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Reversal interest rate and macroprudential policy

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Abstract

Could a monetary policy loosening entail the opposite effect than the intended expansionary impact in a low interest rate environment? We demonstrate that the risk of hitting the rate at which the effect reverses depends on the capitalization of the banking sector by using a non-linear macroeconomic model calibrated to the euro area economy. The framework suggests that the reversal interest rate is located in negative territory of around -1% per annum. The possibility of the reversal interest rate creates a novel motive for macroprudential policy. We show that macroprudential policy in the form of a countercyclical capital buffer, which prescribes the build-up of buffers in good times, can mitigate substantially the probability of encountering the reversal rate, improves welfare and reduces economic fluctuations. This new motive emphasizes also the strategic complementarities between monetary policy and macroprudential policy.

Keywords: Reversal Interest Rate, Negative Interest Rates, Macroprudential Policy, Monetary Policy, ZLB

JEL Codes: E32, E44, E52, E58, G21

Non-Technical Summary

The prolonged period of ultra low interest rates in the euro area and other advanced economies has raised concerns that further monetary policy accommodation could entail the opposite effect than what is intended. Specifically, there is a risk that for very low policy rates a further monetary policy loosening might have contractionary effects. The policy rate enters a "reversal interest rate" territory, in which the usual monetary transmission mechanism through the banking sector breaks down.

In this paper, we show that a less well-capitalized banking sector amplifies the likelihood of encountering the reversal interest rate. This gives rise to a new motive for macroprudential policy. Building up macroprudential policy space in good times to support the bank lending channel of monetary policy, for instance in the form of a countercyclical capital buffer, mitigates the risk of monetary policy hitting a reversal rate territory, or alleviates the negative implications if it does.

A key feature in understanding the potential threat of a reversal rate is the behaviour of different interest rates. The ECB deposit facility rate, which is one of the key policy rates, and the average bank retail deposit rate paid to households co-moved strongly during the 2000s. Afterwards, the two rates decoupled substantially, highlighted by the fact that the deposit rate is still positive in 2019, whereas the policy rate is already negative. In contrast to this, the interest on government bonds followed closely the ECB deposit facility rate. This suggests that in an environment of very low interest rates the impact of the policy rate on retail and market interest rates can have negative repercussions on bank balance sheets through a declining deposit rate pass-through and losses on government bond holdings, which can potentially weaken the effectiveness bank lending channel of monetary policy transmission.

Using a newly developed non-linear macroeconomic model that captures the outlined stylized facts, we demonstrate the conditions where such a reversal rate could materialize. The model contains a carefully designed banking sector with three key features. First, banks are assumed to be capital constrained. Second, the banks have market power in setting the deposit rate. While the banks have market power for the deposit rate in good times, the market power depletes if the policy rate approaches a negative environment. As a consequence, monetary policy affects the deposit rate less if interest rates are low. Third, the banks face requirements to hold low risk government assets for a fraction of their funding based on regulatory constraints. The key prediction of the model is that the effect of a monetary policy loosening is ambiguous in an environment of very low interest rates. We show that the model endogenously determines the reversal interest rate in a region of around minus one percent.

The main novelty in our paper is the role of macroprudential policy in connection with the reversal interest rate. We incorporate macroprudential policy in the form of a countercyclical capital buffer that can impose additional capital requirements. The Basel Committee on Banking Supervision prescribes that the buffer is built up during a phase of credit expansion and can then subsequently be released during a downturn. We demonstrate that macroprudential policy can lower the probability of hitting the reversal interest rate. The banking sector builds up additional equity in good times, which can then be released during a recession. Having accumulated additional capital buffers during good times, the negative impact on the banks' balance sheets of a reduction of monetary policy rates is dampened in a low interest rate environment. Consequently, monetary policy becomes more effective during economic downturns and the reversal interest rate is less likely to materialise, which improves overall welfare. In the context of a "lower for longer" interest rate environment, the risk of entering a reversal interest rate territory creates a new motive for macroprudential policy as it can help to strengthen the bank lending channel.

The analysis has at least two important policy implications. First, building up macroprudential space in good times in the form of a positive countercyclical capital buffer has the potential to alleviate and mitigate the risks of entering into a reversal rate territory. Second, there are important strategic complementarities between monetary policy and a countercyclical capital-based macroprudential policy. The latter can help facilitate the effectiveness of monetary policy, even in periods of ultra low, or even negative, interest rates. Overall, the findings in this paper provide important insights into the relevance of financial stability considerations in monetary policy strategy discussions.

1 Introduction

The prolonged period of ultra low interest rates in the euro area and other advanced economies has raised concerns that further monetary policy accommodation could entail the opposite effect than what is intended. Specifically, there is a risk that a further monetary policy loosening might have contractionary effects for very low policy rates. The policy rate enters a "reversal interest rate" territory to use the terminology of Brunnermeier and Koby (2018), in which the usual monetary transmission mechanism through the banking sector breaks down. We show that a less well-capitalized banking sector amplifies the likelihood of encountering the reversal interest rate. This gives rise to a new motive for macroprudential policy. Building up macroprudential policy space in good times to support the bank lending channel of monetary policy, for instance in the form of a countercyclical capital buffer (CCyB), mitigates the risk of monetary policy hitting a reversal rate territory, or alleviates the negative implications if it does.

A key feature in understanding the potential threat of a reversal rate is the behaviour of different interest rates, which are shown for the euro area in Figure 1. The ECB deposit facility rate, which determines the interest received from reserves, and the average deposit rate paid to households co-moved strongly during the 2000s. In a more technical jargon, there was a (close to) perfect deposit rate pass-through of the policy rate. Afterwards, the two rates decoupled to some degree, highlighted by the fact that the deposit rate is still positive in 2019, whereas the ECB deposit facility rate is negative. Inspecting the distribution of overnight deposit rates across individual euro area banks as shown in Figure 1, there is a significant decrease in interest rates across the entire spectrum of banks since the policy entered negative rates in July 2014. While the banks did not impose negative rates initially in 2014, a substantial fraction of banks charged sub-zero deposit rates in December 2019. This emphasizes the changing nature of the deposit rate pass-through, which becomes increasingly imperfect with low interest rates. In contrast to this, the interest on government bonds, which is shown for the German one year bond yield as example, followed closely the ECB deposit facility rate. This suggests that the return on government bonds and central bank reserves, which together constitute a share of close to 25% of banks asset at the end of 2019, was below the interest rate paid on household deposits. This potentially weakens the balance sheet and limit monetary policy reductions.

Using a newly developed non-linear general equilibrium model that captures the outlined stylized facts, we demonstrate the conditions where a reversal rate could materialize. The proposed model is embedded in a quantitative New Keynesian model. Three key fea-

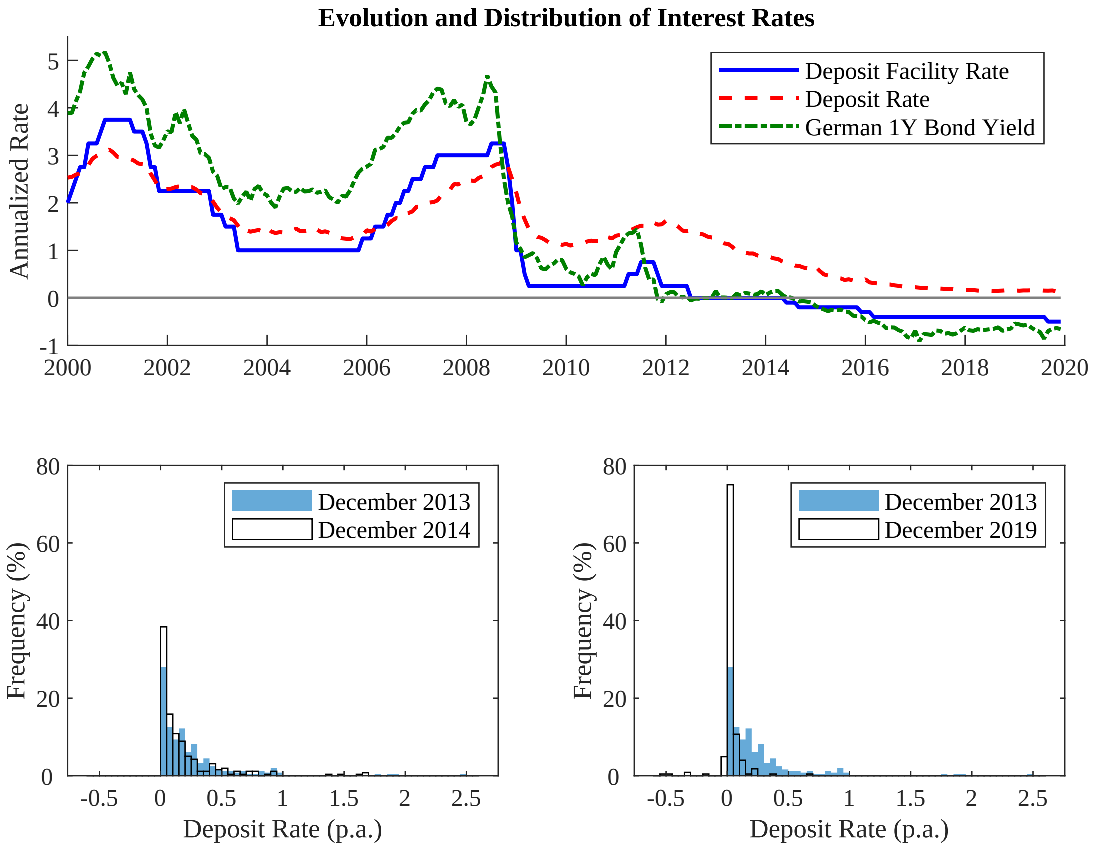

Figure 1: The upper panel shows the ECB deposit facility rate, average household deposit rate in the euro area and the German 1Y bond yield. The lower panel shows the distribution of overnight household deposit rates across banks. Details are in the Appendix B.

tures characterising the banking sector are instrumental for generating situations where a reversal rate may emerge. First, banks are assumed to be capital constrained which implies that shocks to their net worth can give rise to financial accelerator effects through the bank lending channel. Second, the banking sector is assumed to be monopolistic which implies an imperfect deposit rate pass-through of policy rates. Importantly, the degree to which policy rates are passed-through to deposit rates depends on the interest rate level. While the banks have market power for the deposit rate in good times, the market power depletes if the policy rate approaches a negative environment and the pass-through declines, which therefore has a negative impact on banks' net worth. Third, the banks face requirements to hold low risk government assets for a fraction of their funding (i.e. deposits), on which the return is assumed to equal the policy rate. This feature reflects both a reserve requirement for monetary policy purposes and regulatory liquidity requirements. The key implication of these frictions is that effect of a monetary policy loosening is ambiguous in a low interest rate environment.

In particular, the bank lending channel becomes state-dependent and the transmission of shocks is asymmetric. A lowering of the policy rate in a low interest rate environment has only a modest impact on the deposit rates due to the imperfect pass-through. Therefore, the positive impact on aggregate demand is modest. At the same time, a reduction of the policy rate lowers the return on banks' government asset holdings and reduces their net worth. If the latter channel is the dominant one for the banking sector's profitability, lending is reduced despite the monetary policy loosening. Accordingly, the model determines endogenously the level of the reversal interest rate.

The main novelty in our paper is that the role of macroprudential policy in this reversal interest rate environment is discussed. We incorporate macroprudential policy in the form of a countercyclical capital buffer that can impose additional capital requirements. The Basel Committee on Banking Supervision prescribes that the buffer is created during a phase of credit expansion and can then subsequently be released during a downturn. The buffer is asymmetric as it is state-dependent and restricted to be non-negative. We incorporate the outlined non-linear framework in our model. We demonstrate that macroprudential policy can lower the probability of hitting the reversal interest rate and alleviate the impact of the imperfect deposit rate pass-through. The banking sector builds up additional equity in good times, which can then subsequently be released during a recession. Having accumulated additional capital buffers during good times, the negative impact of monetary policy loosening on bank balance sheets is then dampened in a low interest rate environment. Consequently, monetary policy becomes more effective during economic downturns and the reversal interest rate is less likely to materialise, which improves overall welfare. In the context of a "lower for longer" interest rate environment, the risk of entering a reversal interest rate territory creates a new motive for macroprudential policy as it can help to strengthen the bank lending channel. We thereby provide evidence of important strategic complementarities between monetary policy and macroprudential policies.

We calibrate the model to match salient features of the euro area economy for the current low interest rate environment. The model predicts that the reversal interest rate is located around minus one percent per annum. The policy rate enters this territory with a probability of 2.7 percent. Macroprudential policy in the form of a countercyclical capital buffer rule makes it less likely that the reversal interest rate constrains monetary policy. In particular, the welfare optimising capital buffer rule reduces the probability to be at or below the reversal rate by around 26%. It also lowers the frequency of negative rates and the economic fluctuations. This illustrates that macroprudential policy can be a crucial tool in repairing the bank lending channel of monetary policy in a low interest rate environment.

The paper builds on recent theoretical contributions that connect negative interest rates and its impact on the bank lending channel. The model closest to ours is the seminal contribution of Brunnermeier and Koby (2018). We share that the reversal interest rate is endogenously determined in an economy with an imperfect-pass through. The main difference is that our model features macroprudential policy. Therefore, we can assess if macroprudential policy can help to restore the bank lending channel. In addition to this, the mechanism that generates the reversal interest rate differs. While in our model banks' holdings of government assets can generate a reversal interest rate, the maturity structure is the reason in Brunnermeier and Koby (2018). Eggertsson et al. (2019) show the importance of reserve holdings for the bank lending channel with negative interest rates. If the policy rate and deposit rates are disconnected, the bank's profitability is hurt. They consider an environment in which the deposit rates face a zero lower bound instead of an imperfect pass through. Their model implies that a negative interest rate cannot be expansionary, while in our framework the impact of policy rate adjustment depends on the endogenously determined reversal rate. Ulate (2019) emphasizes the trade-off between increasing demand and reducing bank profitability for negative interest rates. We demonstrate that this assessment gives a new motive for countercyclical macroprudential policy. In addition to these studies, De Groot and Haas (2020) show that negative interest rates can be used as a signal about future monetary policy. Balloch and Koby (2019) highlight the long run consequences of low bank run profitability in a low interest rates environment.

This paper is also related to the large literature about the interaction between monetary policy and macroprudential policies.[1] Whereas the role of the CCyB has been one of the main instruments in the literature, as a new feature, we incorporate the asymmetric design of the CCyB in our model using an occasionally binding policy rule. Van der Ghote (2018) shows the importance of a non-linear economy for the coordination of monetary and macroprudential policies. His work focuses on occasionally binding financial constraints, while our model contains a reversal interest rate.[2] Farhi and Werning (2016) consider monetary and macroprudential policy in economies with a zero lower bound. They show the importance of macroprudential policy in an environment with a binding zero lower bound. Korinek and Simsek (2016) consider macroprudential policies that tar-

[1]See for instance Darracq-Pariès, Kok and Rodriguez-Palenzuela (2011), Lambertini, Mendicino and Punzi (2013), Angelini, Neri and Panetta (2014), Quint and Rabanal (2014), Rubio and Carrasco-Gallego (2014), Benes and Kumhof, 2015, Collard et al. (2017), De Paoli and Paustian (2017), Gelain and Ilbas (2017), among many others.

[2]We do not incorporate an occasionally binding financial constraint to clearly identify the impact of the imperfect deposit rate pass-through and the government asset holdings.

get the indebtedness of households in an economy where the interest rates are bounded at zero. They highlight the importance of ex-ante macroprudential policy. Lewis and Villa (2016) demonstrate that a countercyclical capital requirement can mitigate the output contractions in the presence of a zero lower bound. In contrast to these studies, we assess macroprudential policy in a negative interest rate environment, where the intended effect of monetary policy can endogenously reverses. This creates a new strategic complementarity between negative interest rates and macroprudential policy. Macroprudential policy can help to restore the transmission of the bank lending channel.

