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The natural rate of interest: estimates, drivers, and challenges to monetary policy

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Abstract

Using a wide range of models we document a protracted fall in the natural (or neutral) rate of interest in advanced economies, driven by ageing, waning productivity growth, a rise in mark-ups, and a surge in risk aversion in the wake of the global financial crisis. While our neutral rate estimates are highly uncertain and model dependent, most of them have been negative in the wake of the financial crisis. This observation is highly relevant for assessing the monetary policy stance and the risk of monetary policy becoming constrained by the lower bound on nominal interest rates. We highlight model dependence of natural rate estimates by illustrating large differences in their stabilising properties, depending on the context chosen. We also emphasise high statistical uncertainty of natural rate estimates within models. Looking ahead, a return to higher levels would have to come from a reversal in risk aversion and flight to safety and a boost in productivity. To achieve this, structural reforms are crucial.

Keywords: Natural rate of interest, return on capital, demographics, productivity growth, monetary policy

JEL Classification: E52, E43.

There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the natural rate of interest on capital. [...] If it were possible to ascertain and specify the current value of the natural rate, it would be seen that any deviation of the actual money rate from this natural rate is connected with rising or falling prices. ... (Wicksell, 1898)

The natural rate is an abstraction; like faith, it is seen by its works. One can only say that if the bank policy succeeds in stabilizing prices, the bank rate must have been brought in line with the natural rate, but if it does not, it must not have been. (Williams, 1931).

Preface

This report summarises analysis done by the Working Group on Econometric Modelling (WGEM). The WGEM assists the Monetary Policy Committee (MPC) in (i) reviewing the underlying tools for assessing current economic and financial developments and for preparing economic forecasts, thereby contributing to the development of an econometric infrastructure for the ESCB to meet the monetary policy needs of the Eurosystem; (ii) studying and assessing technical economic issues relating to monetary and exchange rate policies.

In early 2017, the WGEM commissioned an expert team to provide an overview of estimates of the natural rate of interest and its drivers, using a wide set of models, and to explore its role in the conduct of monetary policy. In this report the team presents its key findings in a synthetic manner. While the team has not aimed at an exhaustive evaluation of interest rate theory, this report covers theoretical foundations, a wide range of modern modelling applications, and a quantitative, critical appraisal of the role of the natural rate of interest in the conduct of monetary policy.

The expert team would like to thank members of the WGEM and of the MPC for their valuable comments and their support in shaping up this report, in particular, Matteo Ciccarelli, Eva Ortega Eslava, Benoît Mojon, and Frank Smets. We are grateful for comments, suggestions, or technical input by Daniel Buncic, Sandra Gomes, Daniel Kapp, Wolfgang Lemke, and Ralph Setzer.

Executive summary

Since the 1980s, real interest rates in advanced economies have followed a protracted downward trend and, in the wake of the global financial crisis, slumped to exceptionally low levels. This development has often been associated with a decline in the *natural* or *neutral* rate of interest, henceforth r^* .

Conceptually one of the most important variables in modern macroeconomics, r^* is the real rate of interest that brings output into line with its potential or natural level in the absence of transitory shocks (in the case of semi-structural models) or nominal adjustment frictions (in the case of DSGE models). r^* thus closes the output gap and stabilises inflation, either eventually or concomitantly depending on the type of model. Numerous factors, such as demographics or technological progress in the long run, or changes in risk aversion in the short run, affect r^* .

The key challenge is that r^* is unobservable. We explore a wide range of methods to estimate it. These estimates are highly model specific and differences between them reflect assumptions made on methodology, time series properties, and what channels are included or ignored. The stabilising properties of natural rates can differ widely across models. These differences can provide a wealth of information for policy makers but which model to use depends on the policy question at hand. Within structural DSGE models, we confirm that tracking r^* (which can be quite volatile) can considerably improve macroeconomic stability. However, this result cannot be generalised to all DSGE-based r^* measures. We also show that tracking smoother r^* values from semi-structural models fails to deliver sufficient macroeconomic stability. Overall, model-specific differences and statistical uncertainty pose formidable obstacles to using estimates of r^* to gauge the appropriate monetary policy stance.

Our estimates show a declining trend in r^* in the advanced economies starting in the 1980s, driven by lower trend growth and demographic factors. Risk aversion and flight to safety are shown to have contributed to a further decline in the wake of the global financial crisis. Remarkably, most of our estimates of r^* for the euro area have been negative regardless of the type of model used.

We note that, in contrast to this decline in interest rates, estimates of the return on equity and capital have remained fairly constant. We illustrate that the growing wedge between the return on capital and the return on safe and liquid assets can be reconciled with rising risk aversion and increasing mark-ups.

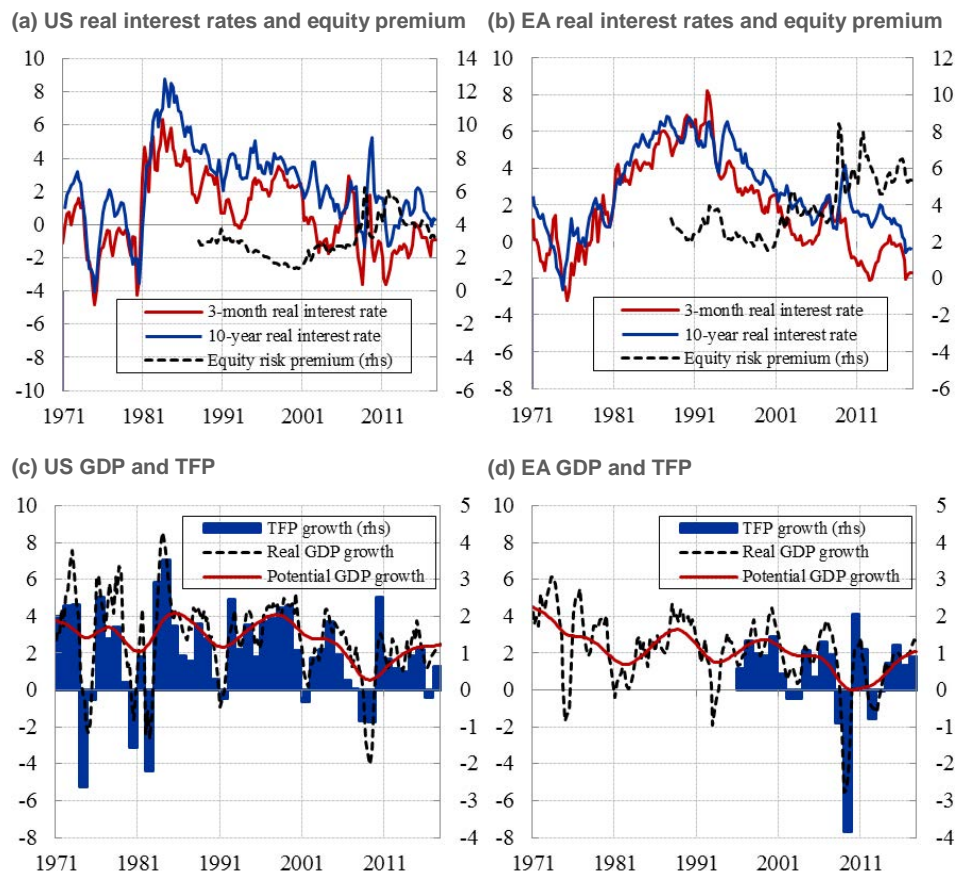
The protracted downward trend in r^* estimates indicates elevated risks of monetary policy becoming constrained by the lower bound on nominal interest rates in the future. Given these challenges, we discuss the advantages and disadvantages of changes in monetary policy strategies and instruments, but emphasise that the drivers depressing real returns cannot be addressed by central banks.

1 Introduction and Overview

The natural rate of interest, henceforth r^* , lies at the heart of modern macroeconomics. As Box 1 explains in more detail, r^* is the fulcrum in the relationship between an IS equation and a Phillips curve equation which jointly explain the evolution of the output gap and inflation. A given real short-term interest rate is inflationary or deflationary depending on the level of r^* . The key insight of Wicksell (1898) was that the natural rate will vary over time. There is not, therefore, a timeless or unconditional benchmark to which the current real policy rate can be compared. This time variation would be unproblematic if it could be observed accurately and in real time, but unfortunately neither of these are the case. The natural rate has to be estimated, often using filters that revise previous estimates significantly as new data are released.

Chart 1

Macroeconomic developments and interest rates



Sources: ECB, Eurostat, European Commission, Federal Reserve Bank of St. Louis, Thomson Reuters and authors' calculations. Notes: Real GDP growth, potential GDP growth, real interest rates and equity risk premia shown as quarterly data. TFP displayed as annual data. Potential GDP growth calculated with a HP filter (bearing caveats about its use in mind) and considering projections for the period from 2018Q1 to 2020Q4. Interest rates expressed as ex-post real interest rates by taking into account realized inflation.

In the long run, output gaps average out to zero (whether looking forwards or backwards) and so average r^* will never be far away from average long-term real

interest rates. Long-term real interest rates have been falling since at least the mid-1980s in the US and the euro area (see Charts 1a and 1b). This gives us our first clue that r^* has fallen significantly over the past thirty years and has been at or below zero since the global financial crisis. Chart 1 displays macroeconomic developments in the US and the euro area since the beginning of the 1970s. The panels at the top illustrate a drawn out decline in money market rates and government bond yields and, at the same time, a rise in the equity risk premium. The panels at the bottom illustrate declining trends in GDP growth and total factor productivity (TFP).

To a large part, the protracted decline in the low-frequency component of real interest rates can be attributed to demographic developments. In Section 2.1 we present evidence that the current demographic transition has reduced real interest rates in the euro area by around 1 percentage point since the 1980s, reflecting low fertility rates, rising life expectancy and changing composition of age cohorts. This demographic transition is incomplete and largely pre-determined and, on current trends, can be expected to depress real interest rates by a further 0.25-0.5 percentage points by 2030. We show that rising income inequality may have had a role to play as well.

Box 2 illustrates that whilst real returns on short-term risk-free interest rates or debt instruments (sovereigns and corporates), have fallen consistently over the past three decades, estimates of the return on equity and capital have remained fairly constant. We reconcile this growing wedge across different return measures with rising risk aversion and higher profit margins.

A broad class of models that estimate the long-run level of r^* using time series techniques is presented in Section 2.2. As we make extensive use of the well-known semi-structural approach by Laubach and Williams (2003), we devote Box 3 to explain the conditions under which such r^* estimates lack robustness and why estimated r^* uncertainty is so high. Econometric models confirm that slowing growth is an important driver of the fall in r^* but indicate that other factors such as risk aversion have also played an important role, particularly since the global financial crisis. We also illustrate sizeable differences in r^* across euro area countries which are an obstacle to the smooth transmission of the single monetary policy (see Box 4).

In Section 2.3 we report estimates of r^* derived from DSGE models. These models estimate r^* to have fallen sharply after the financial crisis and to have stayed largely negative since then. This analysis confirms the importance of productivity, risk premia, and financial factors (i.e. disturbances to financial intermediation) in explaining the exceptional macroeconomic conditions experienced in recent years.

All modelling approaches – notwithstanding their conceptual and statistical differences – testify to a protracted decline in equilibrium real rates, in particular since the end of the 1980s, with a particularly strong and sustained downturn in the wake of the financial crisis, concomitant with declining trend growth and rising risk aversion. The extent by which this low level of r^* is judged to persist will also have far-reaching consequences for how to normalise monetary conditions eventually and for how frequent monetary policy might risk running up against the lower bound on interest rates in the future.

We illustrate model-specific stabilising properties of r^* estimates in Section 3. We show that tracking r^* derived from structural DSGE models (which can be quite volatile) can considerably improve macroeconomic stability. But this result cannot be generalised to all DSGE-based measures. By contrast, tracking smoother (and conceptually different) r^* estimates from semi-structural models may exhibit insufficient stabilising properties.

Overall, model-specific differences and statistical uncertainty pose formidable obstacles to using estimates of r^* to gauge the appropriate monetary policy stance.

We stress doubts about the ability to forecast r^* other than with models that consider demographic transition. Econometric models constructing r^* consistent with the notion of a ‘terminal’ rate contain no relevant forward-looking information. Structural models feature a metric of r^* which is of a purely contemporaneous nature. Demographic models, in turn, show that r^* will if anything continue to fall. In this context we discuss the advantages and disadvantages of permanent changes in monetary policy strategies versus temporary changes in policy instruments in ensuring the effectiveness of monetary policy, but emphasise that the drivers depressing real returns cannot be affected by central banks.

Box 1

The natural rate of interest: theoretical foundations

The notion of the natural rate of interest has evolved in tandem with the theory of capital, money, credit, and economic value. Noticing that in an economy with an advanced financial sector banks create purchasing power, Wicksell (1898) proposed the concept of the natural rate as the real rate that equates the *monetary (or bank loan) rate* determined by the financial sector with the equilibrium rate in the capital market (the latter equating the supply and demand for *real* capital goods).

While the concept itself only recently experienced a revival thanks to Woodford (2003), the idea has been underpinned by a number of contributions in the economic literature of the 20th century. Fisher (1930) describes the optimal inter-temporal choice of a consumer by the Euler equation:

$$c_t^{-1/\sigma} = E_t \left[\frac{1 + r_{t+1}}{1 + \rho} c_{t+1}^{-1/\sigma} \right] \quad (1)$$

where consumption c in the current period is a decreasing function of the real interest rate r adjusted by the elasticity of intertemporal substitution σ and the household’s discount rate, ρ . This negative relationship between consumption and real interest rates underpins all models covered in this report.

The natural rate in the long run

In the Ramsey (1928) growth model with population growth n and no uncertainty, an analogue of equation (1) can be expressed in general equilibrium as:¹

¹ This formula represents the solution of the social planner’s problem who weighs each period’s population equally. When $\rho = 0$ and $\sigma = 1$, equation (2) is equivalent to the golden rule result in the Solow (1956) model.

$$r = \frac{1}{\sigma} g_c + n + \rho \quad (2)$$

Equation (2) shows that equilibrium r , which also corresponds to r^* , moves one-for-one with ρ and n and depends on the growth rate in per capita consumption g_c .

