38) is negative by Assumption 3, Therefore $\frac{\partial X_s^*}{\partial \bar{e}} > 0$ if and only if $N''_{xI} > 0$, i.e. $\tau_s^* \frac{\partial^2 E(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} + \frac{\partial^2 C(X_s^*, I_{1s}^*)}{\partial X_s^* \partial I_{1s}^*} < 0$.

### A.4.3 Proof of Proposition 6

The first order conditions of the regulator with respect to $\bar{e}$ is:

$$u'(A_0^b - \bar{e}) - 1 = + \sum Prob[s = k]\left[\left(\rho - \mu - \gamma_s \frac{\partial E}{\partial I_{1s}} - \frac{\partial C}{\partial I_{1s}}\right)\frac{\partial I_{1s}^*}{\partial \bar{e}} - \left(\gamma_s \frac{\partial E}{\partial X_s} + \frac{\partial C}{\partial X_s}\right)\frac{\partial X_s^*}{\partial \bar{e}}\right] \tag{39}$$

Using the private FOC wrt $X$ and the fact that $r(\gamma_s, X_s^*, I_{1s}^*) = r(\tau_s, X_s^*, I_{1s}^*) + \tau_s \frac{\partial E(X_s^*, I_{1s}^*)}{\partial I_{1s}^*} - \gamma_s \frac{\partial E(X_s^*, I_{1s}^*)}{\partial I_{1s}^*}$, yields (14).

If the financial constraint binds only in the bad state $s = B$ then the regulator's and borrowers FOCs can be restated using the shorthand notation introduced in Appendix A.3.1 as, respectively:

$$u'(A_0^b - \bar{e}) - 1 = \frac{-r(\tau_B)C''_{x^2} + (\gamma_B - \tau_B)[E'_I C''_{x^2} + E'_x N''_{xI}]}{\tilde{r}(\tau_B(1-\psi) + \theta\gamma_B^p)C''_{x^2} + (\psi\tau_B - \theta\gamma_B^p)E'_x N''_{xI}} \tag{40}$$

$$u'(A_0^b - e) - 1 = \frac{-r(\tau_B)}{\tilde{r}(\tau_B)} \tag{41}$$

Thus, borrowers choose a lower level of equity than the regulator if and only if:

$$\frac{-r(\tau_B)C''_{x^2} + (\gamma_B - \tau_B)[E'_I C''_{x^2} + E'_x N''_{xI}]}{\tilde{r}(\tau_B(1-\psi) + \theta\gamma_B^p)C''_{x^2} + (\psi\tau_B - \theta\gamma_B^p)E'_x N''_{xI}} > \frac{-r(\tau_B)}{\tilde{r}(\tau_B)}$$

Since under Assumption 3 $\tilde{r}(\tau_B(1-\psi) + \theta\gamma_B^p)C''_{x^2} + (\psi\tau_B - \theta\gamma_B^p)E'_x N''_{xI} < 0$, and by Assumption 2 $\tilde{r}(\tau) < 0$ the above can be rewritten as:

$$\left(E'_I + E'_x \frac{N''_{xI}}{C''_{x^2}}\right)\left[(\gamma_B - \tau_B) - \frac{r(\tau)}{\tilde{r}(\tau)}(\theta\gamma_B^p - \psi\tau_B)\right] < 0.$$

Borrowers choose a higher level of equity than the regulator if the LHS is larger than zero. This yields condition (15) in Proposition 6.

To see that the borrower's choice of equity corresponds with that of the regulator when $\psi = 1$, plug in the optimal emissions tax $\tau_B^*$ into (15). If $\psi < 1$, there is a motive for leverage regulation as long $E'_I + E'_x \frac{N''_{xI}}{C''_{x2}} \neq 0$ because, as we have shown in Appendix A.4.1, the optimal tax set by the regulator $\tau_B^* \neq \frac{\gamma_B + \lambda_B^* \theta \gamma_B^p}{1+\lambda_B^*}$ in this case.

