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Using structural models to understand macroeconomic tail risks

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Contents

Abstract

Understanding asymmetric risks in macroeconomic variables is challenging. Most structural models used for policy analysis are linearised and therefore cannot generate asymmetries such as those documented in the empirical growth-at-risk (GaR) literature. This report examines how structural models can incorporate non-linearities to generate tail risks. The first part reviews the various extensions to dynamic stochastic general equilibrium (DSGE) models and the computational challenges involved in accounting for risk distributions. This includes the use of occasionally binding constraints and more recent developments, such as deep learning, to solve non-linear versions of DSGEs. The second part shows how the New Keynesian DSGE model, augmented with the vulnerability channel as proposed by Adrian et al. (2020a, b), satisfactorily replicates key empirical facts from the GaR literature for the euro area. Furthermore, introducing a vulnerability channel into an open-economy set-up and a medium-sized DSGE highlights the importance of foreign financial shocks and financial frictions, respectively. Other non-linearities arising from financial frictions are also addressed, such as borrowing constraints that are conditional on an asset's value, and the way macroprudential policies acting against those constraints can help stabilise the economy and generate positive spillovers to monetary policy. Finally, the report examines how other types of tail risk beyond financial frictions – such as the recent asymmetric supply-side shocks – can be incorporated into macroeconomic models used for policy analysis.

Keywords: Tail risks, structural models, non-linearities, vulnerability channel, DSGE, macroprudential policies, asymmetric shocks.

JEL codes: E70, D50, G10, G12, E52

Executive summary

Understanding asymmetric risks in macroeconomic variables is challenging. The time series literature on tail risks is an excellent starting point for understanding determinants of asymmetries, but to grasp the structural sources of tail risks policymakers need models – such as dynamic stochastic general equilibrium (DSGE) models – that can disentangle causal relationships and handle non-linearities.

The literature has provided various methods for incorporating non-linearities in structural macroeconomic models that can generate asymmetric tail risks. For example, the DSGE literature focuses mainly on financial friction elements that can generate those types of risk. This is usually done by incorporating elements such as occasionally binding constraints (OBCs) or heterogeneous agents. Asymmetries may also arise when examining other types of non-linearity such as state-dependent Phillips curves, stochastic volatility, or deviations from normally distributed shocks, also found in structural vector autoregression (VAR) models.

It is still a challenge to solve and estimate non-linear DSGE models. Many current algorithms used to solve and estimate non-linear DSGE models can be applied to smaller models but they may not be useful for policy analysis, which requires medium to large-scale macroeconometric models. Therefore, until researchers apply newer and more efficient algorithms that can take advantage of increased computational power, it is difficult to conclude that DSGE models are a perfect solution to understanding macroeconomic tail risks.

Most structural models used for policy analysis are linearised and therefore cannot generate asymmetries as documented in the empirical growth-at-risk (GaR) literature. Linearised DSGE models have many advantages. These include their capacity to accommodate numerous state variables and the fact that they can be fitted to empirical time series via Bayesian estimation. However, even for routine policy applications such as policy counterfactuals, it would be interesting to obtain GaR effects, for example when evaluating the impact of various monetary policy paths on the lower tail of gross domestic product (GDP) growth ("downward risk").

As proposed by Adrian et al. (2020a, b), a vulnerability channel can generate asymmetries in otherwise linearised models. The authors propose to make the standard deviations of exogenous shocks within the model a function of past financial conditions and the output gap. They propose a small New Keynesian DSGE model augmented by such a vulnerability channel (the New Keynesian vulnerability (NKV) model) in order to introduce conditional heteroscedasticity. This report shows that such an NKV model, matched to euro area dynamics via a simulated method of moments (SMM), replicates key empirical facts from the GaR literature well. These include asymmetries across output growth quantiles and a skewed distribution of that variable, as well as a negative correlation between its conditional mean and volatility. Moreover, the report highlights that the vulnerability channel is a versatile tool to express GaR dynamics in otherwise linearised models by presenting various applications and extensions of the NKV model.

Several caveats to this approach should be borne in mind. First, the vulnerability function is not micro-founded. Although present in all the models discussed here (meaning that agents form rational expectations about its effects), the vulnerability function is fitted to past empirical relationships and will be invariant against policy changes. Second, there is currently no established procedure for fitting the vulnerability function to the data. Exercises across EU countries have shown differences in how well the NKV model fits country-level data: there was clear heterogeneity in the strength of asymmetries across countries, which might reflect differences in the role of financial conditions for these economies. Euro area asymmetries may thus mask differences across Member States and shocks to the common monetary policy in the area may affect output growth in those countries differently, depending on their position in the financial cycle. Third, it is an open question as to whether additional empirical moments not matched in the models (such as those provided by the time series workstream of this Expert Group on Macro at Risk) show a good fit to their empirical counterpart.

Introducing a vulnerability channel into an open-economy set-up and a medium-sized DSGE highlights the importance of foreign financial shocks and financial frictions, respectively. First, the NKV model is extended to a small open economy (SOE) set-up. Here, financial conditions from abroad can be shown to spill over into the home economy and increase GDP tail risks. Private consumption in the SOE, however, appears to be more vulnerable than output, given that the latter is subject to offsetting effects stemming from international variables such as the real exchange rate or the terms of trade. In another application, Angeloni and Faia (2013) introduce a vulnerability function into the medium-sized DSGE model, which features a micro-founded financial friction and a good proxy variable for financial conditions. When this model is fitted to euro area data, it generates the asymmetric quantiles and skewness of the GDP distribution. Further, in addition to the direct negative impact on output growth, it highlights an indirect channel of monetary policy tightening: a reduction of future output growth volatility that may improve the impact of future monetary policy decisions. Both exercises prove that a vulnerability channel can be introduced into existing linearised models with relative ease.

Some of the non-linearities explored in this report arise from financial frictions, as in the case of borrowing constraints which are conditional on the value of an asset. Macroprudential policies acting against those constraints can help stabilise the economy and generate positive spillovers to monetary policy. Occasionally binding borrowing constraints generate non-linearities. The financial accelerator that arises from this friction can have implications for the real economy via macro-financial linkages in the form of more pronounced booms but also deeper recessions, leading to fatter left tails in the distribution of output and inflation. In this case, macroprudential policies are better suited to counter this friction and stabilise macroeconomic fluctuations. Macroprudential policy that acts against the borrowing constraint restrains the financial accelerator and dampens the boom-and-bust cycle, thereby attenuating strong fluctuations in output and inflation. This generates positive spillovers to the conduct of monetary policy, as it reduces the probability of tail risks and hence the need for strong monetary tightening or loosening.

While the literature has focused on the role of financial frictions in generating macroeconomic tail risks, it is important for policymakers to consider other risk sources. Economic developments since 2020 have shown that a number of other factors may indicate asymmetric tail risks, such as the COVID-19 pandemic and subsequent supply bottlenecks, or energy constraints following Russia's invasion of Ukraine, with the associated exceptional increase in inflation. It is, however, a challenge to devise structural models that incorporate all the various sources of risk. The literature has also looked at smaller models that can identify asymmetric distributions for supply-side developments, or non-linear Phillips curves that can generate state-dependent pricing and wage setting.

1 Using structural models to understand macroeconomic tail risks

Extreme events affect many economic variables but are poorly captured by the normal distribution. Since standard macroeconomic models typically assume that economic disturbances follow a normal distribution, they regularly underestimate the frequency of major economic downturns. The empirical literature has made much progress in matching the likelihood of extreme events affecting output and inflation (see companion report on time series methods and its references). Yet few theoretical contributions have explored the conceptual drivers of tail risk.

Several factors may account for deviations from the normal distribution: financial frictions, sectoral shocks, the zero lower bound (ZLB) on nominal interest rates, spikes in uncertainty, natural disasters and government policies (Chart 1.1) among others. Below we discuss examples of each.

Financial frictions are one of the most important sources of non-linear business cycles. In a frictionless economy, funds flow to the most profitable project. Aggregate output is determined by total capital and labour. Financial frictions, however, create an important role for liquidity and the distribution of wealth. According to the seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), financial frictions can amplify small shocks and lead temporary shocks to generate persistent effects. Building on these studies, Brunnermeier and Sannikov (2014) found that financial frictions also lead to asymmetric business cycles if their effects are stronger during downturns. Christiano, Eichenbaum and Trabandt (2015) provided empirical evidence to support this argument. They demonstrate that financial frictions accounted for the vast majority of movements in aggregate real economic activity during the Great Recession. More recently, Adrian et al. (2020a, b) have presented a New Keynesian (NK) model with vulnerability where output volatility depends on the endogenous price of risk, leading to a “vulnerability channel of monetary policy”. In this model, unlike in the standard NK model, monetary policy that focuses only on inflation and output gap stabilisation can lead to instability in the presence of tail risks.

The bank lending channel is generated by frictions within financial institutions: in a downturn, poorly capitalised lenders are often forced to reduce lending and sell their assets at distressed prices. For example, Gertler and Kiyotaki (2010) show how when bank capital contracts, the cost of credit rises and economic growth slows, which in turn further depresses asset prices and bank capital. He and Krishnamurthy (2013) find an important asymmetry: when bank capital is low, losses within the sector cause risk premia to skyrocket, but when bank capital is high, losses have little to no effect on premia. Lastly, Gertler and Kiyotaki (2015) find that the condition of bank balance sheets affects not just the cost of bank credit, but also the likelihood of bank runs. In their model, bank runs lead to a collapse in intermediation and aggregate economic activity.

In contrast, the balance sheet channel is generated by frictions associated with bank borrowers: lenders might become reluctant to extend credit to riskier and poorly capitalised borrowers. For example, in the Eggertsson and Krugman (2012) model "deleveraging crises" arise when abrupt changes in lenders' views about "safe" levels of debt for individual borrowers force highly indebted individuals to reduce debt by slashing spending. On a related issue, Korinek and Simsek (2016) show how individually rational borrowers might take on excessive leverage from a social point of view, making deleveraging episodes more likely. Christiano, Motto and Rostagno (2014) emphasise the empirical relevance of the balance sheet channel. They argue that fluctuations in individual firms' risk are the biggest driver of fluctuations in US GDP.

Sectoral shocks are another source of macroeconomic tail risk. Acemoglu, Ozdaglar and Tahbaz-Salehi (2017) find that sectoral heterogeneity can alter the probability of large economic crises occurring relative to a normal distribution. In a related argument, Baqaee and Farhi (2019) show that non-linearities in production amplify the impact of negative sectoral productivity shocks and mitigate the impact of positive shocks. Their findings suggest that these non-linearities affect the distribution of aggregate output: they lower its mean and generate negative skewness and excess kurtosis.

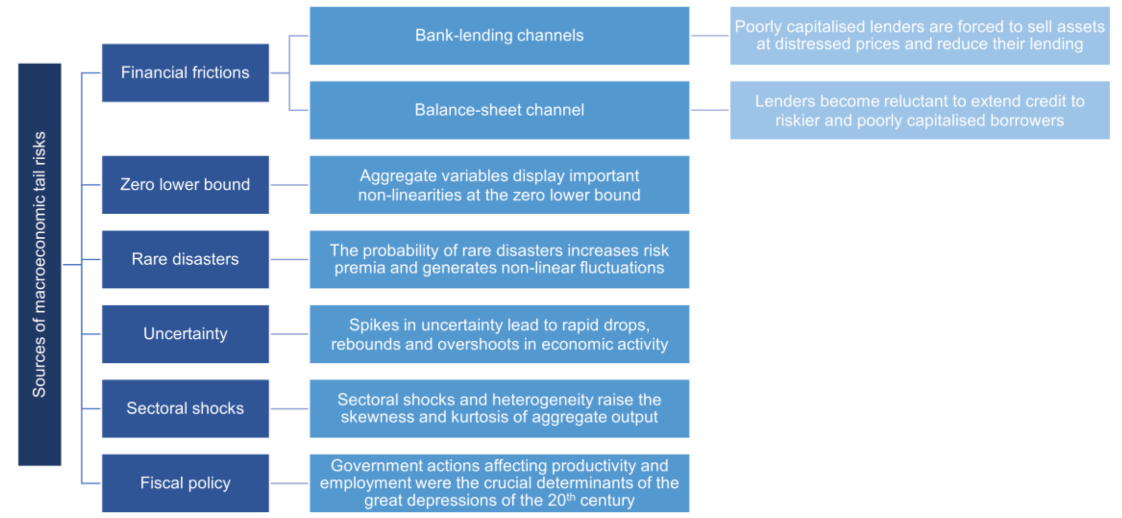
The ZLB on nominal interest rates can also amplify the effects of exogenous shocks. For example, Garín, Lester and Sims (2019) show that an increase in productivity at the ZLB amplifies their impact on output. Similarly, Fernández-Villaverde et al. (2015) show that, in a standard NK model, output and inflation behave differently when the nominal interest rate approaches the ZLB. Linear approximations perform poorly during large downturns, since linearising equilibrium conditions will neglect these non-linear interactions between the ZLB and agents' decision rules. Christiano, Eichenbaum and Rebelo (2011) raise a similar point, arguing that government spending multipliers increase when the economy approaches the ZLB, because central bankers cannot lower the nominal interest rate further in response to an increase in government spending.

Uncertainty also affects the tails of the distribution of macroeconomic variables. In a groundbreaking paper Bloom (2009) uses firm-level data to estimate a model where a spike in uncertainty generates rapid drops, rebounds and overshoots in economic activity. Similarly, Bloom et al. (2018) find that uncertainty is countercyclical, and that recessions are best modelled as being driven by shocks with a negative first moment and a positive second moment. Lastly, Fernández-Villaverde et al. (2011) complement these findings by showing that when real interest rate volatility increases, output and debt fall.

Rare, extreme disasters may also affect the tails of the distribution of economic variables. Barro (2006) and Gabaix (2012) argue that the potential for abnormal negative shocks explains many asset-pricing puzzles, including the high equity premium, low risk-free rate, and volatile stock returns. Likewise, Gourio (2012) develops a real business cycle model where an increase in the probability of disasters causes risk premia to rise and asset prices, employment and output to contract. Both these papers treat the frequency and severity of rare disasters as exogenous.

Finally, Kehoe and Prescott (2002) reviewed how extreme events such as the Great Depression are treated in the older neoclassical literature. For them, the conclusion from growth accounting based on general equilibrium models is clear: government policies that affected total factor productivity and hours per working-age person were the crucial determinants of the great depressions of the 20th century. For example, Cole and Ohanian (2002) find that generous unemployment benefits, combined with large negative sectoral shocks, depressed the UK economy for 20 years between the two world wars.

Chart 1.1
Sources of macroeconomic tail risks (non-exhaustive list)



2 Computational challenges in structural models

Generating asymmetric risk distributions using structural models is a complex task that calls for expensive computational methods. Most of the literature on structural models has focused on linear DSGE models with Gaussian disturbances. A linear DSGE model with a reasonable number of variables can be estimated using a variety of methods and solved relatively quickly. However, a linear DSGE model with normally distributed shocks can only generate symmetric forecasting distributions. Hence, risks are balanced around a mean forecast. To understand this, we can consider a DSGE model that has been linearised in structural form:

$$Ax_t = C + Bx_{t-1} + DE_t x_{t+1} + F\varepsilon_t$$

where:

x_t is a vector of state variables in the model;

ε_t is a vector of structural shocks;

E_t is the expectations operator.

The matrices A, B, C, D and F capture the structural relationships between the state variables, their leads and lags, and how they map onto the structural shocks. Under conditions of full information, rational expectations and some a degree of regularity, the solution to the model is given by:

$$x_t = J + Qx_{t-1} + G\varepsilon_t$$

which is simply a linear process. If ε_t follows a standard Gaussian distribution, $\varepsilon_t \sim N(0, I)$, then:

$$x_{t+1} \sim N(J + Qx_t, GG')$$

hich is also a normal and symmetric distribution.

Structural models can generate asymmetric distributions if there are non-linearities, such as occasionally binding constraints (OBCs), or non-linear behavioural equations, such as non-linear pricing decisions. But if the model is linearised and solved as above it will miss the non-linearities, which in some cases may be key to understanding its transmission channels. Alternative computational methods are needed to solve the model with varying degrees of complexity and accuracy. We discuss some of these alternatives below.