The natural rate and business cycle dynamics

A variety of methods based on equation (2) have been used to estimate r^* . Holsten, Laubach and Williams (2017)'s specification assumes:²

$$r_t^* = g_t + z_t \quad (3)$$

where $g = g_c + n$ and z is a “catch-all” factor that could include the household’s discount rate or risk aversion. Both components are assumed to follow random walks so r^* and g can permanently diverge. But they add further structure to the model with a *backward looking* IS curve formulation of the Euler equation

$$\tilde{y}_t = a_{y,1} \tilde{y}_{t-1} + a_{y,2} \tilde{y}_{t-2} - \frac{a_r}{2} (r_{t-1} - r_{t-1}^* + r_{t-2} - r_{t-2}^*) + \epsilon_{\tilde{y},t} \quad (4)$$

an accelerationist Phillips curve linking inflation dynamics to the real economy

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi) \pi_{t-2,4} + b_y \tilde{y}_{t-1} + \epsilon_{\pi,t} \quad (5)$$

and an underlying growth process

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{y^*,t} \quad (6)$$

where $\tilde{y}_t = y_t - y_t^*$ is the output gap, $\{a_{y,1}, a_{y,2}, a_r, b_\pi, b_y\}$ is a set of structural parameters, $\{\epsilon_{\tilde{y},t}, \epsilon_{\pi,t}, \epsilon_{y^*,t}\}$ are random white noise disturbances and $\pi_{t-2,4}$ denotes the average of the second to fourth lags of inflation.

The state variables g_t and z_t can be extracted using as observables the short-term real rate of interest, r , the logarithm of real GDP, y , and the CPI inflation rate, π . Equations (4) and (5) are consistent with Wicksell’s idea that when short-term real interest rates are set above r^* , the output gap becomes negative, which in turn translates to lower rates of inflation, *ceteris paribus*. A policy of tracking this r^* metric stabilises inflation only asymptotically and, with an accelerationist Phillips curve, at an *indeterminate* level.

Structural business cycle models and the natural rate

Eventually, Wicksell’s idea was integrated into modern macroeconomic theory by Woodford (2003) building on the Real Business Cycle (RBC) literature started by Kydland and Prescott (1982). Woodford defines r^* as the level of real rate required to keep aggregate demand equal to the level of output that would be obtained in a counterfactual economy with full price flexibility.³ r^* evolves according to the RBC core that DSGE models inherit, and thus can be quite volatile in the short run, as the economy is hit by a number of shocks. In the textbook New Keynesian model, the Euler equation (1) is recast in the form of a *forward-looking* IS curve:⁴

² Laubach and Williams (2003) assume that $\sigma = 1$.

³ Due to the complete asset market assumption the Neo-Wicksellian concept departs from Wicksell’s original concept of explaining the natural rate of interest in a developed financial system.

⁴ Within this model consumption and GDP are equal, and they can be used interchangeably.

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^*) \quad (7)$$

where i denotes the nominal interest rate, set by the central bank. The assumption of price stickiness à la Calvo (1983) and optimizing price setters generates the *forward-looking* Phillips curve:

$$\pi_t = 1/(1 + \rho) E_t \pi_{t+1} + \kappa \tilde{y}_t \quad (8)$$

where κ is a combination of structural parameters, including the degree of price stickiness in the economy.

In this very simple setting, if the central bank sets the real interest rate equal to r^* , at all times, this eliminates the distortions arising from nominal rigidities. Both the output and inflation gaps (deviation of inflation from target) are simultaneously closed, a property called the *divine coincidence*.

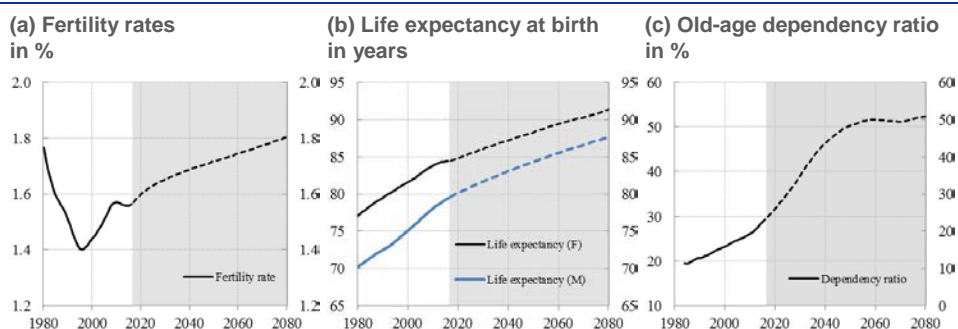
2 Quantitative Results

2.1 The role of demographics as low-frequency determinant of the natural rate

Advanced economies have been undergoing a demographic transition towards low fertility and mortality (Lee, 2016). For Europe, Charts 2a, 2b and 2c display fertility rates, life expectancy at birth and old-age dependency ratios: individuals tend to have fewer children (sub-replacement fertility) and live longer (decreasing mortality), which generates a dramatic increase in the relative number of the elderly (increasing old-age dependency ratio). While at the beginning of the 21st century, the ratio of the elderly (aged 65 and over) to working-age people (aged 15-64) has been 25 to 100, the European Commission projects this proportion to rise to above 50 to 100 by year 2050. Likewise, over the same time span population growth is projected to decline from around 0.45% to -0.4% . This profound shift in demographic patterns has been anticipated for decades, as demographic trends can be predicted with less uncertainty than other economic or social developments.

Chart 2

Past and projected demographic developments in the euro area



Source: Bielecki *et al.* (2018), World Bank, Eurostat. See Appendix A.1 for details. (M) and (F) refer to male and female age cohorts. Note: Projections are dashed lines and marked by shaded areas.

The impact of these demographic developments on the equilibrium rate can be captured by overlapping generations models (OLGs). As in the neoclassical growth model (see Box 1), they embed the logic that in the long run r^* is driven by population growth, technological progress and the discount rate. The OLG literature identifies the following three channels through which the demographic transition can affect r^* :⁵

1. *A downward impact from lower labour input* (depressing capital demand). Labour as a production factor decreases so that *ceteris paribus* capital per worker rises, in turn depressing the marginal product of capital and r^* . This is akin to a permanent slowdown in productivity growth.

⁵ See Krueger and Ludwig (2007); Carvalho, Ferrero and Nechio (2016).

2. *A downward impact from higher life expectancy* (raising capital supply). Lower mortality rates mean that individuals expect to live longer so that *ceteris paribus*, depending on the benefits set in place by the pension scheme and assuming perfect foresight, they increase their saving in anticipation of a longer retirement period. This is akin to a preference shock (decreasing the discount rate, reflecting that individuals become more patient).
3. *An upward impact from a rising proportion of dissavers* (lowering capital supply). Ageing means that the age composition of the population shifts towards relatively older individuals who are dissavers. This is akin to a preference shock, but going in the opposite direction to the second channel.

So far, virtually all studies indicate that ageing has a depressing influence on r^* , with the second channel usually being stronger than the third channel, implying that even if fertility rates were higher, the increase in life expectancy alone would put downward pressure on r^* .

2.1.1 Structural approaches to capture demographic effects

Following the approach pioneered by Auerbach and Kotlikoff (1987), Bielecki *et al.* (2018) and Papetti (2018) construct OLG models to quantify the effects of demographic changes on the natural rate of interest. Both studies attribute the secular decline in the natural rate of interest since the 1980s largely to ageing, with an estimated impact of around one percentage point in Bielecki *et al.* (2018) and around 0.8 percentage points in Papetti (2018). Looking ahead, demographic developments will lower the natural rate of interest even further by 2030, as shown in Chart 3a: by another 0.5 percentage points according to both studies.

These results are in line with previous estimates found in the literature. In a rich, multi-country OLG model, Krueger and Ludwig (2007) estimate a decrease in worldwide r^* from 2005 to 2080 by around 0.86 percentage points. Quantitatively similar results are obtained by Domeij and Floden (2006). Carvalho *et al.* (2016) estimate r^* to have declined by 1.5 percentage points between 1990 and 2014, while Kara and von Thadden (2016) project the natural rate of interest in the euro area to decrease by 0.9 percentage points between 2008 and 2030.⁶

Although there is consensus in the literature regarding the overall effect of ageing on the natural rate of interest, the relative roles of the three specific channels are still debated.⁷ Carvalho *et al.* (2016) find that the second channel (rising life expectancy) is almost uniquely responsible for declining r^* . But they might overestimate this channel, as they rely on a model with only two age-groups and constant mortality risk

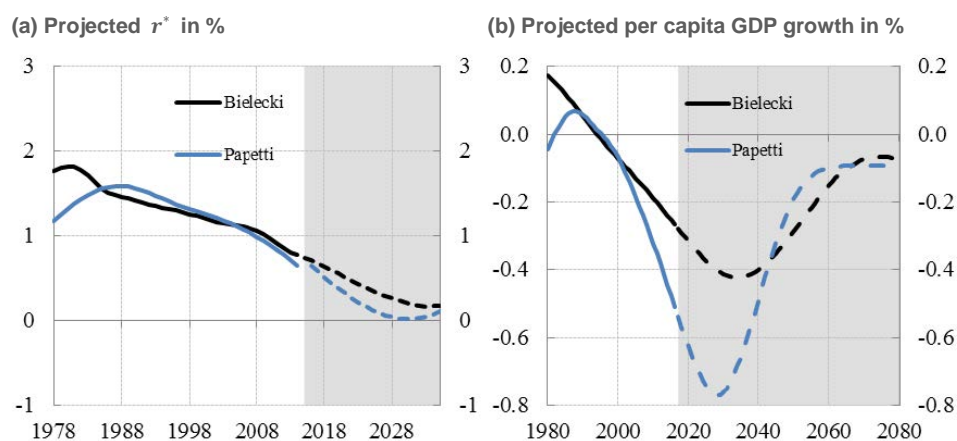
⁶ The influence of the demographic changes on the natural rate of interest is strong enough that ageing has been suspected to be a key factor of the “secular stagnation”, see Eggertsson and Mehrotra (2014); Gagnon, Johannsen and Lopez-Salido (2016); Lisack, Sajedi and Thwaites (2017); Cooley and Henriksen (2017); Sudo and Takizuka (2018); Jones (2018).

⁷ As a note of caution, one needs to bear in mind that the reference year in which one fixes either fertility or mortality rates matters: past fertility and mortality rates imply a certain type of ageing even absent any further changes in fertility and mortality rates.

for the retired (as in Gertler, 1999). In contrast, in their multi-cohort OLG model for the US (as a closed economy) Gagnon *et al.* (2016) find that the first channel (lower labour supply and lower marginal product of capital) is more prominent. Eggertsson, Mehrotra and Robbins (2017) estimate that in the US over the 1970-2015 period the influence of changes in mortality and fertility has been of equal strength.

Chart 3

OLG-based scenarios by Bielecki *et al.* (2018) and Papetti (2018)



Note: Projections are dashed lines and marked by shaded areas.

Such decomposition for the euro area, based on Bielecki *et al.* (2018), is reported in Chart 4a. About half of the change in r^* is due to changes in fertility and another half to changes in mortality. However, their approach cannot distinguish between effects stemming from increasing life expectancy and at the same time increasing share of dissavers in the economy. Using a proximate representation of an OLG model first introduced by Jones (2018), Papetti (2018) derives an expression for r^* , which can be used to quantify the influence of all three channels. These simulations give relatively more prominence to the first channel (lower labour supply), captured by ‘labour quantity’ (see Chart 4b).

Papetti (2018) identifies an important role of the third channel (ageing-induced change in the savers-dissavers composition) in depressing r^* . While this impact is steadily declining, it is not being reversed as an increasing proportion of more populous age-cohorts are projected to approach retirement. This latter result is in line with the literature. Lisack *et al.* (2017) attribute this lack of reversal to the prevalence of effects from the stock of wealth relative to the flow of dissavings generated by the baby boomers.

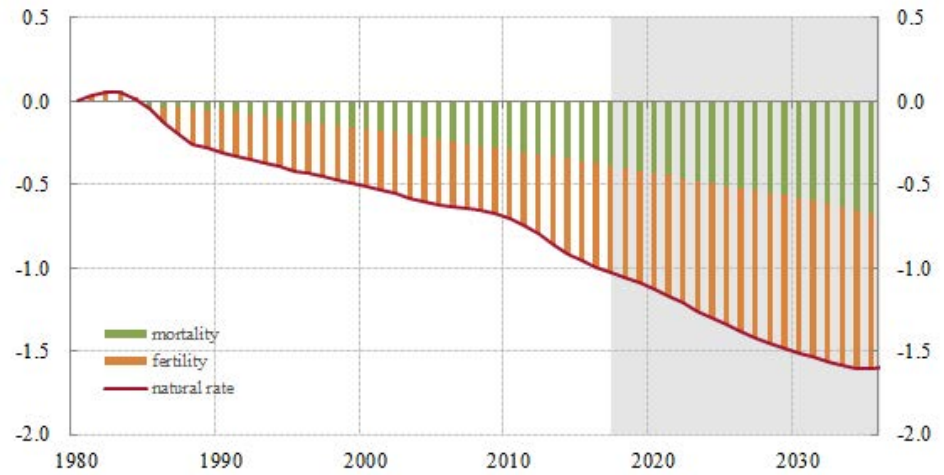
The relative scarcity of labour in the face of ageing is a crucial factor in understanding the impact of ageing not only on r^* , but also on economic growth. Cooley and Henriksen (2017) have confirmed these findings for the US and Japan. Bielecki *et al.* (2018) and Papetti (2018) conclude that the demographic situation exerted positive influence on potential growth rates up to the year 2000, but in the 21st century the influence has turned negative, and is projected to become even more so at least until year 2030. The estimated downward impact on potential output growth is around half a

percentage point (see Chart 3b). This effect is mainly due to the falling share of working-age individuals in the population.⁸

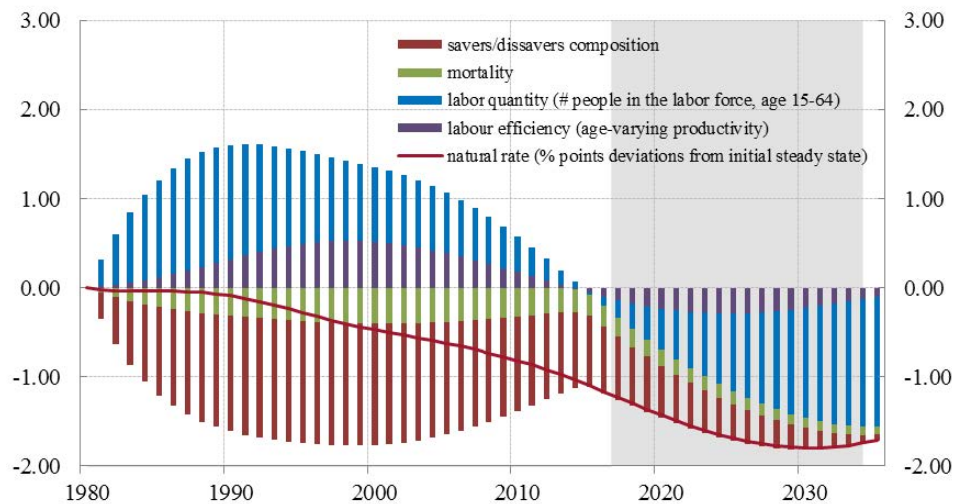
Chart 4
Demographic drivers of euro area r^* estimates

(percentages)

(a) Estimates from Bielecki *et al.* (2018)



(b) Estimates from Papetti (2018)



Note: Projections marked by shaded areas; contributions in percentage points.