The model is based on studies that incorporates financial frictions in dynamic stochastic general equilibrium model. First, we incorporate a bank leverage constraint as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). Second, the framework features imperfect pass through of the policy rate. Imperfect banking sector competition affects the transmission of monetary policy and hence the macroeconomic propagation, as shown in Darracq-Pariès, Kok and Rodriguez-Palenzuela (2011) and Gerali et al. (2010). Finally, we introduce a demand for banks to hold for a certain share of government assets on their balance sheet along the lines of Curdia and Woodford (2011) and Eggertsson et al. (2019). Our contribution is to combine these features in a non-linear general equilibrium framework. Based on this model, the optimal lower bound on monetary policy can be endogenously determined.

The paper is also connected to the empirical literature regarding negative policy rates. Jackson (2015) and Bech and Malkhozov (2016) document the early experiences with negative policy rates and find that a negative policy rate has a limited pass-through. Heider, Saidi and Schepens (2019) document that negative policy rates impact bank lending in the euro area. Banks are reluctant to pass through the policy rates to their depositors, which results in less lending for banks that depend heavily on deposit funding. Basten and Mariathasan (2018) also document the limited pass-through of negative interest rates using supervisory data from Switzerland. Altavilla et al. (2019) and Eisenschmidt and Smets (2019) outline that banks can charge negative interest rates on some portion of their deposits. Ampudia and Van den Heuvel (2018) show that the impact of an unexpected interest rate varies with the level of the interest rate due to the imperfect deposit rate pass-through. Our model incorporates the imperfect pass-through in a low interest rate environment in a structural macroeconomic model to determine the reversal interest rate and its interaction with macroprudential policy.

The paper is organized as follows. In Section 2, the non-linear macroeconomic model is introduced. We calibrate the model and parametrize the imperfect-deposit rate pass through in Section 3. In Section 4, we study the non-linear transmission of shocks and analyze the reversal interest rate. The optimal endogenous lower bound on monetary policy is derived. In Section 5, we incorporate macroprudential policy and study its interaction with the reversal interest rate. We conclude in Section 6.

2 The Model

The setup is a New Keynesian macroeconomic framework with a capital-constrained banking sector giving rise to financial accelerator effects as in Gertler and Karadi (2011). We embed two further financial frictions in this model that enable the possibility of a reversal interest rate: i) an imperfect pass-through of monetary policy to deposit rates as in Brunnermeier and Koby (2018) and ii) a reserve and liquidity requirement for the banking sector which generates substantial government asset holdings as in Eggertsson et al. (2019). The degree of the pass-through of the monetary policy rate to deposit rates depends on the level of interest rate. In particular, the pass-through declines if the monetary policy rate approaches a negative interest rate territory. Consequently, monetary policy is less effective in a low interest rate environment. At the same time, the reserve requirement forces the banks to hold a fraction of their deposits as government bonds. The fact that banks hold liquid government assets is motivated both by the reserve requirements for monetary policy purposes and by regulatory liquidity constraints. The return on government assets is assumed to have a perfect pass-through of the policy rate.

The implication of these two features is that when the policy rate is reduced to a sufficiently low level, the spread between the policy rate and deposit rate turns negative thereby suppressing bank net worth. Likewise, the reserve requirement generates profits during normal times, it can create losses during a recession. As banks become more capital constrained, they start to reduce credit. Therefore, the impact of monetary policy is state-dependent in this setup. The bank lending channel of monetary policy can break down and even reverse. The combination of these elements generates the possibility of a reversal interest rate that can have a sizable impact on the economy. To capture these state-dependencies, we are solving the model in its non-linear specification.

2.1 Model Description

Households The representative household is a family with perfect consumption insurance for the different members. The family consists out of workers and bankers with constant fractions. The workers elastically supply labor to the non-financial firms, while the bankers manage a bank that transfers its proceedings to household. Additionally, the household also owns the non-financial firms and receives the profits.

The household can hold deposits at the bank for which they earn the predetermined nominal rate R_t^D. In addition to this, the return also depends exogenously on the risk premium shock η_t, which follows an AR(1) process and is based on Smets and Wouters (2007). This shock is shown to be empirically very important to explain the great recession and zero lower bound episodes in estimated DSGE.[3] This shock creates a wedge that distorts the choice of deposits as it affects the decision between consumption and saving. At the same time, the risk-premium shock impacts the refinancing costs of the banking sector as it alters the payments on the deposits to the households. Its structural interpretation is further outlined in Appendix C.

The nominal budget constraint reads as follows:

$$P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1} - P_t D_t + P_t \Pi_t^P - P_t \tau_t \tag{1}$$

where P_t is an aggregate price index, C_t is consumption, W_t is the wage, L_t is labor supply, D_t are the deposits and Π_t^P are the real profits from the capital good producers, retailers and transfers with the banks and τ_t is the lump sum tax.

The household maximizes its utility that depends on consumption and leisure:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{2}$$

The first-order conditions are given as:

$$\beta R_t^D \eta_t E_t \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$

$$\chi L_t^{\varphi} = C_t^{-\sigma} W_t$$

where $\Lambda_{t-1,t} = C_t^{-\sigma} / C_{t-1}^{-\sigma}$ and Π_t is gross inflation. The risk premium shock creates a wedge in the Euler equation. An exogenous increase in the risk premium leads to a higher return on deposits. This induces the households to increase their deposit holdings and to postpone consumption, which lowers aggregate demand.

Banking Sector The banks' role is to intermediate funds between the households and non-financial firms. They hold net worth n_t and collect deposits d_t from households to buy securities s_t from the intermediate good producers at the real price Q_t and reserve

[3] For instance Barsky, Justiniano and Melosi (2014) and Christiano, Eichenbaum and Trabandt (2015) show this using linearized medium-sized DSGE models, among others. Gust et al. (2017) and Atkinson, Richter and Throckmorton (2019) are examples of estimated non-linear models featuring this shock.

assets a_t from the government. The flow of fund constraint in nominal terms is

$$Q_tP_ts_t + P_ta_t = P_tn_t + P_td_t \tag{3}$$

where the small letters indicates an individual banker's variable, while the capital letter denotes the aggregate variable. The banker earns the stochastic return R^K_{t+1} on the securities and pays the nominal interest R^D_t as well as risk premium for the deposits. The reserve assets earn the nominal gross return R^A_t, which is the policy rate. Leverage is defined as securities over assets:

$$\phi_t = \frac{Q_ts_t}{n_t}$$

To accrue net worth, the earnings are retained:

$$P_{t+1}n_{t+1} = R^K_{t+1}Q_tP_ts_t + R^A_tP_ta_t - R^D_t\eta_tP_td_t \tag{4}$$

which can be written in real terms as

$$n_{t+1} = \frac{R^K_{t+1}Q_ts_t + R^A_ta_t - R^D_t\eta_td_t}{\Pi_{t+1}} \tag{5}$$

The banker closes its bank with an exogenous probability of $1-\theta$ and transfers the accumulated net worth to households in case of exit. Therefore, the bankers maximize its net worth:

$$v_t(n_t) = \max_{s_t,d_t,a_t}(1-\theta)\beta E_t\Lambda_{t,t+1}\Big((1-\theta)n_{t+1} + \theta v_{t+1}(n_{t+1})\Big) \tag{6}$$

The banker is subject to an agency problem, which imposes a constraint on the leverage decision. The banker can divert a fraction λ of the banks assets as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). Since this fraction cannot be recovered by the households, funds are only supplied if the banker's net worth exceeds the fraction λ of bank assets. Furthermore, the banker faces a requirement to hold a certain amount of government assets that cover at least a fraction δ^B of the deposits. This requirement is meant to capture both regulatory liquidity constraints and the reserve requirements for monetary policy purposes.[4] The two constraints can be summed up as:

$$v_t(n_t) \geq \lambda(Q_ts_t + a_t) \tag{7}$$

[4]Curdia and Woodford (2011) and Eggertsson et al. (2019) use a function in which reserves lower the intermediation costs of the banks. The regulatory liquidity requirement is not explicitly modelled but provides an additional motivation for banks to hold substantial amounts of liquid government bonds and other assets on their balance sheets.

$$a_t \geq \delta^B d_t \tag{8}$$

The banker's problem is given as:

$$\psi_t = \max_{\phi_t} \mu_t \phi_t + \nu_t \tag{9}$$

$$\text{s.t.} \quad \mu_t \phi_t + \nu_t \geq \lambda\Big(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\Big) \tag{10}$$

where we define $\psi_t = \frac{v_t(n_t)}{n_t}$ and assume that the reserve ratio $a_t = \delta^B d_t$ is binding ad discussed later. μ_t is expected discounted marginal gain of expanding securities for constant net worth, ν_t the expected discounted marginal gain of expanding net worth for constant assets and R_t is the deposit rate adjusted for the holding of reserve assets:

$$\mu_t = \beta E_t \Lambda_{t,t+1} \left(1-\theta+\theta\psi_t\right) \frac{R^K_{t+1} - R_t}{\Pi_{t+1}} \tag{11}$$

$$\nu_t = \beta E_t \Lambda_{t,t+1} \left(1-\theta+\theta\psi_t\right) \frac{R_t}{\Pi_{t+1}} \tag{12}$$

$$R_t = (\eta_t R^D_t)\frac{1}{1-\delta^B} - R^A_t \frac{\delta^B}{1-\delta^B} \tag{13}$$

The banker's leverage maximization results in an optimality condition:

$$\xi_t = \frac{\lambda/(1-\delta^B) - \mu_t}{\mu_t} \tag{14}$$

where ξ_t is the multiplier on the market-based leverage constraint in the banker's problem. This constraint is binding if $0 < \mu_t < \lambda/(1-\delta^B)$, which requires that the return on the security is larger than the combined interest rate adjusted for inflation $E_t(R^K_{t+1} - R_t)/\Pi_{t+1} \geq 0$. The reserve asset ratio is binding as long as the expected return of the security is larger than the policy rate adjusted for inflation $E_t(R^k_{t+1} - R^A_t)/\Pi_{t+1} \geq 0$. Both constraints are binding at the relevant state space, which we verify numerically.

The individual leverage ϕ_t does not depend on bank specific components so that it can be summed up over the individual bankers, that is:[5]

$$Q_t S_t = \phi_t N_t \tag{15}$$

The aggregate evolution of net worth N_t is the sum of the net worth of surviving bankers N^S_t and newly entering banks that N^N_t that receive a transfer from the households:

$$N_t = N^S_t + N^N_t \tag{16}$$

[5]Similarly, the leverage ratio associated with reserve assets does not depend on bank specific components.

$$N_t^S = \theta N_{t-1} \frac{R_t^K - R_{t-1}\phi_{t-1} + R_{t-1}^D}{\Pi_t} \tag{17}$$

$$N_t^N = \omega^N \frac{S_{t-1}}{\Pi_t} \tag{18}$$

Non-financial Firms The non-financial firms are the intermediate good producers, retailers subject to Rotemberg pricing and capital good producers.

Intermediate good producers produce output using labor and capital:

$$Y_t = A^P K_{t-1} L_t \tag{19}$$

where A^P is the productivity. It sells the output at price P_t^M to the retailers. It pays the labor at wage W_t. The firm purchases capital at market price Q_{t-1} in period $t-1$, which is financed with a loan from the bank. It pays the state-contingent interest rate R_t^K to the banks. Thus, the maximization problem of the firm can be written as

$$\max_{K_{t-1}, L_t} \sum_{i=0}^{\infty} \beta \Lambda_{t,t+1} \left[P_t P_t^M Y_t + P_t Q_t (1-\delta) K_{t-1} - R_t^K P_{t-1} Q_{t-1} K_{t-1} - P_t W_t L_t \right] \tag{20}$$

This gives the nominal rate of return on capital:

$$R_t^k = \frac{(P_t^m \alpha Y_t / K_{t-1} + (1-\delta) Q_t)}{Q_{t-1}} \Pi_t$$

The final good retailers, which are subject to Rotemberg pricing, buy the intermediate goods and bundle them to the final good using a CES production function:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{21}$$

where $Y_t(f)$ is the demand of output from intermediate good producer j. Cost minimization implies the following intermediate good demand:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon} \tag{22}$$

where the price index P_t of the bundled good reads as follows

$$P_t = \left[\int_0^1 P_t(f)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{23}$$

The retailer then maximizes its profits

$$E_t\left\{\sum_{t=0}^{\infty}\left[\left(\frac{P_t(f)}{P_t}-MC_t\right)Y_t(f)-\frac{\rho^r}{2}Y_t\left(\frac{P_t(f)}{P_{t-1}(f)\Pi}-1\right)^2\right]\right\} \tag{24}$$

where $MC_t = P_t^M$ and Π is the inflation target of the central bank. This gives us the New Keynesian Phillips curve:

$$\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi}=\frac{\epsilon}{\rho^r}\left(P_t^m-\frac{\epsilon-1}{\epsilon}\right)+\beta E_t\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\frac{Pi_{t+1}}{\Pi_t}-1\right)\frac{\Pi_{t+1}}{\Pi}$$

Capital good producers have access to the function $\Gamma(I_t, K_{t-1})$ which they can use to create capital out of an investment I_t. The capital is then sold so that the maximization problem reads as follows:

$$\max_{I_t} Q_t\Gamma(I_t, K_{t-1})K_{t-1} - I_t \tag{25}$$

The real price of capital is then given as

$$Q_t = [\Gamma'(I_t, K_{t-1})K_{t-1}]^{-1}$$

The stock of capital evolves then as:

$$K_t = (1-\delta)K_{t-1} + \Gamma(I_t, K_{t-1})K_{t-1} \tag{26}$$

Monetary Policy and Imperfect Deposit Rate Pass Through The central bank sets the nominal interest rate for the reserve asset. It responds to inflation and output deviations, while it faces iid monetary policy shock ζ_t.[6] Furthermore, the central bank can set a lower bound $\tilde{R}^A$ that restricts the level of the interest rate. The policy rule reads as follows:

$$R_t^A = \max\left[R^A\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{Y_t}{Y}\right)^{\theta_Y}, \tilde{R}^A\right]\zeta_t \tag{27}$$

The lower bound gives the central bank the opportunity to endogenously restrain itself from lowering the policy rate below a specific rate as the model features a potential reversal interest rate. This level could be a negative or positive net interest rate as we will later determine based on welfare considerations. In contrast to this, a zero lower

[6] The advantage of an iid monetary policy shock is to avoid that the monetary policy shock could be used as a device to keep interest rates low for long and influence the economy via future expectations. De Groot and Haas (2020) discuss such a signalling channel in a negative interest rate environment.

bound exogenously restricts the central bank from setting a negative net interest rate.

However, there is an imperfect pass-through of the policy instrument to retail deposit rates as in Brunnermeier and Koby (2018). The margin on the deposit varies with the level of the policy rate R_t^A. The nominal interest rate on deposits is given as

$$R_t^D = \omega(R_t^A) \tag{28}$$

where $\omega(R_t^A)$ is a flexible functional form that can be fitted to the observed pass-through in the data. This approach can capture the varying market power of banks in setting the deposit rate. In particular, it can help to match the declining pass-through if the policy rate approaches low and negative interest rates. The functional form and the parametrization with the help of non-linear least squares are described in Section 3.

Government and Resource Constraint The government has a balanced budget constraint. It holds the reserve assets and taxes the households with a lump sum tax:

$$P_t \tau_t + P_t A_t = R_{t-1}^A P_{t-1} A_{t-1} \tag{29}$$

The resource constraint is:

$$Y_t = C_t + I_t + \frac{\rho^r}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2 Y_t \tag{30}$$

2.2 Competitive Equilibrium

The competitive equilibrium is defined as a sequence of quantities $\{C_t, Y_t, K_t, L_t, I_t, D_t, S_t, \Pi_t^P, N_t, N_t^E, N_t^N\}_{t=0}^{\infty}$, prices $\{R_t, R_t^D, R_t^A, R_t^K, Q_t, \Pi_t, \Lambda_{t,t+1}, w_t, i_t, i_t^D, P_t^M\}_{t=0}^{\infty}$, bank variables $\{\psi_t, \nu_t, \mu_t, \phi_t\}$, and exogenous variable $\{\eta_t\}_{t=0}^{\infty}$ given the initial conditions $\{K_{-1}, R_{-1}D_{-1}, \eta_{-1}\}$ and a sequence of shocks $\{e_t^{\eta}, \zeta_t\}_{t=0}^{\infty}$ that satisfies the non-linear equilibrium system of this economy provided in Appendix A.