2.1 Traditional solution methods for DSGE models

The most popular method for solving DSGE models is perturbation. Here, the model's decision rules are approximated using linear, log-linear or quadratic approximations around a steady state using Taylor series expansions. These local approximations typically suffice, since the decision rules in the standard real business cycle and NK models are close to linear in the vicinity of the steady state. Therefore, we expect the dynamics implied by local approximations for small deviations to be reasonably close to the true ones. This makes perturbation a good solution method as it is fast and can handle models with a large state space. In addition, the approximation order can be set such that the dynamics include the effects of risk arising from precautionary behaviour and time-varying shock variances, as well as the effects of other non-linearities in the behavioural equations of the model (for example, downward pricing rigidities).[1]

Therefore, the technique can be used to develop large structural models that can be solved, estimated and simulated while avoiding the curse of dimensionality that plagues other techniques such as global solution methods.

Ajevskis (2017) proposes an approach based on a perturbation technique to construct global solutions to DSGE models. The key idea is to expand a solution in a series of powers of a small parameter, scaling uncertainty in the economy around a solution to the deterministic model but not necessarily around a steady state. Global solutions to deterministic models can be obtained reasonably fast by effective numerical algorithms incorporated in software like Dynare (Adjemian et al., 2022) – even for large models like the Euro Area and Global Economy (EAGLE) model (Gomes, Jacquinot and Pisani, 2012). Based on the assumption of a known deterministic path, the higher-order terms in the expansion are obtained recursively by solving linear rational expectations models with time-varying parameters.

The approach is quite effective when an economy (after large and persistent shocks, for example), or its initial conditions (in developing countries, for instance), are far from steady state. In these situations, the method is more accurate overall than the conventional approach based on perturbations around the steady state. On the other hand, semi-global solutions are more computationally effective in terms of computation time than non-linear methods based on projection. What is more, unlike deterministic solutions this model takes in Jensen's inequality, so the solution reflects the impacts of risks on agents' behaviour.

Standard perturbation is not always a suitable solution technique. It can yield unreliable approximations when the underlying decision rules are highly non-linear in the vicinity of the steady state or for shocks that move the system far from that state. Crucially, it can only be used on continuously differentiable functions: it cannot approximate decision rules that feature kinks such as slack borrowing constraints and the ZLB, referred to as OBCs. Nor can it handle the solution in the presence of

[1] Using popular toolboxes such as Dynare (Adjemian et al., 2022), users can easily control the approximation up to order 3, with higher orders handled using C++ implementation due to the exponential increase in the number of operations required.

discontinuities such as financial default. These are important sources of non-linearity that can give rise to macroeconomic tail risks.

There are several alternative solution methods for this scenario, which we expand on below. As with most numerical methods, these alternatives all have their strengths and weaknesses, which we highlight where relevant.

In the presence of OBCs, the extended path method proposed by Fair and Taylor (1983) can be used to solve models with non-linearities This is a perfect foresight algorithm that solves the non-linear version of the model over a fixed time period, subject to boundary conditions. The solution is appealing, since the algorithm can be used to simulate a model with kinks in the state space while preserving the full non-linearity of the original model. Its drawback is its failure to take account of precautionary behaviour driven by future uncertainty. It assumes that agents, despite observing current uncertainty, do not anticipate future shocks. A partial remedy for this is the stochastic extended path (Adjemian and Juillard, 2013), an algorithm that integrates future uncertainty up to a horizon S. Therefore, this algorithm is not fully consistent with rational expectations, as agents' expectations are only bounded up to the integration horizon S. Beyond this, the model assumes that agents believe all shocks will be zero. Nevertheless, it is readily available in Dynare and is quick to set up. The downside to this algorithm is that computational complexity increases exponentially over horizon S and with the number of shocks. As a result, the algorithm is not feasible even for small models with just a handful of shocks (Swarbrick, 2021).

A modification of the extended path proposed by Ajevskis (2019) reduces computational time compared with the standard approach by a preliminary linear transformation of the model. Once this change is made, the solutions lying on the approximate stable manifold can be obtained by mapping iterations. The speed of convergence for the algorithm turns out to be quadratic but, in contrast to Newton-like methods, there is no need to compute derivatives. In addition, this approach also deals with near-unit root processes, which are problematic for the extended path model (Laffargue, 1990).

Drawing on recent developments, DSGE models with OBCs can be solved using piecewise linear approximations of the model's equations around the steady state, as implemented in the OccBin toolbox (Guerrieri and Iacoviello, 2015). This gives an approximation of the behaviour of the different regimes possible for an economy, in which the constraint can be either binding or slack.[2] The solution derived from this model can be highly non-linear, as the coefficients in the decision rules are a function of the expected time away from the constraint. Guerrieri and Iacoviello (2015) show that the approximation holds up well compared with a global solution (derived from value function iteration). The approach relies on first-order perturbation and can therefore be scaled to models with a large number of state variables. Since it is based on perturbation it is also typically fast. Yet, as it assumes – like the extended path – that no shocks will hit in the future, it too misses the

[2] The original application is the work of Jung, Teranishi and Watanabe (2005).

potentially important role of precautionary behaviour in shaping the dynamics of the economy.

DynareOBC combines higher-order perturbation and mixed integer linear programming techniques to capture non-linearities associated with OBCs and preserve arbitrary moments associated with risk. Although the perfect foresight methods discussed above can capture non-linearities, they miss higher-order effects relating to the effect of risk. Holden (2016, 2022) developed a toolbox, DynareOBC, that captures these effects. Although the core of the toolbox is a perfect foresight solver, as in the other approaches above, DynareOBC enforces the different constraints by searching for a sequence of anticipated news shocks (Swarbrick, 2021). When using a higher-order approximation, the toolbox first finds the stochastic steady state using the Lan and Meyer-Gohde (2013) method. It then approximates the decision rules around it, rather than around the deterministic steady state. The next step is to run the stochastic extended path, which becomes feasible since in this perturbation approximation the order of integration increases as a polynomial in periods of uncertainty only, independently of the number of shocks. This contrasts with the stochastic extended path on the fully non-linear model, where the order of integration increases exponentially both over the integration horizon and with the number of shocks. Hence, the toolbox can accommodate models with higher-order uncertainty, such as the NKV (Adrian et al, 2020a, b), for which a third-order solution is required to track the effect of stochastic volatilities. Nor does DynareOBC place limits on the number of OBCs in the model. When these higher-order effects are not needed and the effects of future uncertainty are ignored, the toolbox returns a solution that is identical to that obtained using OccBin. This feature makes this toolbox highly flexible. However, it can be slower than either extended path or OccBin without additional MATLAB toolboxes and an MEX compiler. Computation time increases significantly when the effects of uncertainty up to horizon S are captured (see the application in Section 4), which can be important in applied work that requires frequent updating.

2.2 Recent developments: deep learning to solve macroeconomic models

While the methods discussed in the previous section can help solve many macroeconomic models, the “curse of dimensionality” makes it challenging – if not impossible – to solve large dynamic programming problems. Simply put, the computational requirements of solving a dynamic programming problem grow exponentially with the dimensions of the problem.

Neural networks can help solve large macroeconomic models as they appear to avoid the curse of dimensionality. By breaking the curse, neural networks can provide global solutions to large macroeconomic models, including large-scale DSGE models or heterogeneous agent NK models (Fernández-Villaverde and Guerrón-Quintana, 2021). (A deep dive into how this is achieved is outside the scope of this paper).

Neural networks are universal approximators. They are composed of simple functions that can be combined and tuned to approximate more complicated ones (Hornik et al, 1989).[3] Other simple functions that fall into the category of universal approximators are monomials, Chebyshev polynomials or sine waves. The building blocks of neural networks are called “nodes” or “neurons”, which are functions of the formula:

$$m(\boldsymbol{x};\boldsymbol{\gamma}) = f\left(y_0 + \sum_{i=1}^{N} \gamma_i x_i\right).$$

Each node $m(\boldsymbol{x};\boldsymbol{\gamma})$ takes $\boldsymbol{x} \in \mathbb{R}^N$ as input and is parameterised by the weights $\boldsymbol{\gamma} \in \mathbb{R}^{N+1}$. The activation function $f(\cdot)$ defines the node output. Activation functions are often non-linear functions, such as the hyperbolic tangent function or the sigmoid function.

Individual nodes are combined in a “layer”, which is a function of the formula:

$$M(\boldsymbol{x};\boldsymbol{\Gamma}) = \left(m(\boldsymbol{x};\boldsymbol{\gamma_1}), \dots, m(\boldsymbol{x};\boldsymbol{\gamma_P})\right)^T,$$

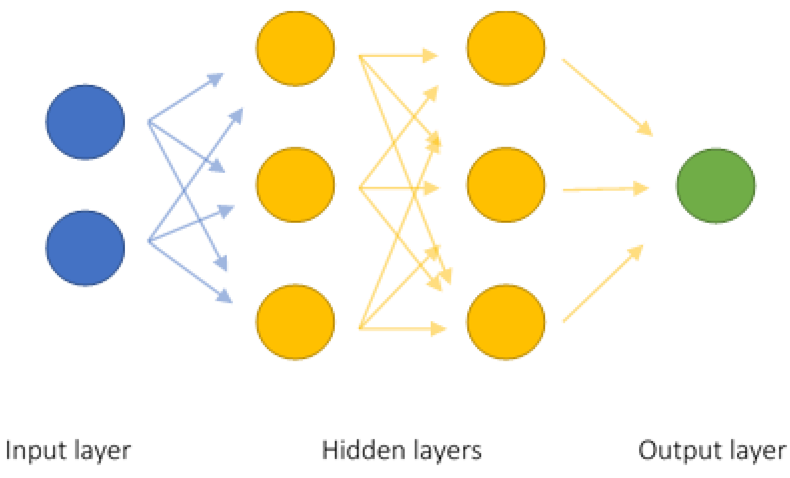
where $\Gamma = (\boldsymbol{\gamma_1}, \dots, \boldsymbol{\gamma_P})$ contains all the node parameters. Crucially, all nodes share the same inputs $\boldsymbol{x} \in \mathbb{R}^N$, but not the same parameters. As a result, the layer yields as many outputs as nodes.

As shown in Chart 2.1, neural networks combine several of these layers. They contain an input layer, one or more hidden layers, and an output layer. In the hidden layers, each node receives data from the previous layer, transforms it according to its weights and activation function, and sends its output to the next layer of the network.

Since neural networks are just one among many universal approximators, it is vital to understand what sets them apart. For a given level of approximation error in a neural network, the number of parameters required to approximate a function increases on a linear basis with the number of inputs. For other classes of approximators the increase is exponential (Fernández-Villaverde, Hurtado and Nuño (2019)). Moreover, neural networks are robust when dealing with multicollinearity – unlike monomials, for example – and can perform well even when approximating functions with kinks and discontinuities (Maliar, Maliar and Winant (2021)).

[3] Although the terms deep learning and neural networks are often used interchangeably, they are not the same thing. Technically, the term “deep” refers to the number of layers in a neural network. A neural network that consists of more than three layers is considered a deep learning algorithm. A neural network with only two or three layers is a basic neural network.

Chart 2.1
A deep neural network



2.2.1 Using deep learning to solve macroeconomic models

Deep learning can approximate the decision rules in large dynamic programming problems using neural networks instead of the conventional polynomial functions in standard projection methods. To illustrate how to solve macroeconomic models using deep learning, we follow Maliar, Maliar and Winant (2021). Taking a standard dynamic model, we approximate some of the model's decision rules using a neural network $\varphi(\boldsymbol{x}; \boldsymbol{\Gamma})$. Here $\boldsymbol{x}$ is a vector containing the model's state variables (e.g. capital), and $\boldsymbol{\Gamma}$ contains the neural network parameters to be set.

Solving a model boils down to minimising the objective function $\chi(\boldsymbol{\Gamma})$ with respect to the vector of parameters $\boldsymbol{\Gamma}$:

$$\chi(\Gamma) = \min_{\Gamma} E_{\varpi} F(\boldsymbol{\varpi}; \Gamma).$$

Here $\boldsymbol{\varpi} = (\boldsymbol{x}, \boldsymbol{\epsilon})$ includes the state variables x, and the future shocks $\boldsymbol{\epsilon}$. $F(\cdot)$ is a user-defined residual function that captures the environment and the optimality conditions of the model.

Treating $\boldsymbol{\varpi}$ as a vector of random variables, we can simulate the model to generate a set of random draws $\{\boldsymbol{\varpi}\}_{i=1}^{n}$ and then replace the expectation operator with the sample average of $F(\cdot)$ across *n* random draws. The resulting optimisation problem is given by:

$$\chi(\Gamma) = \min_{\Gamma} \frac{1}{n} \sum_{i=1}^{n} F(\varpi; \Gamma)^2.$$

This transforms our dynamic programming problem into a standard non-linear regression model. We get a solution $\mathbf{\Gamma}^*$ by "training" the neural network to minimise $\chi(\mathbf{\Gamma})$ on the simulated data.[4]

Training a neural network refers to the process of adjusting its parameters to minimise the objective function. Gradient descent and its many variants are the most common training algorithms for deep learning. The idea behind gradient descent is simple: calculate the gradient of the loss function at the current point and move the parameters in the opposite direction. This provides a new point with a lower loss function value, at which point we repeat the procedure until no further material improvements can be made.

In summary, the steps involved in training a neural network using gradient descent are as follows:

1. pick a starting point for the parameters, $\mathbf{\Gamma}$;
2. compute the gradient of the loss function at this point, $\nabla\chi(\Gamma)$;
3. take a small step in the opposite direction to the gradient, $\Gamma' = \Gamma - \lambda\nabla\chi(\Gamma)$, where $\lambda > 0$;
4. repeat the last two steps until a given stopping criterion is reached.

There are constraints associated with deep learning. It is a computationally intensive and time-consuming method, while coding deep learning algorithms involves much trial and error, given the many hyper-parameters: number of nodes and layers, type of activation functions, or learning rates (the size of step λ in the gradient descent procedure).

Therefore, deep learning is not always preferable to previous methods, but can deliver significant improvements for high-dimensional non-linear problems.

2.3 Tail risks in medium and large-scale models

2.3.1 Macroeconomic asymmetries in a medium-scale model

Guerrieri and Iacoviello (2017) derive and estimate an NK macroeconomic model that embeds housing and financial frictions in the form of collateralised borrowing constraints that introduce a financial accelerator.

In their model, financial frictions play a bigger role in explaining developments during recessions, when borrowing limits are more likely to be binding. This mechanism gives rise to asymmetries. The authors show that while increased

4 Note that the above discussion implicitly assumes that the dynamic programming problem is framed in discrete time. But from a numerical point of view, continuous time has an advantage: handling derivatives (gradients and hessians in the Hamilton-Jacobi-Bellman equations) rather than integrals (expectations), as in Fernández-Villaverde et al. (2019).

housing wealth before the Great Recession of 2008 contributed very little to consumption growth, the subsequent fall in house prices and contraction in housing wealth exacerbated the decline in consumption. They also demonstrate the role of the ZLB on nominal interest rates in driving this asymmetry.

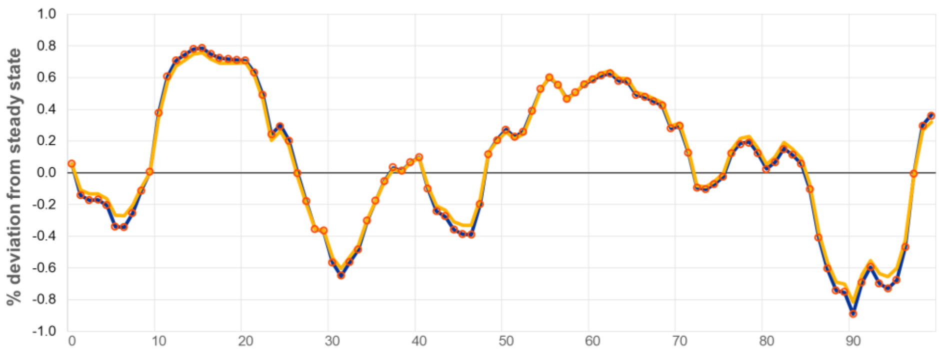
The model is of medium scale, which makes it ideal for an exploration of different solution algorithms and their output. Guerrieri and Iacoviello (2017) estimate the model using US data. In the model, six structural shocks drive macroeconomic fluctuations, including housing preference (demand) shocks. This model is solved and simulated for 500 periods using three of the methods discussed above, OccBin, extended path and DynareOBC. The parameters are posterior mode. While OccBin uses a perfect foresight solver, DynareOBC allows for integration over future uncertainty S periods ahead. When $S = 0$ (perfect foresight), DynareOBC delivers results which are identical to OccBin. Chart 2.2 below shows a sub-sample of 100 periods for the consumption and shadow price paths reflecting the borrowing constraint in the model when the economy suffers a series of housing demand shocks. Agents factor uncertainty into their decisions up to 16 periods (four years) into the future when DynareOBC is used.