To summarise, both theoretical reasoning and empirical analysis suggest that the current demographic transition is playing a key role in driving low-frequency fluctuations in real interest rates, due to the influence of low fertility rates, rising life expectancy, and changing composition of age cohorts. Looking ahead, as ageing is

⁸ Jones (2018), who fits business cycle fluctuations around the trends generated by demographic change, finds that in the US, ageing alone explains about one-third of the gap between output per capita and its long run trend, while the remainder of the gap is explained by real and financial factors (preference, investment and mark-up shocks) and to a lesser extent by nominal frictions and a binding zero-lower-bound constraint for the nominal interest rate.

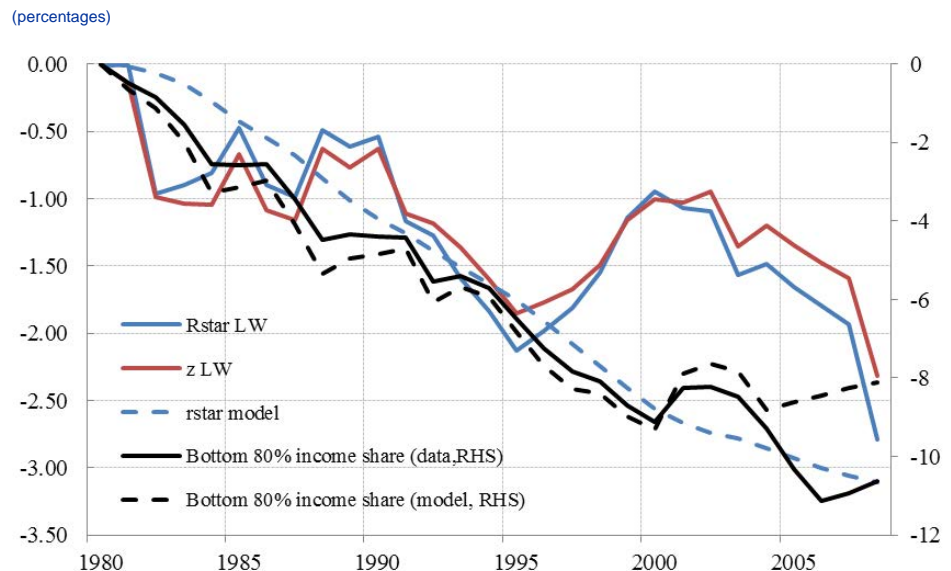
set to continue, the influence of demographic changes will tend to decrease r^* in the foreseeable future as well.

2.1.2 The role of rising inequality

Besides ageing, rising income inequality could also be an important factor in lowering equilibrium real interest rates (see e.g. Rachel and Smith, 2015). In fact, there is a striking coincidence in the dynamics of the non-growth component of r^* in the Laubach and Williams (2016) approach (z_t in Box 1) and income inequality in the US (see Chart 5).⁹ For instance the lower 80% income share declined by 11% from 1980 to 2007.

Chart 5

Non-growth component of Laubach and Williams (2016) r^* and inequality-related decline in r^* for the US (Rannenberg, 2018)



A secular increase in income inequality could depress the natural rate of interest if rich households save part of an increase in their permanent income, as found by Dynan, Skinner and Zeldes (2004) using US micro data. Rannenberg (2018) (see Annex Section A.3) formalises this mechanism in a model with two types of households supplying distinct types of labour, one of which represents the top 20% of the income distribution (referred to as “the rich”). Crucially, the rich have “capitalist-spirit” type preferences over wealth (Francis, 2009) resulting in a positive marginal propensity to

⁹ Arguably, developments in wealth inequality appear to be more relevant in affecting the natural rate. There is evidence suggesting an increase in wealth inequality in the post-crisis period in some OECD countries (Murtin, Lena-Nozal and Le Thi, 2015). Global trends in income inequality might matter too given the high degree of financial globalisation. While between-country income inequality has been declining over the last three decades, mainly due to rising incomes in populous countries like China and India, income inequality within countries has been increasing (World-Bank-Group, 2016).

save out of permanent income changes.¹⁰ To replicate the decline in the lower 80% income share over the 1980 to 2004 period, Rannenberg (2018) shocks the production elasticities of the two labour types. This period precedes the recent downward trend in the US labour share, suggesting that the increase in income inequality can be attributed to an increase in wage inequality.¹¹

Chart 5 illustrates that the increase in inequality (black dashed line) can contribute to a decline in r^* (blue dashed line) by more than three percentage points. The simulation reproduces part of the increase in the lower 80% of households' debt-to-income ratio, the rise in the value of the housing stock relative to GDP, and the household debt-to-GDP ratio since the 1980s (not shown in Chart 5). The simulated effect of inequality on the natural rate exceeds the estimated decline in the z -component of r^* post 1995, as it abstracts from the role of other offsetting factors (e.g. the rise in US government debt, or the decline in lending standards during the US housing market boom, both of which are held constant in the simulation).

Box 2

Drivers of the wedge between the returns on capital and on safe assets

The return on capital

While real interest rates have declined for more than 30 years across advanced economies, rates of return on capital have not. In fact, the return on capital in the euro area and US has been broadly stable over time and highly correlated across euro area countries (see Chart A).

¹⁰ Specifically, the rich derive utility from their holdings of real and financial assets (housing, government bond and bank deposits) over and above the future consumption opportunities they entail. The non-rich borrow from the rich subject to credit market frictions. Kumhof, Ranci re and Winant (2015) use preferences over wealth to create a link between the increase in inequality and the pre-crisis increase in the debt-to-income ratio of the bottom 95% of the income distribution in the US.

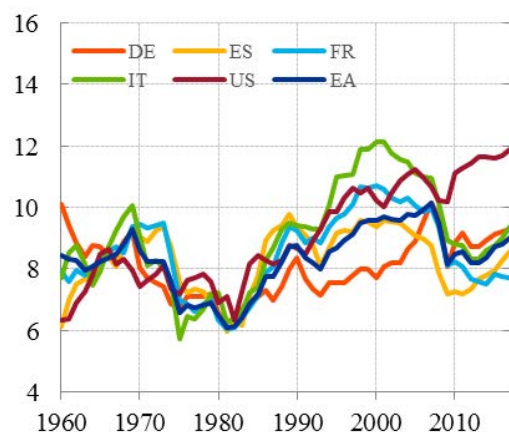
¹¹ At the microlevel, the increase in US labor earnings inequality has been documented using different data sources for instance by Kopczuk, Saez and Song (2010) and DeBacker, Heim, Panousi, Ramnath and Vidangos (2013).

Chart A

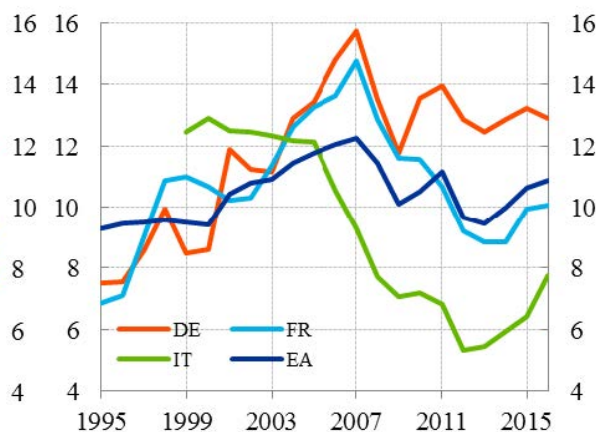
Return on capital

(percentages)

(a) Pre-tax return on all capital



(b) After-tax return on business capital



Source: AMECO database and national accounts.

A variety of assumptions are needed to measure the return on capital.¹² In particular, assumptions need to be made in relation to which assets should be examined (e.g. financial assets, the housing sector, the total economy or the productive sector of the economy) as well as how key measures such as the capital stock, operating surplus and corporate earnings are constructed.

Both panels in Chart B report post-tax measures of the return on capital based on German non-financial corporate sector data. While visibly different in terms of levels, all measures indicate a similar pattern of a non-declining return on capital.¹³ The richness of the data allows for estimating the marginal return on productive capital – arguably more relevant for investment decisions than the average return – which after experiencing a catching-up process from the mid-1990s after German reunification, does not display a significantly different profile to that of the average return (see left panel in Chart B).

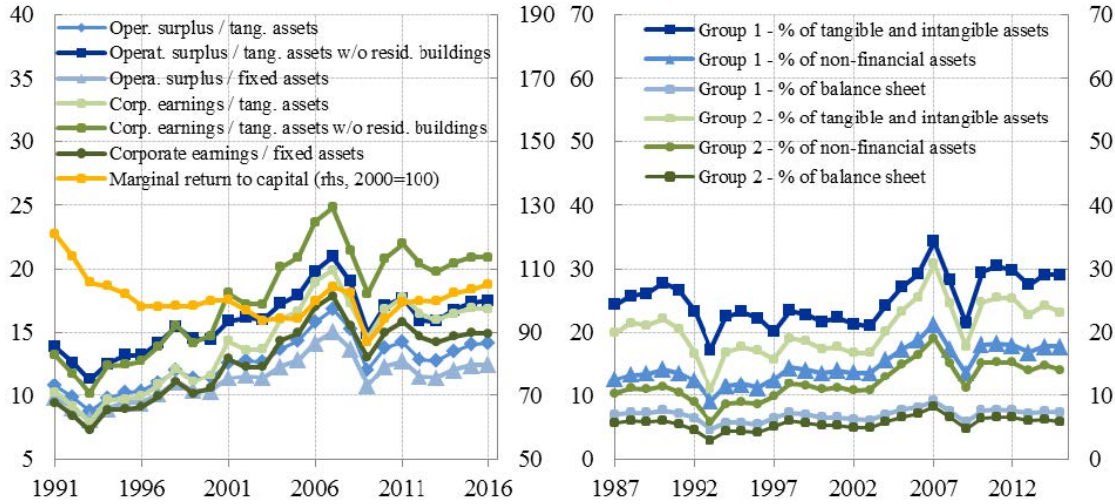
¹² See for example Gomme, Ravikumar and Rupert (2011), Piketty (2015), Caballero, Farhi and Gourinchas (2017), and Jordà, Knoll, Kuvshinov, Schularick and Taylor (2017).

¹³ The return on the entire productive capital for the non-financial corporations sector is calculated as the ratio of the operating surplus to the productive capital stock, with the productive capital stock being approximated across all fixed assets. For details, see Deutsche Bundesbank (2017).

Chart B

Average return on corporate capital based on financial statements in Germany

(percentages)



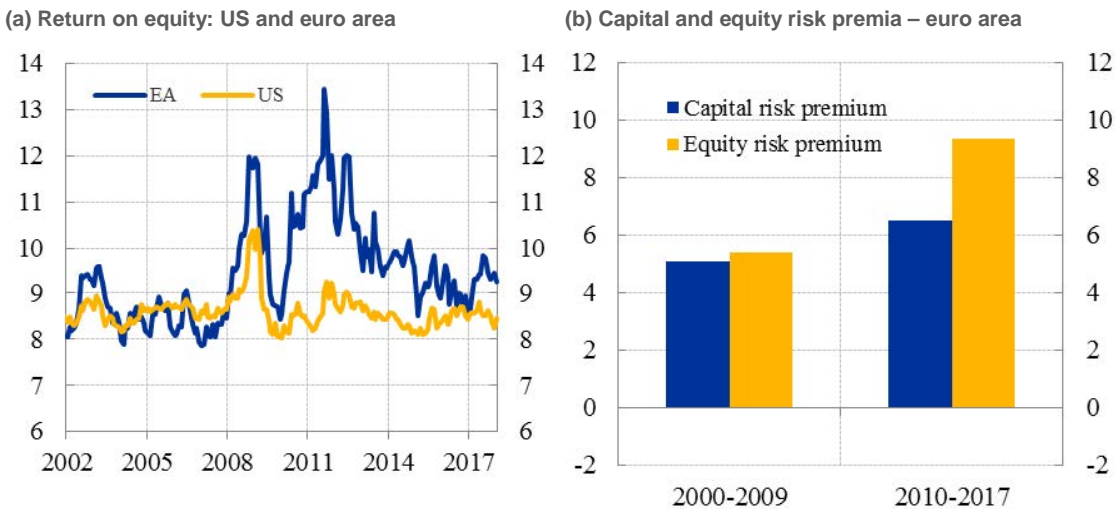
Note: Group 1 := Annual result before taxes on income plus interest and similar expenses minus interest and similar income.
 Group 2 := Annual result before taxes on income.
 Source: Deutsche Bundesbank (2017).

A cross-checking exercise using market-based measures (captured by the return on equity depicted in Chart Ca) indicates broadly stable returns in the euro area and US, albeit with higher volatility in the euro area. In the years leading up to the global financial crisis, the return on equity was stable and of similar magnitude in both the euro area and US. The equity risk premium has been increasing over time in both jurisdictions and thereby reflects the increasing wedge between the return on equity and the risk-free rate (see Chart Cb for the euro area).

Chart C

Return on equity and capital and equity risk premia

(percentages)



Source: Thomson Reuters and ECB calculations.

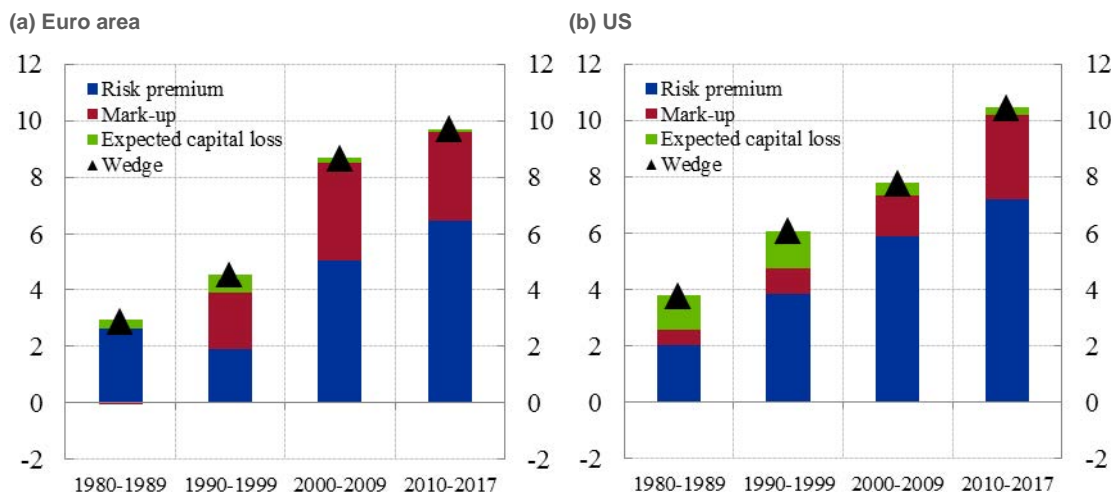
The role of the capital risk premium, risk aversion, mark-ups, and private debt

The drivers of the wedge between the return on capital and safe assets can be estimated using the framework of Caballero *et al.* (2017) that focuses on the potential roles of the labour share, the capital risk premium, expected capital loss, and mark-ups (see Annex B.5).