2.3 Global Solution Method

The model is solved in its non-linear specification with global methods. This approach is necessary to capture the state-dependency of the monetary policy pass-through. In particular, this setting allows monetary policy to have a different quantitative as well as qualitative impact depending on the state of the economy. Another advantage of the non-linear approach is that agents take future uncertainty into account, which is particularly relevant due to the highly non-linear region of low and negative interest rates. The solution method is time iteration with piecewise linear policy functions based on Richter, Throckmorton and Walker (2014). The algorithm is described in more detail in Appendix E.

3 Calibration

The model is calibrated to the euro area economy with a particular emphasis on the current low interest rate environment. The considered horizon begins with 2000Q1 and ends in 2019Q4. The data to parametrize the model is mostly based on the ECB's statistical data warehouse and the AWM database, which is built for the ECB's large scale DSGE Model the Model II.[7] Appendix B contains the details regarding the data sources and construction.

Table 1 summarizes the calibration. The discount factor is set set to 0.9975 which corresponds to a risk free rate of 1% per annum. This is in line with the average estimate of 1.27 for the euro area from Holston, Laubach and Williams (2017).[8] The inflation target is set to 1.9 % percent to match the ECB's inflation target of close but below 2 percent. The Frisch Labor Elasticity $1/\psi$ equals 0.75 to match the evidence provided in Chetty et al. (2011). The disutility of labor aims that agents work 1/3 of their working time. The parameter α is set to 0.33 in line with the capital share of production. The depreciation rate is 0.025 to match an annualized depreciation rate of 10%. The elasticity of the asset price is parametrized to 0.25 as in Bernanke, Gertler and Gilchrist (1999). We target a mark-up of 10% so that $\epsilon = 11$. The Rotemberg parameter $\rho^r = 1000$ implies a 1% slope of the New Keynesian Phillips curve. The inflation and output response are set to 2.5 and 0.125, which are standard values in the literature. The endogenous lower bound $\tilde{R}^A = 0.995$ limits the potential interest rate cuts. The monetary authority does not lower the systemic component of the policy rate below minus two percent per annum.

Deposit Rate Pass-Through The pass-through is parametrized using data of bank retail deposit rates and the policy rate for the euro area. We use a weighted measure of different deposit rates to take into account the different maturities in the data. The policy rate is defined as the deposit facility rate. The evolution of both series can be seen in the upper panel of Figure 1. The imperfect deposit rate pass-through in the model is captured in the equation $R_t^D = \omega(R_t^A)$. For this mapping, we follow the functional form

[7] The AMW database provides data only until 2017Q4.

[8] Even though our value is slightly lower, this accounts for the trend of falling real interest rates.

Table 1: Calibration

Parameters	Sign	Value	Target
a) Preferences, Technology and Monetary Policy			
Discount Factor	β	0.9975	Risk free rate = 1% p.a.
Risk Aversion	σ	1	Risk Aversion = 1
Disutility of labor	χ	12.38	SS Labor Supply = 1/3
Inverse Frisch labor elasticity	φ	1.5	Frisch Elasticity = 1.5
Capital production share	α	0.33	Capital income share = 33%
Capital depreciation rate	δ	0.025	Annual depreciation rate = 10%
Elasticity of asset price	η_i	0.25	Elasticity of asset price = 25%
Investment Parameter 1	a_i	0.5302	$Q = 1$
Investment Parameter 2	b_i	-0.0083	$\Gamma(I/K) = I$
Elasticity of substitution	ϵ	11	Market power of 10%
Rotemberg adjustment costs	ρ^r	1000	1% slope of NK Phillips curve
Inflation	Π	1.0047	Inflation Target = 1.9% p.a.
Inflation Response	κ_π	2.5	Standard
Output Response	κ_Y	0.125	Standard
Endogenous Lower Bound	$\tilde{R}^A$	0.995	Lower bound of -2% p.a.
b) Deposit Rate Pass Through			
Pass Through Parameter 1	ω_1	-0.0008	Perfect pass through at SS
Pass Through Parameter 2	ω_2	0.0027	Markdown $R^A = \bar{R}^A = 0.056\%$ p.a.
Pass Through Parameter 3	ω_3	124.73	Imperfect pass through if $R^A < \bar{R}^A$
Banks Market Power	ς	0.001	Markdown if $R^A > \bar{R}^A = 0.056\%$ p.a.
c) Financial Sector			
Reserve Asset Requirement	δ^B	0.2545	Government asset share = 23% if $R^A < 1$
Survival Probability	θ	0.9	$R_K - R_D = 2\%$ p.a.
Diversion Banker	λ	0.1540	Leverage = 8
Proportional transfer to new banks	ω^N	0.00523	Uniquely determined from θ and λ
d) Shocks			
Persistence Risk-Premium Shock	ρ^η	0.75	Probability of negative policy rate
Std. Dev. Risk Premium Shock	σ^η	0.125%	Standard deviation of detrended output = 0.021
Std. Dev. Monetary Policy Shock	σ^ζ	0.0001	Small value to avoid distortion

in Brunnermeier and Koby (2018). This function separates the connection between the two rates in a region with an imperfect pass through ($R_t^A < \bar{R}^A$) and a region with a perfect pass through ($R_t^A \geq \bar{R}^A$), where the threshold parameter $\bar{R}^A$ is the deterministic steady state of the policy rate. The functional form is given as

$$R_t^D = \omega(R_t^A) = \begin{cases} \omega^1 + \omega^2 \exp(\omega^3(R_t^A - 1)) + 1 & \text{if } R_t^A < \bar{R}^A \\ R_t^A - \varsigma & \text{else} \end{cases} \tag{31}$$

where ω^1, ω^2 and ω^3 determines the shape of the imperfect deposit pass through and ς is related to banks market power.

We parametrize this functional form to capture the varying deposit rate pass-through for the euro area economy. Figure 2 shows the fit of the functional form with the actual data, where we use an approach that also incorporates non-linear least squares. Specifically, the shape parameters are calibrated to minimize the distance between the connection of the policy and deposit rate. This approach uses the observations that are below the threshold $\bar{R}^A$. Furthermore, we impose two restrictions on this minimization. First, there is a perfect deposit rate pass-through at the steady state.[9] Second, the markdown at the steady state is 0.56% in annualized terms. For the markdown, we use the measured average spread between the deposit rate and the deposit rate facility conditional on being at or above the steady state. This also gives the markdown for the region with perfect pass-through $\varsigma = 0.0014$. We then fit the curve using a non-linear least square approach that incorporates the descrobed constraint. The details are in the Appendix B.2. The fitted values of ω^1, ω^2 and ω^3 are -0.0008, 0.0027 and 124.73.

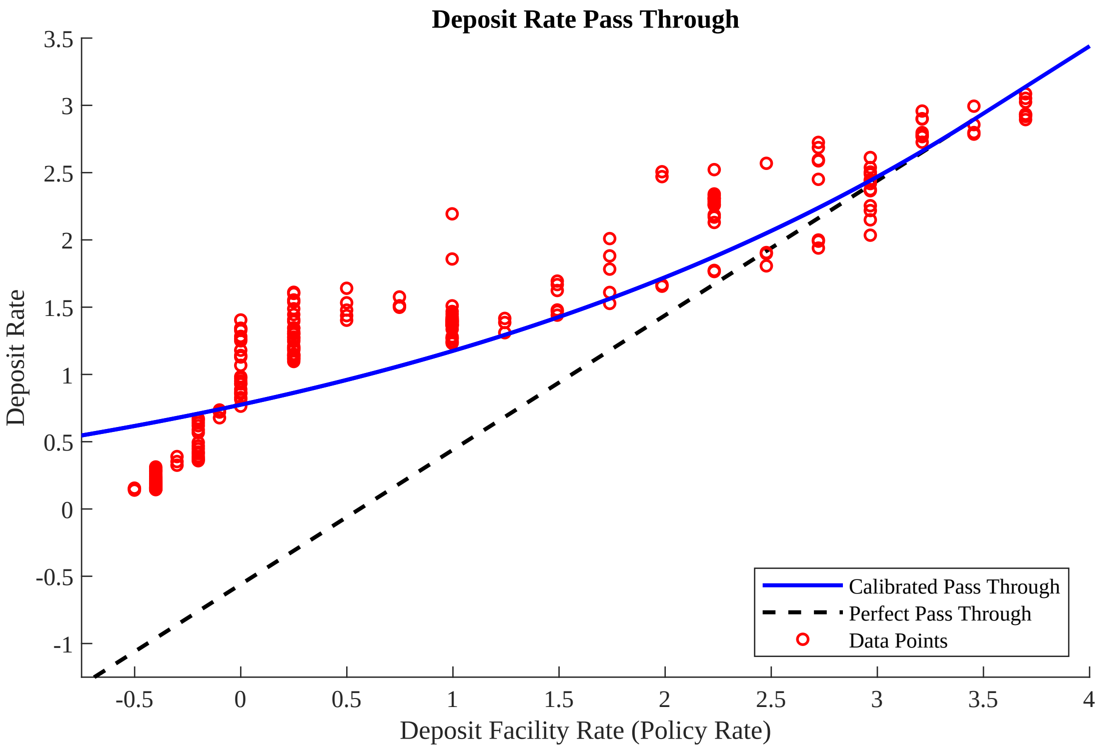

Figure 2: Figure shows the deposit rate pass-through estimated with non-linear least squares. The blue line is the imperfect pass-through, the black dashed line is a scenario with a perfect pass-through and the red dots refer to the data points.

Banking sector We calibrate the financial friction parameter λ to match a leverage ratio of 8. The banks have to hold at least a fraction δ^B of their deposits as government assets. Different measures of government asset shares in the banks' balance sheet can be compared in the lower panel of Figure 3. The different shares are government bonds

[9]This implies that the derivative of the function at the steady state equals 1.

only, government bonds plus required reserve assets and government plus reserve assets. We match the model to the broadest measure as our requirement captures government bonds as well as reserve assets. According to this measure, since the introduction of negative interest rates in the euro area in 2014 the share of government assets to total banking sector assets has edged up to almost 25%. In line with this, we target that banks have a government asset share of 23% during periods of negative interest rates. The corresponding value for the fraction of deposits is then 0.2545. The banker's survival rate θ is set to 0.9 to get an average spread between the return on capital and deposit rate of 2% p.a. at the steady state similar to the New Area Wide Model II. The average spread between the lending rate and deposit rate is around 2.5% p.a. in the data. However, there is a maturity mismatch in the data as loans are more long-term. Moreover, the survival probability θ and the financial friction parameter λ uniquely determine the endowment to new bankers ω^N.

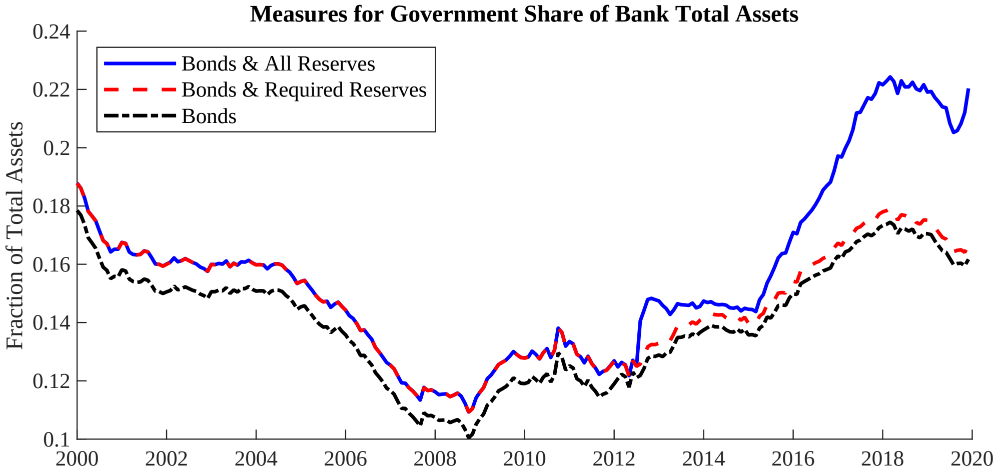

Figure 3: Figure shows different measures of the share of government assets in the banks balance sheet.

Shocks The risk premium shock is parametrized to match the fluctuations in output and the frequency of a negative interest rate environment. We set the standard deviation σ^η to 0.125% and the persistence to 0.75. The model prediscts a standard deviation of 2.2% for output in line with the data.[10] The policy rate falls below minus one percent with a 2.7% probability. A negative policy rate occurs with a probability of 5% in the model. A caveat is that the model underestimates the materialization of a negative policy rate compared to the recent experience in the euro, where the policy rate entered negative territory for the first time in June 11 in 2014 and is still below zero in the last quarter of

[10]The standard deviation of detrended real GDP is 2.1%. As the model does not have a trend, we detrend the logarithm of real output linearly.

2019. To increase substantially the episodes with negative interest rates poses a problem for a model featuring a zero lower bound as shown in Bianchi, Melosi and Rottner (2019) and Fernández-Villaverde et al. (2015). The reason is that episodes in which monetary policy is not effective affect the stability of the model. [11] The standard deviation of the monetary policy shock is set to negligible value. This ensures that this shock does not affect the moments of the model.

4 Non-Linear Transmission, Reversal Interest Rate and Optimal Lower Bound

This section deals with the non-linear transmission of the shocks and its implications for monetary policy conduct. In particular, the conditions that give rise to a reversal interest rate are discussed. The shocks have asymmetric effects as the deposit rate pass-through is state-dependent. The quantitative and qualitative impact of an innovation depends on the size of the shock, the sign of the shock, and the current state of the business cycle when the shock materializes. Specifically, the model predicts that accommodative monetary policy becomes contractionary, which is the reversal interest rate, conditionally on being in a severe recession. Finally, the optimal lower bound on the policy rate is assessed since it can be used to avoid that monetary policy reverses.

4.1 Impulse Response Functions and Non-Linearities

We begin with an impulse response analysis to demonstrate the non-linearities of the model.

Risk-Premium Shock Figure 4 shows the impulse response functions of a risk premium shock. The different lines are associated with different sizes and signs of the shock ϵ_t^η. We consider negative and positive shocks with the size of one and two standard deviations. The starting point of the economy is the risky steady state, which is the point to which the economy would converge if future shocks are expected and the realizations turn out to be zero (Coeurdacier, Rey and Winant, 2011). To begin with, the model has the standard financial accelerator which amplifies the impact of financial shocks. An increase

[11] Bianchi, Melosi and Rottner (2019) show that a high frequency of being at the zero lower bound can result in deflationary spirals so that there does not exist an equilibrium anymore. The probability of a constrained monetary policy leads to a vicious circle of low inflation, rising real interest rates, which in turn leads to lower inflation. Fernández-Villaverde et al. (2015) show that for instance a tax that affects the Euler equation can help to match the duration and frequency of a zero lower bound episode.

in the risk premium, which is a contractionary shock, affects the consumption and saving decision of the households as well as the refinancing costs of the banks. The households postpone consumption so that output drops. This affects banks as their return on assets is lower and asset prices falls. In addition, the funding costs of the banks increase. Both effects reduce the net worth and weaken the balance sheet of the banks which amplifies the shock via the financial accelerator mechanism. Monetary policy lowers the interest rate to mitigate the bust. However, the impact of such a policy is non-linear due to the imperfect deposit rate pass-through and the reserve requirement.

The stronger relative impact of a contractionary risk premium shock compared to an expansionary one demonstrates that monetary policy can lose its effectiveness. As can be seen in Figure 4, this asymmetry is visible from the reaction of output, the policy rates, bank net worth and leverage which all have a more pronounced response for a risk premium increase. Monetary policy is less effective in stabilizing the economy in a downturn as deposit rates move less than one-to-one due to the imperfect pass-through. This stems from two different channels that operate via the households and banks. First, the deposit interest rates offset less the increase in the wedge in the household's Euler equation. This results in a stronger drop in consumption. Second, the funding costs of the banking sector do not decrease much as the deposit rates are decoupled from the policy rate. At the same time, the spread of the reserve assets also diminishes. This together implies that the banks' net worth losses are comparatively more severe so that there is a strong contraction of lending and output. Importantly, the financial accelerator increases such effects.