All three solution algorithms deliver almost identical dynamics for consumption and the implied shadow price for the borrowing constraint. Solutions using extended path approach at times differ slightly from those using the other two. A possible reason is that the extended path simulates the non-linear model, while OccBin and DynareOBC simulate a linearised version of the model. Note that the role of future uncertainty over the subsequent four years in the model contributes very little to the dynamics. In addition, the consumption paths produced by DynareOBC and OccBin differ by an average of less than 0.01% of steady-state consumption over the entire simulation, while they both underestimate the value of consumption by an average of 1% relative to extended path.

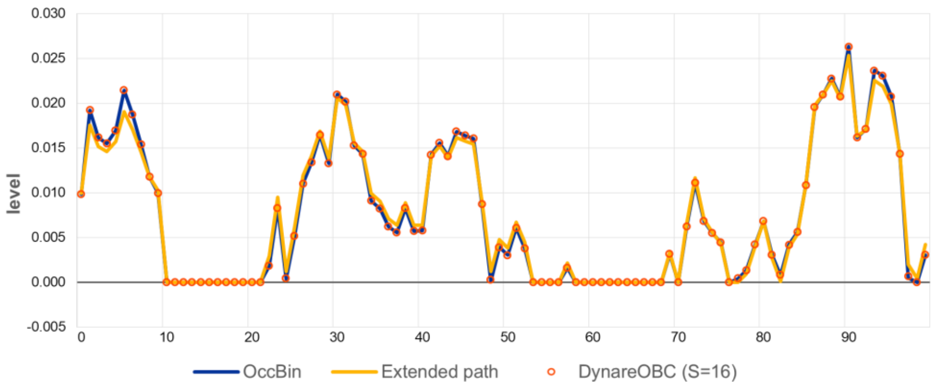
Chart 2.2

Sample paths following a sequence of housing demand shocks in the Guerrieri-Iacoviello model

a) Consumption



b) Shadow price



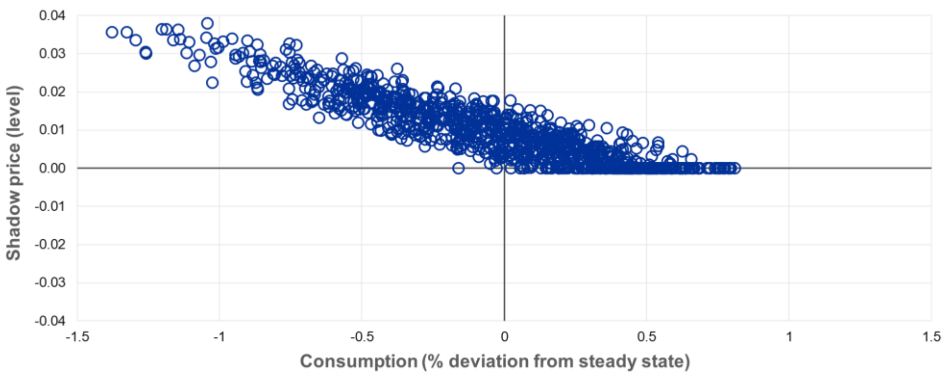
Source: Authors' calculations.

Nevertheless, the solutions capture the role of asymmetries produced by the borrowing limit in the model: the constraint becomes slack when consumption rises but is very binding when consumption falls.

Chart 2.3 highlights this asymmetry: consumption barely rises above 0.5% relative to the steady state in this hypothetical simulation, as the borrowing constraint slackens and the financial accelerator is dampened. On the other hand, consumption falls significantly below the steady state when negative housing demand shocks hit, as they lower the value of collateral (housing). This tightens the borrowing limit and prompts an adjustment by borrower households.

Chart 2.3

Simulations for consumption and the shadow price on the borrowing constraint

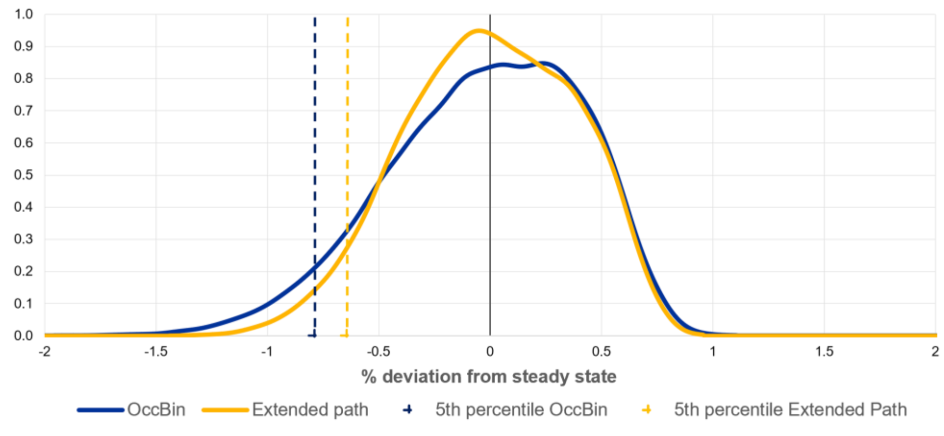


Source: Authors' calculations.
Note: Scatter plot based on a simulation of 1000 periods using OccBin.

Ultimately, these dynamics result in a skewed distribution of consumption, as shown in Chart 2.4. As we discussed above, while different algorithms might produce solutions that differ slightly in numerical terms, this application highlights the possibility that the macroeconomic models typically used by policy institutions could provide risk estimates that incorporate non-linearities. In Chart 2.4 the shaded region denotes the estimate of "consumption-at-risk", which is a useful metric to quantify the cost of downturns or adverse developments.

Chart 2.4

Consumption distribution under housing demand shocks



Source: Own lculations.
Note: The distributions are based on 100 simulations for the consumption path over 500 periods, after discarding the first 50 periods to remove the effect of the initial value, using OccBin and extended path. The shaded regions denote the lower 5% of the distribution.

Finally, it is instructive to look at the computation time required by different algorithms to produce an output. Table 2.1 shows the average run times to solve and simulate the Guerrieri-Iacoviello model described above. DynareOBC with perfect foresight $(S = 0)$ and OccBin, which produce identical results, are very fast to run and yield their output in under 10 seconds. Extended path is slower and takes about three times as long to finish. Run times get longer on DynareOBC as the integration order rises. It takes about 42 seconds when integrating over four periods of future uncertainty, but shoots up to just under 2.5 hours when the order of integration rises to 16. As shown above, this partial relaxation of perfect foresight adds very little to the results, at least when working with a linearised version of the model, and therefore may not justify the additional computing time needed.

Table 2.1

Run times across different solution algorithms

	OccBin	Extended path	DynareOBC (S = 0)	DynareOBC (S = 4)	DynareOBC (S = 16)
Run times (s)	9.9	33.7	8.2	42.5	8860.9

Note: Run times are expressed in seconds and record the time taken to simulate the model for 500 periods. The simulations were conducted on a Windows 11 machine with an Intel Xeon W-2145 processor running at 3.7 (4.5) GHz, with 32Gb of RAM. S is the horizon up to which agents in the model are uncertain about shocks.

3 Macroeconomic models with financial vulnerability channels

3.1 Introduction

Generating asymmetric distributions of variables in structural models is a difficult task. While asymmetries can be obtained in models with global solutions, these are difficult to solve and maintain (see previous section). Even for more versatile models like linear models with OBCs, it remains challenging to generate asymmetries like those observed in the data. For the most part, asymmetries have so far proved elusive in the linearised structural models commonly used by central banks for policy analysis.

To introduce GaR dynamics into otherwise linear models, Adrian et al. (2020) suggest using a financial vulnerability channel. The resulting NKV model makes the standard deviation of exogenous disturbances a function of past financial conditions. The model can replicate several key GaR features that are absent from fully linear DSGE models, such as the asymmetry in conditional output gap quantiles or the skewness in its distribution.

This chapter looks at GaR dynamics across a range of models with vulnerability. The NKV model and the procedure to fit it to the data are presented in detail. The model fitted to euro area data gives GaR dynamics such as asymmetric quantiles of conditional output gap growth and a left-skewness of its distribution, much as empirical models do. The same is true when the models are applied to three European System of Central Banks (ESCB) Member States (Czech Republic, Germany and Portugal), although the strength of GaR varies across countries. We present two extensions of the NKV model: one including open-economy dynamics and one with vulnerability in the larger, medium-sized DSGE model proposed by Angeloni and Faia (2013). A vulnerability channel is a versatile tool to elicit GaR dynamics from a wide range of models.

Structural models like these have significant potential for policy analysis, such as the GaR implications of policy counterfactuals. For example, models of this class could be useful to assess the financial stability implications of accommodative monetary policy and interactions with macroprudential policies. Hence, they could be a valuable methodological tool to support an enhanced role for financial stability in quarterly monetary and financial assessments following the monetary policy strategy review of the ECB.

3.2 The New Keynesian Vulnerability (NKV) model

Adrian et al. (2020) extend the small NK model with a “vulnerability function” to introduce GaR dynamics. Starting from the textbook 3-equation NK model

(Woodford, 2003; Galí, 2008), they add endogenous risk through a financial accelerator (Bernanke, Gertler and Gilchrist, 1999) and a financial vulnerability channel. The three NK equations: the investment and savings (IS), Phillips curve, and Taylor rule (see equations (1)-(3) below) are enriched with an explicit role for financial conditions and vulnerability. Financial conditions are assumed to be forward-looking. In other words, they endogenously depend on the contemporaneous and expected levels of the output gap (see equation 4). Due to the introduction of the extra wedge $V(X_t)\varepsilon_t^{y^{gap}}$ in the IS curve, the variance of the shock to that equation becomes conditionally heteroscedastic: while $\varepsilon_t^{y^{gap}}$ is i.i.d. $\mathcal{N}(0, \sigma_y^2)$, $V(X_t)$ represents vulnerability to exogenous shocks, which depends on past values of financial conditions and of the output gap and varies with past state variables (see equation 5). Moreover, the fact that financial conditions η_t depend indirectly on the interest rate i_t through the IS curve opens up the "risk-taking channel" of monetary policy. As in Adrian et al. (2020), the model has just one exogenous driver, $\varepsilon_t^{y^{gap}}$.

$$
\begin{aligned}
y_t^{gap} &= E_t y_{t+1}^{gap} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}) - \gamma_\eta \eta_t - V(X_t)\varepsilon_t^{y^{gap}} && (1)\\
\pi_t &= \beta E_t \pi_{t+1} + \kappa y_t^{gap} && (2)\\
i_t &= \phi^\pi \pi_t + \phi^y y_t^{gap} + \nu_t && (3)\\
\eta_t &\equiv \lambda_\eta \eta_{t-1} + \lambda_{\eta\eta}\eta_{t-2} - \theta_y y_t^{gap} - \theta_\eta E_t y_{t+1}^{gap} && (4)\\
V(X_t) &= \nu_c + \varrho_{\eta_1}\eta_{t-1} + \varrho_{\eta_2}\eta_{t-2} + \varrho_{gap} y_{t-1}^{gap} && (5)
\end{aligned}
$$

The resulting NKV model is non-linear, as conditional second moments are functions of state variables. Therefore, higher-order approximations are needed to allow conditional second moments to vary over time. The model is thus simulated based on a third-order approximation (perturbation).

The simulated method of moments (SMM) is used to fit the NKV model to euro area data. While some NKV model parameters are calibrated to standard textbook values (Table 3.1), those central to the vulnerability function – and therefore to the non-linearity – are estimated on euro area data using the SMM (Table 3.2).[5] The model moments implied by the estimated parameters remain close to the empirical ones (Table 3.3).

Table 3.1
Calibrated NKV parameters

σ	β	α	ε	θ	φ	ϕ^π	ϕ^y
1	0.99	1/3	6	2/3	1	1.5	0.125

Note: See equations (1) to (5) in the text. The calibration largely follows Galí (2008). Some standard composite parameters follow as $\omega = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$, $\lambda = \frac{(1-\theta)(1-\beta\theta)\omega}{\theta}$ and $\kappa = \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha})$.

5 The composite indicator of systemic stress (CISS, Hollo et al (2012)) (Figueres and Jarociński, 2020) is used as a proxy for financial conditions (η). Inflation is calculated based on the euro area (changing composition) Harmonised Index of Consumer Prices (HICP), while GDP is the real euro area 19 chain-linked, seasonally-adjusted series. As in Coenen et al. (2019), to construct the output gap series annual figures of the EU Commission's potential output series (AMECO database) have been converted into quarterly figures using cubic splines.

Table 3.2

SMM-estimated NKV parameters

γ_η	v_c	ϱ_{η_1}	ϱ_{η_2}	ϱ_{gap}	λ_η	$\lambda_{\eta\eta}$	θ_y	θ_η
0.008	0.005	-0.019	0.010	0.452	1.980	-0.100	0.091	0.434

Note: See equations (1) to (5) in the text. Also see Table 3.3 for which moments are matched to obtain these values.

Table 3.3

Empirical and model-based moments

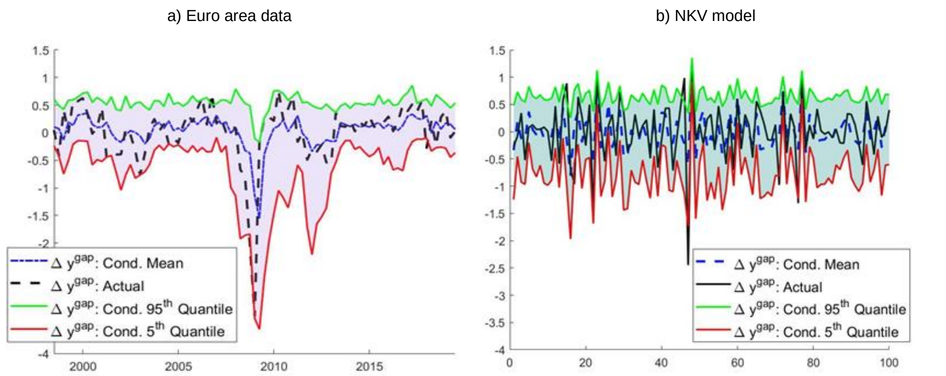
	Data	Model
$AC1\left(E\left(y^{gap}\right)\right)$	0.956	0.811
$AC3\left(E\left(y^{gap}\right)\right)$	0.772	0.741
$AC5\left(E\left(y^{gap}\right)\right)$	0.563	0.605
$corr\left(E\left(y^{gap}\right), cond.vol.\left(\Delta y^{gap}\right)\right)$	-0.820	-0.976
$var\left(\Delta y_{Q5}^{gap}\right)$	0.485	0.262
$var\left(\Delta y_{Q95}^{gap}\right)$	0.022	0.033
$\frac{var\left(\Delta y_{Q5}^{gap}\right)}{var\left(\Delta y_{Q95}^{gap}\right)}$	21.842	7.901
$corr(\eta_{t-1}, \eta_{t-1})$	0.887	0.993

Note: Sample period is 1997Q1 to 2019Q3. See Table [T_NKV2] for which parameters are estimated based on matching these moments. AC(l) is autocorrelation at lag l.

The NKV model replicates well the asymmetry of output (gap) quantiles. Chart 3.1 shows the 5th (red), median (blue) and 95th (green) quantile of the distribution of conditional output gap growth. As also documented in the empirical literature for the euro area (see Figueres and Jarociński, 2020, for example), lower output growth quantiles are more responsive to financial conditions than the median, while the upper quantiles are almost constant (panel a) of Chart 3.1). This finding is replicated when simulated data obtained from the NKV model are fitted to euro area data (panel b).