Chart D

Decomposition of the wedge between return on capital and risk free rate

(percentages)

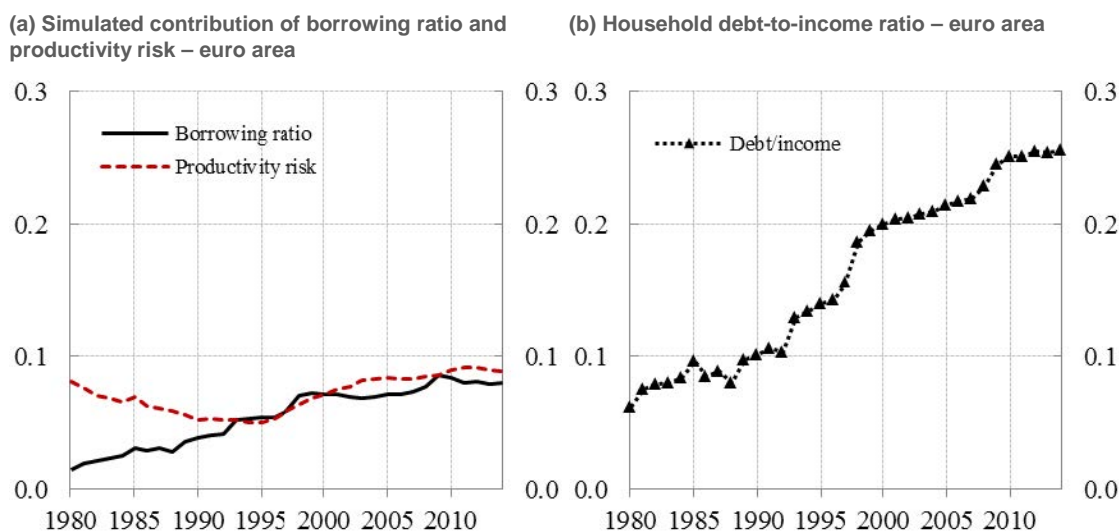


Source: ECB Calculations.

Chart E

Simulated contribution of borrowing ratio and productivity risk and household debt-to-income ratio – euro area

(percentages)



Source: Marx, Mojon, and Velde (2017).

Chart D shows that the increasing wedge between the safe rate and the return on capital can be reconciled – through the lens of a growth and production model – by rising mark-ups and surging

premia on capital risk (defined as the expected return on physical capital in excess of the risk-free rate).

Using data for the euro area and the US, this framework can be calibrated to match the observed wedge between the return on capital and the risk free rate over 1970-2016. , The pre-tax return to all capital for the euro area is used to estimate the real average return on capital, similar to the estimate of Gomme *et al.* (2011) for the US. The safe real interest rate r^s is the real 3-month OIS for the euro area¹⁴ and the real 3-month Treasury bill for the US.

Marx, Mojon and Velde (2017) perform a similar type of exercise to Caballero *et al.* (2017) in trying to explain jointly the movements of risky and risk-free interest rates using a small set of structural parameters. They impose certain inputs such as working-age population growth, productivity growth and the price of investment goods, but leave their model free to estimate the importance of the leverage ratio and productivity risk.

The results of their analysis for the euro-area are presented in Chart E. By construction, the model can fully explain the evolution of the risk-free rate and the risk-premium. Chart Ea illustrates that the model rationalises the data with a secular increase in the borrowing ratio since the mid-1980s. (In the absence of this rise in the borrowing ratio, risk-free rates would be even lower than they actually are for a given risk premium.) This result is consistent with the increase in the private debt to GDP ratio in the euro area observed over the same period (Chart Eb).¹⁵ In the model, riskiness of capital returns needs to decline gradually during the 1980s and mid-1990s before rising gradually again thereafter.

2.2 Estimates and drivers of the natural rate in econometric models

The secular trends discussed so far play a key role in determining equilibrium real rates at very low frequencies. But more flexible approaches are required to model r^* at frequencies that are more relevant for monetary policy. Econometric approaches commonly track time-variation in macroeconomic equilibria, stripping cyclical measures out of macroeconomic time series.

A range of such methods are employed in this section to estimate the interaction of r^* with a potentially large set of explanatory variables reflecting, saving-investment imbalances due to demographics or global imbalances, the pricing of risk, unconventional policies, and growth and productivity trends. All the following models are described in more technical details in the Annex.

Rolling window regressions: Jarocinski (2017) produces long-range macroeconomic forecasts as proxies for their time-varying equilibrium values, based on rolling regressions from a Bayesian VAR (see Annex B.1 for further technical details). Using these forecasts, r^* is constructed as a *terminal rate* of interest. Such a

¹⁴ The real short term rate is then backcasted over the entire sample using the real short term rate from the AWM database.

¹⁵ The debt ratio has been adjusted for generations lasting 10 years.

rolling-regression approach has also been employed by Hamilton, Harris, Hatzius and West (2015).

Error correction models: Multivariate error-correction models can be used to estimate time-varying equilibria using long-run relationships between macroeconomic variables. Estimates from this approach are presented, as adopted in Fiorentini *et al.* (2018), that are obtained for a long time span using a broad set of macroeconomic information, including total factor productivity and demographic developments (see Annex B.4 a for detailed model and data description).

Semi-structural models: As expounded in Box 1, the Laubach and Williams (2003) framework has become the econometric workhorse approach when it comes to estimate real equilibrium rates at lower-than-business-cycle frequencies. A range of different variants have been implemented for the euro area. Geiger and Schupp (2017) and Kupkovic (2017) use the set-up to estimate r^* for Germany and Slovakia (see Annex B.2.4 and B.2.3 for details). Hledik and Vlcek (2018) embed the approach into a fully-fledged multi-country model for the euro area, but drop the non-growth z_t component and instead model r_t^* as tracking potential output growth with inertia (see Box 4 and Annex B.3 for further details). Pedersen (2015) develops an open-economy version for Denmark (see Annex B.2.5). For the euro area, Brand and Mazelis (2018) close the model using a Taylor rule, drop the unit root from the Phillips curve and model inflation to be stable around some norm, so that r^* is consistent with the inflation objective (see Annex B.2.1). Bragoudakis (2018) replaces the short-term rate by a bank lending rate and unobserved components of output by their corresponding official estimates to estimate equilibrium levels of *bank lending rates* for Greece (see Annex B.2.6). Krustev (2018) also models inflation to be stationary, but additionally takes into account the financial cycle – arguably an omitted variable from the original model – to present r^* estimates for the US (see Annex B.2.2).

Macro-finance models: Christensen and Rudebusch (2017) have extracted similarly slow-moving estimates of real interest rates from financial markets data using affine term structure models capturing macro-economic variables. In this report, results for the euro area from the macro-finance model by Ajevskis (2018) are reported (see Annex B.6).

Box 3

Identification and uncertainty of natural rate estimates using Laubach and Williams (2003)

While widely used, the unobserved-components approach of Laubach and Williams (2003) has been reported to produce estimates of the natural rate of interest that are highly imprecise (Kiley, 2015; Beyer and Wieland, 2017; Lewis and Vazquez-Grande, 2017; Holston, Laubach and Williams, 2017). Clark and Kozicki (2005) have, in addition, pointed to significant real-time measurement problems that amplify uncertainty about natural rate estimates.

This box seeks to illustrate potential sources of estimation uncertainty, first emerging from filtering uncertainty depending on the slopes of both the Phillips and the IS curve, and second, from a Bayesian perspective, resulting from the degree to which the data can inform the parameters governing the natural rate process.

Identification of latent factors when the IS and the Phillips curve are flat

To assess the source of uncertainty, Fiorentini *et al.* (2018) investigate the relationship between the theoretical mean squared errors of the unobserved states and the ‘true’ parameters of the model.

Importantly, the high imprecision in estimating r^* is due to large filter uncertainty (the problem of estimating the values of the state variables, rather than the values of the model coefficients). Only in cases where both the IS and the Phillips curve are relatively steep can the model produce accurate estimates of r^* .

Chart A

Filter uncertainty and slopes of the IS and Phillips curves

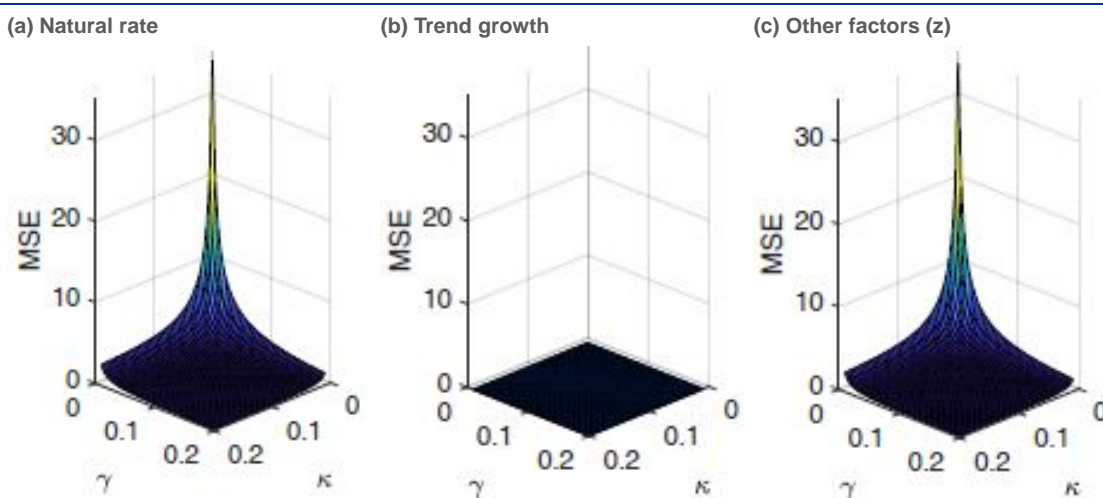
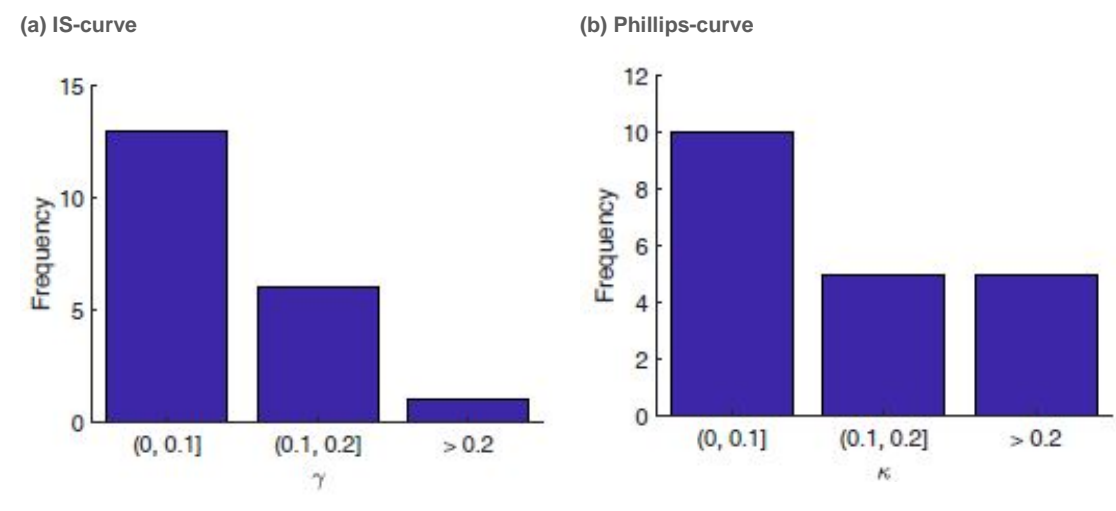


Chart A illustrates this point by plotting the mean squared error of the unobserved states as a function of the steepness of the IS and Phillips curves, denoted here by the coefficients γ and κ , respectively. Under the assumption that all model coefficients are known, the mean squared error provides an indication about the extent of filter uncertainty. Notice that for relatively steep IS and Phillips curves (i.e. γ and κ large), the filter uncertainty associated to the natural interest rate is small, thus confirming that the model is generally able to produce accurate estimates of r^* . However, as either γ or κ approach zero (and particularly when both do), the uncertainty associated with r^* dramatically increases, principally due to a rise in the uncertainty associated with the z_t component. Hence the relevant component which affects the uncertainty of the natural interest rate is the z_t process, while the process for trend growth features relatively small uncertainty regardless of the steepness of the IS and Phillips curves.

Chart B

Estimated steepness



The data tend to support rather flat IS and Phillips curves: Chart B reports the frequencies of estimates of γ and κ across several studies which estimate the Laubach and Williams model or variants of it using data for several advanced economies. The distributions of estimated sensitivities are skewed towards values which are small and close to zero. It appears to be more the rule than the exception to find estimated IS and Phillips curves which are close to be flat. In these circumstances, imprecision about r^* is large even with perfect knowledge of the true values of the model parameters.

The 'pile-up' problem

Laubach and Williams (2003) refer to the 'pile-up problem' (in this context¹⁶ the problem that, if the model was estimated simultaneously, using maximum-likelihood, the variance of one of the shocks to the latent state variables would peak at zero) to motivate their use of the multi-step Stock and Watson (1998) approach.

In principle, a Bayesian approach should not be subject to this problem, as the posterior distribution of the true parameter is a property of the data and not of the sampling distribution of the estimator (see Sims and Uhlig, 1991; Kim and Kim, 2013). Kiley (2015) and Lewis and Vazquez-Grande (2017) demonstrate challenges to parameter identification – and thus identification of the natural rate – when using uninformative priors.

For the closed-model version estimated in Brand and Mazelis (2018), the data are not sufficiently informative to identify the growth component and only weakly informative of the non-growth component of Laubach and Williams (2003). While tight priors on the variance of shocks to the growth component σ_g^2 may be used to align the model-specific output gap with official estimates, there is no prior information at all about the variance of shocks to the non-growth component σ_z^2 . Data are only weakly informative of σ_z^2 , so working with flat priors will produce large estimates of σ_z^2 . Accordingly,

¹⁶ In finite samples, the literature documents non-invertibility problems (with the likelihood function 'piling up' at one) when estimating a moving-average process when the true process is invertible, i.e. the coefficient is smaller than one, see Kim and Kim (2013).