Furthermore, another non-linear feature can be discerned from the fact that the size of the contractionary shock matters for how forcefully it is transmitted to the economy. The economy responds considerably more than twice as strong in case of a two-standard deviation compared to a one standard deviation shock increase. The reason is that the deposit rate pass-through becomes more sluggish the deeper the recession. This effect is reinforced through the government asset requirement. In contrast to this, the size of a decrease in the risk premium has less of an effect if the economy is initially at the steady state. There is a perfect pass-through in this part of the state space so that the size of the shock does not matter.

Monetary Policy Shock The transmission of monetary policy shocks with distinctive sizes and signs are shown in Figure 5. The economy is initially again at the risky steady state. A lowering of the monetary policy rate boosts the economy and vice versa. Reducing the policy rate affects the deposit rate, which induces households to consume more and

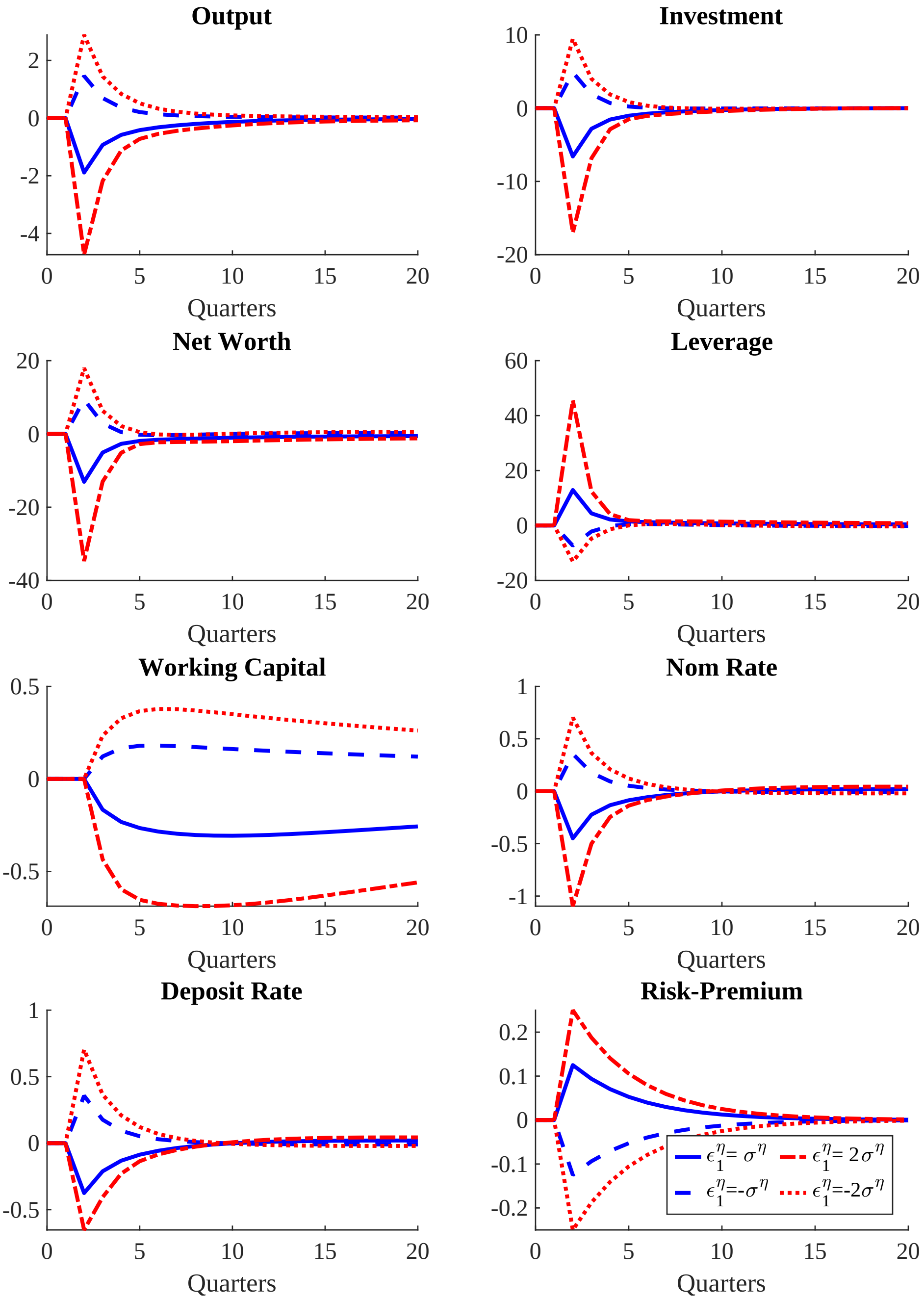

Figure 4: Impulse response functions of the risk premium shock that differ in the size and sign of the innovation. A one standard deviation increase (blue solid) and decrease (blue dashed) as well as a two standard deviation increase (red dash-dotted) and decrease (red dotted) for the innovation ϵ_t^η is shown. Deviations are in percentages. The economy is initially at the risky steady state.

reduces the refinancing costs of banks. This leads to an increase in aggregate demand and increases credit supply. Compared to the risk-premium shock, the non-linearities are less pronounced. As the relative impact of the monetary policy shock is small, it does not push the economy far away from the initial point. In this area, there is then almost perfect deposit rate pass-through so that monetary policy is very effective.

4.2 Reversal Interest Rate

The previous simulation suggests at first glance that accommodative monetary policy is effective and there is no reversal interest rate. This is due to the fact that the starting point of the simulations are the risky steady state which implies that the economy is in a region with normal interest rates and close to perfect pass-through of deposit rates. However, the impact of the monetary policy shock is asymmetric for varying interest rate environment. Therefore, combining the monetary policy shock with simultaneously occurring risk-premium shocks allows to assess the monetary policy shock at different points of the cycle.

Figure 6 shows the impulse responses of a negative one standard deviation monetary policy shock depending on different risk-premium innovations ϵ_1^η. The starting point is still the steady state, but the risk premium shock contracts the economy. The displayed paths show the percentage deviations between a path with and without the monetary policy shock for varying risk premium innovations. Depending on the size of the contractionary risk-premium shock, the monetary policy shock becomes less powerful. The expansionary impact of monetary policy shock decreases with the strength of the risk premium shock as can been in the responses of output, inflation, net worth and leverage. In fact, its impact even reverses for a scenario with $\epsilon_t^\eta = 3\sigma_t^\eta$. In this case, monetary policy, which is intended to be accommodative, actually reduces output, inflation and bankers' net worth. The reason is that the nominal interest rate is so low when the risk premium shock occurs that monetary policy does not only become less effective, but even harmful for the economy. It turns out that an increase in the nominal rate would actually be beneficial in such a state. The reason is that the reduction in the interest rate hurts the net worth of the banks sufficiently strongly due to their substantial government asset holdings. At the same time, the refinancing costs and aggregate demand of households are mostly unaffected as the deposit rate is very sticky in this state of the economy.

To better understand when and how the impact of the shock reverses, the solid line in Figure 7 shows the first period impact of an exogenous one-standard deviation monetary policy shock for varying risk premium shocks. If the risk premium shock is negative

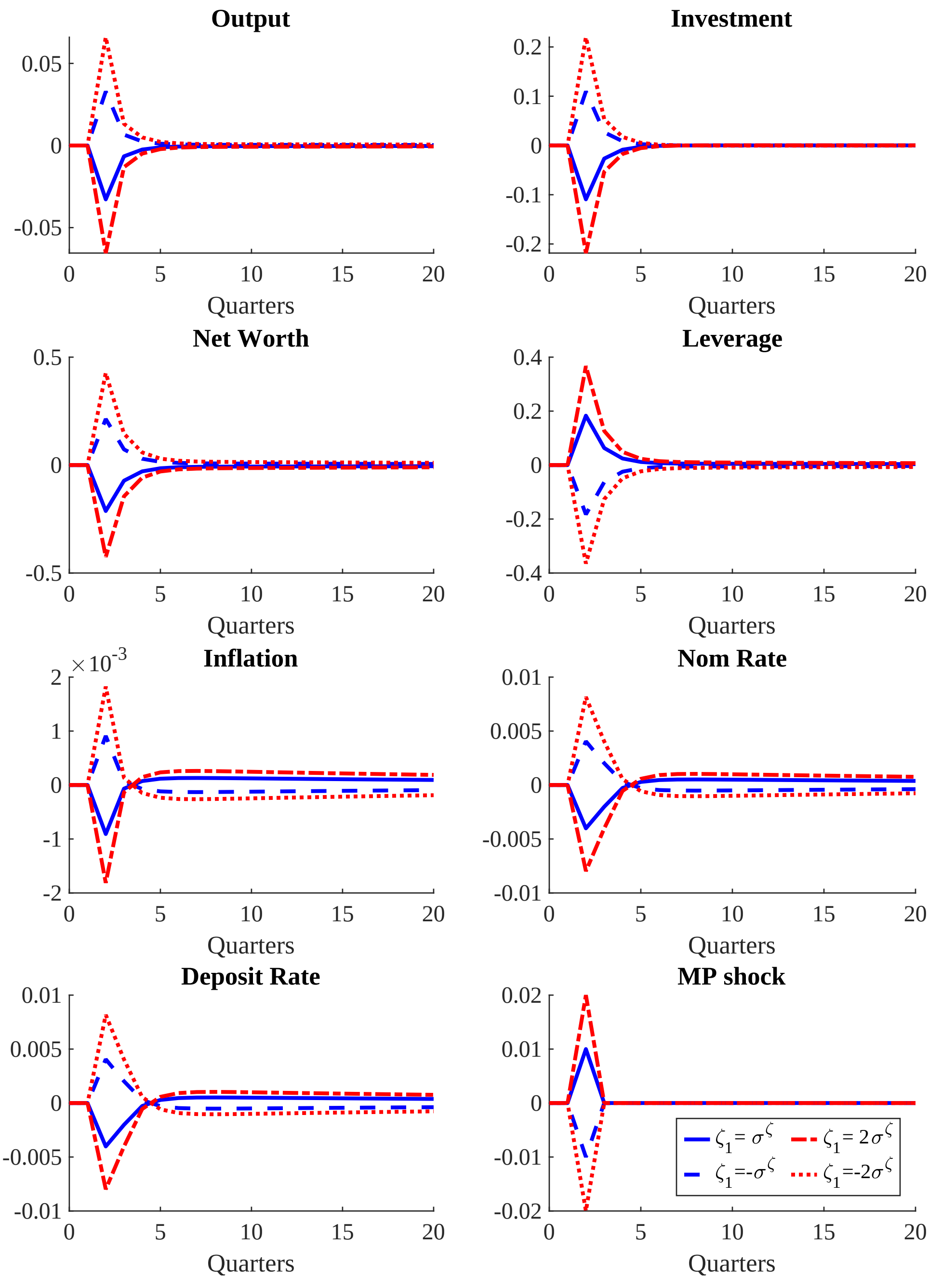

Figure 5: Impulse response functions of the monetary policy shock that differ in the size and sign of the innovation. A one standard deviation increase (blue solid) and decrease (blue dashed) as well as a two standard deviation increase (red dash-dotted) and decrease (red dotted) for the innovation ζ is shown. The responses are displayed in percentage deviations from the risky steady state, which is the initial point of the economy.

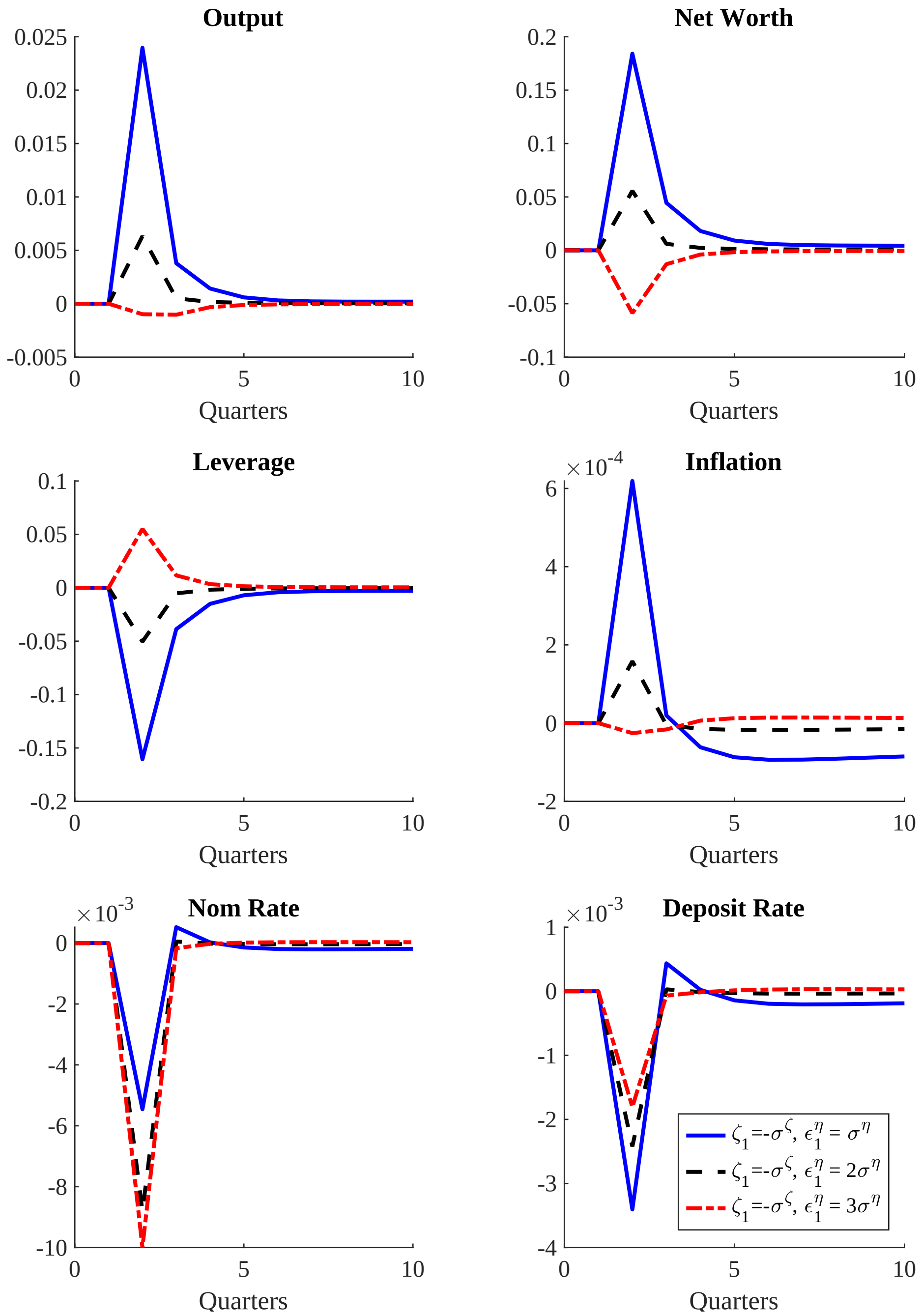

Figure 6: Response to a monetary policy shock combined with different sized risk premium shocks. IRFs display the difference between a shocked path, which introduces a negative one-standard deviation innovation for the monetary policy shock in quarter 1 $\zeta_1 = \sigma^\zeta$, and a path, in which the monetary policy innovation does not occur. Each line correspond to a different sized innovation in the risk premium shock that occurs simultaneously in the first period: $\epsilon_1^\nu = \sigma^\nu$ (solid); $\epsilon_1^\nu = 2\sigma^\nu$ (dashed), $\epsilon_1^\nu = 3\sigma^\nu$ (dash-dotted). The risky steady state is the initial point of the economy. The deviations are in percent.

or around zero, which can be interpreted as an expansion respectively tranquil times, monetary policy is very effective. Importantly, the nominal interest rate is high and is efficiently passed-through. In this case, there is no strong state-dependency. In contrast to this, monetary policy is less powerful in recessions than in booms. Around a risk premium shock of $\epsilon^{\eta} = 1.003$, which is around 3 standard deviations, output and inflations fall when monetary policy expands. This is explained by the strong drop in bank net worth in this state of the economy. Furthermore, we can see that the deposit rate pass-through declines as the drop in the nominal interest rate increases while the impact on the deposit rate becomes weaker. This ineffectively increases in the severity of the economic contraction. Hence, a sufficiently strong contraction implies that loose monetary policy not only becomes ineffective but potentially even harmful. Finally, we can see a flat line on the nominal rate and deposit rate, which indicates the lower bound of monetary policy.