Chart 3.1

Conditional output gap growth quantiles

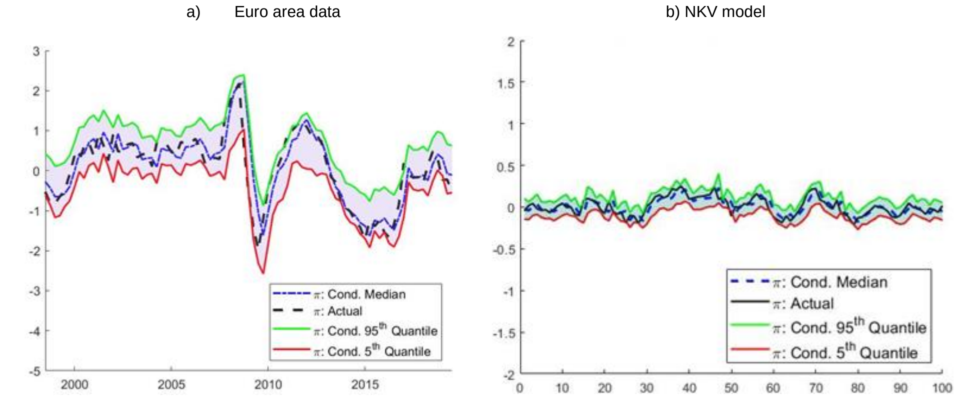


Note: Selected quantiles of output gap growth, conditional on past state variables, from euro area data (panel a) and simulated from the NKV model fitted to euro area data (panel b). "Actual" refers to realised values (empirical values in panel a), simulated in panel b).

As in the euro area data, this asymmetry in quantiles does not hold for inflation. Instead, the lower and upper quantiles of inflation are equally responsive and vary little over time (Chart 3.2).

Chart 3.2

Conditional inflation quantiles

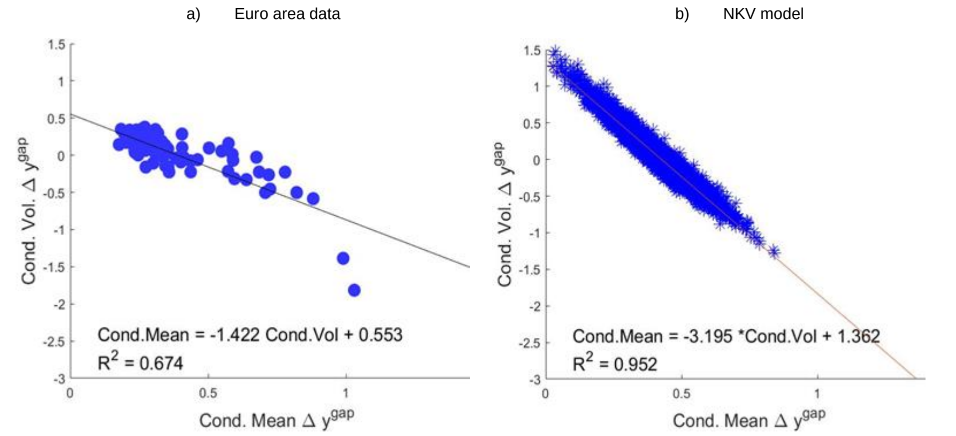


Note: Selected inflation quantiles, conditional on past state variables, from euro area data (panel a) and simulated from the NKV model fitted to euro area data (panel b). “Actual” refers to realised values (empirical values in panel a), simulated in panel b).

Unlike in standard linear models, the conditional mean and volatility of output gap growth correlate negatively in the NKV model. In standard linear DSGE models with uncorrelated shocks, when exogenous shocks cause significant output gap growth, volatility is expected to increase as the output gap returns to its steady state. The implied positive correlation between the conditional mean and variance of the output gap (growth) contradicts empirical findings for the euro area (Chart 3.3 right). But in a model with vulnerability, periods of benign financial conditions and a relatively closed output gap will heighten the economy’s vulnerability to shocks via the added $V(X_t)$ channel and therefore increase conditional output volatility. Hence, the negative correlation between the conditional mean and variance of output gap growth is empirically valid (Chart 3.3).

Chart 3.3

Correlation of the conditional mean and conditional volatility of output gap growth

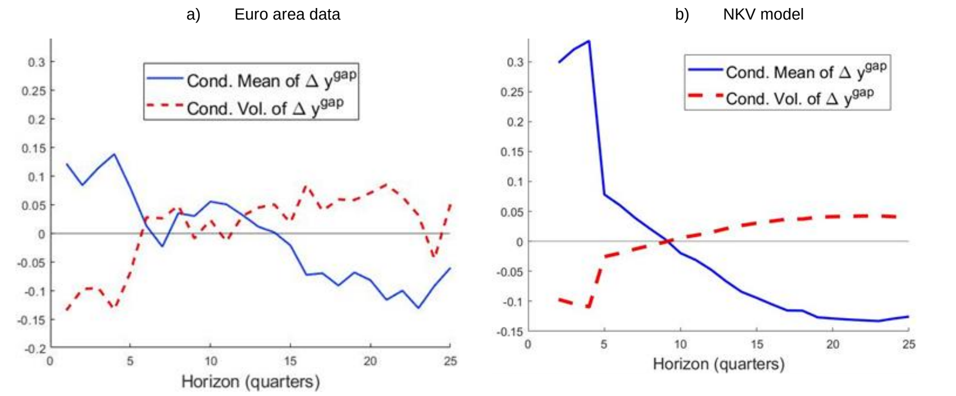


Note: Mean and volatility of output gap growth conditional on past state variables from euro area data (panel a) and simulated from the NKV model fitted to euro area data (panel b).

The NKV model also replicates a volatility paradox observed in the euro area data. The volatility paradox refers to the observation that future risks build during good times, when contemporaneous risk is low while output growth is high (Brunnermeier and Sannikov, 2014). In good times, financial conditions are loose and GDP volatility is low. But this effect eventually reverts because the increased risk-taking in good times leads to higher vulnerability. These volatility paradox dynamics are shown in Chart 3.4 by the elasticity of the conditional mean and conditional volatility of the output gap to financial conditions. In both the euro area data and the NKV model, the elasticity of conditional output gap volatility to financial conditions is negative in the short term but becomes positive as the projection horizon lengthens. The opposite is true for the elasticity of the conditional mean of the output gap.

Chart 3.4

Volatility paradox (elasticity of conditional moments to financial conditions)



Note: The graphs depict the time-varying elasticity of the conditional mean and volatility of the output gap growth to financial conditions.

3.3 Country-specific NKV studies: Czech Republic, Germany and Portugal

By fitting NKV model dynamics to data from ESCB Member States, we compare GaR dynamics across countries. Three national central banks (in the Czech Republic, Germany and Portugal) used a common set of routines to fit the NKV model to national macroeconomic data (demonstrating the relative ease of applying the model to a range of settings). The results show a degree of heterogeneity across the three economies.

Table 3.4

Different model specifications tested for the three Member States

	Model 1	Model 2	Model 3
Volatility kernel	$\nu + \tau_a \eta_{t-1} + \tau_b \eta_{t-2} + \boldsymbol{\tau_c y_{t-1}^{gap}}$	$\nu + \tau_a \eta_{t-1} + \tau_b \eta_{t-2}$	$\nu + \tau_a \eta_{t-1} + \tau_b \eta_{t-2} + \boldsymbol{\tau_c y_{t-1}^{gap}}$
Moments targeted by SMM	$\mathrm{E_t}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1})]$ $\mathrm{Var}[y_t^{gap}]$ $\mathrm{Cov}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1}), \mathrm{E}_{t-1}(\Delta y_t^{gap})]$ $\mathrm{Var}[\Delta y_t^{gap}]$ $\mathrm{Cov}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1}), \eta_{t-1}]$	$\mathrm{E_t}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1})]$ $\mathrm{Var}[y_t^{gap}]$ $\mathrm{Cov}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1}), \mathrm{E}_{t-1}(\Delta y_t^{gap})]$ $\mathrm{Var}[\Delta y_t^{gap}]$ $\mathrm{Cov}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1}), \eta_{t-1}]$	$\mathrm{E_t}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1})]$ $\mathrm{Var}[y_t^{gap}]$ $\mathrm{Cov}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1}), \mathrm{E}_{t-1}(\Delta y_t^{gap})]$ $\mathrm{Var}[\Delta y_t^{gap}]$ $\mathrm{Cov}[\mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1}), \eta_{t-1}]$ $\boldsymbol{\mathrm{Cov}[y_{t-1}^{gap}, \mathrm{Var}(\Delta y_t^{gap} \vert \Omega_{t-1})]}$

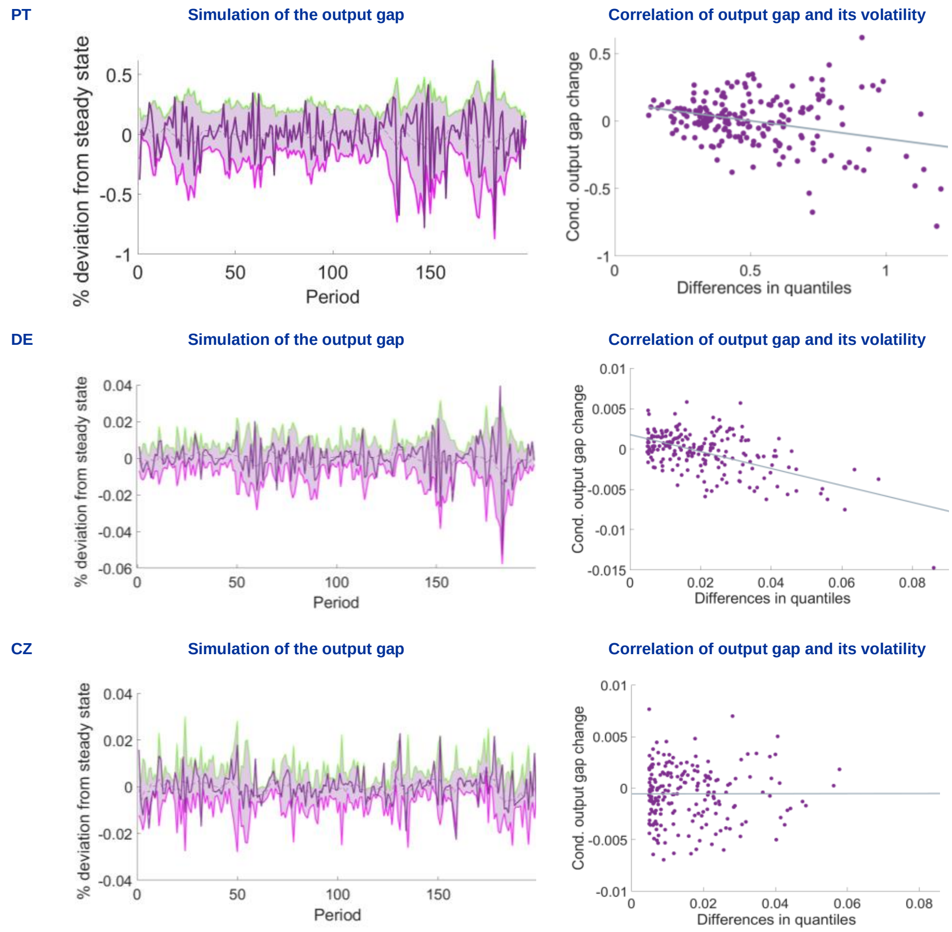
Note: For each of the three countries (CZ, DE, PT), all three model variants were tested. Model 1 corresponds to choices in Adrian et al. (2020); model 2 leaves the lagged output gap out of the volatility function; and model 3 matches an additional empirical moment.

The trade-off here is between the number of parameters fitted to country data and the quality of the estimation. Even though the model is small, estimating the linear part was challenging for all three countries, due to the large number of parameters and moments to be matched contemporaneously. An alternative (which was not pursued) would be to use textbook calibration for the linear model and focus instead on estimating the non-linear part for each country.

Different vulnerability set-ups work best for different countries. Three different model specifications are used (see Table 3.4): one with a volatility function specified as in the original NKV model and targeting five moments; one with a volatility function that is not dependent on the lagged output gap; and one targeting a sixth moment. Model 1 yields the strongest GaR results for Germany and Portugal, while model 2 works best for the Czech Republic.

Chart 3.5

Quantiles of simulated changes in the output gap across countries

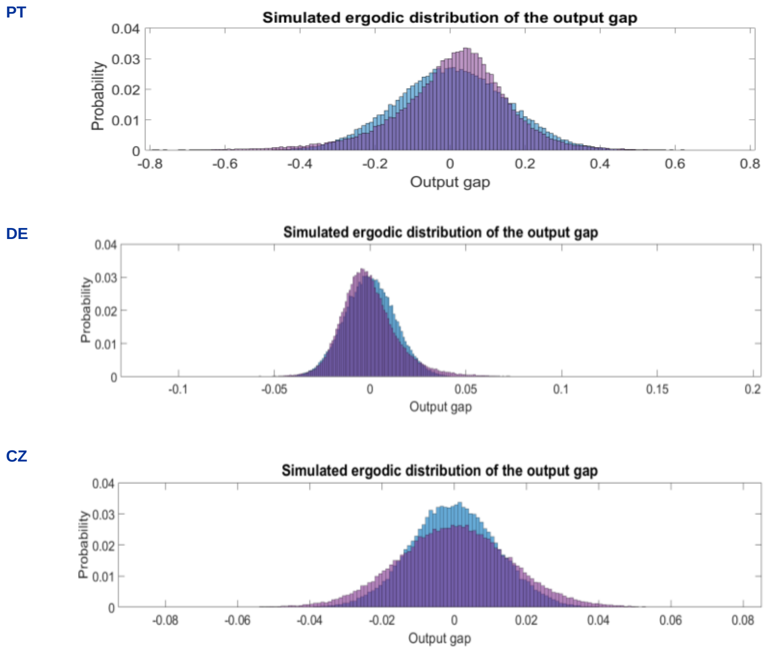


Note: In the left column, the violet line gives the median and the green and pink lines the 95th and 5th quantiles of conditional output gap growth. In the right column, output gap volatility is approximated by the difference between its 95th and 5th quantiles.

We found differences among countries in the effectiveness of the specifications and the magnitude of GaR effects. For both Germany and Portugal the 5th quantile of output gap growth (pink line in the left-hand panels of Chart 3.5) is much more volatile than the 95th quantile (green line). This is not the case for the Czech Republic. Allowing for vulnerability also leads to a negative correlation between the conditional mean and variance of output gap growth for Germany and Portugal, while it is flat at best for the Czech Republic (right row). Similarly, the left-skewness of the ergodic distribution of the simulated output gap in models with vulnerability (violet bars in Chart 3.5 relative to a model without vulnerability (blue bars) is very prominent for Portugal, but less so for the Czech Republic and Germany.

Chart 3.6

Simulated ergodic distributions of the output gap across countries



Note: The blue bars give results from the linear and homoscedastic model without vulnerability; the violet bars give results from the non-linear model with heteroscedasticity resulting from a vulnerability function.

3.4 A small open economy extension of the NKV model

In this section we explore how fluctuations in financial conditions are transmitted internationally. To do this, we extend the closed economy model proposed by Adrian et al. (2020) to a small open economy framework as in the benchmark models of Galí and Monacelli (2005) and De Paoli (2009).

There are two economies in the extended model: an SOE ("home") and the rest of the world. Households in the home economy buy domestic and international bonds. Home bonds are traded only within the domestic economy. The supply side of the economy consists of producers of consumer goods that set their prices infrequently, as in Calvo (1983), and engage in producer currency pricing (PCP).[6] The monetary policy rate in each region is set according to a Taylor rule, while fiscal authorities run a balanced budget. In line with Adrian et al. (2020), in each region the standard deviation of demand shocks is a time-varying function of financial conditions. Financial conditions in turn are a function of own lags, the domestic private consumption growth rate and the monetary policy stance.