If $\psi = 0$ and $\gamma_B^p < \hat{\gamma}^p(\tau_B^*)$ then borrowers choose too little equity if $E'_x N''_{xI} + E'_I C''_{x^2} < 0$ and too much equity if $E'_x N''_{xI} + E'_I C''_{x^2} > 0$.

If $\tau_B^* = \gamma_B$ (which is optimal when $\gamma_B = 0$ or $\lambda_B^* = 0$) the regulator's FOC (14') is identical to the borrower's FOC (9'), so the regulator does not regulate leverage.

If $\tau_B^* < \gamma_B$ the RHS of regulator's FOC (14') is higher than the RHS of borrower's FOC (14') if and only if:

$$\frac{\partial E(X_B^*, I_{1B}^*)}{\partial I_{1B}^*} + \frac{\partial E(X_B^*, I_{1B}^*)}{\partial X_B^*} \frac{\frac{\partial^2 N(X_B^*, I_{1B}^*, \tau_B)}{\partial X_B \partial I_{1B}^*}}{\frac{\partial^2 C(X_B^*, I_{1B}^*)}{\partial (X_B^*)^2}} < 0 \tag{42}$$

If the RHS of regulator's FOC (14') is higher than the RHS of borrower's FOC (14') then the regulator prefers a higher level of equity than the borrower. In this case, regulator implements binding leverage regulation.

### A.4.4 Optimal Price of Permits

The first order conditions of the borrower's problem are given by:

$$(1+\lambda_s)\left(p_s\frac{\partial E(X_s,I_{1s})}{\partial X_s}+\frac{\partial C(X_s,I_{1s})}{\partial X_s}\right)=0, \tag{6'}$$

$$\rho(1+\lambda_s(1-\theta))-(1+\lambda_s)\left[\mu+\frac{\partial C(X_s,I_{1s})}{\partial I_{1s}}+p_s\frac{\partial E(X_s,I_{1s})}{\partial I_{1s}}\right]-\overline{\kappa}_{Is}+\underline{\kappa}_{Is}=0, \tag{7'}$$

$$u'(A_0^b-e)-1-(1-q)\lambda_G-q\lambda_B=0. \tag{9'}$$

The complementary slackness condition of borrower's problem is now

$$\lambda[\tilde{R}(I_{1s},E_s^a,\gamma_s^p)+I_0+\mu(I_0-I_{1s})+e-C(X_s,I_{1s})+p_s(\phi Q_s-E(X_s,I_{1s}))]. \tag{8'}$$

The first order condition of the regulator is:

$$r(\gamma_s,X_s^*,I_{1s}^*)\frac{\partial I_{1s}^*}{\partial p_s}-(\gamma_s-p_s)\frac{\partial E(X_s^*,I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial p_s}+\kappa_p=0 \tag{11'}$$

To find $\frac{\partial X_s^*}{\partial p_s}$, we take a total derivative of (6') with respect to $p_s$. This yields:

$$\frac{\partial X^*}{\partial p}=\frac{\frac{\partial E(X^*,I_1^*)}{\partial X^*}-\frac{\partial^2 N(X^*,I_1^*,p_s)}{\partial X^*\partial I_1^*}\frac{\partial I_1^*}{\partial p}}{\frac{\partial^2 C(X^*,I_1^*)}{\partial (X^*)^2}} \tag{19'}$$

To find $\frac{\partial I_{1s}^*}{\partial p_s}$ take a total derivative of (8') with respect to $p_s$, keeping in mind that $Q_s^f=\phi Q_s=\phi E_s^a$.

$$\frac{\partial I_{1s}^*}{\partial p_s}=\frac{(1-\phi)E(X_s^*,I_{1s}^*)-(\phi p_s-\theta\gamma_s^p)\frac{\partial E(X_s^*,I_{1s}^*)}{\partial X_s^*}\frac{\partial X_s^*}{\partial p_s}}{\tilde{r}(p_s(1-\phi)-\theta\gamma_s^p,X_s^*,I_{1s}^*)} \tag{20'}$$