Deposit Rate Pass-Through and Government Asset Holdings The deposit rate pass-through and the banking sector's government asset holdings are the key factors that generate state-dependent monetary policy in our framework. To analyse their impact, the frictions are relaxed one at a time.

First, a model featuring perfect deposit rate pass-through is considered. Accordingly, the deposit rate equals the policy rate adjusted for the mark down:

$$R_t^D = R_t^A - \varsigma \tag{32}$$

As a consequence, the pass-through is not state-dependent. Consequently, monetary policy transmission is equally effective in a expansion as well as in a recession. Thus, the central bank can stimulate demand and lower the refinancing costs for the banking sector also during a downturn. Simultaneously, the negative effects via the government bonds are shut down as the government spread is fixed, that is $R_t^A - R_t^D = \varsigma$. To show this, Figure 7 contrasts this setup with the full model for the first period response of a monetary policy shock. There are almost no state-dependencies anymore and monetary policy shock has almost the same impact over the same cycle. This can be seen in the relatively flat line. Consequently, monetary policy is always effective and this specification does not feature a reversal interest rate. This highlights the importance of including imperfect pass-through in the model as observed in the data.

The second experiment is to alter the amount of reserve assets. In particular, we consider a calibration in which the banks only hold half the share of government assets than what is assumed in the benchmark model calibration. Monetary policy is still assumed to be state dependent and is less powerful in recessions due to the imperfect deposit rate

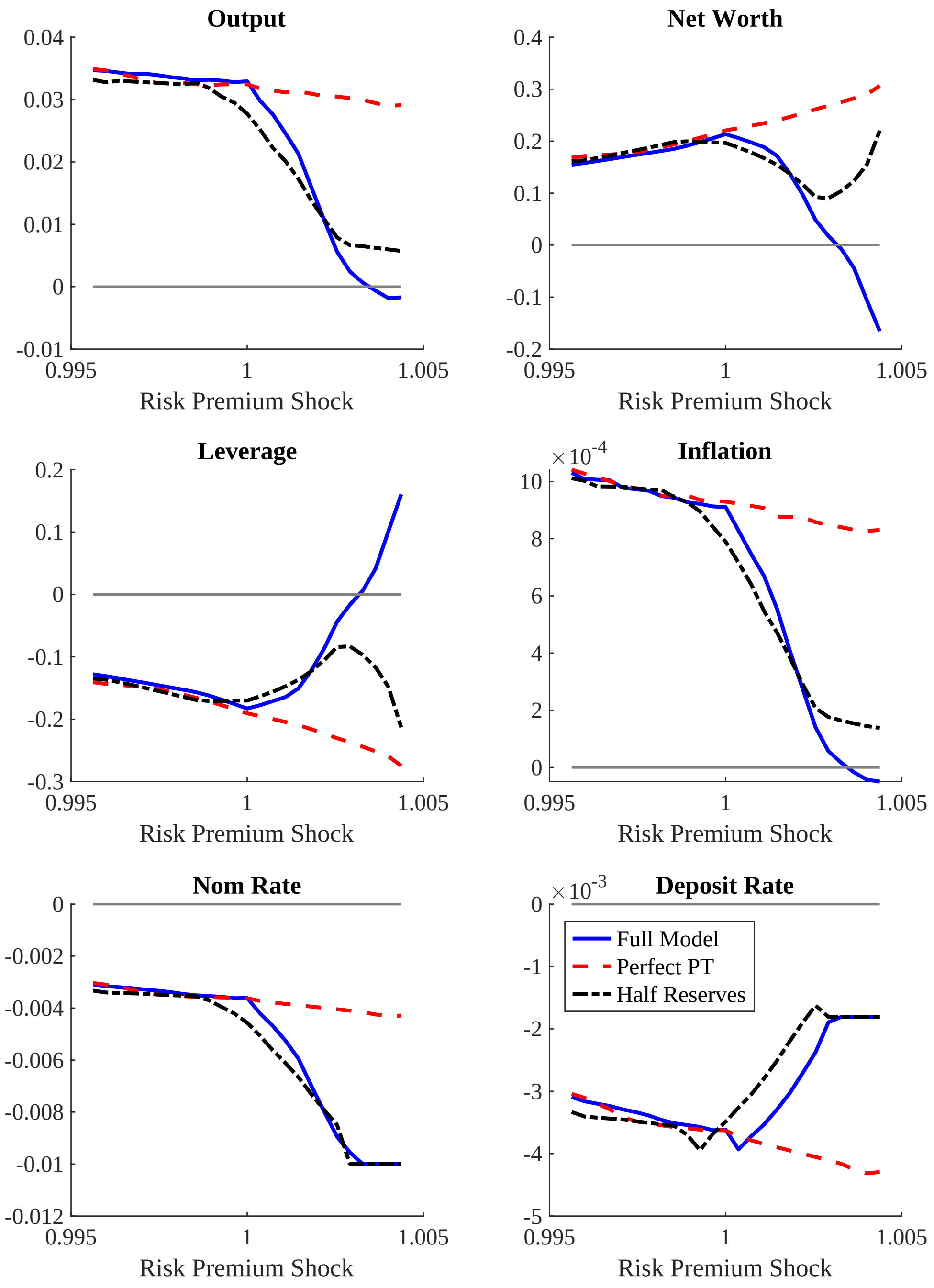

Figure 7: First period response to a monetary policy shock combined with different sized risk premium shocks. The vertical axis display the state-dependent difference for the period $t = 1$ response between a shocked path, which introduces a negative one-standard deviation innovation for the monetary policy shock $\zeta_1 = \sigma^\zeta$, and a path, in which the monetary policy innovation does not occur. The state-dependence results from the different sized risk premium shock that occurs simultaneously in the first period, which is displayed on the horizontal axis.

pass-through. However, a reversal rate does not materialize in this setting because monetary policy does not result in net worth losses of bankers as can be seen in Figure 7. While monetary policy becomes less effective for low interest rates, it does not become contractionary. In fact, monetary policy can stabilize the banking sector now even in a severe recession. This result can be seen in the increase on net worth for a risk premium shock around a value of 1.003. From this point onward, the optimal lower bound is binding so that the policy rate is capped. However, a policy shock can lower the interest rate further. A monetary policy accommodation is useful in this setup as the net worth of the banks increases. Thus, the overly restraining lower bound explains the increase in the effectiveness of a monetary policy shock.

4.3 Optimal Lower Bound of Monetary Policy

The model can generate a reversal interest rate, in which an exogenous lowering of the interest rate contracts the economy. Importantly, the same mechanism holds for the lower bound of monetary policy. A very loose lower bound can have adverse effects. The endogenous lower bound R^A can avoid such adverse effects. At the same time, setting a too conservative bound would restricts monetary policy unnecessarily. We evaluate the optimal lower bound in our model using the welfare of the households, which is given by:

$$W_0 = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{33}$$

In addition to this, we consider the distributional impact on output, financial sector variables and inflation.

Figure 8 shows the shape of welfare depending on the variation in the lower bound. The optimal lower bound for the interest is around -1% per annum. At this rate, the trade-off between lowering the interest rate with diminishing deposit rate pass-through and lowering banks' income on their government asset holding is optimally balanced. This is the endogenously determined reversal interest rate in our model. It should be noted that an overly restrictive lower bound such as keeping the policy rate at positive levels lowers welfare as the central bank forgoes potentially beneficial monetary accommodation. This highlights the problem with monetary policy accommodation when approaching a reversal interest rate territory. Monetary policy needs to balance inflation stabilization and the stability of the banking sector

We can compare the impact of the lower bound on the moments of the model. Table 2 shows the different selected moments for a very negative lower bound at -5% , the baseline

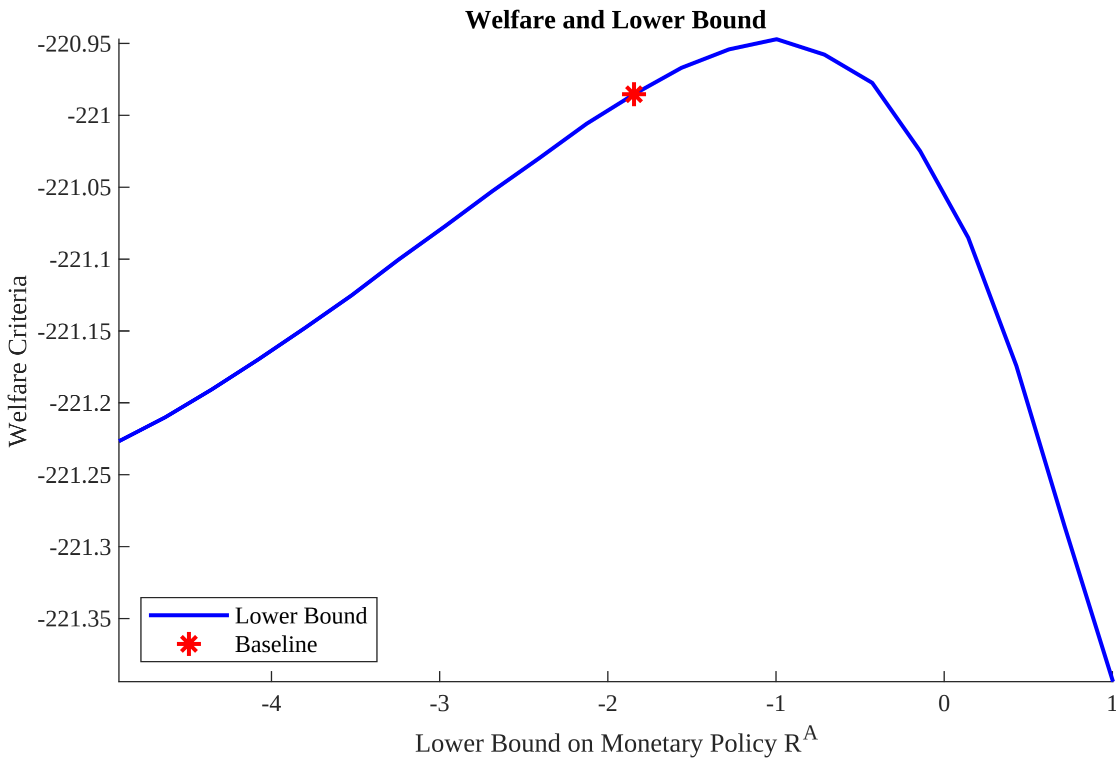

Figure 8: Welfare for different lower bounds of the policy rule R^A (in annualized net interest rate). The x-axis shows the interest rate in percent per annum. The star marks the baseline of $R^A = 0.995$.

case with -2% and a rather large and positive lower bound at 1% using a simulation of 200000 periods (after a burn-in period). The differences between a very negative lower bound and the baseline case are rather small. In particular, we can see that output and leverage is slightly larger in the economy with a lower bound at -2%. The banking sector is allowed to be more levered up as the banks do not face potential losses through the reversal interest rate. The strongest difference is in the behaviour of inflation, where a very low lower bound leads to increased inflation. In addition to this, leverage is much more volatile for a lower bound with $R_t^A = -5\%$. Nevertheless, the differences are rather small because interest rates are rarely so negative. If the economy would be more often in such a severe recession that can trigger very low rates, the differences in the moments would be stronger. At the same time, we see stronger response of the moments if the lower bound is set very tight. A lower bound of 1% results in considerably lower average output. We also see much more deflation as the central bank does not respond to deflationary pressure sufficiently. In addition to this, the economy is also much more volatile as monetary policy intervenes less.

The observation that the differences are larger for a high lower bound compared to a

Table 2: Selected Moments fo Varying Monetary Policy Lower Bound R^A

Moment	Model I: $R^A = -5$	Model II: $R^A = -2$	Model III: $R^A = 1$
a) Mean			
$\overline{Y}$	1.0040	1.0042	1.0015
$\overline{N}$	1.1477	1.1465	1.1517
$\overline{\phi}$	8.1943	8.2076	8.2282
$\overline{\pi}$	2.0157	1.9835	1.9727
b) Standard Deviation			
$\sigma(Y)$	0.0219	0.0223	0.02462
$\sigma(N)$	0.1675	0.1712	0.1907
$\sigma(\phi)$	5.1945	4.237	6.2787
$\sigma(\pi)$	0.4057	0.4152	0.4564

very low is a result from the fact that the economy only infrequently encounters very low interest rates where the reversal rate affects the economy. Therefore, an overly restricted monetary policy does not stabilize the economy for macroeconomic outcomes that occur frequently, while the occurrence of the reversal interest rate hurts the economy, but this is more of a tail event. This suggests that the decision between setting the optimal lower bound is a decision between financial stability and inflation stabilization if interest rates are low.

5 Macroprudential Policy

Macroprudential policy is an important tool that can be used to restore the transmission of monetary policy. It can help to improve the banking sector's capital position and hence, the resilience over the cycle. This is especially important in our setup as monetary policy loses efficiency and can even have a reverse impact due to the imperfect deposit rate pass-through and the requirement of holding government assets. A stronger capitalized banking sector could remedy this problem, which creates a role for macroprudential regulation in addition to the market-based requirement.

The macroprudential regulator can impose restrictions on the the bank capital ratio, which is defined as the inverse of leverage $1/\phi$. In particular, the regulator can require the banks to build additional capital buffers and release them subsequently. This policy instrument is based on the countercyclical capital buffer (CCyB) that was introduced as part of the Basel III requirements. The CCyB is build up during an expansion and can then be subsequently released, although never below 0%, during a downturn.

We incorporate this asymmetry using an occasionally binding macroprudential rule. The policy cannot reduce the capital requirements below the market-based capital demands. The regulator could theoretically set capital ratios below the market ones, but the market-based constraint would be the binding constraint for the banks. Thus, the market enforces a lower bound on regulatory capital requirements. This restriction diminishes the welfare gains of macroprudential policy as the scope of policy interventions during a downturn is limited.[12] This in particular highlights the importance of building up buffers in good time in order to create sufficient macroprudential space that can be employed to relax capital requirements in bad times and thus ensure macroprudential policy efficiency.

5.1 Macroprudential Policy Rule

The macroprudential regulator can set a time-varying capital buffer τ_t that imposes additional capital requirements. We use the following functional form:

$$\tau_t = \min\left\{(\phi^{MPP} - \phi_t^M)\tau^{MPP}, 0\right\} \tag{34}$$

where τ^{MPP} is the responsiveness and ϕ^{MPP} is the anchor value of the buffer. The rule responds to deviations of the market-based leverage ϕ_t^M from the anchor value ϕ^{MPP}. The asymmetry of the buffer depends directly on ϕ^{MPP}. For this reason, we consider different potential anchor values. The alternative approach would be to impose the non-negativity at a pre-imposed point such as the steady state. However, this would unncessarily restrict how macroprudential policy space is build-up and released. The min operator ensures that the buffer can only have non-negative values, which creates an asymmetry in the buffer in line with the Basel III requirements.

The market-based capital constraint stems from the agency problem of the banker (see equation (10)) and is repeated for convenience:

$$\phi_t^M = \frac{\nu_t + \frac{\delta^B}{1-\delta^B}}{\frac{\lambda}{1-\delta^B} - \mu_t} \tag{35}$$

This implicitly ensures that the buffer is countercyclical in our model if $\tau^{MPP} > 0$ since market-based bank leverage is countercyclical in the model. As the buffer is additive to the market-based equity requirements, the banks capital ratio reads as follows

$$\frac{1}{\phi_t} = \frac{1}{\phi_t^M} + \tau_t \tag{36}$$

[12] The usual approach in the DSGE literature is based on unrestricted rules without a lower bound in assessing countercyclical capital requirements. An exception is for instance Van der Ghote (2018), where the market-based leverage constraint restricts optimal macroprudential regulation.