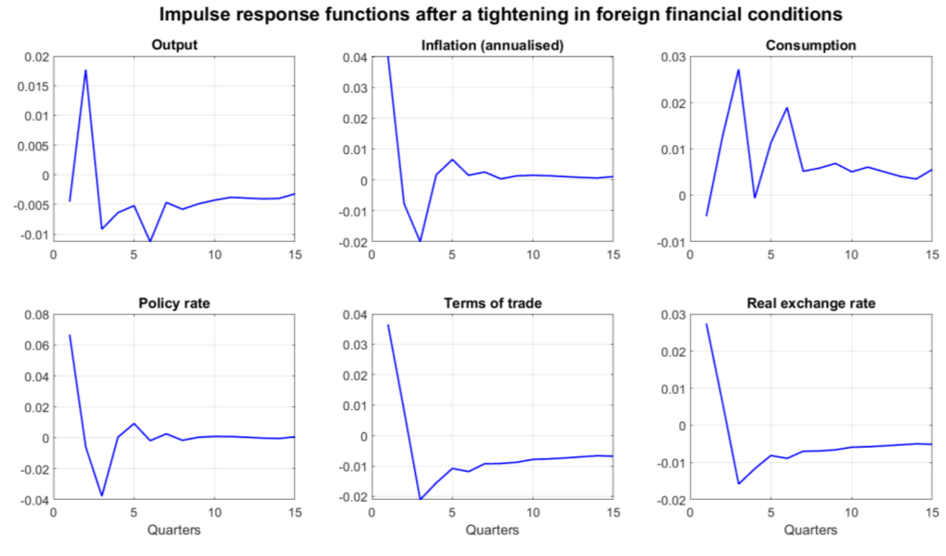
6 The assumption of PCP clearly magnifies the effects of exchange rate fluctuations on domestic inflation and thereby on output. The alternative of local currency pricing (LCP), for instance, would imply incomplete exchange rate pass-through and would mitigate the effects on inflation and output presented later in this section. However, accounting for the properties of open economy models under PCP and LCP, and because all the transmission channels are demand side, the pricing assumptions would not impact the qualitative conclusions (i.e. the direction of effects and the transmission channels) presented in this section.

The SOE set-up gives rise to four transmission channels of financial conditions not found in the closed economy configuration: one domestic and three international channels. First, as in the closed economy set-up, fluctuations in financial conditions impact consumption and output directly via the Euler equation, a mechanism defined as the domestic channel. Second, since households hold home and foreign assets, financial conditions affect expected exchange rate fluctuations via the modified uncovered interest rate parity condition. Third, because the home economy has a non-zero net foreign asset position, financial conditions affect the domestic current account and hence home output. And fourth, since foreign bonds are traded internationally and international financial markets are incomplete, financial conditions affect foreign demand for home goods through the imperfect risk-sharing condition.

Note that these transmission channels hold regardless of whether the model features a homoscedastic or a heteroskedastic demand-side shock – although the quantitative conclusions clearly depend on the assumptions about the second moments of this shock. In this case, the transmission channels that are directly impacted are those operating through the Euler equation and imperfect risk sharing.

Chart 3.7

Dynamic responses of key variables after a tightening in foreign financial conditions (SOE model)

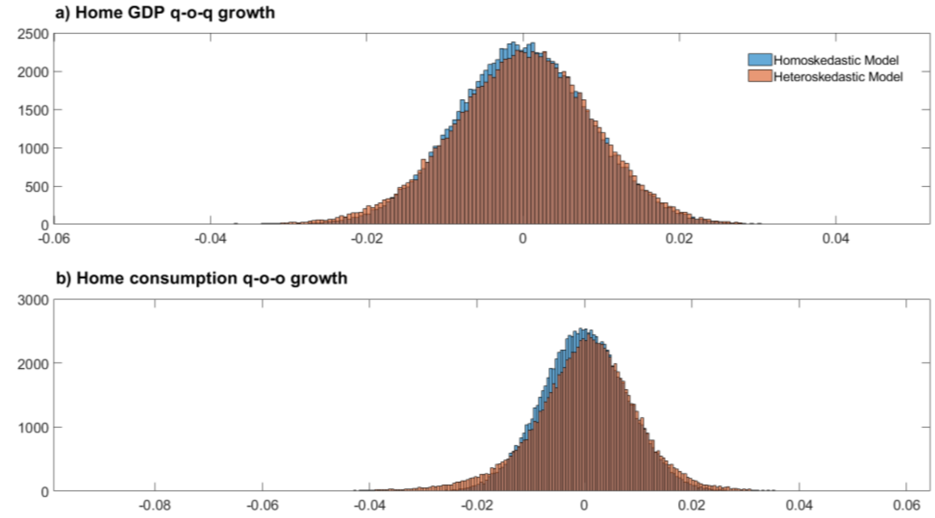


Note: X-axis: quarters post shock. Y-axis: percentage deviations from the steady state.

This section has two main goals: to measure the vulnerability of an SOE to fluctuations in foreign financial conditions and to explore downside risk to growth for such an economy. For the first of these goals the model features home and foreign supply and demand shocks, as well as a shock to home financial conditions and a shock to foreign financial conditions. Chart 3.7 shows impulse responses of the home SOE to an exogenous tightening in foreign financial conditions. Foreign financial tightening causes the real exchange rate to depreciate on initial impact, to then appreciate in subsequent quarters. The depreciation is associated with an initial deterioration in the terms of trade. As a result, global demand for goods produced in the SOE rises and output expands in the initial quarters after the shock. Importantly, the effects of the depreciation more than offset the downward pressures on output that stem from the initial decline in private consumption. Inflation increases on initial impact, which is clearly due first to the perfect exchange rate pass-through and then to the initial increase in global demand for domestically produced goods.[7]

Chart 3.8

Ergodic distributions of key variables (SOE model)



Note: The blue bars give results from the linear and homoscedastic model without vulnerability; the orange bars show results from the non-linear model with heteroscedasticity resulting from a vulnerability function.

We gain further insights when we focus on the real exchange rate. Specifically, and given price stickiness, the behaviour of the real exchange rate is similar to that of the nominal exchange rate. At the same time, PCP implies strong exchange rate pass-through, which provides the rationale for the jump in home inflation on impact. The initial inflationary pressures prompt the home central bank to raise its policy rate, which in turn slows home consumption and tightens domestic financial conditions. The subsequent lower rates (higher expected present value of wealth) and the appreciation (higher purchasing power) in the medium term cause private consumption to increase in subsequent quarters. Finally, the current account channel explains the gradual real depreciation in the medium term until the real exchange rate reverts to the steady state. Tighter foreign financial conditions lead to a gradual rise in the stock of net foreign assets, which in turn increases demand for foreign currency.

The second goal of this section is to explore downside risk to growth for SOEs. A model simulation including all the shocks above and a comparison with the homoscedastic counterpart shows that private consumption growth in the SOE is

[7] Note that under the alternative LCP, the depreciation in the domestic currency would not be passed through to domestic inflation, at least not directly. In this case, therefore, domestic monetary tightening, and thereby the medium-run contraction, would be milder. Note that the depreciation in the domestic currency is not the result of pricing assumptions. The currency depreciates because the tightening of foreign financial conditions leads to higher foreign spreads and higher interest rates in the foreign block. Through the UIP condition, this causes the domestic currency to depreciate on impact.

subject to downside risks. Specifically, the left tail of the distribution of private consumption growth extends beyond the homoscedastic model's left tail, while the right tail is affected less (panel b) in Chart 3.8). There are visible downside risks for home output also, but not to the same extent as for private consumption (panel a). One explanation is that downside risks stemming from private consumption are offset by the behaviour of the terms of trade and the real exchange rate, both of which have a non-negligible effect on the SOE's output.

3.5 The Angeloni-Faia with vulnerability (AFV) model

We can easily transfer the vulnerability channel from the NKV model to larger models with an explicit financial friction. There are limits to the 5-equation model based on the set-up described in Adrian et al. (2020) due to its somewhat ad hoc specification of financial frictions and the financial conditions variable. The alternative is a medium-sized DSGE model with an explicit financial friction to generate GaR results with a vulnerability function, such as the Angeloni and Faia (2013) model used here. Besides containing well-established frictions like nominal rigidities and capital adjustment costs, this model introduces "financial fragility" to generate a financial accelerator. It embeds a micro-founded banking sector in which banks bundle funds from depositors and bank capitalists to finance productive investments. This model draws on the financial literature (Diamond and Rajan, 2000). Banks choose an optimal deposit ratio (i.e. leverage) with a classic risk-return trade-off: more deposit financing increases bank returns, while more bank capital financing lowers the risk of bank runs. Banks are subject to idiosyncratic returns on their investments which, together with the assumption of a sequential servicing constraint for deposits, leads to runs on some banks in equilibrium. The aggregate "bank-run probability" (proportion of banks likely to be run) depends on aggregate leverage in the economy and the realisation of the exogenous shocks in the model.

The bank-run probability gives a micro-founded and model-consistent indicator of financial conditions (or financial stress). Taking this indicator and following Adrian et al. (2020), we assume that the conditional volatility of the exogenous shocks in the model depends directly on the aggregate run-probability: in other words, our financial stress indicator. Although the assumption is ad hoc, it is fully consistent with the evidence provided by the empirical GaR literature, which suggests a strong relationship between indicators of financial stress and the conditional distribution of GDP growth.

The AFV model is obtained by adding a vulnerability term dependent on past financial conditions. Specifically, the model is extended by adding the vulnerability function:

$$V(X_{i,t}) = \sigma_i \tau_a \exp\{\tau_{b,i}(\phi_{t-1} - \phi_{ss})\},$$

where:

ϕ_t is the aggregate bank-run probability

ϕ_{ss} is the steady-state value

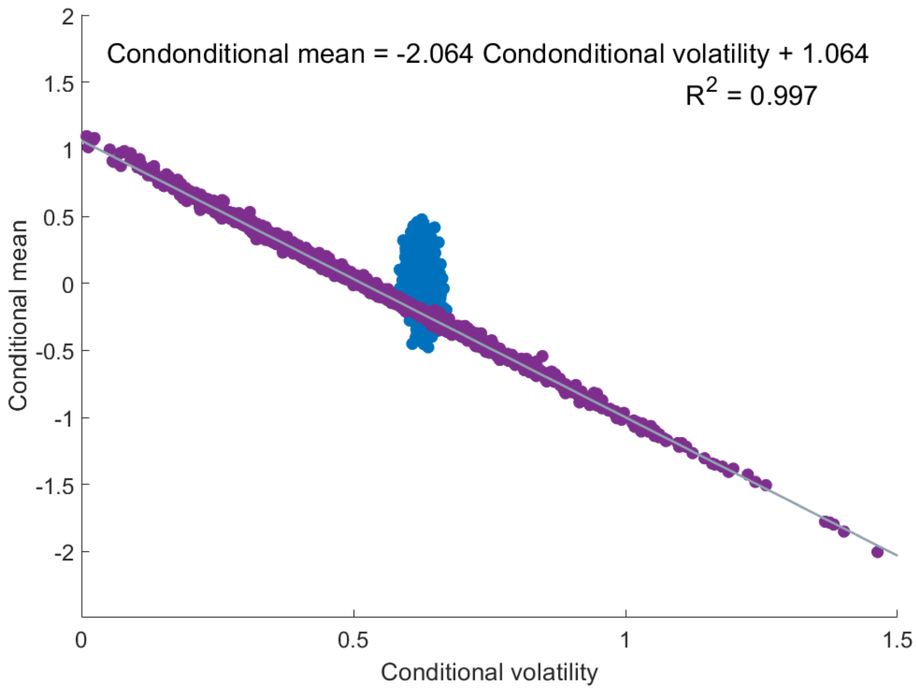
σ_i is the standard deviation of shock i,

τ_a and $\tau_{b,i}$ are scaling parameters configured using SMM.

Technically, $V(X_{i,t})$ gives the time-varying conditional standard deviation of the exogenous model shocks. While in Adrian et al. (2020) $V(X_{i,t})$ affects the variability of just one shock, in our version of the Angeloni-Faia model it scales the standard deviation of all eight shocks i: monetary policy; TFP technology; government spending; investment technology; time preference; bank capital; price markups; and the dispersion of idiosyncratic firm returns. Note that as $\tau_{b,i} > 0$, any of the above shocks that ease financial conditions (and increase ϕ_t) will automatically increase vulnerability and thus stochastic heteroscedasticity, similarly to the set-up for only one shock in Adrian et al. (2020).

Chart 3.9

Correlation of conditional mean and conditional volatility of output growth, with and without $V(X_t)$

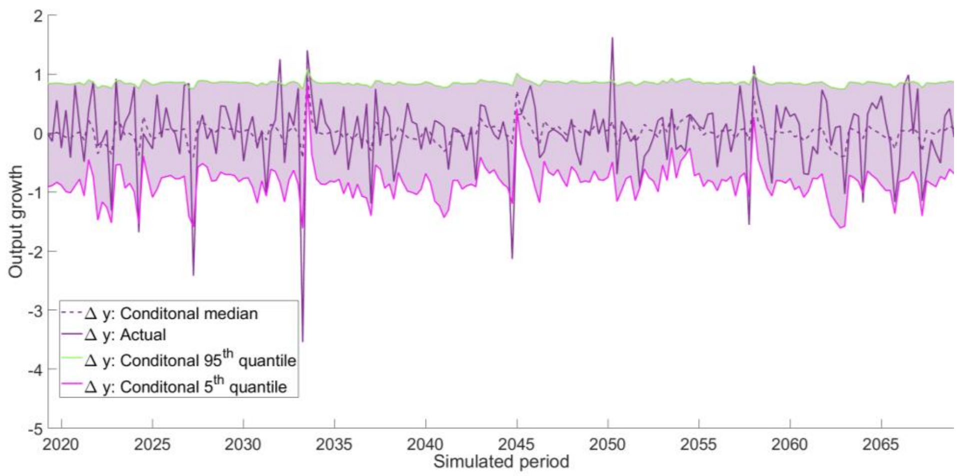


Note: Results from a third-order approximation of the Angeloni-Faia model with vulnerability (violet) and without (blue). The graph plots the difference between the 95% and 5% quantiles as a proxy for volatility conditional on past state variables.

The model parameters are based on a linear estimation of the AFV model on euro area data. Quantitative results should therefore be interpreted with caution. Still, several regular features in the empirical GaR literature can be replicated in an otherwise largely “off-the-shelf” linear structural model with micro-founded financial friction. The vulnerability parameter is set at an intermediate value of $\tau_a = 0.5$. Chart 3.9 shows the negative correlation between the conditional mean and conditional variance of output growth for the model with vulnerability (violet dots). This replicates the GaR finding for developed economies for long periods of tranquil growth punctuated by abrupt crises. This feature is absent from linear DSGE models without a vulnerability component, where exogenous disturbances are equally likely to increase or decrease economic growth (see blue dots).

Chart 3.10

Simulation of conditional quantiles of output growth

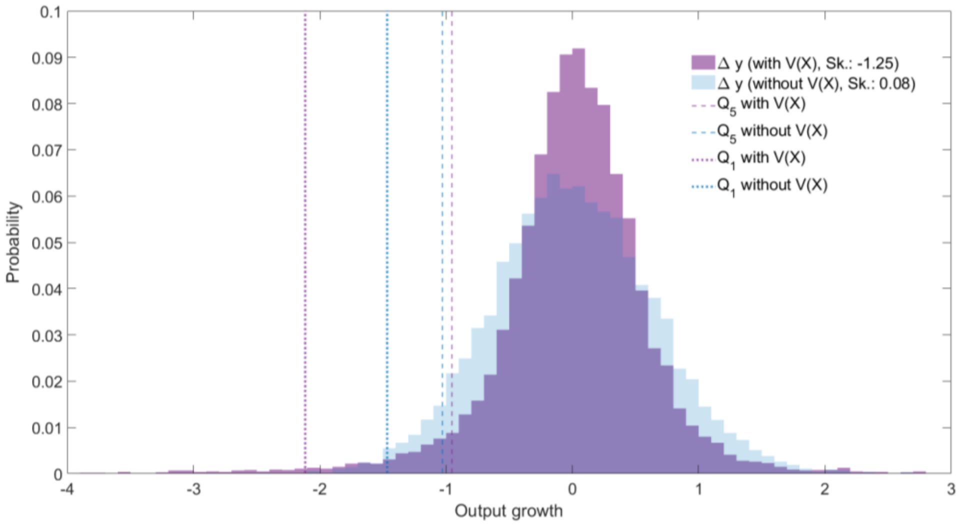


Note: Results from a third-order approximation of the AFV model with vulnerability function $V(X_{i,t})$ parameters obtained by SMM on euro area data. Quantiles are based on a one-step-ahead forecast. 50 simulated periods are plotted.

Chart 3.10 shows that the AFV model also performs well in replicating the GaR feature of asymmetric growth rates. The upper (95th) quantile of output growth is much more stable than the corresponding lower (5th) quantile. Thus, while periods of positive growth fall into a tighter range, making them easier to predict, recessionary periods vary more widely as to how much they reduce growth.