Let's define:

$$\frac{\partial X_s^*}{\partial \tau_s}=g_X(\tau_s)$$
$$\frac{\partial I_{1s}^*}{\partial \tau_s}=g_I(\tau_s)$$

Comparing (19) with (19') and (20) with (20'), it is straightforward that $\frac{\partial X_s^*}{\partial p_s} = g_X(p_s)$ and $\frac{\partial I_{1s}^*}{\partial p_s} = g_I(p_s)$ Thus, the first order condition of the regulator's problem in the baseline model (11) is equivalent to the first order condition of the problem of choosing $Q_s$ to implement $p_s$ taking as given $\phi$, given by (11').

#### A.4.5 Hedging

With hedging as described in Section 4.6, the borrower's problem can be written as the following Lagrangian:

$$\max_{X_s, I_{1s}, d_1, e, h_s} \mathcal{L} = u(A_0^b - e)$$
$$+ \sum_{k \in \{G,B\}} Pr[s = k] \left[\mu(I_0 - I_{1k}) + e + h_k - I_0 - C(X_k, I_{1k}) + R(I_{1k}, E_k^a, \gamma_s^p) - \tau_k E(X_k, I_{1k}) + T_k\right]$$
$$+ \sum_{k \in \{G,B\}} Pr[s = k] \left\{\lambda_k \left[\tilde{R}(I_{1k}, E_k^a, \gamma_s^p) - \tau_k E(X_k, I_{1k}) + \psi T_k + h_k - d_{1k}\right] + \underline{\kappa}_{Ik} I_{1k} + \overline{\kappa}_{Ik}[I_0 - I_{1k}]\right\}$$
$$+ \sum_{k \in \{G,B\}} Pr[s = k] \kappa_{c_{1k}} \left[d_{1k} + \mu(I_0 - I_{1k}) + e + h_k - I_0 - C(X_k, I_{1k})\right] \tag{43}$$

The problem and first order conditions are equivalent to the problem in the main text (18), except that now additionally borrowers choose $h_s$ subject to the fair pricing condition (16). Using (16) to substitute $h_B = -\frac{(1-q)h_G}{q}$, the first order condition w.r.t. $h_G$ is given by

$$\lambda_G = \lambda_B.$$

**Constrained Efficiency** With hedging, the problem of a constrained social planner is similar to Eq. (25), but with $h_s$ as an additional choice variable, analogous to the updated borrower problem (43).

$$\max_{X_s, I_{1s}, d_{1s}, e, h_s} \mathcal{L} = A_0^i + A_1^i + u(A_0^b - e) + e - I_0$$
$$+ \sum_{k \in \{B,G\}} Pr[s=k] \left\{ R(I_{1k}, E_k^a, \gamma_k^p) + \mu(I_0 - I_{1k}) + h_k - 2\gamma_k^u E(X_k, I_{1k}) - C(X_k, I_{1k}) \right\}$$
$$+ \sum_{k \in \{B,G\}} Pr[s=k] \lambda_l^{SP} \left\{ \tilde{R}(I_{1k}, E_k^a, \gamma_k^p) + h_k + \mu(I_0 - I_{1k}) - C(X_k, I_{1k}) + e - I_0 \right\}$$
$$+ \sum_{k \in \{B,G\}} Pr[s=k] \left[ \underline{\kappa}_{Ik} I_{1k} + \overline{\kappa}_{Ik}(I_0 - I_{1k}) \right]. \quad (44)$$

Using (16) to substitute $h_B = -\frac{(1-q)h_G}{q}$, the first order condition w.r.t. $h_G$ is equivalent to the borrower's first order condition:

$$\lambda_G^{SP} = \lambda_B^{SP}.$$

All other first order conditions are the same as in the model without hedging. This implies the efficiency properties of the equilibrium allocation are the same as in the baseline model without hedging, as outlined in Proposition 5.