Due to the non-negativity restrictions of the buffer, the policy instrument occasionally affects leverage. If the buffer is at zero, leverage is determined directly from ϕ_t^M. Therefore, the regulatory capital buffer is an occasionally binding constraint. It affects asymmetrically the capitalization of the banking sector depending on the state of the world. It imposes additional capital requirements if the banks hold many securities.

The buffer also impacts the transmission of the risk premium shock, which Figure 9 highlights. We compare the economy with and without the policy rule. The regulated economy uses $\tau^{MPP} = 0.016\%$ and $\phi^{MPP} = 9.75$.[13] This parametrization ensures the build-up of the buffer in good times and its subsequent release. The starting point for both economies is their respective risky steady state. Once the risk premium shock arrives in period 1, the economy with the buffer responds much less to a contractionary shock as the impact of the net worth channel is reduced. This emphasizes the dampening effect of the capital buffer in downturns. The initial response to an expansionary shock is very similar despite the additional requirements from the buffer. Thus, macroprudential policy has the potential to impact the reversal interest rate.

5.2 Macroprudential Policy and Reversal Interest Rate

We have shown and highlighted the importance of the reversal interest rate for economic outcomes. As the impact of monetary policy on banking sector leverage is key for the possibility to enter a reversal rate territory, a better capitalized banking sector can compensate losses and reduce the asymmetry of monetary policy shocks. To illustrate the beneficial role of macroprudential policy we compare the impact of the capital buffer rule on the reversal interest rate.

Figure 10 shows the initial impact of a negative one-standard deviation monetary policy shock for varying risk premium shocks. We compare the same macroprudential policy as before to the baseline scenario without a buffer. This clearly shows that macroprudential policy can be used to avoid reaching a territory with a reversal interest rate. As the buffer dampens contractionary shocks, the economy encounters less severe recessions and fewer interest rate reductions. This implies that monetary policy retains more of its efficiency for large ϵ_t^η and is less likely to enter the region with a reversal interest rate. The lower bound in the nominal interest rate plot demonstrates this. Macroprudential policy does not only stabilize output, it also affects the response on inflation. One feature of the reversal interest rate is that it lowers inflation. However, inflation response is pushed outwards depending on the strength of the buffer.

[13]The values are optimal regarding welfare as shown later.

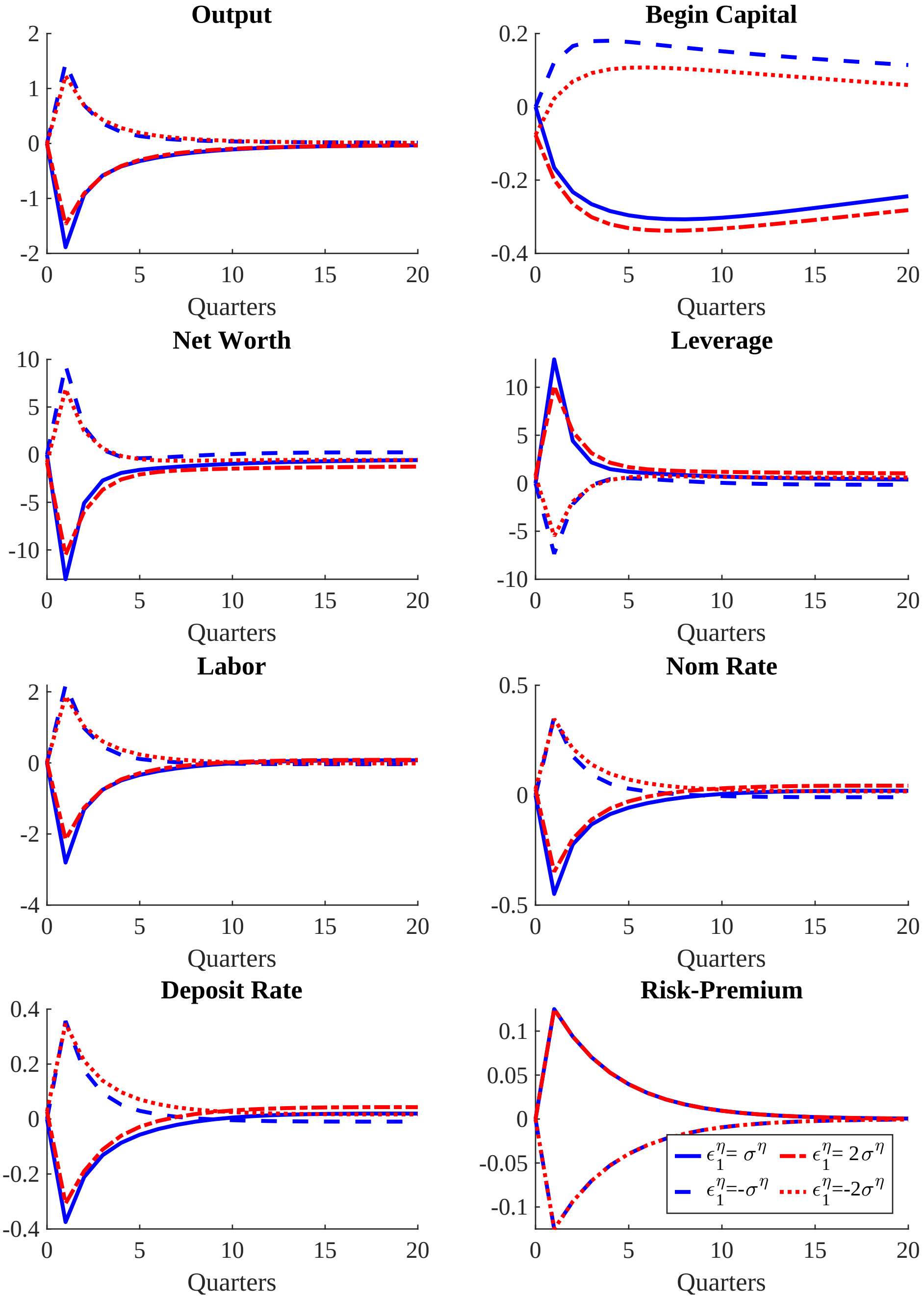

Figure 9: Impulse response functions of different the risk premium shock depending on the capital buffer is shown. A one standard deviation increase and decrease is shown for an economy without buffer (blue solid dotted resp. blue dashed) and an economy with a buffer $\tau^{MPP} = 0.016\%$ and $\phi^{MPP} = 9.75$ (red dash-dotted resp. red dotted). Starting point is the risky steady state of each economy Deviations are in percent relative to the risk steady state of the economy without a capital buffer rule.

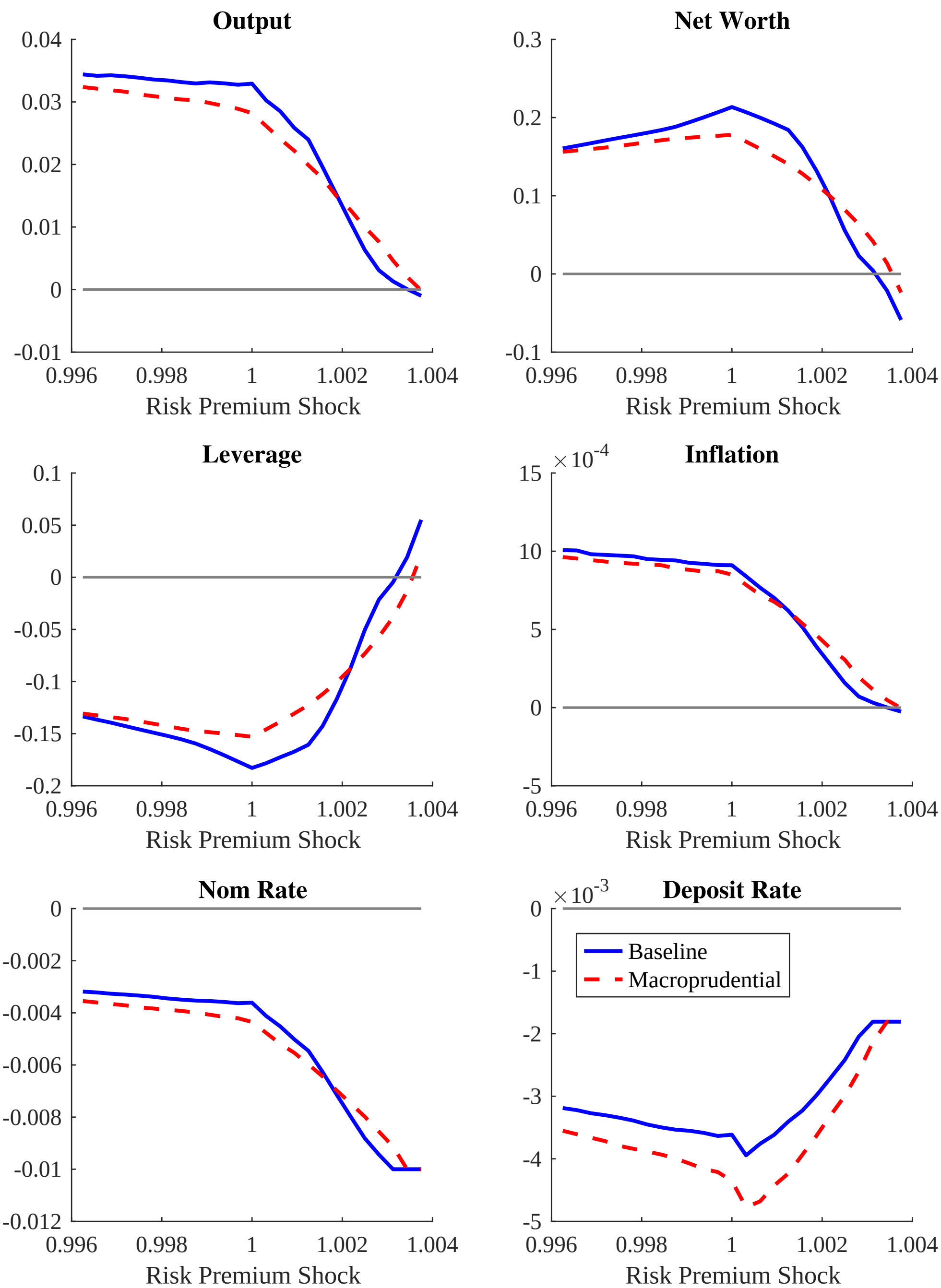

Figure 10: First period response to a monetary policy shock combined with different sized premium shocks to compare the baseline with the macroprudential rule. Vertical axis display the state-dependent difference for the period $t = 1$ response between a shocked path, which introduces a negative one-standard deviation innovation for the monetary policy shock $\zeta_1 = \sigma^{\zeta}$, and a path, in which the monetary policy innovation does not occur. The state-dependence results from the different sized risk premium shock that occurs simultaneously in the first period, which is displayed on the horizontal axis.

While the countercyclical capital buffer rule helps to restore the monetary policy transmission mechanism in case of large contractionary shocks, it also affects it in normal times. As the banking sector is better capitalized, monetary policy is less powerful during an expansion. For instance, the increase in output or net worth is smaller in an economy with an active macroprudential policy.

5.3 Optimal Macroprudential Policy

Macroprudential policy affects the distribution and can reduce the threat of the reversal interest rate. This notwithstanding, a too large capital requirement could also depress the economy. We evaluate this trade-off using the same welfare criteria as before, which is specified in equation (33). Figure 11 shows the welfare depending on the variation in the rules. We show the changes in welfare using different anchor values. For each anchor value, the optimal level of responsiveness is calculated and used. Macroprudential policy can improve welfare as can be seen in the hump-shaped welfare function. It is also above the baseline scenario without the buffer. The optimal macroprudential policy rule has $\phi^{M} = 9.75$, where $\tau^{MPP} = 0.016\%$. In this exercise, we jointly maximize over the two parameters related to the buffer. Appendix D contains more details about the interactions between the parameter ϕ^{MPP} and the responsiveness of the rule τ^{MPP}.

In setting the rule, the regulator faces a trade-off between stabilizing the economy and imposing too large buffers. While buffers are costly in good times, they stabilize the economy in bad times. A too low buffer does not create enough macroprudential space that can be used during a severe downturn. It should be noted that the positive impact of this rule results from the reversal interest rate and the imperfect deposit rate pass-through. For instance, in an economy with a perfect pass-through, the proposed macroprudential policy rules to welfare losses. In fact, it would be optimal to not have the capital rule (or to set $\tau^{MPP} = 0$) as the the costs of building-up the buffers outweigh the benefits in this economy without a reversal rate.

Figure 12 compares the impact of the buffer on the distribution of economic variables.[14] In particular, the optimal policy is contrasted to an economy without macroprudential policy. This shows the trade-off between stabilization in crisis times and the potential costs in good times. The optimal buffer reduces the risk of large output contraction since the left tail of output is much less fat with macroprudential policy. The standard deviation of output falls by 11 percent due to the buffer. The reason is that the banking sector

[14] The density functions are estimated with an Epanechnikov Kernel based on a simulation of 200000 periods after a burn-in period.

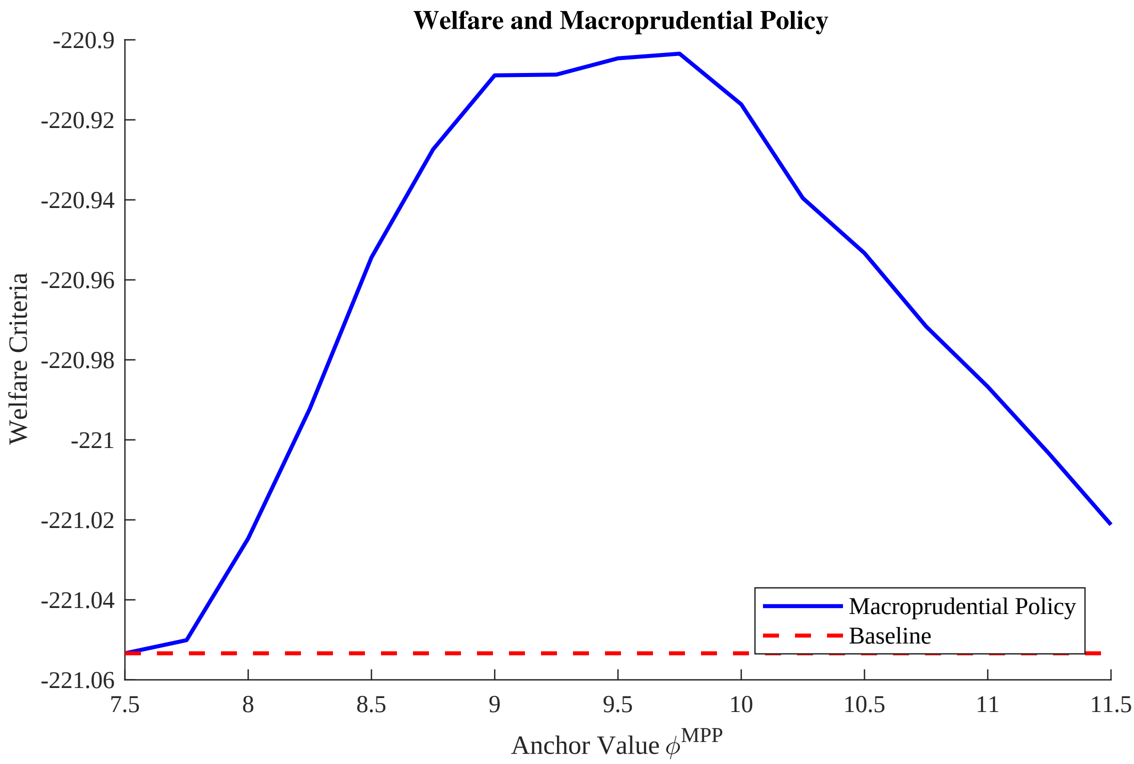

Figure 11: Welfare for different anchor values ϕ^{MPP}, which is varied on the horizontal axis. The response to deviations τ^{MPP} is set optimally to maximize welfare for each value of ϕ^{MPP}.

with a buffer is better capitalized. It can be seen that this economy has a lower right tail for leverage. This also implies that the economy is less likely to encounter negative interest rates. In particular, the buffer decreases the likelihood of negative interest rates by around 23 percent. The interest rate is less likely at or below minus one percent, which is the optimal lower bound. The bank capital rule lowers the probability of a policy rate below -1% by 26 percent.