Chart 3.11

Simulated ergodic distribution of output growth, with and without $V(X_t)$



Note: Δy is output growth. Results from a third-order approximation of the AFV model with parameters of the vulnerability function $V(X_{i,t})$ obtained by SMM on euro area data.

Lastly, vulnerability leads to a skewed ergodic output growth distribution (Chart 3.11). Without vulnerability (blue bars), the ergodic distribution approximates a symmetric normal distribution. However, in the model with vulnerability (violet bars), the distribution has a higher kurtosis (more mass in the tails). Therefore, growth outliers (both to the upside and downside) are more likely, a fact also alluded to in the GaR, and other, literature. Moreover, the distribution under vulnerability visibly skews to the left: GDP grows predominantly at intermediate positive rates but relatively rare recessionary periods may presage rather large GDP contractions.

In conclusion, a version of the Angeloni and Faia (2013) model with a vulnerability function produces several empirical GaR regularities. In particular, the conditional heteroscedasticity introduced by the $V(X)$ function is based on a financial condition variable stemming from a micro-founded financial sector. In general, many linear DSGE models used for policy analysis that include such a variable summarising financial conditions could be augmented with a vulnerability function to account for GaR dynamics.

3.6 Conclusion

GaR features can be introduced into otherwise linear models through a vulnerability channel of financial conditions and conditional heteroscedasticity. A version of the NKV model fitted to euro area data replicates several key empirical findings from the GaR literature, from asymmetry in quantiles of output growth to the left-skewness of its distribution. When fitted to macro data from three significantly different ESCB Member States (the Czech Republic, Germany and Portugal), the NKV model replicates important stylised facts and non-linearities from the empirical literature. Nevertheless, we found variances in the results in the three versions of the model. These may stem from the limited size of the NKV 5-equation model. Two model extensions help to address these limitations: i) a version of the NKV model in an open-economy setting, and ii) a version of the medium-sized Angeloni and Faia model (2013) with vulnerability. In addition, when financial vulnerability is included, these otherwise linearised models can replicate key GaR features.

A vulnerability channel appears to be a versatile tool to elicit GaR dynamics in widely used policy models. Linear DSGE models are often used for counterfactual or normative analysis. One disadvantage of this model class is the lack of asymmetries in distributions, such as output growth, documented in the empirical GaR literature. Vulnerability functions are one potential tool to introduce GaR features into a number of otherwise linear DSGE models.

Box 1
Macroprudential policy versus extended Taylor rule in the NKV model

Prepared by Matthieu Darracq Pariès, Rouven Geismar and Niki Papadopoulou

The NKV model, with its focus on endogenous output risk, is an appropriate tool for evaluating monetary and cyclical macroprudential policies simultaneously, since the purpose of cyclical macroprudential tools is to mitigate downside risks to the output gap. For this type of scenario analysis, the euro area NKV model is extended and complemented by a hypothetical policy instrument and/or an extended monetary policy rule that impacts the level of financing conditions.

In the first case, tighter macroprudential policy is assumed to increase the price of risk and affect output growth through the financial accelerator. More specifically, we assume that a state-contingent countercyclical macroprudential policy instrument, μ_t, is dependent on financing conditions with a lag, $\mu_t = \nu_\eta \eta_{t-1} + \nu_{\eta\eta}\eta_{t-2}$, and can also contemporaneously affect financing conditions (note that these significant assumptions might overstate how swift and effective macroprudential decisions are in practice):

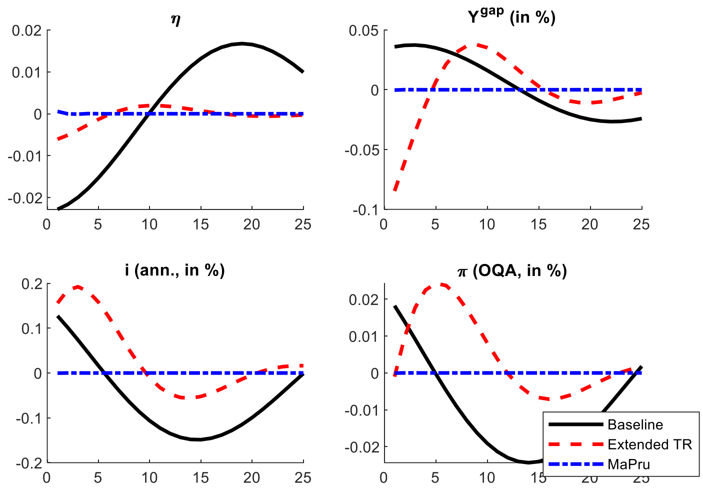
$$\eta_t \equiv \mu_t + \lambda_\eta \eta_{t-1} + \lambda_{\eta\eta}\eta_{t-2} - \theta_y y_t^{gap} - \theta_\eta E_t y_{t+1}^{gap}$$

In the second case, policymakers may need to evaluate the benefits of alternative policy rules that account directly for expected financial conditions, in the following manner:

$$i_t = \phi^\pi + \phi^y y_t^{gap} - \phi^\eta E_t \eta_{t+1} + \nu_t$$

Chart B1.1

Impulse response functions to demand shocks with alternative policy tools



Note: Starting at regime-specific η-volatilities. Baseline: std(η) = 0.23; extended Taylor Rule: std(η) = 0.23; MaPru: std(η) = 0.23

In Chart B1.1 the above policies are evaluated separately and compared in an environment of loose financial conditions. These are associated with a positive output gap and higher levels of inflation that prompt central banks to tighten rates by around 0.1 basis points. This eventually brings down inflation, reduces the output gap, and leads to a gradual tightening of financial conditions. Under the standard Taylor rule, financial conditions overshoot, leading to elevated vulnerability after the tenth quarter. And under the extended Taylor rule, policymakers also account for fluctuations in financial conditions, thereby effectively eliminating periods of very tight or very loose financial conditions. Lastly, when financing conditions are directly affected by state-contingent countercyclical macroprudential policy dependent on financing conditions with a lag, policymakers can do more to eliminate periods of fluctuations in financial conditions and deliver lower volatility in both the output gap and inflation.

Box 2

A detailed look at the bank-run mechanism in Angeloni and Faia (2013)

Prepared by Nikolay Hristov, Benedikt Kolb and Leonardo Urrutia

The financial friction in Angeloni and Faia (2013) builds on Diamond and Rajan (2000), in which a sequential servicing constraint for deposits and idiosyncratic returns for banks create a role for runs on some banks in equilibrium.

The model features a continuum of banks that combine funds from depositors and bank capitalists to fund productive investments. Depositors are served sequentially, after which bank capitalists receive their returns pro-quota. Banks have unique knowledge of the investment concerned and can thus extract a higher return from it than outsiders (due to relationship lending). Let $0 < \lambda < 1$ be the share of the project value to outsiders relative to banks, bearing in mind that banks are also subject to idiosyncratic returns on their investments. The return on investments is $R_t^A + x_t$, where R_t^A is the expected return and x_t is an observable bank-specific return with a mean of zero. Each bank takes R_t^A and the gross deposit rate R_t as given. For a given deposit ratio d_t, there are three cases to consider to determine the probability of a run, depending on the idiosyncratic return $R_t^A + x_t$ and the bank recovery ratio $\lambda(R_t^A + x_t)$ relative to deposit returns $R_t d_t$.

Case A. $R_t d_t > R_t^A + x_t$: run for sure. If the return on the bank's investment is too low to cover even the payments on deposits $R_t d_t$, depositors make a run on the bank and bank capitalists get no returns. Depositors obtain the remaining returns on the investment depending on the extent of the bank's unique knowledge.

Case B. $R_t^A + x_t \geq R_t d_t > \lambda(R_t^A + x_t)$: run only without the bank. The return on investment is high enough to pay all depositors in full, but only provided the bank uses its unique knowledge to extract the full return. Bank capitalists rely on the bank to avoid a run and depositors get paid in full. Bank capitalists and banks bargain over the remainder. The return without the bank's unique knowledge (minus deposit returns) is a lower bound of what bank capitalists receive. It is assumed that the rest is split equally between banks and bank capitalists.

Case C. $\lambda(R_t^A + x_t) \geq R_t d_t$: no run for sure. Depositors are repaid in full and bank capitalists have a stronger bargaining position than the banks, as they now have a lower bound in net returns without the risk of a run.

Given these three cases, each bank chooses its optimal deposit ratio d_t by trading off higher bank returns against a higher run-probability – a classic risk-return trade-off. More equity financing (and hence lower leverage) will reduce the risk of a bank-run, while more deposit financing (and higher leverage) will increase expected returns. Banks will benefit more from the intermediate Case B, in which their unique knowledge of the project increases their return. We can rule out Case C, as some risk will always be optimal for banks.

We can calculate the probability of a run on one bank by integrating idiosyncratic returns x_t. This run-probability decreases in the spread $R_t^A - R_t d_t$ and increases in the standard deviation of its idiosyncratic return x_t. As all banks are symmetric, the run risk of one bank, ϕ_t, is equal to the aggregate share of banks subject to runs in each period, or the "bank-run probability". In our AFV model, we take this variable as the financial conditions variable that drives the vulnerability function.

Box 3
In models with vulnerability, monetary tightening tends to reduce downside risks to output growth

Prepared by Nikolay Hristov, Benedikt Kolb and Leonardo Urrutia

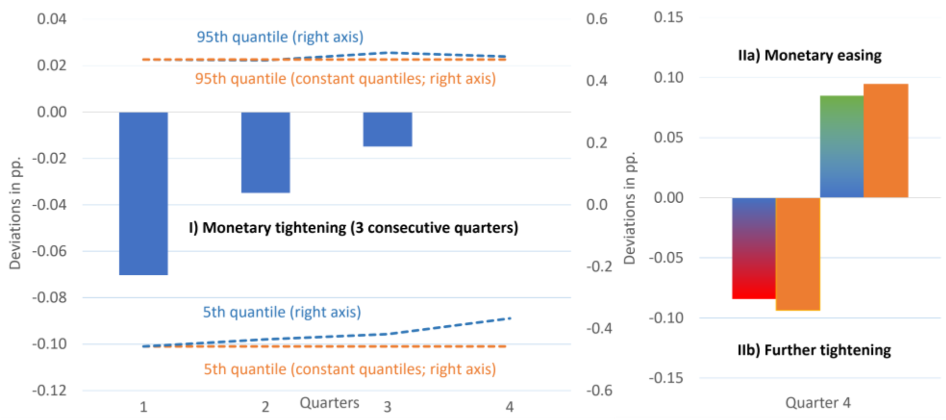
The AFV model (see Section 3.4) fitted to euro area data is used to highlight the effects of monetary policy tightening on the output growth mean and quantiles. The model simulates the effects on output growth of contractionary monetary policy shocks followed either by further tightening or by easing. It should be noted that the model and its estimation are works in progress; results should only be interpreted qualitatively. In addition, sudden monetary policy tightening might increase downside risks via other channels not included in the model. For example, the model does not take into account the medium-term build-up of financial risks on bank balance sheets by maturity mismatch, which might overstate the GaR benefits of monetary policy tightening via tighter financial conditions.

Monetary policy tightening depresses aggregate demand and thus output growth (blue bars in Chart B3.1) and leads to tighter financial conditions: banks deleverage when borrowing costs go up, which makes them less fragile. This in turn lowers endogenous volatility via the vulnerability channel. Given the swift deleveraging in the model, monetary policy tightening therefore implies an increase in output growth in the 5th quantile, i.e. lower downside risk, while there is little or no impact on the 95th quantile (lower and upper dashed blue lines). The model thus predicts costs in the mean, but benefits in the quantiles representing a monetary policy contraction in line with the volatility paradox (Brunnermeier and Sannikov, 2013).

An important additional result is that after the initial tightening the effects of future monetary policy actions are more benign. The additional tightening in period 4 (blue-red bar) is less detrimental and a policy reversal in the form of monetary policy easing (blue-green bar) is more beneficial for output growth than in a world without the previous tightening (symmetric orange bars).

Chart B3.1

Effects on mean output growth (bars, left axis) and output growth quantiles (dashed lines, right axis) for monetary tightening (I) and future monetary policy actions (II) in the AFV model



Note: The underlying model is a work in progress; therefore, results should only be interpreted qualitatively. The figure on the left shows the effects of unexpected monetary tightening for three quarters (three contractionary monetary shocks of 1.5 standard deviation) on the output growth mean (blue bars; left axis) and quantiles (blue dashed lines; right axis). Note that the negative growth impact recedes as output reverts to its mean after the initial shock. Orange dashed lines represent the quantiles without the tightening. The figure on the right shows the effects of additional (unexpected) monetary easing (blue-green bar) or tightening (blue-red bar) of two standard deviations each, after the initial tightening for three quarters. Here we show output growth deviations from steady state, which can be directly contrasted with the output growth effects of the same policy actions in quarter four without the previous tightening for three quarters (orange bars).

Box 4

The role of macroprudential policy in dampening tail risks

Prepared by William Gatt

Macroprudential policy can play a significant role in reducing tail risks when these arise as a consequence of borrowing limits. The role of macroprudential policy in this case is to offset or weaken the financial accelerator that is prevalent when credit is tied to the value of collateral, like housing. A time-varying loan-to-value (LTV) ratio can be one such tool to influence borrowing limits over a financial cycle. It can be lowered when credit rises above a given fundamental level to reduce the volume of credit a household can take on, thus limiting its leverage.

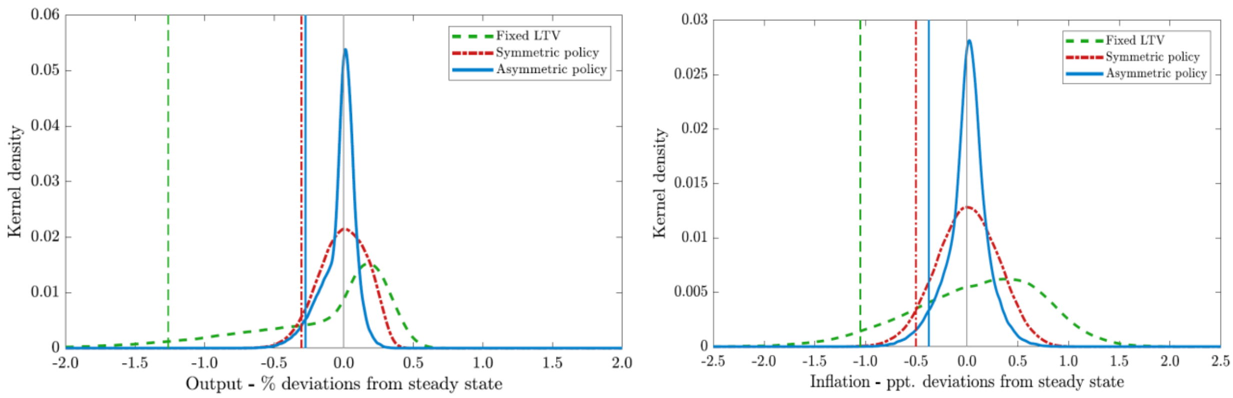
When credit cycles are driven by non-fundamental forces, a rise in house prices, credit and leverage can be expansionary, raising output and inflation. The ensuing correction forces households to deleverage and cut back on spending, which leads to a protracted downturn in the real economy. A countercyclical LTV policy that tightens borrowing limits during credit booms and loosens them during downturns can dampen episodes of boom and bust in the housing and credit markets, and lead to better macroeconomic stabilisation (Mendicino, 2012; Lambertini, Mendicino and Punzi, 2013).

Gatt (2024) shows that while a symmetric LTV ratio policy reduces left tail risks, this outcome can be improved. Since the borrowing constraint can become slack during periods of strong house-price growth (but not during downturns), a stronger macroprudential response during the boom phase may be warranted. Therefore, policymakers tighten borrowing limits relatively more during credit booms and unwind them relatively less during credit busts – creating a kink in the policy response function. This asymmetric macroprudential policy response further reduces tail risks, as it prevents collateral constraints from becoming slack and helps prevent excessive household leverage. Chart B4.1 shows the ergodic distribution of output and inflation when a sequence of housing demand shocks hits the economy in an NK DSGE model with a collateral constraint, as in the Guerrieri and Iacoviello model discussed above. When macroprudential policy is passive (fixed LTV), output and inflation have a negative skew with a corresponding fat left tail – “output-at-risk” and “inflation-at-risk”. A symmetric countercyclical LTV rule greatly reduces the extent of these two parameters, while an asymmetric countercyclical LTV policy can stabilise both even further. The reduction in both the 5th and 95th percentiles in the distribution of output following the use of LTV policy is consistent with the empirical evidence presented in Franta and Gambacorta (2020).