# B Extensions and Additional Results

## B.1 Implementation of the Capital Mandate through Taxes on Leverage

This appendix shows that a capital mandate $\bar{e}$ derived in Section 4.3 can alternatively be implemented through a tax $\tau_d$ on $t = 0$ debt (or a subsidy if $\tau_d < 0$). Given that capital requirements in the Basel Accord apply to financial institutions, leverage taxes and subsidies may be a more likely tool seen in the real world if borrowers in the model are interpreted as non-financial firms (such as manufacturing firms). Tax proceeds are fully rebated to borrowers via a lump-sum rebate $T_0^b$.

With a leverage tax $\tau_d$, the $t = 0$ budget constraint is given by $I_0 = e + d_0(1-\tau_d) + T_0^b$, which can be re-arranged to $d_0 = \frac{I_0 - e - T_0^b}{(1-\tau_d)}$. With this budget constraint, the borrower's problem (18) is now given by the following Lagrangian:

$$
\begin{aligned}
&\max_{X, I_1, d_1, e} \mathcal{L} = u(A_0^b - e) \\
&+ \sum_{k \in \{G,B\}} Pr[s=k] \left[ \mu(I_0 - I_{1k}) - \frac{I_0 - e - T_0^b}{1-\tau_d} - C(X_k, I_{1k}) + R(I_{1k}, E_k^a, \gamma_s^p) - \tau_k E(X_k, I_{1k}) + T_k \right] \\
&+ \sum_{k \in \{G,B\}} Pr[s=k] \left\{ \lambda_k \left[ \tilde{R}(I_{1k}, E_k^a, \gamma_s^p) - \tau_k E(X_k, I_{1k}) + \psi T_k - d_{1k} \right] + \underline{\kappa}_{Ik} I_{1k} + \overline{\kappa}_{Ik}[I_0 - I_{1k}] \right\} \\
&+ \sum_{k \in \{G,B\}} Pr[s=k] \kappa_{c_{1k}} \left[ d_{1k} + \mu(I_0 - I_{1k}) - \frac{I_0 - e - T_0^b}{1-\tau_d} - C(X_k, I_{1k}) \right],
\end{aligned}
\tag{45}
$$

The first order conditions with respect to $X_s$ and $I_{1s}$ are equivalent to those in the main text and given by (6) and (7), respectively. By contrast, the first order condition with respect to equity $e$ is different from the main text Eq. (9), and is now given by

$$
u'(A_0^b - e) = \frac{1 + (1-q)\lambda_G + q\lambda_B}{1 - \tau_d}.
$$

From this equation it is clear that a higher tax on debt induces borrowers to choose a higher level of $e$, i.e., lower leverage. By fully rebating the taxes, such that $T_0^b = \tau_d d_0$, a regulator can ensure that the tax does not affect any constraints. Consequently, a equity mandate $\bar{e}^*$ can be implemented by setting a leverage tax $\tau_d^*$ such that

$$u'(A_0^b - \bar{e}^*) = \frac{1 + (1-q)\lambda_G + q\lambda_B}{1 - \tau_d^*}.$$

## B.2 Interpretation of Borrowers as Financial Institutions

This appendix derives a version of the model in which borrowers are banks that make loans to non-financial firms. A continuum of firms run by risk-neutral owners have access to the same investment project as described in Section 2. Firms have no own funds and must obtain a loan from a bank. Banks have the same preferences and the same limited endowment $A_0^b$ as borrowers in the baseline model. Banks can also raise financing from investors as in the baseline model. In contrast, each firm is matched with a bank and can only obtain financing through a loan from its bank, i.e., firms cannot obtain funding from other investors or banks. There is no friction between a firm and its bank, but banks are constrained by the same financial constraint (4) as borrowers in the baseline model. That is, banks can fully seize the firm's assets at $t = 2$ but can only pledge $\tilde{R}(I_1, E^a)$ of the seized asset returns to outside investors. In this version of the model, "borrowers" are split into a financial and a real sector, where banks finance loans to bank-dependent firms through bank equity and outside financing, and firms use loans to finance real investment and abatement. We assume that firm owners are risk-neutral and bank owners have the same quasi-linear utility as borrowers in the baseline model. For simplicity, we focus on the case $\psi = 0$.