At the same, the buffer can be costly in good times as the buffer is build-up in good times. Therefore, an expansion is smaller as can be seen in the left tail of output and net worth. The impact on inflation is very small. The macroprudential policy reduces the left tail slightly.

5.4 Interaction with Lower Bound on Monetary Policy

Macroprudential and monetary policy are strategic complementaries in the model. Therefore, it is important to understand the interaction of macroprudential policy with different lower bounds for monetary policy. To address this question, we compare the different lower bounds for an economy without and with macroprudential policy, which can be

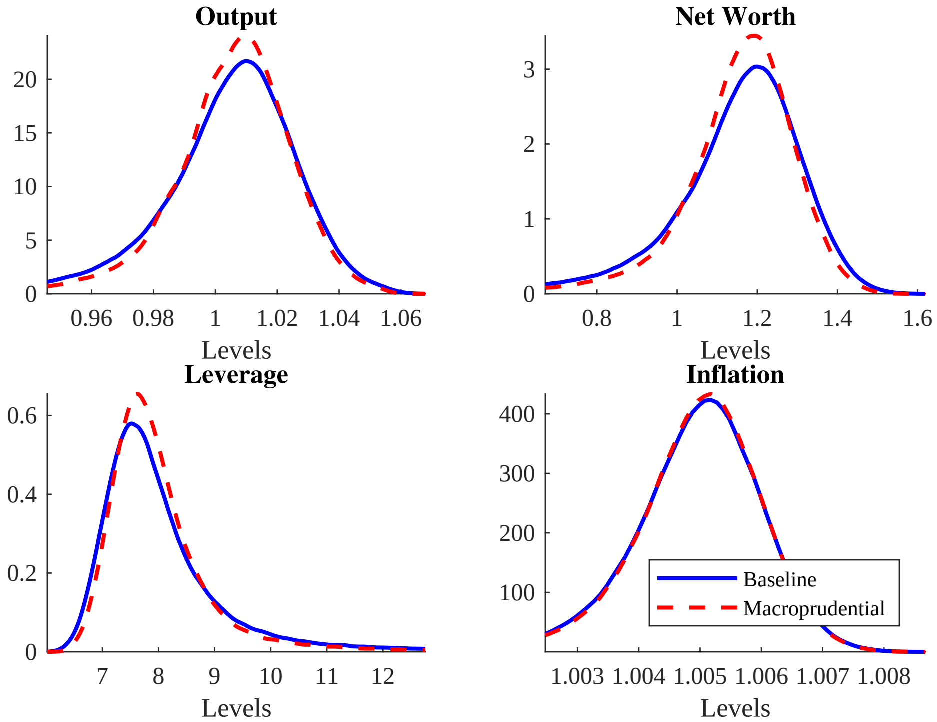

Figure 12: Density functions for varying macroprudential rules: baseline economy without macroprudential versus the optimal rule Each distribution is estimated using an Epanechnikov kernel function based on a simulation of 200000 periods (after burn-in).

seen in Figure 13. For each lower bound, we choose the optimal macroprudential policy to calculate welfare.[15] While both welfare curves are hump-shaped, welfare under the macroprudential rule is higher. Macroprudential policy helps to avoid that the economy enters reversal rate territory. As it stabilizes the banking sector, the recession and the threat of ultra low interest rates is less severe. Via this channel, the welfare optimising capital rule improves welfare independent of the specific lower bound.

We can also see that the capital buffer does not affect directly the choice of the optimal lower bound. The reason is that the macroprudential policy space is already released once the policy rate is lowered to such a negative territory. If the macroprudential policy space would affect also the capital holdings in a negative region of -1%, the lower bound would adjust. This could be the case if the central bank would require very large buffer holdings or increase the general level of capital requirements.

In addition to the increase in welfare, the macroprudential policy results in a more flat curve. The capital buffer rule smoothes the fluctuations and the economy is less often in such a low interest rate area. A suboptimal lower bound, which either restricts monetary policy very much or allows a too negative policy rate, has then less of an impact. In other

[15]This implies that we maximize ϕ^{MPP} and τ^{MPP} for each value of τ^{MPP}.

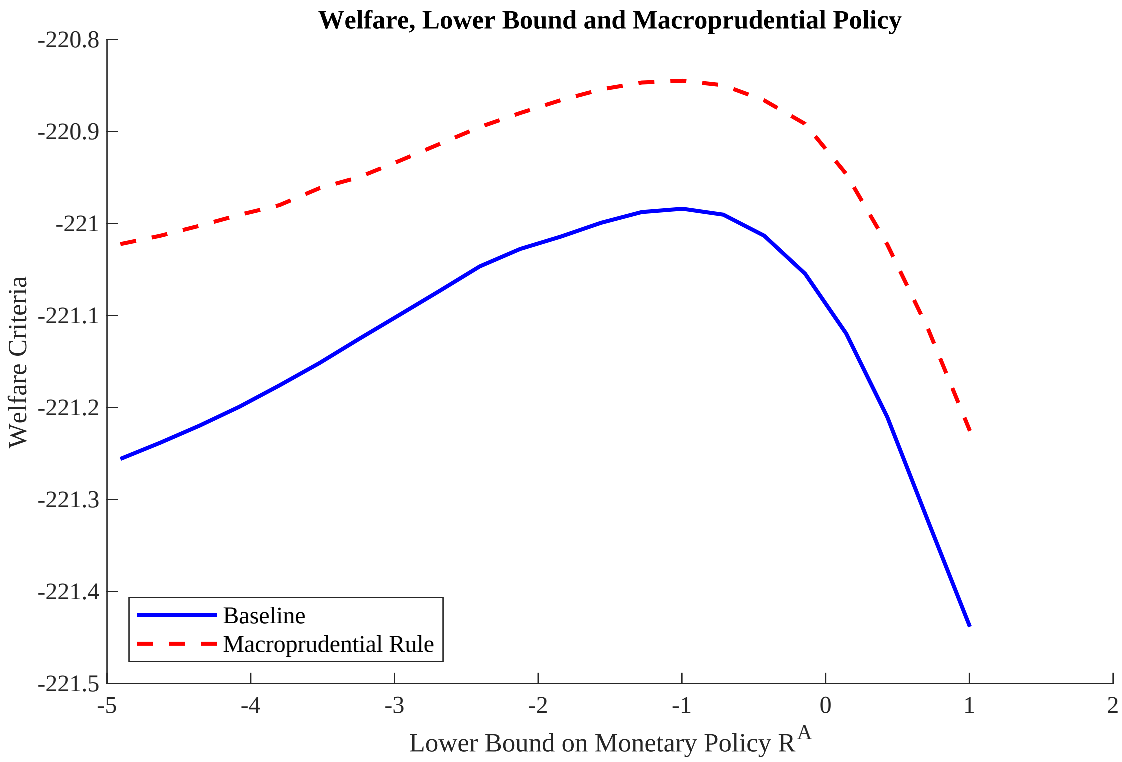

Figure 13: Welfare with and without macroprudential policy for different lower bounds on monetary policy. The macroprudential policy rule parameters ϕ^{MPP} and τ^{MPP} are optimized seperately for each lower bound.

words, macroprudential policy mitigates the danger of either too loose or too restrictive monetary policy in a very deep recession. This connection adds further to the strategic complementarity between macroprudential and monetary policy in a low interest rate environment.

6 Conclusion

In this paper, using a novel non-linear general equilibrium model for the euro area, we have shown how shocks hitting the economy may give rise to asymmetric effects depending on the state of the economy. Conditional on being in a severe recession, our model predicts the possibility of a reversal rate where an accommodative lowering of the policy rate may give rise to a contraction of output. This also allows us to derive an optimal lower bound for the policy rate below which monetary policy loses its effectiveness.

We also demonstrated an important link between the role of banks and bank leverage for the effectiveness of monetary policy transmission and the reversal rate. Specifically, there are two financial frictions in the model that enables the possibility of a reversal rate: (i) an imperfect deposit rate pass-through due to a monopolistic banking sector which becomes more sluggish as policy rates approach zero or become negative and (ii) a reserve requirement which may create losses during recessions. Furthermore, as banks are capital constrained negative shocks affect their net worth and amplify via the financial accelerator. We show that a less well-capitalized banking sector enhances the likelihood that monetary policy loses its potency and also the risk of entering reversal rate territory.

The analysis has at least two important policy implications. First, macroprudential policy using a countercyclical capital buffer approach has the potential to alleviate and mitigate the risks of entering into a reversal rate territory. Second, there are important strategic complementarities between monetary policy and a countercyclical capital-based macroprudential policy in the sense that the latter can help facilitate the effectiveness of monetary policy, even in periods of ultra low, or even negative, interest rates. Overall, the findings in this paper provide important insights into the relevance of financial stability considerations in monetary policy strategy discussions.

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A Non-Linear Equilibrium Equations

Households

$$C_t = W_t L_t + D_{t-1}\frac{R^D_{t-1}}{\Pi_t}\eta_{t-1} - D_t + \Pi^P_t - \tau_t$$

$$\beta R^D_t \eta_t E_t \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$

$$\chi L^{\varphi}_t = C^{-\sigma}_t W_t$$

Banks

$$\mu_t\phi_t + \nu_t \geq \lambda\Big(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\Big)$$

$$\psi_t = \mu_t\phi_t + \nu_t$$

$$\mu_t = \beta E_t\Lambda_{t,t+1}\left(1-\theta+\theta\psi_t\right)\frac{R^K_{t+1}-R_t}{\Pi_{t+1}}$$

$$\nu_t = \beta E_t\Lambda_{t,t+1}\left(1-\theta+\theta\psi_t\right)\frac{R_t}{\Pi_{t+1}}$$

$$Q_t S_t = \phi_t N_t$$

$$R_t = (\eta_t R^D_t)\frac{1}{1-\delta^B} - R^A_t\frac{\delta^B}{1-\delta^B}$$

$$N_t = N^S_t + N^N_t$$

$$N^S_t = \theta N_{t-1}\frac{R^K_t - R_{t-1}\phi_{t-1} + R_{t-1}}{\Pi_t}$$

$$N^N_t = \omega^N\frac{S_{t-1}}{\Pi_t}$$

Production, Investment and New Keynesian Phillips Curve

$$Y_t = A^P K^{\alpha}_{t-1} L^{1-\alpha}_t$$

$$W_t = P^m_t(1-\alpha)Y_t/L_t$$

$$R^k_t = \frac{(P^m_t\alpha Y_t/K_{t-1} + (1-\delta)Q_t)}{Q_{t-1}}\Pi_t$$

$$Q_t = \frac{1}{(1-\eta_i)a_i}\Big(\frac{I_t}{K_{t-1}}\Big)^{\eta_i}$$

$$K_t = (1-\delta)K_{t-1} + (a_i(I_t/K_{t-1})^{(1-\eta_i)} + b_i)K_{t-1}$$

$$\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi} = \frac{\epsilon}{\rho^r}\left(P^m_t - \frac{\epsilon-1}{\epsilon}\right) + \beta E_t\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\frac{Pi_{t+1}}{\Pi_t}-1\right)\frac{\Pi_{t+1}}{\Pi}$$

Policy Rule, Interest Rates, Government Budget Constraint and Aggregate Resource Constraint

$$R_t^A = \max\left[R^A\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{Y_t}{Y}\right)^{\theta_Y}, \tilde{R}^A\right]\zeta_t$$

$$R_t^D = R_t^A - \omega(R_t^A)$$

$$R_t^D = \mathbf{1}_{R_t^A \geq R^{A\,SS}}\left[R_t^A - \varsigma\right] + (1 - \mathbf{1}_{R_t^A \geq R^{A\,SS}})\left[\omega^1 + \omega^2 \exp(\omega^3(R_t^A - 1)) + 1\right]$$

$$\tau_t + A_t = \frac{R_{t-1}^A}{\Pi_t}A_{t-1}$$

$$Y_t = C_t + I_t + \frac{\rho^r}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2 Y_t$$

A.1 Occasionally Binding Regulatory Constraint

The non-negative capital buffer is

$$\tau_t = \min\left\{\tau^{MPP}(\phi^{MPP} - \phi_t^M), 0\right\} \tag{37}$$

The market imposed leverage constraint is given from the run-away constraint

$$\phi_t^M = \frac{\nu_t + \frac{\delta^B}{1-\delta^B}}{\frac{\lambda}{1-\delta^B} - \mu_t}$$

Banks leverage is then given as

$$\phi_t = \left(\frac{1}{\phi_t^M} + \tau_t\right)^{-1} \tag{38}$$

B Data and Calibration

B.1 Data Sources and Construction

This section describes the data source and construction. Table 3 shows all used series and their source. We use euro area data from 2002Q1 until 2019Q4.[16]

Deposit Rate The deposit rate weights the different lending rates for varying maturities, where the rates are from ECB SDW MIR data and the volume is based on the ECB SDW - BSI data. The used rates are the overnight deposit rate, deposit rate up to 1 year

[16] The data from the euro area has a changing composition.

for new business, deposit rate over 1 and up to 2 years for new business and the deposit rate over 2 years for new business. Their contribution is weighted with their relative outstanding amount in the balance sheet. All different rates and outstanding amounts are for deposits from households. The constructed deposit rate R_t^D reads then as follows:

$$R_t^D = \frac{DS0_t \times RD0_t + DS1_t \times RD1_t + DS2_t \times RD2_t + DS3_t \times RD3_t}{DS0_t + DS1_t + DS2_t + DS3_t} \tag{39}$$

Lending Rate The lending rate uses data from the ECB SDW - MIR data and the volume to weight is based on BSI data. For the lending rate, we use up to 1 year, over 1 year and below 5 years, and over 5 years to non-financial corporates and outstanding amounts. The volume data has the same maturity and is the outstanding amount to all non-financial corporations. The constructed lending rate R_t^K is the weighted index of the different rates:

$$R_t^K = \frac{LR1_t \times LS1_t + LR2_t \times LS2_t + LR3_t \times LS3_t}{LS1_t + LS2_t + LS3_t} \tag{40}$$

Policy Rate The main policy rate is the ECB's deposit facility rate. Euriobor 3-month and the Eonia rate are the typical alternatives in the New Keynesian literature for the Euro Area.

Government Assets The share of government assets uses data from the ECB SDW - BSI data. We use loans to Euro area government hold by Monetary Financial Institutions (MFIs), Euro area government debt securities hold by MFIs, required reserves hold by credit institutions and excess reserves hold by credit institutations.[17] This is compared to the total assets held by the MFIs. The consolidated balance sheet of the euro area MFIs is used for each series. The different measures include to a different extent the reserves:

$$\frac{A_t^1}{S_t + A_t^1} = \frac{LG + LS}{TA} \tag{41}$$

$$\frac{A_t^2}{S_t + A_t^2} = \frac{LG + LS + RR}{TA} \tag{42}$$

$$\frac{A_t^3}{S_t + A_t^3} = \frac{LG + LS + RR + ER}{TA} \tag{43}$$

The different series can be seen in the lower panel of Figure 3 in the main text.

[17] There are two important regulatory changes for the reserve requirement. Initially, the reserve requirement was 2% of the deposit base, which was lowered to 1% from 18 January 2012. Furthermore, a two-tier system takes effect rom 30 October 2019. This system exempts credit institutions from remunerating part of their excessive holdings.

Bank Level Deposit Rates The deposit rates for different banks are based on the ECB IMIR data.