Chart C illustrates this point by displaying prior (red lines) and posterior densities (blue lines) from Brand and Mazelis (2018). Still, the Charts also illustrate that posterior densities do not run up against zero (and the same is true for all other innovation variances of the model), unlike the pile-up feature arising from maximum likelihood.

Chart C

Posterior and prior densities on σ_g^2 and σ_z^2

(a) Prior/posterior densities on sigma-g

(b) Prior/posterior densities on sigma-z

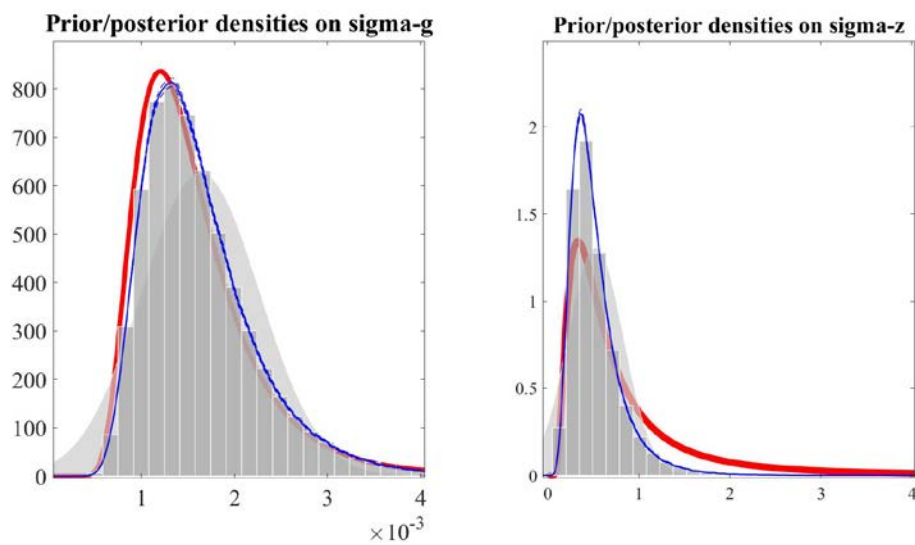


Chart 6 presents in-sample estimates of r^* for the euro area and the US across different econometric methods. These are based on the semi-structural approach of Holston *et al.* (2017) as applied by Brand and Mazelis (2018) and Krustev (2018). Results from the macro-finance model by Ajevskis (2018), the error-correction model by Fiorentini *et al.* (2018), and the rolling regression BVAR by Jarocinski (2017) are also reported. The estimates display a decline in r^* that has accelerated in the wake of the global financial crisis, with r^* dipping into negative territory and subsequently inching higher only tepidly.

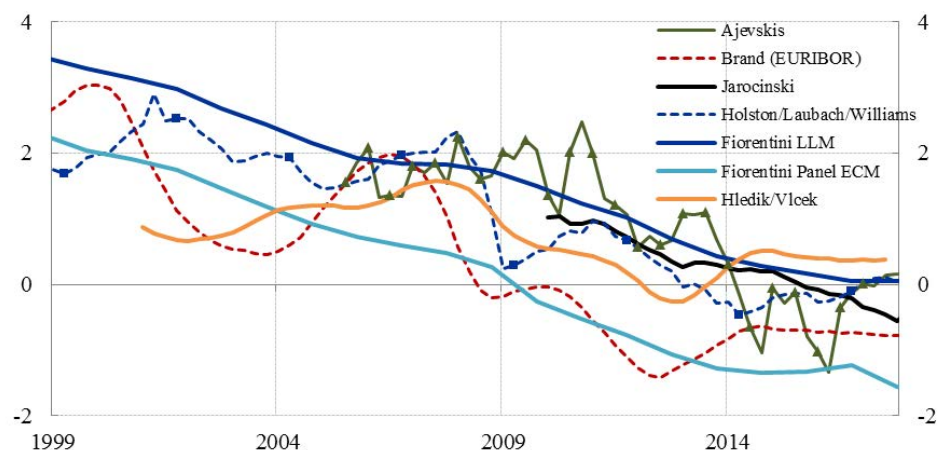
The differences amongst the semi-structural models need to be interpreted in the light of how they were constructed, with the one by Laubach and Williams (2003) reflecting equilibrium rates being consistent with *stable* inflation (independent of its level), the one in Brand and Mazelis (2018) being consistent with price stability, and Krustev (2018) integrating a financial cycle.

Chart 6

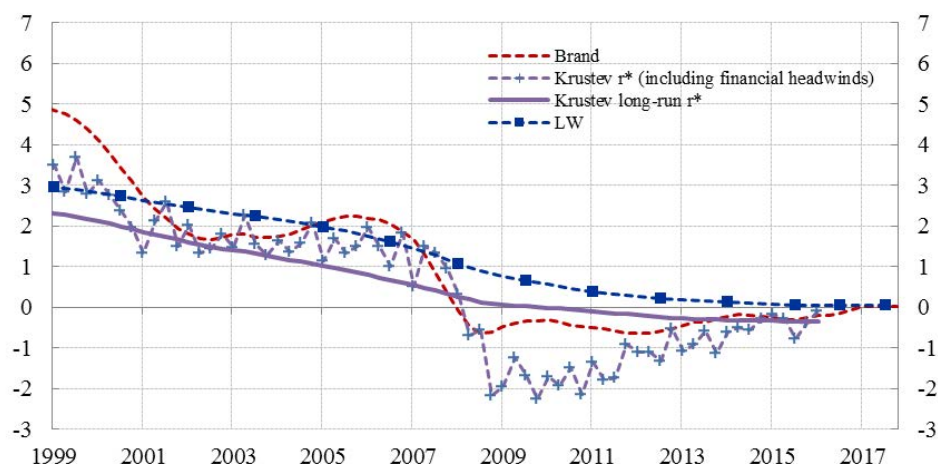
Econometric estimates of r^*

(percentages)

(a) Econometric estimates for euro area



(b) Econometric estimates for US



Notes: Both euro area estimates from Holston *et al.* (2017) and (updated) US estimates from Laubach and Williams (2003) are obtained from the homepage of the Federal Reserve Bank of San Francisco with latest observation being 2017Q4 in both cases. Holston *et al.* (2017) based on filtered estimates and Brand and Mazelis (2018) based on smoothed estimates of states.

Underlying growth trends

Chart 7 displays key macroeconomic trends underlying econometric r^* estimates. Specifically Chart 7a documents how, in the semi-structural model class, lower trend growth has underpinned the decline in r^* , especially in the wake of the financial crisis, with an estimated drop in potential output growth by nearly one percentage point.

The finding of a slump in potential output growth in unobserved components models (even prior to the crisis) corroborates evidence documented earlier. Garnier and Wilhelmsen (2005), using four decades of post-WW II data for Germany, and Fries, Mésonnier, Mouabbi and Renne (2016), who provide a joint estimation for Italy, Spain, France, and Germany using monthly data, document a sustained decline in r^* on the

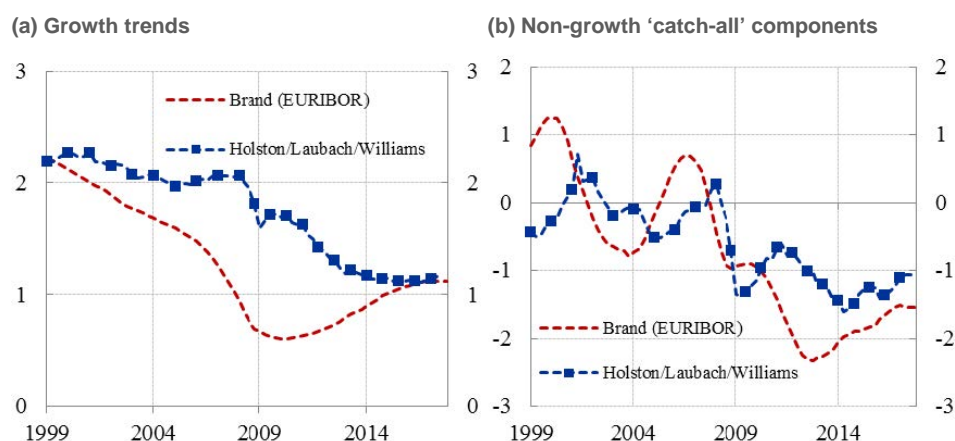
back of flagging growth trends. Fries *et al.* (2016) identify significant discrepancies in potential output growth rates at national level and r^* across major euro area countries, in particular for Spain and Germany before and after the financial crisis. Similar country discrepancies based on the Czech National Bank’s multi-country, unobserved components model for the euro area are illustrated in Box 4. Common to all these studies is that slowing trend growth is identified to be an important driving factor behind the decline in r^* – but not exclusively so.

These declining growth trends appear to have been factored into return measures at longer maturities too, suggesting expectations of low growth for an extended period of time. Applying arbitrage-free term-structure models to inflation-linked bonds in the euro area, Ajevskis (2018) estimates a drawn-out decline in risk-neutral equilibrium real rates across the maturity spectrum, similar as obtained by Christensen and Rudebusch (2017) for the US. This declining trend is attributed to a gradual turn lower in the expectations component prior to the implementation of the Eurosystem’s asset purchase programme and to a falling term premium component thereafter.

Chart 7

Key euro-area macroeconomic trends underlying r^*

(percentages)



Note: Holston *et al.* (2017) based on filtered estimates and Brand and Mazelis (2018) based on smoothed estimates of states.

Financial factors, demographics, and productivity

The non-growth component of r^* (often associated to safe-haven flows and the pricing of risk, but typically not modelled explicitly here) is estimated to have contributed to an additional fall in the natural rate, particularly following the financial crisis, by 150-200 basis points in the euro area (see Chart 7b) and up to 70 basis points in the US. This component highlights the importance of a large persistent wedge between trends in growth and in the return on safe assets. The existence of this wedge may well explain the weak correlation between growth and real interest rates as observed by Hamilton *et al.* (2015) in their econometric analysis of long-run macroeconomic data going back to the 19th century.

Incorporating financial variables into the semi-structural framework can also affect *low-frequency* estimates of r^* . Kiley (2015), for example, incorporates credit

conditions, as “demand shifters” into the cyclical component, implying a more limited decline in equilibrium rates than estimated by other methods. This is likely so because financial conditions capture aspects of monetary transmission that, in their absence, would have to be explained by higher real rate gaps which, given a specific path in observed short-term rates, can only be achieved by more volatile r^* measures.

The estimates by Krustev (2018) for the US illustrate differences in r^* at *higher frequencies* when incorporating financial information. Consistent with the mechanism in Benigno, Eggertson and Romei (2014), Krustev (2018) finds that the global financial crisis and persistent deleveraging have temporarily lowered r^* by around one percentage point below its long-run trend. By incorporating the financial cycle the model delivers more plausible business cycle dynamics too. This evidence supports the argument that the omission of financial imbalances may lead to biases in the estimation of both r^* and potential output growth, as claimed by Juselius, Borio, Disyatat and Drehmann (2016), Cukierman (2016), and Taylor and Wieland (2016).

Importantly, the financial cycle has also a global dimension (e.g. Borio, 2014; Rey, 2018). In the Laubach and Williams (2003) framework, Pescatori and Turunen (2015) have illustrated an increasing role for excess global savings in depressing r^* . Global factors, i.e. excess global savings proxied by current account surpluses in emerging markets and an increase in the equity premium (as estimated by Duarte and Rosa, 2015) after the global financial crisis appear to play a prominent role.

Using a panel of 17 advanced economies, Fiorentini *et al.* (2018) capture demographic developments and other global macroeconomic factors from the end of the 19th century, tracking developments in actual real rates. The effect of demographic developments is estimated to have been significant. The hump-shaped increase and subsequent fall in real returns from the 1970s until the past decade (see Chart 8) is estimated to have been mainly due to demographics, rising risk aversion (proxied as the spread between long and short-term interest rates), and only marginally to total factor productivity growth. Yet, since mid-last decade, alongside productivity and risk, the share of young-age cohorts plays an increasingly important role in depressing r^* , by magnitudes comparable to those obtained from calibrated OLG models.

Similar empirical evidence on the role of demographics has previously been documented in Lunsford and West (2017) for the US, but with a dominant role for demographics and an unclear role for productivity trends. Using long historical time series for 19 countries Borio, Disyatat, Juselius and Rungcharoenkitkul (2017) challenge the view that real interest rates are driven by variations in desired saving and investment and rather assign variations in real yields to differences in monetary policy regimes. These exercises illustrate challenges with capturing the complexity of channels through which demographics affect economic trends in an empirical manner, especially when relying on single and separate age-related proxies.

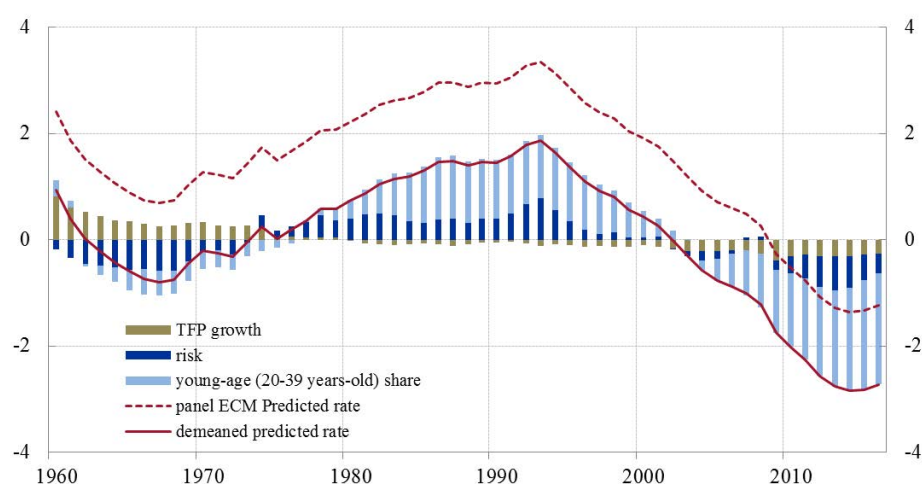
Unconventional policies

Neither theory nor empirical models of r^* were developed to deal with unconventional monetary policy or frictions in the transmission of short-term interest rates to output and inflation. As explained in Box 1, monetary policy is typically assumed to work only

through the gap between a one-period risk-free real interest rate and the benchmark of r^* . Complicating matters further, unconventional policy comes in a variety of forms including quantitative easing, forward guidance on interest rates, and various credit easing policies such as, in the case of the Eurosystem, the TLTROs.

Chart 8

Econometric estimates of r^* drivers in the euro area based on Fiorentini *et al.* (2018)



Note: Contributions in percentage points.