**Firm problem.** Banks make a take-it-or-leave-it offer to firms, offering a loan $l_t$ at $t = 0$ and $t = 1$, and repayment $D$ due at $t = 2$. Firms can decide to accept or reject the loan but conditional on accepting take $l_t$ and $D$ as given. When rejecting the loan, the outside option for firms is not to finance the project.

Firms have no own funds, so that $I_0 = l_0$. At $t = 1$ firms can liquidate some initial investment to generate a liquidation value $\mu(I_0 - I_{1s})$, and invest in abatement $X_s$ at a cost $C(X_s, I_{1s})$. Firm owner's consumption is given by

$$
\begin{aligned}
c_0^f &= l_0 - I_0 \\
c_{1s}^f &= \mu(I_0 - I_{1s}) - C(X_s, I_{1s}) + l_{1s} \\
c_{2s}^f &= R(I_{1s}, E_s^a, \gamma_s^p) - \tau E(X_s, I_{1s}) + T_s - D
\end{aligned}
$$

The firm's problem is to choose $I_{1s}$ and $X_s$ so as to maximize $c_0^f + c_1^f + c_2^f$ subject to $I_0 \geq I_{1s} \geq 0$ and non-negativity constraints on consumption. This problem can be written as follows:

$$
\begin{aligned}
&\max_{X_s, I_{1s}, l_{1s}, l_0} \mathcal{L} = l_0 - I_0 \\
&+ \sum_{k \in \{G,B\}} Pr[s=k] \left[ R(I_{1k}, E_k^a, \gamma_k^p) - \tau_k E(X_k, I_{1k}) + T_k - D + l_{1k} + \mu(I_0 - I_{1k}) - C(X_k, I_{1k}) \right] \\
&+ \kappa_{c_0^f}(l_0 - I_0) + \sum_{k \in \{G,B\}} Pr[s=k] \kappa_{c_1^f k} [\mu(I_0 - I_{1k}) - C(X_k, I_{1k}) + l_{1k}] \\
&+ \sum_{k \in \{G,B\}} Pr[s=k] \Big[ \kappa_{c_2^f k} [R(I_{1k}, E_k^a, \gamma_k^p) - \tau_k E(X_k, I_{1k}) + T_k - D] + \underline{\kappa}_{Ik} I_{1k} + \overline{\kappa}_{Ik}(I_0 - I_{1k}) \Big].
\end{aligned} \tag{46}
$$

The first order conditions with respect to $I_{1s}$ and $X_s$ are, respectively,

$$
(1 + \kappa_{c_2^f s}) \left( \rho - \tau \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}} \right) - (1 + \kappa_{c_1^f s}) \left( \mu + \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}} \right) + \underline{\kappa}_{Is} - \overline{\kappa}_{Is} = 0, \tag{47}
$$

$$
-\tau_s \frac{\partial E(X_s, I_{1s})}{\partial X_s} - \frac{\partial C(X_s, I_{1s})}{\partial X_s} = 0. \tag{48}
$$

The first order condition with respect to $X_s$ is the same as in the baseline model, cf. Eq. (6). By Assumption 2 (liquidations are inefficient) and the fact that $\kappa_{c_2^f s} \geq 0$, it also follows that $(1 + \kappa_{c_2^f s}) \left( \rho - \tau \frac{\partial E(X_s, I_{1s})}{\partial I_{1s}} \right) - \left( \mu + \frac{\partial C(X_s, I_{1s})}{\partial I_{1s}} \right) > 0$. This implies that either $\overline{\kappa}_{Is} > 0$ or $\kappa_{c_1^f} > 0$, so that $I_{1s}$ is either $I_{1s} = I_0$ or is pinned down by $c_1^f = 0$, which defines $I_{1s}(l_{1s})$.