Government bond yield The government bond yield is shown for the German 1 year bond, where the data is extracted from Datastream

Table 3: Data Sources

Data	Name	Source
a) Deposit Rate		
Overnight Deposit Rate, Households (HH)	RD0	ECB SDW - MIR
Deposit rate, maturity up to 1 year, HH, New Business	RD1	ECB SDW - MIR
Deposit rate, maturity over 1 and up to 2 years, HH, New Business	RD2	ECB SDW - MIR
Deposit rate, maturity over 2 years, HH, New Business	RD3	ECB SDW - MIR
Overnight deposits, Total, HH	DS0	ECB SDW - BSI
Deposits, maturity up to 1 year, HH, Outstanding	DS1	ECB SDW - BSI
Deposits, maturity over 1 and up to 2 years, HH,Outstanding	DS2	ECB SDW - BSI
Deposits, maturity over 2 years, HH, Outstanding	DS2	ECB SDW - BSI
b) Lending Rate		
Lending rate, maturity up to 1 year, NF-Corp., Outstanding (Out)	LR1	ECB SDW - MIR
Lending rate, maturity over 1 and up to 5 years, NF-Corp., (Out)	LR2	ECB SDW - MIR
Lending rate, maturity over 5 years, NF-Corp., Outstanding	LR3	ECB SDW - MIR
Loans, maturity up to 1 year, NF-Corp., Outstanding	LS1	ECB SDW - BSI
Loans, maturity over 1 and up to 5 years, NF-Corp., Outstanding	LS2	ECB SDW - BSI
Loans, maturity over 5 years, NF-Corp., Outstanding	LS3	ECB SDW - BSI
c) Policy Rate		
ECB Deposit facility rate	PR1	ECB SDW - FM
Euribor 3-month	PR2	ECB SDW - FM
Eonia rate	PR3	ECB SDW - FM
d) Government Asset		
Loans to government, MFI, Stock	LG	ECB SDW - BSI
Government debt securities, MFI, Stock	LS	ECB SDW - BSI
Reserve Maintenance Required Reserves, Credit Inst.	RR	ECB SDW - BSI
Reserve Maintenance Excess Reserves, Credit Inst.	ER	ECB SDW - BSI
Total Assets, MFI	TA	ECB SDW - BSI
e) Bank Level Data		
Overnight Deposit Rate, Households	RDi	ECB SWD - IMIR
f) Government bond yield		
German government 1 year bond yield	G1Y	Datastream

B.2 Non-Linear Least Squares

The model function that relates the deposit rate data dd_i and the policy rate data pd_i (conditional on being below the threshold) is given as

$$dd_i = (\eta 1 + \eta 2 \exp(\eta_3 pd_i)$$

We impose two restrictions, which allow us to express η_1 and η_2 in terms of η_3. First, the markdown at the threshold value corresponds to ς. Second, the pass-through at the threshold value is 1, which implies perfect pass-through. Thus, the shape parameters η_1 and η_2 can be written as:

$$\eta_1 = i^{SS} - \varsigma - \frac{1}{\eta_3}$$
$$\eta_2 = \frac{1}{\eta_3 \exp(\eta_3 i^{SS})}$$

where i^{SS} is the threshold parameter.

The non-linear least squares finds now the parameter η_3 that minimizes the squared residuals r_i from the model function:

$$r_i = dd_i - \left(i^{SS} - \varsigma - \frac{1}{\eta_3} + \frac{\exp(\eta_3 pd_i)}{\eta_3 \exp(\eta_3 i^{SS})} \right)$$

C Structural Interpretation of the Risk Premium Shock

The risk premium shock of Smets and Wouters (2007) is empirically very important in structural DSGE models, and can explain the zero lower bound episodes. However, its structural interpretation as a risk premium shock is heavily criticized in Chari, Kehoe and McGrattan (2009). They argue that it is best to be interpreted as a flight to quality shock that affects the demand for a safe and liquid asset such as government debt. Fisher (2015) microfounds this argument and indeed shows that this shock can be interpreted as a preference shock for treasury bills.

We show that the risk premium shock in our model can be interpreted as a a flight to quality shock in government bonds in line with the argument above. For this reason, we incorporate government debt as an additional asset that earns the one period ahead nominal gross interest rate R_t^G. Following Fisher (2015), the government bond enters the household utility function as additive term and is subject to an exogenous preference shock Ω_t so that the household problem is given as:

$$\max_{C_t, L_t, D_t, B_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} + \Omega_t U(B_t) \right]$$
$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1} + P_{t-1} B_{t-1} R_{t-1}^B - P_t D_t - P_t B_t + P_t \Pi_t^P - P_t \tau_t$$

where $U(\cdot)$ is positive, increasing and concave. η_t is not an exogenous innovation in the model in this setup. Instead, the nominal gross interest is now artificially divided as $R_{t-1}^D \eta_{t-1}$ to better illustrate the mapping between the flight to quality shock and the risk-premium shock. The first-order conditions with respect to deposits and government bonds are

$$\beta R_t^D \eta_t E_t \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} = C_t^{-\sigma}$$
$$\beta R_t^G E_t \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} = C_t^{-\sigma} - \Omega_t U'(B_t)$$

which can combined to:

$$R_t^D \eta_t = R_t^G \frac{1}{1 - \Omega_t U'(B_t)}$$

This equation suggests that η_t captures changes in the preference for the safe asset Ω_t. In particular, an exogenous increase in the demand for the government bond would require that either the nominal deposit rate would increase or the return on government bonds would fall. If R_t^G does not respond to offset entirely the impact of the shock, then there is a direct mapping from the flight to quality preference shock to our risk premium shock. η_t accounts for the rise in the nominal interest rate shock that resulted from a change in the risk premium. The rise in the nominal interest rate resulting from the preference shock can be accounted by an adjustment in η_t ,which we can then use as the risk premium shock. To avoid any impact on the households budget constraint, the government bond can be in zero net supply. [18]

Regarding the bankers, their maximization problem is not directly affected from the flight to quality preference shock. The only impact on them is on the change in the nominal interest rates on deposits exactly as in the model. However, the increased funding costs for the banks via deposits are taken into account.

To conclude, there is a direct mapping of our version of the risk premium shock to the interpretation in Chari, Kehoe and McGrattan (2009) and Fisher (2015). An increase in the risk premium of deposits captures an increased demand in government bonds via a substitution effect.

Flight to quality and deposits Since our original model abstracts from government bonds for simplicity, an alternative approach would be to introduce a preference of holding deposits in the utility function instead of government bonds. The exogenous shock ω_t

[18] One other potential caveat could be that this shock could actually also capture potential heterogeneities in the pass-through of deposits and governments. Nevertheless, the shock would still capture the impact of flight to quality just adjusted for the different pass-through.

targets now the preference for deposits:

$$\max_{C_t,L_t,D_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} + \omega_t U(D_t) \right]$$

$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1} - P_t D_t + P_t \Pi_t^P - P_t \tau_t$$

where η_t is not an exogenous innovation in this setup, but part of the interest rate as before. The first-order condition can be written as

$$\beta R_t^D \eta_t E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} = 1 + \omega_t^\star U(D_t)$$

where the shock is normalized with respect to marginal utility of consumption $\Omega_t^\star = \omega_t / C_t^{-\sigma}$. Thus, the shock can be interpreted as a preference shifter of deposits: $\eta_t = 1 + \omega_t U(D_t)$. To capture the idea of a flight to safety to government bonds that increases the nominal interest rate of deposits, it is important to realize that the shocks Ω_t and ω_t are inversely related. A flight to safety scenario implies an increase Ω_t and a reduction ω_t so that eta_t increases. As before, this setup is consistent with our modelling of the banking sector

Bank Default Finally, an alternative could be that the wedge accounts for the probability of default of the banks as our model abstracts from idiosyncratic default and bank runs. If the default probability of deposits is p_t, then the budget optimization problem would be:

$$\max_{C_t,L_t,D_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{44}$$

$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1}(1-p_t) - P_t D_t + P_t \Pi_t^P - P_t \tau_t \tag{45}$$

where η_t should again be interpreted as part of the nominal interest rate. The Euler equations reads as:

$$\beta R_t^D \eta_t E_t (1-p_{t+1}) \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$

Therefore, our risk premium shock would be a proxy for the impact of the probability of default of the bank. It is important to note that the difference in timing between the risk shock and the probability of default. While η_t is known in period t, the probability of default is uncertainty and we have $E_t p_{t+1}$. This approach requires that the problem of the bank side is adjusted behind the increased in nominal rates. Rational bankers would take the probability of (idiosyncratic) default into account in their maximization framework. Thus, the model could be extended to include banking default.

D Macroprudential Policy Rule Parameters

The rule consists of two parameters that interact with each other. Figure14 shows the impact on welfare for different combinations of ϕ^{MPP} and τ^{MPP}. The optimal rule has a rather large anchor value with a small response parameter. This ensures the build-up of a small buffer that can then be released during a crisis. If the anchor value is too large, the economy has on average too many buffers that it never releases.

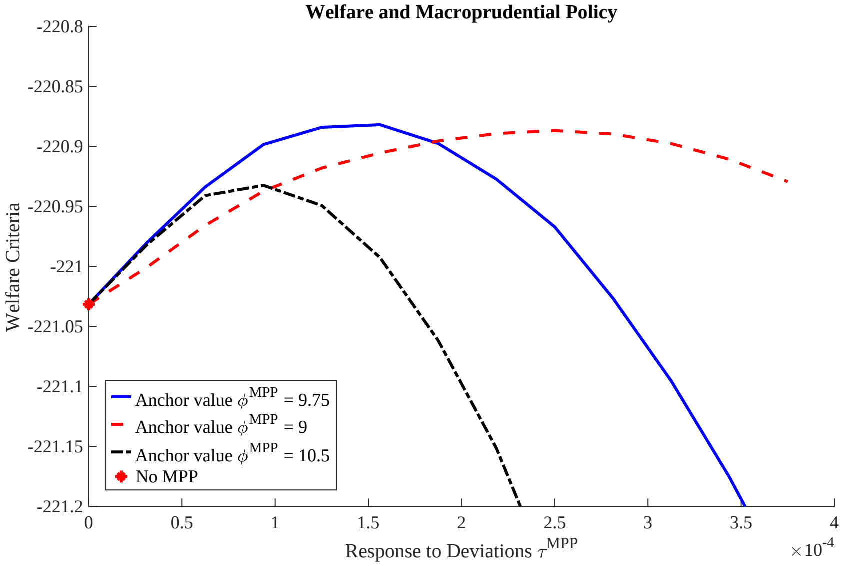

Figure 14: Welfare for response to deviations τ^{MPP} and anchor values ϕ^{MPP}. τ^{MPP} is varied on the horizontal axis. Welfare is on the horizontal axis

E Solution Method

The non-linear model is solved with policy function iterations. In particular, we use time iteration (Coleman, 1990) and linear interpolation of the policy functions as in Richter, Throckmorton and Walker (2014). We solve for the policy functions and law of motions. We rewrite the model to use net worth N_t as state variable instead of $D_{t-1}R_{t-1}$ to ease the computation.

The algorithm has the following steps:

1. Define the state space and discretize the shock with the Rouwenhorst method

2. Use an initial guess for the policy functions

3. Solve for all the time t variables for a given state vector and a law of motion of net worth. Given the state vector $K_{t-1}, N_t, \eta_t, \zeta_t$, the policy variables Q_t, C_t, ψ_t, Π_t and the law of motion of the net worth, we can solve for the following variables in period t

$$I_t = (Q_t(1-\eta_i)a_i)^{\frac{1}{\eta_i}} K_{t-1}$$

$$Y_t = \frac{C_t + I_t}{\left(1 - \frac{\rho^r}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2\right)}$$

$$L_t = \left(\frac{Y_t}{K_{t-1}^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

$$W_t = \chi L^{\varphi} C^{\sigma}$$

$$MC_t = \frac{W_t}{1-\alpha}\frac{L}{Y}$$

$$R_t^A = R^A\left(\frac{\Pi_t}{\Pi}\right)^{\kappa_\Pi}\left(\frac{Y_t}{Y}\right)^{\kappa_Y}$$

$$R_t^D = \mathbf{1}_{R_t^A \geq R^{A\,SS}}\left[R_t^A - \varsigma\right] + (1 - \mathbf{1}_{R_t^A \geq R^{A\,SS}})\left[\omega^1 + \omega^2 \exp(\omega^3(R_t^A - 1)) + 1\right]$$

The endogenous state variables are capital and net worth, which are given from the law of motion of capital and the guess for the law of motion of net worth

$$K_t = (1-\delta)K_t + \left(a_i\left(\frac{I_t}{K_t}\right)^{1-\eta_i} + b_i\right)K_{t-1}$$

$$N_{t+1} = \mathcal{T}(K_{t-1}, N_t, \zeta_t, \eta, \zeta_{t+1}, \eta_{t+1})$$

Note that capital is predetermined, while net worth depends on the shocks. Therefore, we have a net wroth at each integration node for the shocks. At each node i, we then now the policy function $Q_{t+1}^i, C_{t+1}^i, \psi_{t+1}^i, \Pi_{t+1}^i$. At this step, we linear interpolate the policy functions

$$I_{t+1}^i = \left(Q_{t+1}^i(1-\eta_i)a_i\right)^{\frac{1}{\eta_i}} K_t$$

$$Y_{t+1}^i = \frac{C_{t+1}^i + I_{t+1}^i}{\left(1 - \frac{\rho^r}{2}\left(\frac{\Pi_{t+1}^i}{\Pi} - 1\right)^2\right)}$$

$$L_{t+1}^i = \left(\frac{Y_{t+1}^i}{K_t^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

$$W_{t+1}^i = \chi\left(L_{t+1}^i\right)^{\varphi}\left(C_{t+1}^i\right)^{\sigma}$$

$$MC^i_{t+1} = \frac{W^i_{t+1}}{1-\alpha}\frac{L^i_{t+1}}{Y^i_{t+1}}$$
$$R^{k,i}_{t+1} = \frac{MC^i_{t+1}\alpha Y^i_{t+1}/K_t + Q^i_{t+1}(1-\delta)}{Q_t}\Pi^i_{t+1}$$

We can now calculate the following items:

$$\phi_t = \frac{Q_t K_t}{N_t}$$
$$R_t = R^D_t \eta_t \frac{1}{1-\delta^B} - R^A_t \frac{\delta^B}{1-\delta^B}$$
$$\mu_t = \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} (1-\theta+\theta\psi_t) \left(\frac{R^K_{t+1} - R_t}{\Pi_{t+1}}\right)$$
$$\nu_t = \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} (1-\theta+\theta\psi_t) \left(\frac{R_t}{\Pi_{t+1}}\right)$$

where the expectations are based on the weighting of the different integration nodes. The Rouwenhorst method discretizes the shocks and gives the weighting matrix. Finally, we can calculate the errors for the four remaining equations

$$err_1 = \left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} - \left(\frac{\epsilon}{\rho^r}\left(MC_t - \frac{\epsilon-1}{\epsilon}\right) + \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{\Pi_{t+1}}{\Pi_t} - 1\right)\frac{\Pi_{t+1}}{\Pi}\right)$$
$$err_2 = \beta R^D_t \eta_t E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+1}}$$
$$err_3 = \psi_t - (\mu_t\phi_t + \nu_t)$$
$$err_4 = \psi_t - \left(\lambda\Big(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\Big)\right)$$

We minimize the errors using a root solver the policy functions in period t. The policy functions for period $t+1$ are taken from the previous iteration.

4. This step is only relevant for the extension with the countercyclical capital rule. Otherwise, it can be skipped. Check if the occasionally binding constraint is binding. If we introduce the capital requirement, it is occasionally binding. Therefore, we have to check if

$$\phi^R > \phi^M$$

where ϕ^M is the market based leverage that we calculated as ϕ in the previous step. If this is the case, the capital constraint is binding. We now replace two equations from before, namely we impose directly

$$\phi = \phi^R$$

Furthermore, one of the remaining equations is now adjusted as the market based leverage constraint is not binding anymore. Therefore, we remove $\phi_t = \frac{Q_t K_t}{N_t}$ from the calculations and actually minimize the error:

$$err_4 = \phi_t - \frac{Q_t K_t}{N_t}$$

Note that we do not need $\psi_t \geq \left(\lambda\left(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\right)\right)$ from the previous step as it is not binding.

5. Update the law of motion for net worth. We have assumed that we know the actual law of motions. Using the policy functions, we improve our guess of the policy function. Using the result from the previous steps (depending on the binding of the constraint), we update it as follows

$$N^i_{t+1} = \theta\left(\left(R^{k,i}_{t+1} - R_t\right)\phi_t - R_t\right) + \omega K_t$$

We have to update the law of motion for each possible shock realizations next period.

6. Check convergence for the policy functions and the law of motion of net worth for a predefined criteria

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