In theory, any unconventional monetary policy measure that succeeds in raising the output gap for a given real interest rate will require r^* to rise. This is mechanical from the inversion of the IS equation. Of course this somewhat blurs the role of r^* as a benchmark interest rate that is independent of monetary policy. Therefore, theoretically, unconventional monetary policy supports conventional policy because it widens the gap between r^* and the current real policy rate, *ceteris paribus*. A related mechanism is at work in the model of Benigno *et al.* (2014) in which r^* co-varies *positively* with policy accommodation through leverage, leading to higher measures of r^* as leverage is built up, and lower estimates as households and firms delever. Thereby, r^* would respond to monetary policy, including unconventional policies, in a virtuous manner enhancing policy transmission.

This neat theoretical story could be overturned in the pathological case in which quantitative easing amplifies a scarcity channel. Under this hypothesis, proposed by Filardo and Nakajima (2018), bonds are considered safer than central bank reserves (because the former can be pledged in contracts while the latter cannot) so quantitative easing reduces the supply of effective risk-free assets and pushes down on r^* instead. Acharya, Eisert, Eufinger and Hirsch (2017) document negative side effects from unconventional policies lowering sovereign bond yields, thereby (indirectly) re-capitalising banks, possibly perpetuating misallocation of credit due to ‘zombie lending’ (i.e. banks with high share of nonperforming loans continue to lend to impaired borrowers in order to defer loss recognition), and ultimately undermining productive capacities. Such side effects may arise, but interest rates are typically considered too blunt a tool to counter forbearance, which should rather be addressed

using appropriate micro- and macro-prudential instruments (Jiménez, Ongena, Peydró and Saurina, 2017).

Measuring the impact of *unconventional* policies on r^* is much harder and subject to additional uncertainty. This measurement issue is to be distinguished from concerns about an estimation bias incurred from omitting regulatory factors or systematic policy errors, e.g. relative to Taylor-rule prescriptions, as expressed by Taylor and Wieland (2016). Such biases, if relevant, are obviously best mitigated directly for example if the semi-structural framework captures additional relevant information and if it is closed using a policy rule but these yield different r^* estimates.

Box 4

Natural rates at euro area country level

Evidence from a multi-country model

This box first presents estimates of country-specific natural rates for Austria, Belgium, Germany, France, Italy, the Netherlands, and Spain using the structural multi-country model of the euro area developed in Hledik and Vlcek (2018) (see Annex B.3 for further details). This multi-country model encompasses seven country blocks linked through trade and subject to the ECB's monetary policy. Compared to Fries *et al.* (2016), all national country blocks are modelled as open economies, interacting with other union members via a fixed exchange rate regime. There are also trade links with the rest of the world, proxied by the US. The natural rate of interest follows, with some inertia, the q-o-q change in potential GDP.

Chart A shows estimates of the natural rate for these euro area countries. In line with Fries *et al.* (2016) the results suggest that the natural rate of interest peaked prior to the global financial crisis, i.e. in 2006-2007, and turned sharply lower for most countries afterwards. The peak-to-trough slump at a country level ranges between one and two and a half percentage points.

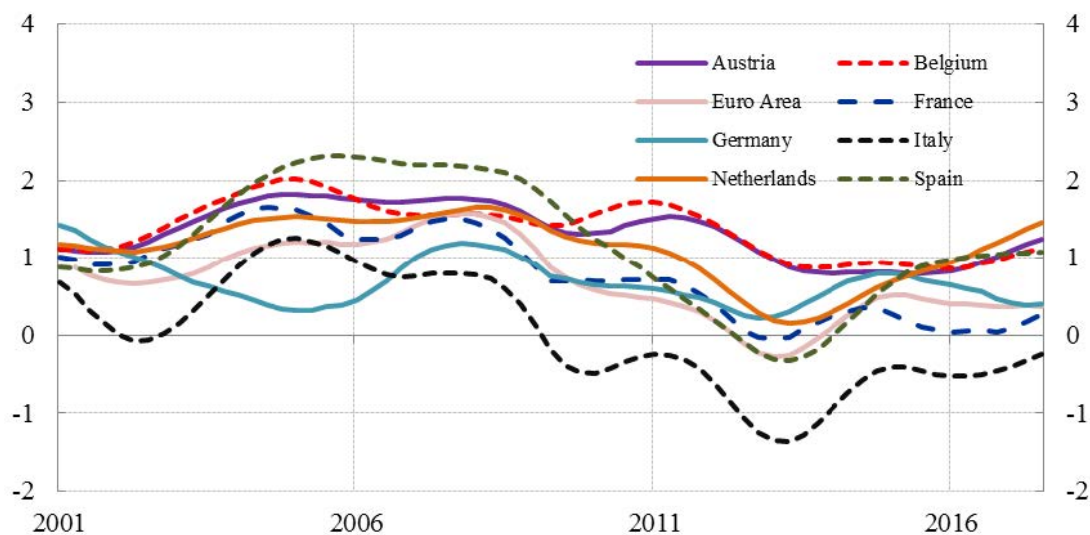
Country specific differences do not take financial factors into account. Some of the estimated increase in potential output growth in the run up to the crisis may have been financially unsustainable in some countries. Likewise, the subsequent slump, in turn, may have to be corrected for deleveraging pressure. Taking these aspects of the financial cycle into account would likely lead to smaller differences across countries.

Currently our estimates suggest that r^* for Austria, Belgium, Germany, the Netherlands, and Spain are between 0.5% to 1.5%. The estimate for France is on the low side of this range, and Italy is below zero, with the latter standing out because estimates of potential growth have been negative since the start of the crisis.

Chart A

Euro-area country estimates of r^*

(percentages)



Source: Hledik and Vıcek (2018).

Note: Euro area r^* has been calculated using GDP weights.

Applications of the semi-structural approach to individual countries

Charts Ba and Bb show country-specific r^* estimates using the Laubach and Williams (2003) approach for Germany and Slovakia, respectively (see Annex B.2.3 for further details about the open-economy version for Slovakia by Kupkovic, 2017). The estimates for Germany are qualitatively comparable to the ones obtained by Hledik and Vıcek (2018) and Fries *et al.* (2016) and reflect that, in this methodology, r^* has more scope to deviate from low-frequency growth trends.¹⁷

¹⁷ Annex B.2 illustrates that versions of Laubach and Williams (2003) typically posit that the real natural rate can deviated infinitely from potential output growth.

Chart B

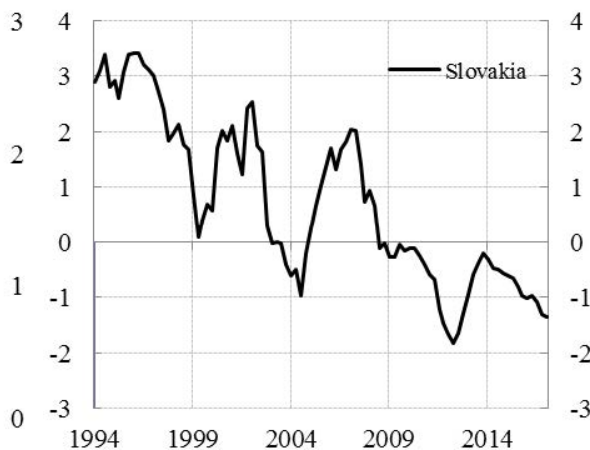
Estimates of r^* and equilibrium lending rates

(percentages)

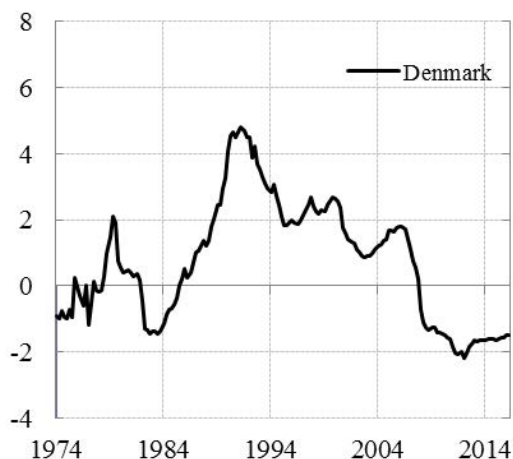
(a) Estimates of r^* for Germany



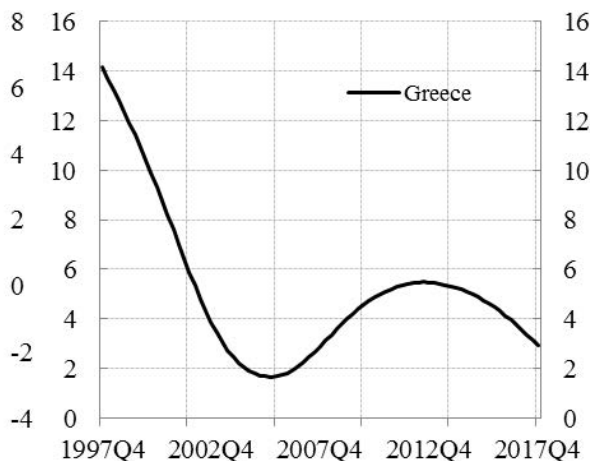
(b) Estimates of r^* for Slovakia



(c) Estimates of r^* for Denmark



(d) Estimates of equilibrium lending rates for Greece



Sources: (a) Geiger and Schupp (2017); (b) Kupkovic (2017); (c) Pedersen (2015); (d) Bragoudakis (2018).

Chart Bc presents estimates from Pedersen (2015)'s open-economy version for Denmark – reporting negative equilibrium rates ever since the start of the crisis. Chart Bd applies the approach to bank lending rates for Greece (and using official output gap estimates and the Hodrick-Prescott filter to approximate the corresponding latent factors, see Model Annex B.2.6 for further details). Throughout the sample, estimates are positive as bank lending rates are significantly higher than risk-free short-term money market interest rates.

Challenges from country differences in r^*

These differences in r^* estimates across euro area countries pose challenges to the transmission of the single monetary policy, as – abstracting from all other factors – a single real short-term interest rate is translated into different real rate gaps and thereby possibly diverging monetary conditions. There is nothing that monetary policy can do about divergences in potential output growth rates at national levels. Only effective structural reforms can attenuate these country differences (see

Section 3.3.3 in this report and, in particular, Masuch, Anderton, Setzer and Benalal (editors) (2018) for more details).

Another example where measurement of r^* at first blush appears to give counter-intuitive results is the incorporation of unconventional policies in arbitrage-free term-structure models of inflation-linked bonds. Ajevskis (2018) applies a similar methodology to the one used by Christensen and Rudebusch (2017) to the euro area and finds a drawn-out decline in risk-neutral equilibrium real rates across the maturity spectrum. But in this case, because large-scale asset purchases push down on market interest rates by extracting term premia (as the instrument is intended to work), the corresponding r^* metric (capturing both the expectations and the term-premium component) also falls. This result emphasises the importance of understanding how measurement and theory interact.

Overall, econometric approaches identify a declining trend in the natural rate of interest, particularly so in the wake of the financial crisis. Lower growth and productivity, also driven by demographic factors, and, in addition, financial factors (such as persistent deleveraging) have contributed to this development. There is broad evidence that the precipitous fall in r^* during the crisis is likely driven by an increase in risk aversion.

2.3 The Neo-Wicksellian approach: the business cycle perspective

While the semi-structural models provide information of the medium-term behaviour of the natural rate of interest and its evolution at lower frequencies as the transitory shocks die out, structural DSGE models can capture its changes over the business cycle, which is arguably more important from the point of view of monetary policy. r^* is the level of real interest rate that is consistent with *contemporaneously* stabilising both the output gap and inflation at the central bank's target, provided that the divine coincidence property holds in the model used to derive r^* (see Box 1).

In DSGE models, the natural rate of interest is an unobservable variable that can be extracted by estimating a fully structural model, in practice using a rich set of macroeconomic data. Unlike in the textbook New Keynesian model case, there is not a unique, unambiguous definition of the natural interest rate. Following Woodford (2003) and Galí (2008), most papers¹⁸ adopt a definition of the natural rate of interest as the real interest rate that would prevail in a counterfactual economy under flexible prices and wages, and absent shocks to the mark-ups on goods and labour markets.

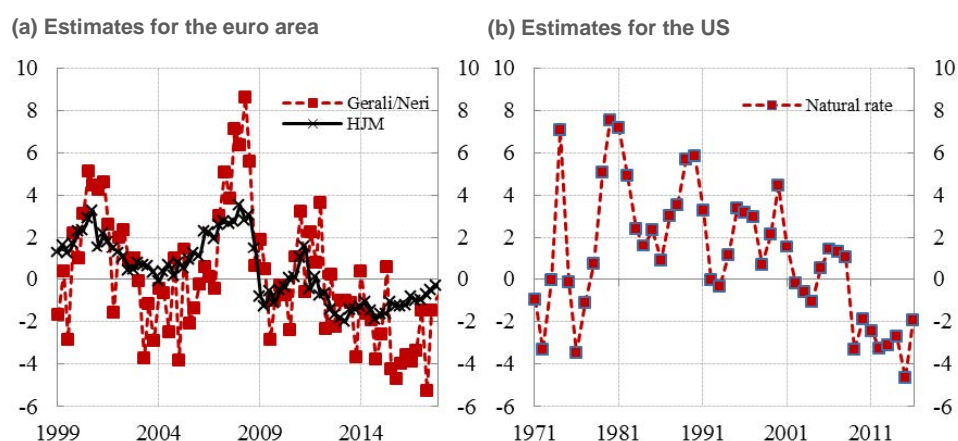
Using the approach of Smets and Wouters (2003, 2007), but adding certain forms of financial frictions, Gerali and Neri (2017) and Haavio *et al.* (2017) construct and

¹⁸ See e.g. Giammarioli and Valla (2003); Edge, Kiley and Laforte (2008); Barsky, Justiniano and Melosi (2014); Curdia (2015); Hristov (2016).

estimate medium scale DSGE models to extract r^* . Chart 9 depicts their results. There is a high degree of consistency in the dynamics of these estimates, including the slump in r^* following the Lehman Brothers bankruptcy in late 2008 and, after recovering from the low levels in years 2009 and 2010, the sovereign debt crisis in 2011/12. Both models agree that at least since 2015 the euro area natural rate of interest is negative.

Chart 9
DSGE estimates of r^*

(annualised; percentages)



Sources: (a) Haavio *et al.*, 2017 (HJM) ; Gerali and Neri, 2017; (b) Gerali and Neri, 2017.

These results are in line with estimates obtained for other developed countries. A wealth of studies, including Justiniano, Primiceri and Tambalotti (2013); Barsky *et al.* (2014); Curdia (2015); Hristov (2016); Del Negro, Giannone, Giannoni and Tambalotti (2017); Gerali and Neri (2017) all find strikingly similar patterns for the US, with a sizeable decline after the Lehman Brothers bankruptcy into negative territory.