**Bank problem.** The bank chooses $l_0$, $l_{1s}$, $D$, $d_{1s}$ and $d_0$, subject to the financial constraint (4).

$$c_0 = A - e$$
$$c_1 = d_{1s} - d_0 - l_{1s}$$
$$c_2 = D - d_{1s}$$

Firm participation requires that $c_t^f \geq 0$. Banks optimally choose $D$, $l_{1s}$ and $l_0$ such that the participation constraints bind, which implies $l_0 = I_0 = e + d_0$, $l_{1s} = -\mu(I_0 - I_{1s}) + C(X_s, I_{1s})$, and $D = R(I_{1s}, E_s^a) - \tau E(X_s, I_{1s}) + T_s$.

If the firm's investment is pinned down by $I_{1s}(l_{1s})$ (defined by $c_1^f = 0$), the bank's problem can be expressed as:

$$\begin{aligned}
&\max_{l_{1s},d_{1s},e} \mathcal{L} = u(A-e) - I_0 + e \\
&+ \sum_{k\in\{G,B\}} Pr[s=k]\left[\mu(I_0 - I_{1k}(l_{1k})) - C(X_k, I_{1k}(l_{1k})) + R(I_{1k}(l_{1k}), E_k^a, \gamma_k^p) - \tau_k E(X_k, I_{1k}(l_{1k})) + T_k\right] \\
&+ \sum_{k\in\{G,B\}} Pr[s=k]\lambda_k \left(\tilde{R}(I_{1k}(l_{1k}), E_k^a, \gamma_k^p) - \tau_k E(X_k, I_{1k}(l_{1k})) - d_{1k}\right) + \kappa_{c_0}(A-e) \\
&+ \sum_{k\in\{G,B\}} Pr[s=k]\left[\kappa_{c_{1k}}\left(d_{1k} - I_0 + e + \mu(I_0 - I_{1k}(l_{1k})) - C(X_k, I_{1k}(l_{1k}))\right)\right] \\
&+ \sum_{k\in\{G,B\}} Pr[s=k]\left[\kappa_{c_{2k}}\left(R(I_{1k}(l_{1k}), E_k^a, \gamma_k^p) - \tau_k E(X_k, I_{1k}(l_{1k})) + T - d_{1k}\right)\right].
\end{aligned} \tag{49}$$

The first order conditions read:

$$u'(A-e) = 1 - \kappa_{c_0} + (1-q)\kappa_{c_{1G}} + q\kappa_{c_{1B}} \tag{50}$$

$$\kappa_{c_{1s}} - \kappa_{c_{2s}} - \lambda_s = 0 \tag{51}$$

$$-(1+\kappa_{c_{1s}})\left(\mu + \frac{\partial C}{\partial I_{1s}}\right) + (1+\kappa_{c_{2s}})\left(\frac{\partial R}{\partial I_{1s}} - \tau_s \frac{\partial E}{\partial I_{1s}}\right) + \lambda_s\left(\frac{\partial \tilde{R}}{\partial I_{1s}} - \tau_s \frac{\partial E}{\partial I_{1s}}\right) = 0 \tag{52}$$

Due to the assumptions on $u'(c_0)$, it is never optimal to have $A - e = 0$, so $\kappa_{c_0} = 0$. Because $d_{1s} \leq \tilde{R}(I_{1s}, E_s^a) - \tau_s E(X_s, I_{1s})$, $c_{2s} > 0$ and $\kappa_{c_{2s}} = 0$. It follows that $\lambda_s = \kappa_{c_{1s}} > 0$, so the FOCs simplify to:

$$u'(c_0) = 1 + (1-q)\lambda_G + q\lambda_B \tag{53}$$

$$\lambda_s = -\frac{r(\tau_s, X_s, I_{1s})}{\tilde{r}(\tau_s, X_s, I_{1s})} \tag{54}$$

which are the same as the conditions as (7') and (9) in the baseline model. Since also Eq. (48) is equivalent to Eq. (6), in this case all first order conditions and therefore the equilibrium allocations are the same as in the baseline model.

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