Output and inflation have an upward bias when macroprudential policy is passive, which reflects a credit-fuelled consumption boom when borrowing constraints are slack. Countercyclical policy significantly reduces this bias as it lowers the probability of the borrowing constraint becoming slack. And, by reducing the conditional variance of inflation, macroprudential policy has positive spillovers to monetary policy and reduces the likelihood of an economy hitting the ZLB – itself a source of left tail risk. Therefore, macroprudential policy can play an important role in mitigating tail risks.

Chart B4.1

Output and inflation-at-risk under different macroprudential policy regimes



Source: Gatt (2024).
Note: The dashed vertical lines denote the 5th percentile of each distribution of the same colour.

4 Beyond financial frictions: risk analysis in structural models

The previous sections in this report focused on the risks arising from a range of financial frictions in structural models.

Financial risks have traditionally been considered in the literature as major drivers of macroeconomic tail risks. However, recent developments such as the long period of low interest rates (effectively at the lower bound), the COVID-19 pandemic and subsequent supply bottlenecks, and the Russian invasion of Ukraine highlight many other sources of tail risks that policymakers should consider. This section presents examples of how to incorporate other types of tail risk into macroeconomic models for policy analysis.

4.1 Supply, demand and alternative sources of macroeconomic disturbances

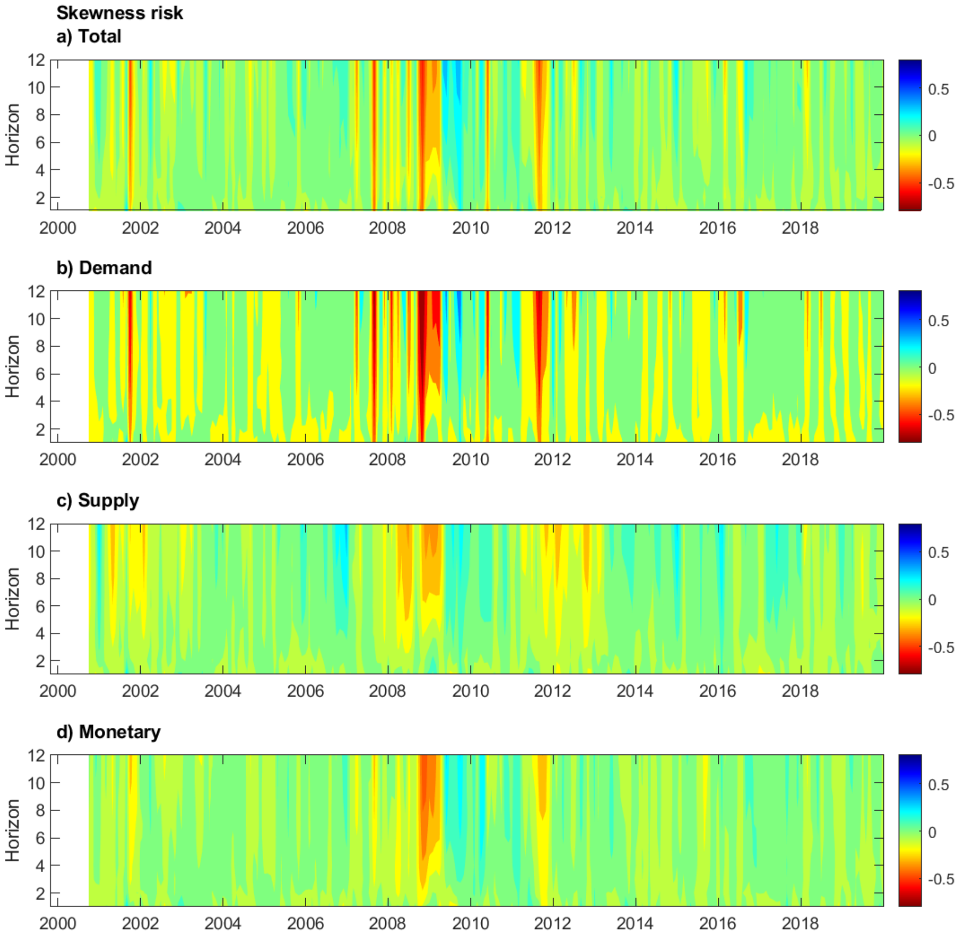
For a better understanding of multivariate tail risks in economic activity and prices, it is vital to distinguish between risks associated with supply-side and demand-side factors. The impact of financial frictions on the economy is similar to the effects of shifts in demand: they influence GDP and inflation in the same direction as changes in demand. It follows, therefore, that asymmetric negative tail risks stemming from financial factors also produce asymmetric downside risks to inflation. But supply shocks generate a different impact: economic activity and inflation usually move in opposite directions. Hence, if supply-side developments like shortages or lockdowns generate asymmetries, we would expect tail risks to move in different directions from each other. While not usually covered in standard GaR models, this has been an important driver of macroeconomic developments in recent years.

Including all sources of tail risks in structural macroeconomic models is not feasible in view of the computational challenges. There are several different approaches in the literature to the task of incorporating different sources of macroeconomic tail risks in macroeconomic models. The first is to use micro-founded non-linearities capable of generating asymmetric distributions in different macroeconomic variables. These include not only the financial frictions discussed in the previous sections but also non-linearities such as downward wage rigidities, non-linear Phillips curves, or the effective lower bound. But models with these types of non-linearities are extremely challenging to estimate and solve, especially if they incorporate all types.

Another possibility is to capture the non-linearities via the disturbances or residuals incorporated in the models. The distribution of residuals in macroeconomic models is assumed to be Gaussian. As we discussed in Section 2, this implies that the model-based forecasting distributions are also Gaussian and therefore cannot capture asymmetric tail risk. But by relaxing this assumption and estimating the models using alternative assumptions where the model shocks follow a time-varying asymmetric distribution, deviations from the norm can be captured in the forecasting distributions. Montes-Galdón and Ortega (2022) introduce a time-varying skewed distribution in the residuals of a macroeconomic model (structural VAR) in which the shocks have an economic interpretation and can be related to demand, supply or monetary policy developments. While not a fully structural model, it is easily estimated. The model can capture empirical facts about changing structural tail risks and provide guidance about the structural drivers of tail risks in the economy.

Chart 4.1 shows the estimated skewness that the model generates in forecasting distributions for the euro area. It shows how the skewness of the different distributions has evolved over a time horizon of up to one year. Episodes with negative skewness risks appear more prevalent than those with positive risks, although the latter are not negligible. The episodes with negative skewness are associated with recessions in the euro area. In the main, they can be explained by the negative skewness in demand shocks, but also by the impact of contractionary monetary policy shocks during this period. The chart also shows periods of positive asymmetries, like in 2009–2010. These positive asymmetries mostly arise from supply shocks and accommodative monetary policy shocks.

Chart 4.1
Skewness in forecasting distributions, real GDP



Note: The figure shows how the skewness in the forecasting distributions of annual real GDP growth evolves over time for different forecasting horizons. Negative skewness is shown in dark red and positive skewness in blue. Colours closer to green indicate that the forecasting distribution is close to symmetric. The first row considers skewness in all shocks, while the remaining rows only consider skewness in one shock.

4.2 Non-linearities beyond financial frictions – the Phillips curve

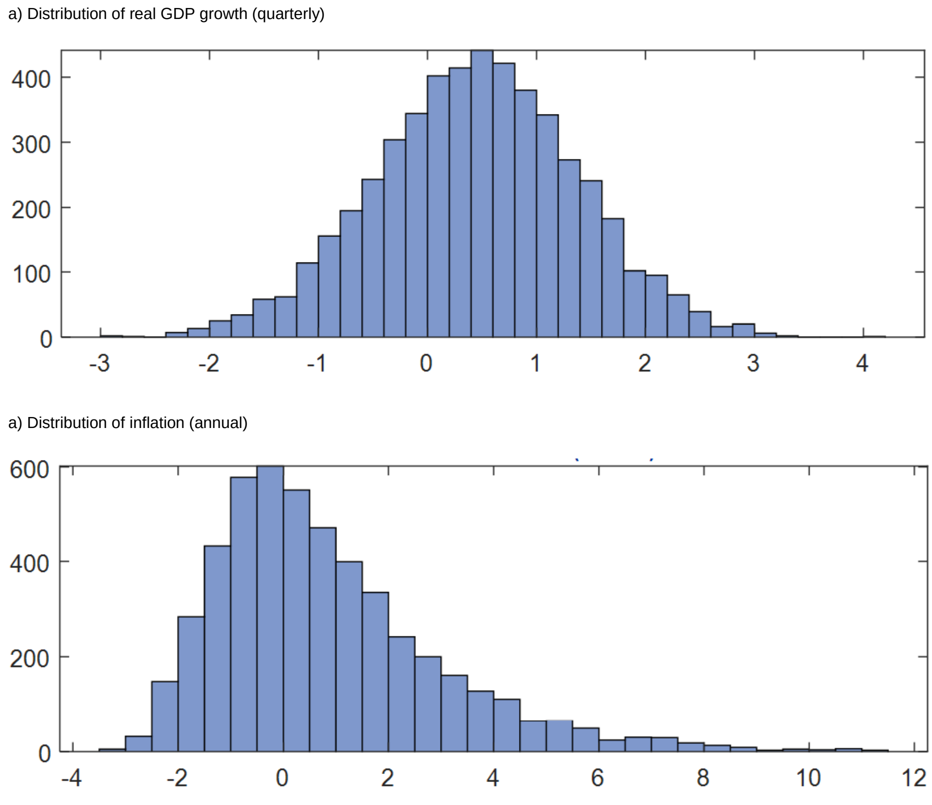
Non-linear Phillips curves may also be an important source of asymmetric tail risks. Normally, the Phillips curve in structural models must have a constant slope. In DSGE models, this constant is a convolution of different structural parameters, such as the probability that firms can adjust their prices, indexation to past inflation, a discounting parameter on the future, or a menu cost parameter. However, it is likely that the slope has been changing over time and that it may be state dependent. The literature on structural models has focused primarily on two sources of non-linearities in the Phillips curve. First, downward wage rigidities bend the curve. During recessions, the rigidities become more binding and the labour market adjustment happens more through the unemployment margin than through wages. The result is an increase in the sacrifice ratio between unemployment and inflation (see Daly and

Hobijn, 2014). Another possibility is to consider that firms' demand elasticity is state dependent. In other words, demand elasticity is an increasing function of its relative price. Harding, Trabandt and Lindé (2022) introduce this mechanism into a non-linear DSGE model, which implies that the demand curve for a firm's product is non-linear. Now, take a recession when marginal costs fall. Since its demand curve is non-linear, a firm has only limited ability to increase demand by cutting prices – and the deeper the recession, the more restricted that ability, as demand becomes less elastic. Large price cuts lower profits because demand increases by very little at the margin, while revenues fall substantially. On the other hand, in booms and periods of high inflation, firms increase their prices much faster and there are large price adjustments.

Chart 4.2 shows stochastic simulations using the non-linear model with state-dependent Phillips curves. In the simulations, all the shocks included in the model are randomly drawn to generate artificial data. The model generates a significant positive skewness in inflation, similar to what is found in the data (for the US, in the Harding, Trabandt and Lindé (2022) model), while there is a small negative skewness in the distribution of real GDP growth.

Chart 4.2

Distribution of real GDP and inflation with non-linear Phillips curves, all shocks

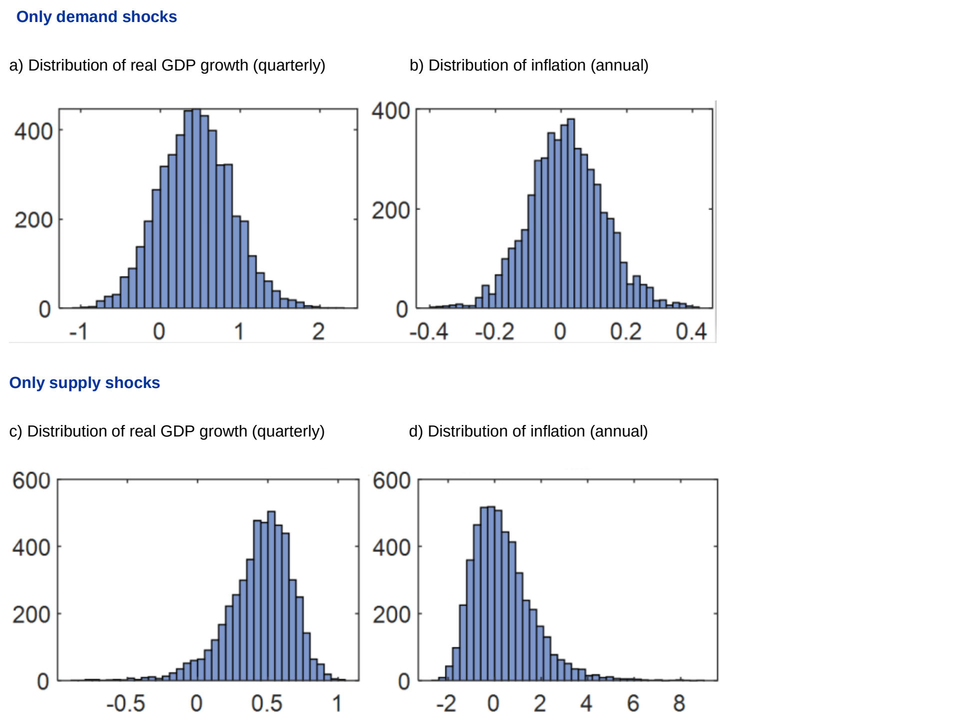


Note: Stochastic simulations using all shocks in the Harding et al. (2022) model.

Supply-side shocks are the biggest drivers of skewness in the model with state-dependent Phillips curves. The simulations above (Chart 4.2) include the impact of both supply and demand shocks. Chart 4.3 shows the distributions with only demand-side (panel a) or supply-side (panel b) shocks to pinpoint the transmission channels in the model. The results show that most of the aggregate skewness in Chart 4.2 stems from the impact of supply-side shocks. The tail risks are positive for inflation and negative for real GDP growth. But it is important to remember that this is still a non-linear DSGE model, even though it can generate asymmetric distributions. Estimating the degree of non-linearities remains a computationally challenging task.

Chart 4.2

Distribution of real GDP and inflation with non-linear Phillips curves – only demand or supply shocks



Notes: Stochastic simulations using all shocks in the Harding et al. (2022) model.

4.3 Sensitivity analysis to sources of risk

While it remains challenging to work with large non-linear DSGE macroeconomic models such as those typically used for policy analysis, it is possible to use linearised models for risk analysis. Many central banks use DSGE models to analyse their baseline projections. As emphasised earlier in this report, most of these models are linear and generally large scale. While implementing the non-linearities discussed here is difficult in those models, linear models can be used to explore risks around baseline projections In this section, we focus on two separate applications that feature regularly in (broad) macroeconomic projection exercises ((B)MPEs).

4.3.1 Risk distributions and changing parameters in structural models

The first application consists of constructing counterfactual forecasting densities around the projection baseline using macroeconomic models. The counterfactuals are constructed by modifying certain model parameters. First, uncertainty bands are constructed around the (B)MPE baseline using the original models and positing random shocks. Second, the models are modified and the same shocks imposed to construct a new forecasting density. The differences in the forecasting densities provide guidance on risks associated with the change in the model. This can be done by computing the difference in risk events, such as the probability of high inflation in both forecasting densities.

To give an example, the ECB-BASE model and the New Area-Wide Model II (NAWM II) explore the risks associated with higher wage indexation in a context of high inflation, as well as the risks of de-anchoring long-term inflation expectations. Both models feature a wage Phillips curve where nominal wages react to past inflation. In the models, the original indexation parameter is close to 0.4 and the counterfactual densities assume that the parameter increases to 0.5 to reflect possible changes in the labour market when inflation is high. In the baseline versions of the models, long-term inflation expectations are firmly anchored. The counterfactual assumes that long-term inflation expectations are endogenous and react to past inflation, so that:

$$\pi_t^* = 0.75\pi_{t-1}^* + 0.25\delta\pi_{t-1}$$

where $\delta = 0.32$.