A unique advantage of the DSGE over the semi-structural approach is their ability to trace the impact of structural shocks on the natural rate of interest.¹⁹ As Charts 10a and 10b show, both Gerali and Neri (2017) and Haavio *et al.* (2017) find that a risk premium shock is responsible for a significant part of the total variance of the natural rate of interest. In both papers, the risk premium shock modifies the households' effective discount rate for one-period risk-free (government) bonds, and following Fisher (2015) is given the interpretation of a shock to demand for safe and liquid assets. In Gerali and Neri (2017), the economy is also subject to a shock to the marginal efficiency of investment, which aims to capture disturbances in financial intermediation. Both shocks contribute to the upward shift in r^* prior to the financial crisis and to a subsequent series of declines.

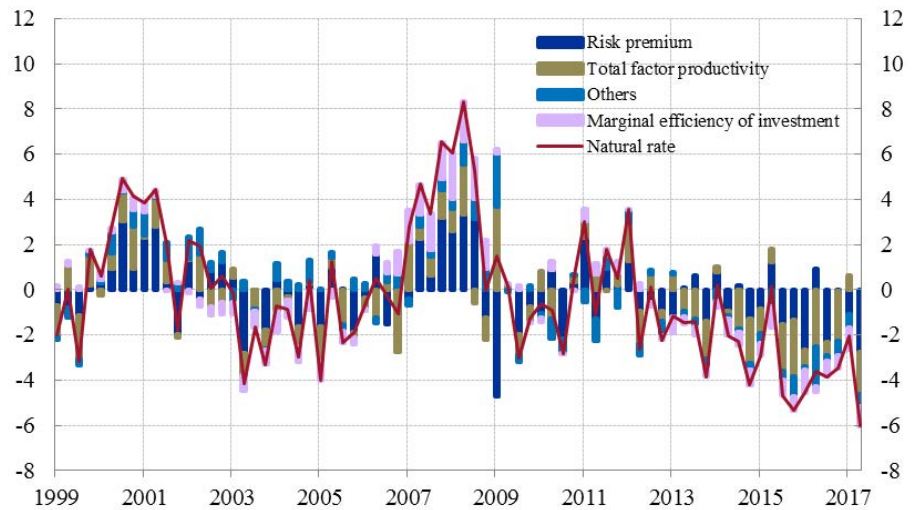
¹⁹ Naturally, caution is needed in interpreting the results, as the structural models also suffer from "omitted shock bias" and will split the disturbances only across the categories consciously built in by the modelers. As Hristov (2016) shows, models that treat the financial side of the economy differently produce visibly different measures of output gaps and the natural interest rate.

Haavio *et al.* (2017), in contrast, build in an explicit treatment of financial intermediation following Gertler and Karadi (2011), which allows them to distinguish between interest rates on riskless government bonds and riskier corporate bonds. Since they observe the resulting spread directly, their model attributes a bigger part of the total deviations of r^* from its steady state level to their interpretation of the risk premium shock. This result is in line with conclusions reached by Del Negro *et al.* (2017) for the US: in the recent years the decline of the natural rate of interest was driven by the increased preference for holding safe and liquid assets.

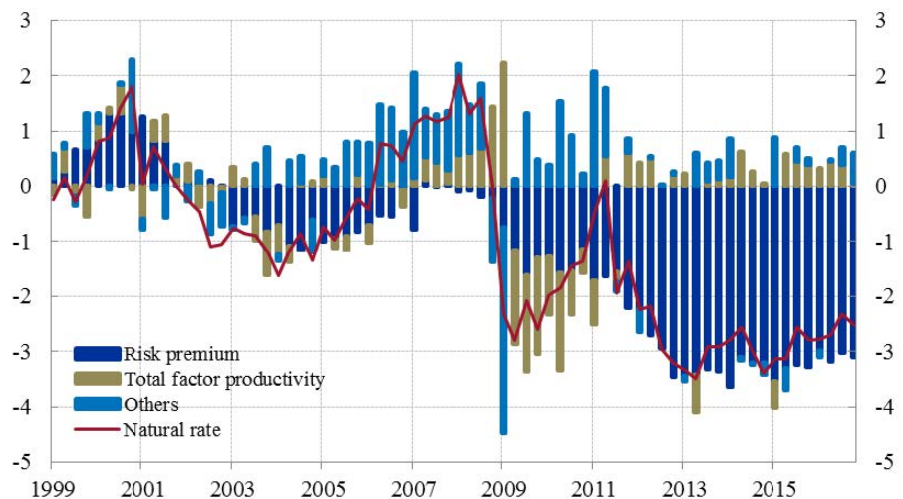
Chart 10
Structural shock decompositions for r^*

(percentage deviation from steady state)

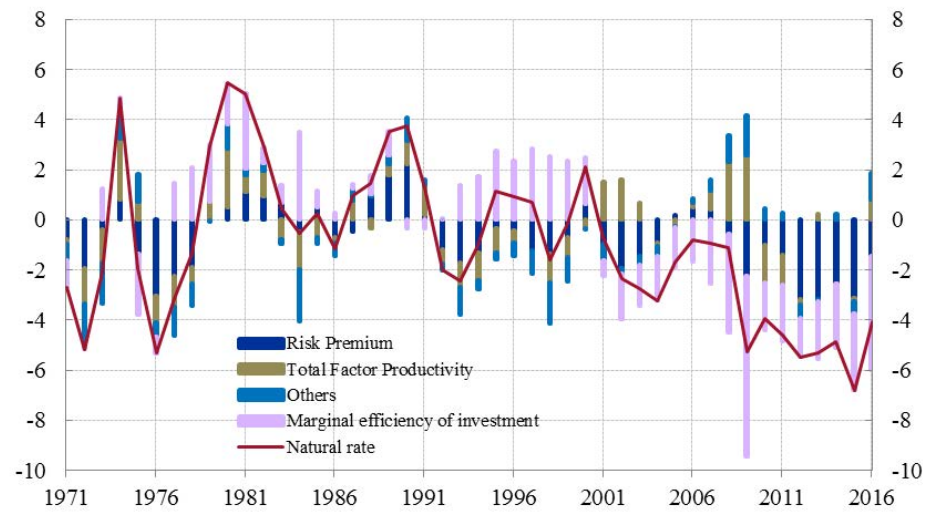
(a) Euro area: Decomposition based on Gerali and Neri (2017)



(b) Euro area: Decomposition based on Haavio *et al.* (2017)



(c) US: Decomposition based on Gerali and Neri (2017)



The prominent role of financial factors is also borne out by the shock decomposition exercise for the US version of the model by Gerali and Neri (2017). Alongside the risk premium, the marginal efficiency of investment is identified to be an important driver of r^* (see Chart 10c). The observation that r^* has declined on both sides of the Atlantic and driven by similar factors point to its global dimension, as also previously discussed in the comprehensive analysis by Rachel and Smith (2015) who emphasise demographic factors, rising inequality, and the savings glut as important global factors.

3 Assessment and Outlook

3.1 Where do we stand? What to expect?

The previous sections have illustrated a ubiquitous downward trend in equilibrium real interest rate estimates across a range of different methods. Yet, there is a high degree of uncertainty surrounding mid-point estimates at specific points in time. Chart 11 provides an overview of in-sample estimates and displays model-specific projections. Charts 11a-d show selected econometric estimates (from Section 2.2), while Charts 11e and 11f show business-cycle-model-based estimates (from Section 2.3), with their respective, estimation-specific 90% error bands and forecasts of r^* until 2025. There are large differences in in-sample confidence bands, capturing different statistical sources of uncertainty. The large error bands displayed in Charts 11a-d are due the high persistence of shocks affecting natural rate estimates and specifically for Chart 11b very high filtering uncertainty as discussed in Box 3.

Due to its equilibrium nature r^* is often also considered as a *terminal* rate of interest. But with the exception of models that consider the impact of the ongoing demographic transition, we can glean very little policy-relevant information on future r^* .

Demographic factors are largely predetermined and would suggest that r^* is more likely to fall than to rise in the medium term. Scenarios obtained from OLG-models (see again Chart 3a) project r^* to stay low, reflecting the past slump in fertility rates and a continued rise in longevity and old-age dependency ratios. Fiorentini *et al.* (2018) also project r^* to fall further still (see Chart 11a), until the middle of the coming decade, reflecting pre-determined information on demographics.

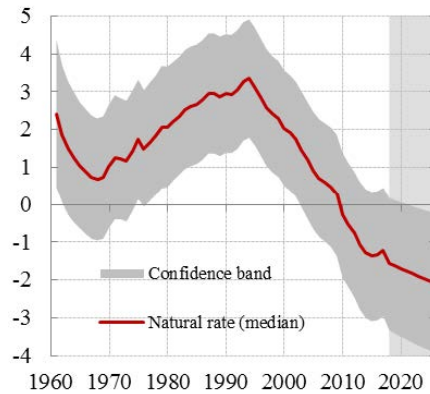
Econometric approaches provide very little policy-relevant information on future r^* . In the semi-structural approach, as adapted by Brand and Mazelis (2018), r^* corresponds to the level of real rates reached when all shocks have dissipated and monetary policy is neither accommodative nor restrictive. But it is entirely backward-looking. Since the two core components of r^* – the growth and a catch-all, non-growth, factor – are assumed to be random walks, the semi-structural model is agnostic about the future. As a result, the last in-sample estimate is the best predictor of what the terminal rate will be. The central model-consistent projection is reflected by a horizontal line at the level of the last point estimate of r^* (see Chart 11b). Still, as soon as new data become available, r^* will be revised. As we extend the projection horizon, the accumulation of shocks in the random walk components of r^* leads to staggering increases in uncertainty about the future level of the natural rate.

Chart 11

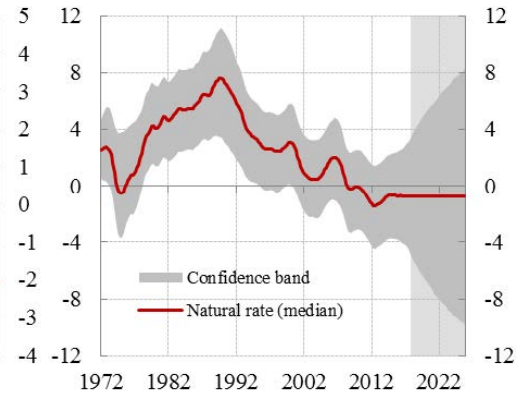
Confidence bands (90%) around euro area r^* estimates

(percentages)

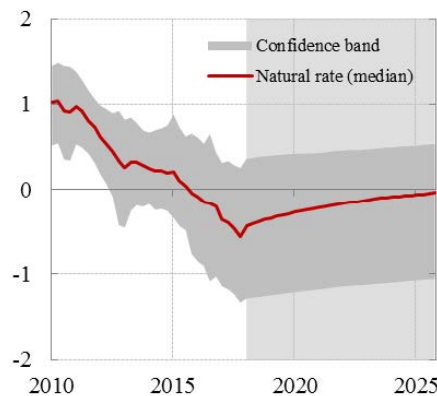
(a) Estimates by Fiorentini *et al.* (2018)



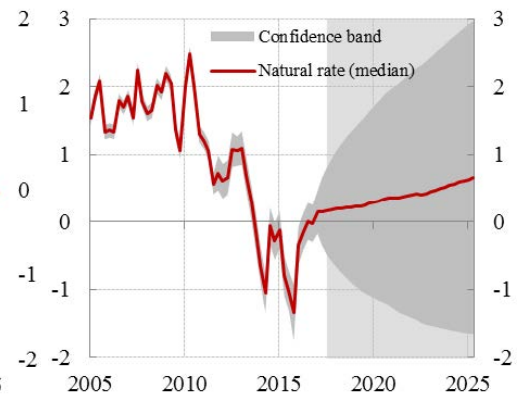
(b) Estimates by Brand and Mazelis (2018)



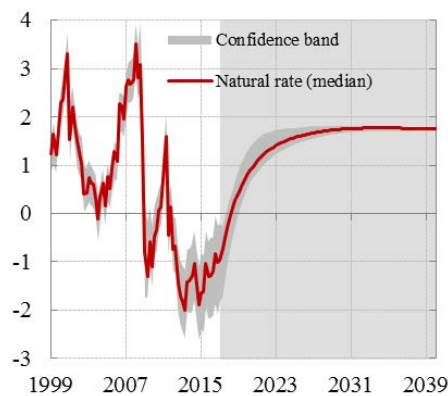
(c) Estimates by Jarocinski (2017)



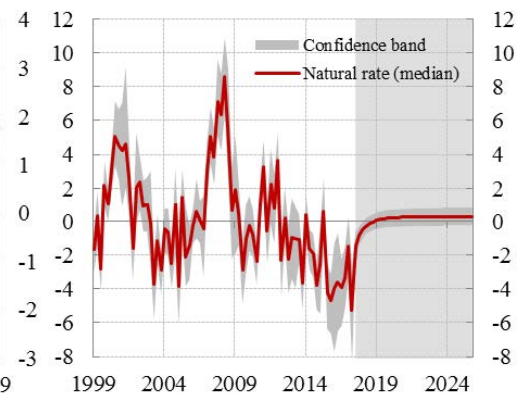
(d) Estimates by Ajevskis (2018)



(e) Estimates by Haavio *et al.* (2017)



(f) Estimates by Gerali and Neri (2017)



The time-series model by Jarocinski (2017) constructs the natural rate directly as a terminal rate. Still, this forecast-based rolling-regression method also strongly relies on past historical trends (Chart 11c).

Macro-finance models already incorporate forward-looking information from financial market participants and could therefore reflect market expectations of the terminal

rate. Since Ajevskis (2018)'s macro-finance model defines r^* as the average real short-term rate forecast over a five-years period starting five years ahead, the projection converges slowly towards the time-varying equilibrium interest rate which follows a random walk process. The forecast error bands around this scenario again expand very quickly (see Chart 11d).

By construction, in structural models the *current* and the *future* level of the natural rate of interest do not coincide at all. DSGE-based measures are strictly period-specific. Unconditional forecasts of r^* can be produced to display the adjustment of r^* towards the model's steady state, which in turn reflects historical patterns and thereby contains no forward-looking information *per se*: Charts 11e and 11f display a swift rebound towards the time-invariant steady state, reflecting the autoregressive structure of the underlying exogenous shocks.

Bearing in mind all the uncertainty, we find that mechanical projections attach a higher probability to r^* staying at levels around zero, or slightly below, in the coming years, rather than rebounding. Looking ahead, it appears as if most of the trends captured by our models will remain intact for years to come. This view certainly holds in the light of the current demographic inflection point and the productivity and growth trends emerging from it. Over the short-term a turn higher in r^* would have to come from a lower degree of risk aversion or a technology-driven boost to productivity.