Table 4.1 shows how the probabilities of high inflation change under the different counterfactuals, around the June 2023 BMPE baseline.

Table 4.1

Probabilities of high inflation under alternative risk scenarios

	2024		2025	
	HICP between 1.75% and 2.25%	HICP > 2.25%	HICP between 1.75% and 2.25%	HICP > 2.25%
ECB-BASE				
Baseline	**9%**	**87.6%**	**23.4%**	**42.3%**
Higher wage indexation	7.4%	88.3%	19.9%	47.7%
De-anchoring long term inflation	9.5%	83.6%	20.6%	45.5%
Higher wage indexation and de-anchoring	6.7%	89.5%	18.5%	55.5%
NAWM II				
Baseline	**10.2%**	**76.4%**	**13.8%**	**50.1%**
Higher wage indexation	8.2%	82.7%	12.8%	55.4%
De-anchoring long term inflation	8.4%	81.4%	13.8%	52.8%
Higher wage indexation and de-anchoring	6.5%	87.4%	11.8%	60.8%

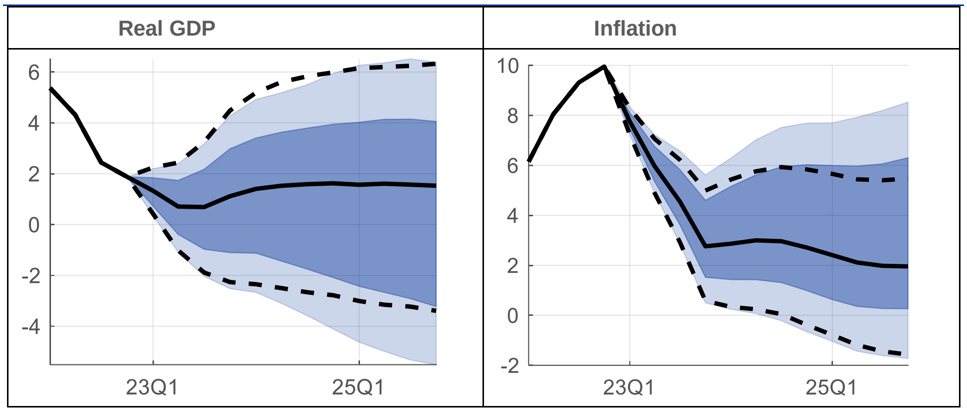
4.3.2 Introducing market-based risks into structural models

Structural models can also help understand the impact of expected market-based risks. Market-based options provide insightful data into how markets expect variables such as oil prices to evolve. These options can be used to construct forecasting densities that in many cases entail asymmetric risks, following the methodology in Vincent-Humphreys et al. (2010). This information can be incorporated in macroeconomic models through conditional density forecast techniques (see Montes-Galdón, Paredes and Wolf, 2022)). By imposing the forecasting density of one variable in the model, we can explore how the densities of other variables react. If the imposed density has some asymmetric risks, they will be inherited by the model.

Chart 4.3 shows that information contained in the full range of option-implied forecasting densities for oil prices around the March 2023 MPE imply upside risks to inflation and downside risks to real economic activity. We use the NAWM II to analyse the impact of introducing market-based information on oil prices into the model-based density forecasts. In the March 2023 MPE, option-based densities for oil prices suggested some positive skewness in the medium term: markets anticipated a higher probability of oil prices rising than falling. When this information is input to the model, the forecasting distribution for inflation (blue shaded areas) shows upside risks compared with the original distribution (black dashed lines). Since oil prices are a type of supply-side shock in the model, the upside risks to inflation and oil prices translate into downside risks to economic activity.

Chart 4.3

Distribution of real GDP and inflation with option-based oil price data



Note: The chart shows alternative model-based densities using a NAWM II version with enhanced transmission of energy prices. The black lines show the 16% and 84% quantiles of the model-based densities centred around the March 2023 MPE baseline. The blue densities show the results of a conditional density forecasting exercise in which the forecasting densities of energy prices (oil and gas) come from options. The methodology used is from Montes-Galdón, Paredes and Wolf (2022): "Conditional Density Forecasting: a Tempered Importance Sampling approach". The densities show the 5%, 16%, 84% and 95% quantiles of the forecasting distributions. The transmission of energy prices in the model is enhanced by assuming a faster pass-through from energy prices to import prices.

Overall, this section shows that, while it is challenging to explore DSGE models with non-linearities, there are still meaningful ways of using current workhorse models to assess macro tail risks.

5 References

Acemoglu, D., Ozdaglar, A. and Tahbaz-Salehi, A. (2017), “Microeconomic origins of macroeconomic tail risks”, *American Economic Review*, Vol. 107, No 1, pp. 54-108.

Adjemian, S. et al. (2022), “Dynare: Reference Manual, Version 5”, *Dynare Working Paper*, No 72.

Adjemian, S. and Juillard, M. (2013), “Stochastic Extended Path Approach”, Centre for Economic and International Studies, Tor Vergata University of Rome.

Adrian, T., Boyarchenko, N. and Giannone, D. (2019), “Vulnerable Growth”, *American Economic Review*, Vol. 109, No 4, pp. 1263-1289.

Adrian, T., Duarte, F., Liang, N. and Zabczyk, P. (2020a), “NKV: A New Keynesian Model with Vulnerability”, *AEA Papers and Proceedings*, Vol. 110, pp. 470-76.

Adrian, T., Duarte, F., Liang, N. and Zabczyk, P. (2020b), “Monetary and Macroprudential Policy with Endogenous Risk”, *IMF Working Paper*, No 2020/236.

Ajevskis V. (2017), “Semi-global solutions to DSGE models: perturbation around a deterministic path”, *Studies in Nonlinear Dynamics & Econometrics*, Vol. 21, No. 2, pp. 1-28.

Ajevskis, V. (2019), “Nonlocal Solutions to Dynamic Equilibrium Models: The Approximate Stable Manifolds Approach”, *Macroeconomic Dynamics*, Vol. 23, No 6, pp. 2544-2571.

Angeloni, I. and Faia, E. (2013), “Capital regulation and monetary policy with fragile banks”, *Journal of Monetary Economics*, Vol. 60, No 3, pp. 311-324.

Baqaee, D.R. and Farhi, E. (2019), “The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem”, *Econometrica*, Vol. 87, No 4, pp. 1155-1203.

Barro, R.J. (2006), “Rare disasters and asset markets in the twentieth century”, *The Quarterly Journal of Economics*, Vol. 121, No 3, pp. 823-866.

Bernanke, B. and Gertler, M. (1989), "Agency Costs, Net Worth and Business Fluctuations", *American Economic Review*, Vol. 79, No 1, pp. 14-31, March.

Bernanke, B., Gertler, M and Gilchrist, S. (1999), “The financial accelerator in a quantitative business cycle framework”, in Taylor, J.B. and Woodford, M. (eds.), *Handbook of Macroeconomics*, Elsevier, Amsterdam, pp. 1341-1393.

Beutel, J., Lorenz, E., Metiu, N., Prieto, E. and Schüler, Y. (2020), “The Global Financial Cycle and Macroeconomic Tail Risks”, *Deutsche Bundesbank Discussion Papers*, No 43.

Bloom, N. (2009), “The impact of uncertainty shocks”, *Econometrica*, Vol. 77, No 3, pp. 623-685.

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten and I., Terry, S.J. (2018), “Really Uncertain Business Cycles” *Econometrica, Vol. 86, No. 3 (May, 2018), 1031–1065.*

Brunnermeier, M.K. and Sannikov, Y. (2014), “A macroeconomic model with a financial sector”, *American Economic Review*, Vol. 104, No 2, pp. 379-421.

Christiano, L.J., Eichenbaum, M. and Rebelo, S. (2011), “When is the government spending multiplier large?”, *Journal of Political Economy*, Vol. 119, No 1, pp. 78-121.

Christiano, L.J., Eichenbaum, M. and Trabandt, M. (2015), “Understanding the great recession”, *American Economic Journal: Macroeconomics*, Vol. 7, No 1, pp. 110-167.

Christiano, L.J., Motto, R. and Rostagno, M. (2014), “Risk shocks”, *American Economic Review*, Vol. 104, No 1, pp. 27-65.

Coenen, G., Karadi, P., Schmidt S. and Warne, A. (2019), “The New Area-Wide Model II: An Extended Version of the ECB’s Micro-Founded Model for Forecasting and Policy Analysis with a Financial Sector”, *Working Paper Series*, No 2200, ECB, Frankfurt am Main, December.

Cole, H.L. and Ohanian, L.E. (2002), “The Great U.K. Depression: A Puzzle and Possible Resolution”, *Review of Economic Dynamics*, Vol. 5, No 1, pp. 19-44.

Daly, M. C., & Hobijn, B. (2014). Downward nominal wage rigidities bend the Phillips curve. Journal of Money, Credit and Banking, 46(S2), 51-93.

De Paoli, B.(2009), “Monetary Policy and Welfare in a Small Open Economy”, , *Journal of International Economics*, Vol. 77, No 1, pp. 11-22.

Diamond, D.W. and Rajan, R.G. (2000), “A theory of bank capital”, *Journal of Finance*, Vol. 55, No 6, pp. 2431-2465.

Eggertsson, G.B. and Krugman, P. (2012), “Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach”, *The Quarterly Journal of Economics*, Vol. 127, No 3, pp. 1469-1513.

Fair, R.C. and Taylor, J.B. (1983), “Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models”, *Econometrica*, Vol. 51, No 4, pp. 1169-1185.

Fernández-Villaverde, J. and Guerrón-Quintana, P. (2021), "Estimating DSGE models: Recent advances and future challenges”, *Annual Review of Economics*, Vol. 13, pp. 229-252.

Fernández-Villaverde, J., Guerrón-Quintana, P., Kuester, K. and Rubio-Ramírez, J. (2015), "Fiscal volatility shocks and economic activity", *American Economic Review*, Vol. 105, No 11, pp. 3352-3384.

Fernández-Villaverde, J., Guerrón-Quintana, P., Rubio-Ramirez, J. and Uribe, M. (2011), "Risk matters: The real effects of volatility shocks", *American Economic Review*, Vol. 101, No 6, pp. 2530-2561.

Fernández-Villaverde, J. Hurtado, S. and Nuño, G. (2019), "Financial frictions and the wealth distribution", *National Bureau of Economic Research Working Papers*, No 26302.

Figueres, J.M. and Jarociński, M. (2020), "Vulnerable Growth in the Euro Area: Measuring the Financial Conditions", *Working Paper Series*, No 2458, ECB, Frankfurt am Main, August.

Franta, M. and Gambacorta, L. (2020), "On the effects of macroprudential policies on Growth-at-Risk", *Economics Letters*, Vol. 196.

Gabaix, X. (2012), "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance", *The Quarterly Journal of Economics*, Vol. 127, No 2, pp. 645-700.

Galí, J., Monacelli T., (2005). "Monetary Policy and Exchange Rate Volatility in a Small Open Economy" The Review of Economic Studies Ltd, vol. 72(3), pages 707-734.

Galí, J. (2008), *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework and its Applications*, Princeton University Press, Princeton.

Garín, J., Lester, R. and Sims, E. (2019), "Are Supply Shocks Contractionary at the ZLB? Evidence from Utilization-Adjusted TFP Data", *Review of Economics and Statistics*, Vol. 101, No 1, pp. 160-175.

Gatt, W. (2024), Housing Boom-Bust Cycles and Asymmetric Macroprudential Policy. Journal of Money, Credit and Banking. https://doi.org/10.1111/jmcb.13097

Gertler, M. and Kiyotaki, N. (2010), "Financial intermediation and credit policy in business cycle analysis", *Handbook of Monetary Economics*, Vol. 3, pp. 547-599.

Gertler, M. and Kiyotaki, N. (2015), "Banking, liquidity, and bank runs in an infinite horizon economy", *American Economic Review*, Vol. 105, No 7, pp. 2011-2043.

Gomes, S., Jacquinot, P. and Pisani, M. (2012), "The EAGLE. A model for policy analysis of macroeconomic interdependence in the euro area", *Economic Modelling*, Vol. 29, No 5, pp. 1686-1714.

Gourio, F. (2012), "Disaster risk and business cycles", *American Economic Review*, Vol. 102, No 6, pp. 2734-2766.

Guerrieri, L. and Iacoviello, M. (2015), “OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily”, *Journal of Monetary Economics*, 2015, vol. 70, issue C, 22-38.

Guerrieri, L. and Iacoviello, M. (2017), “Collateral constraints and macroeconomic asymmetries”, *Journal of Monetary Economics*, Vol. 90, pp. 28-49.

Harding, M., Lindé, J., & Trabandt, M. (2022). Resolving the missing deflation puzzle. Journal of Monetary Economics, 126, 15-34.

He, Z. and Krishnamurthy, A. (2013), “Intermediary asset pricing”, *American Economic Review*, Vol. 103, No 2, pp. 732-770.

Holden, T.D. (2016), “Computation of Solutions to Dynamic Models with Occasionally Binding Constraints”, ZBW – Leibniz Information Centre for Economics, *EconStor Preprints*, No 144569.

Holden, T.D. (2022), “Existence and Uniqueness of Solutions to Dynamic Models with Occasionally Binding Constraints”, *Deutsche Bundesbank Discussion Paper*, No 09/2022.

Holló, D., Kremer, M. and Lo Duca, M. (2012), “CISS - A Composite Indicator of Systemic Stress in the Financial System”, *Working Paper Series*, No 1426, ECB, Frankfurt am Main, March.

Hornik, K., Stinchcombe, M., White, H., (1998), “Multilayer feedforward networks are universal approximators”, *Neural Networks, Vol. 2, Issue 5, 1989, Pages 359-366.*

Jung, T., Teranishi, Y. and Watanabe, T. (2005), “Optimal Monetary Policy at the Zero-Interest-Rate Bound”, *Journal of Money, Credit, and Banking*, Vol. 37, No 5, pp. 813-835.

Kehoe, T.J. and Prescott, E.C. (2002), “Great Depressions of the 20th Century”, *Review of Economic Dynamics*, Vol. 5, No 1, pp. 1-18.

Kiyotaki, N. and Moore, J. (1997), “Credit cycles”, *Journal of Political Economy*, Vol. 105, No 2, pp. 211-248.

Korinek, A. and Simsek, A. (2016), “Liquidity trap and excessive leverage”, *American Economic Review*, Vol. 106, No 3, pp. 699-738.

Laffargue, J.-P. (1990), “Résolution d’un modèle macroéconomique avec anticipations rationnelles”, *Annales d’Économie et Statistique*, Vol. 17, pp. 97-119.

Lambertini, L., Mendicino, C. and Punzi, M.T. (2013), “Leaning against boom–bust cycles in credit and housing prices”, *Journal of Economic Dynamics and Control*, Vol. 37, pp. 1500-1522.

Lan, H. and Meyer-Gohde, A. (2013), “Pruning in perturbation DSGE models: Guidance from nonlinear moving average approximations”, *SFB Discussion Paper*, No 649.

Maliar, L. Maliar, S. and Winant, P. (2021), “Deep learning for solving dynamic economic models”, *Journal of Monetary Economics*, Vol. 122, pp. 76-101.

Mendicino, C. (2012), “Collateral requirements: macroeconomic fluctuations and macro-prudential policy”, *Banco de Portugal Working Papers*, No 11-2012.

Montes-Galdón, C. and Ortega, E. (2022), “Skewed SVARS: Tracking the structural sources of macroeconomic tail risks”, in Dolado, J.J., Gambetti, L. and Matthes, C. (eds.), Essays in Honour of Fabio Canova, Emerald Publishing Limited, Leeds. pp. 177-210.

Montes-Galdón, C., Paredes, J. and Wolf, E. (2022), “Conditional density forecasting: a tempered importance sampling approach”, *Working Paper Series*, No 2754, ECB, Frankfurt am Main, December.

Swarbrick, J. (2021), “Occasionally binding constraints in large models: A review of solution methods”, *Bank of Canada Staff Discussion Paper*, No 2021-5.

Vincent-Humphreys, R., Noss, J., (2012), “Estimating Probability Distributions of Future Asset Prices: Empirical Transformations from Option-Implied Risk-Neutral to Real-World Density Functions” Bank of England Working Paper No. 455.

Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton.

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