3.2 The role of r^* in the conduct of monetary policy

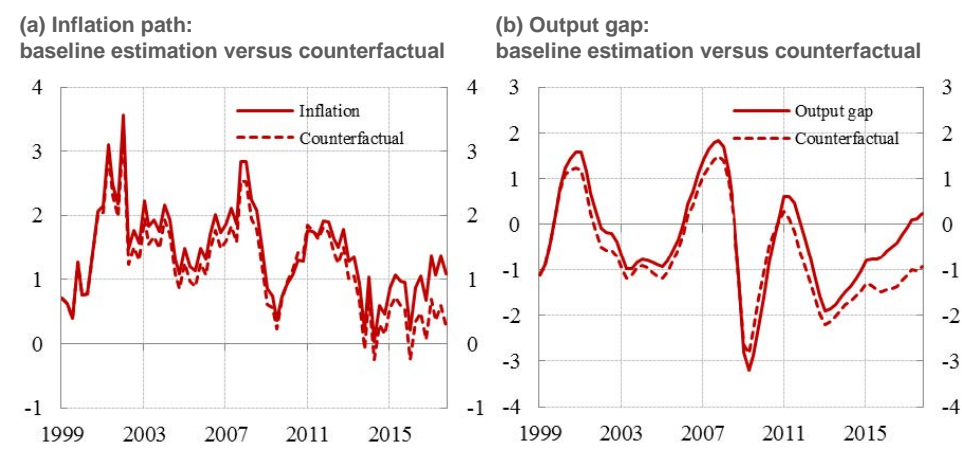
3.2.1 The perils of tracking smooth econometric measures of r^*

Econometric and especially semi-structural metrics of r^* tend to evolve smoothly. Setting the real policy interest rate equal to the semi-structural estimate of r^* will close the output gap and stabilise inflation asymptotically. But the output gap could be closed quicker and inflation stabilised faster through an active monetary policy. Passively tracking the smoothly evolving medium-term trend without responding to deviations of inflation from target or measures of slack is unlikely to anchor inflation in an effective way.

Chart 12 illustrates this point. It displays counterfactual results obtained from monetary policy exclusively tracking r^* in Brand and Mazelis (2018). In line with textbook reasoning, following such a policy incurs higher and more protracted deviations of inflation from its objective – especially since 2015. Conversely, if the same semi-structural model is estimated using a Taylor rule with a *constant intercept*, the parameters on the inflation and output gap terms of the rule would come out to be significantly more aggressive (so as to yield the same stabilising properties as observed in the data).

Chart 12

Tracking r^* in semi-structural model



We conclude, therefore, that in the semi-structural framework, tracking r^* only without responding to the inflation and output gaps has inferior stabilising properties. Consequently, monetary policy needs to respond to output and, in particular, inflation gaps in addition to taking variations in the natural rate into account, to be sufficiently stabilising.

3.2.2 The volatility of DSGE-based measures: vice or virtue?

A striking feature of the DSGE-based r^* estimates shown in Charts 11e and 11f, compared to others displayed in Chart 11, is their high volatility. While semi-structural models base their measure of r^* on a statistical definition of *potential* output, which evolves gradually over time, DSGE models produce estimates of r^* based on their measures of *natural* output, which change in response to disturbances at the business cycle frequency. Therefore they are more volatile than their counterparts obtained from semi-structural models.

This volatility is often taken as a cause for concern: first, policy makers may have a prior that r^* should be quite smooth and treat cyclical variations in r^* as if they are errors; and second, they might be uncomfortable with (and sceptical of) estimates with a relatively low signal-to-noise ratio. However, as explained in subsection 2.3, setting the real policy rate equal to r^* closes the output gap and *contemporaneously* restores the flexible-price equilibrium.

Chart 13 reproduces the DSGE estimates of the euro-area natural interest rate by Gerali and Neri (2017) and Haavio *et al.* (2017), this time together with their corresponding natural rate gaps. The measure of r^* interacts with the output gap, as depicted in Charts 14c and 15c (blue lines, denoted “baseline”). As Gerali and Neri (2017) allow the underlying productivity growth trend to shift over time, it should come as no surprise that this output-gap measure is more concentrated around zero than the measure by Haavio *et al.* (2017), which identifies a persistently negative output

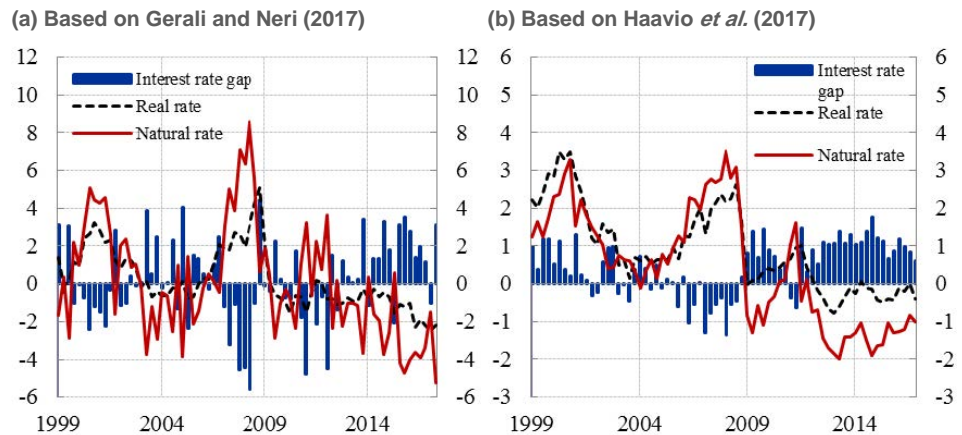
gap as a consequence of the recessions in the euro area. Accordingly the interest rate gap in Haavio *et al.* (2017) also exhibits a more persistent pattern.

It is important to understand whether closing the interest rate gap (by tracking DSGE-based r^* at all times) would yield outcomes that are consistent with policy objectives. If the economy is not subject to significant nominal rigidities nor affected by mark-up shocks, then closing the output gap simultaneously stabilises inflation at its target. Otherwise, there is a trade-off between stabilising inflation and stabilising output, and a single policy instrument is unable to achieve both goals. Consequently, tracking DSGE-based r^* does not constitute optimal policy anymore, as it disregards the deviations of inflation from target, while an optimal policy would strike a balance between stabilizing both gaps.

Chart 13

Euro area natural rate gaps based on structural models

(percentages)



It is an empirical question, whether nominal rigidities and mark-up shocks give rise to output-inflation stabilisation trade-offs. Justiniano *et al.* (2013, on a basis of a DGSE model for the US) argue that the tradeoff between stabilization of output and inflation is negligible. Barsky *et al.* (2014) claim that the US Fed would have “accomplished a substantial degree of macroeconomic stability” had they set the real Federal Funds rate equal to the natural rate derived from their DSGE model. Curdia (2015) goes further and claims that the US Federal Reserve has actually tracked the natural rate of interest fairly closely. These claims may neither hold true at all times in the US nor be valid for the euro area where adjustment frictions are commonly judged to be more important.

We illustrate the stabilising properties of tracking r^* in the models by Gerali and Neri (2017) and Haavio *et al.* (2017) in Charts 14 and 15.²⁰ In both models, tracking r^* would have closed the output gap. However, they differ in terms of inflation outcomes.

²⁰ For simplicity and for the purpose of exposition, the lower bound on nominal interest rates is not assumed to be binding. The counterfactual Taylor rule used in Haavio *et al.* (2017) is $i_t = r_t^* + 1.01 \cdot E_t \pi_{t+1}$ and in Gerali and Neri (2017) $i_t = r_t^* + E_t \pi_{t+1} + 0.1(\pi_t - \bar{\pi})$ with $\bar{\pi}$ denoting the steady state inflation rate.

While Gerali and Neri (2017) suggest that the inflation rate would have been very close to 2%, this is not the case for Haavio *et al.* (2017). This difference is due to nominal frictions and mark-up shocks playing a negligible role in Gerali and Neri (2017) compared to Haavio *et al.* (2017). As a consequence, shocks involve a much smaller trade-off between inflation and output stabilisation in Gerali and Neri (2017). In Haavio *et al.* (2017), tracking r^* stabilises the output gap, but due to the importance of wage stickiness and real labour market rigidities employment can deviate persistently from its natural counterpart, causing large and protracted deviations of inflation from target and giving rise to a significant trade-off between inflation and output stabilisation.

Even if tracking r^* was desirable, policy makers might not consider it a feasible approach. Within the model, the level of the natural rate of interest is easily defined and observable, but requires perfect information on the nature and size of incoming shocks. Policy makers will therefore likely prefer to eschew model-specific considerations about time-variation in r^* in the light of incoming shocks. Yet ignoring variations in the natural rate would lead to inferior policy outcomes.

Chart 14

Performance of monetary policy if tracking r^* in Gerali and Neri (2017)

(percentages)

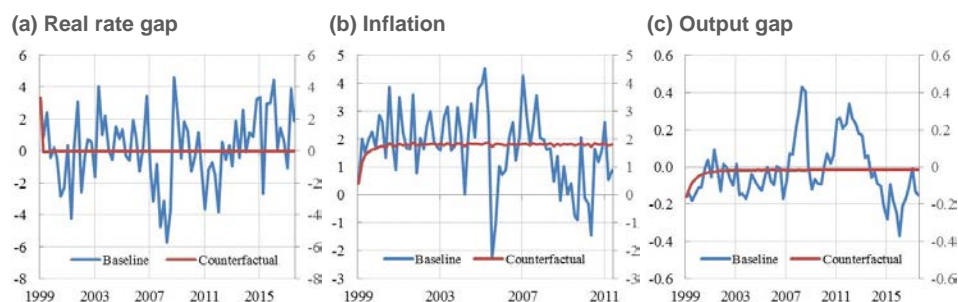
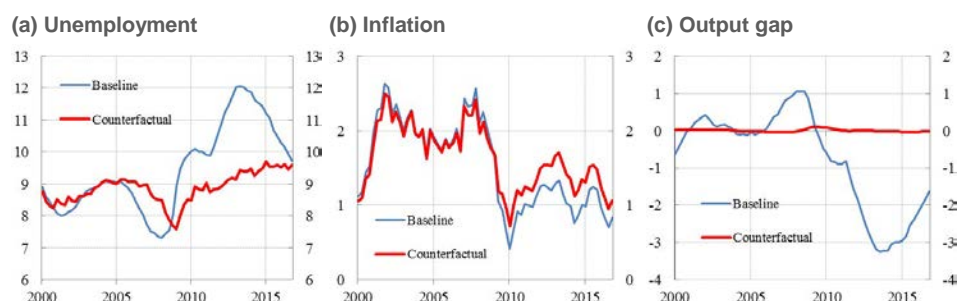


Chart 15

Performance of monetary policy if tracking r^* in Haavio *et al.* (2017)

(percentages)



To illustrate the stabilising properties of a policy predicated on the assumption of r^* being constant, an alternative counterfactual analysis based on Gerali and Neri (2017)

assumes that the central bank had continued to follow a standard Taylor-type interest rate rule (with a *constant* intercept), but reacted more aggressively to deviations of inflation from target.²¹ As the Chart 16 suggests, responding aggressively to inflation yields similar stabilising properties as tracking DSGE-based r^* , while reducing the problem of knowing the true level of r^* to knowing next quarter's inflation. This finding may not be generally applicable, but underscores the result of Orphanides and Williams (2007) who show that when the uncertainty regarding the natural rate of interest and output gaps is high, inertial policy coupled with aggressive responses to inflation gap can be optimal.

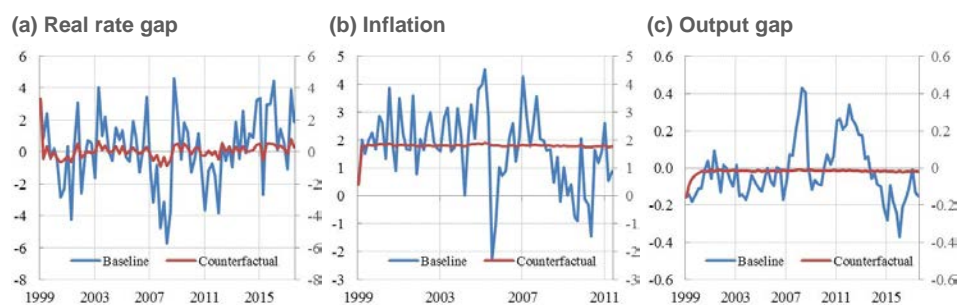
Both within and across the semi-structural and structural model classes, r^* can exhibit entirely different inflation-stabilising properties. Therefore, these estimates should neither be used interchangeably nor in isolation – and only to the extent that their stabilising properties are fully understood.

The differences in volatility or smoothness of structural or semi-structural estimates are rooted in differences in their corresponding definition of potential output. Smooth estimates build on an atheoretical notion of potential output *beyond* the business cycle. Accordingly, semi-structural measures of r^* do not respond to shocks over the business cycle and, in turn, cannot *per se* serve as an optimal guide to monetary policy at that frequency.

Chart 16

Counterfactual II: What if policy (ECB) was more aggressive? (based on Gerali and Neri, 2017)

(percentages)



Policy makers should not be left with the impression that smoothness of estimates implies certainty. Even low frequency estimates are highly uncertain and can be significantly revised in the light of new data, particularly for recent estimates.

Overall, structural, high-frequency measures of r^* convey different information than very low frequency ones. If this distinction can be drawn, then there is a case for presenting these estimates in the light of their stabilising virtues.

²¹ The counterfactual Taylor rule is now $i_t = \bar{r} + 100(\pi_t - \bar{\pi})$, with \bar{r} and $\bar{\pi}$ corresponding to the constant, model-implied steady states of the real rate and inflation.

3.2.3 Natural rate uncertainty

Estimates of r^* are uncertain no matter which approach is used (see the confidence bands as displayed in Chart 11). The width of these confidence bands differs significantly across models and reflects highly specific modelling and estimation assumptions which are not necessarily informative of the nature of r^* uncertainty that policy makers may be concerned about.²² From a policy perspective, the difference in estimates emerging across different methods is probably more relevant than model-specific statistical estimation uncertainty. These differences conceal fundamental conceptual differences with profound policy implications: The difference between smooth econometric and volatile DSGE-based measures is a case in point: Specifically, semi-structural estimates exhibit weak stabilising properties. Conversely, DSGE-based measures may exhibit stronger stabilising properties, but – somewhat heroically – require instantaneous and full information of the nature of shocks hitting the economy.

Overall, model-specific differences and statistical uncertainties of estimates pose formidable obstacles to passing a judgement on the level of the natural rate of interest in real time.

3.3 Policy challenges in a low r^* environment

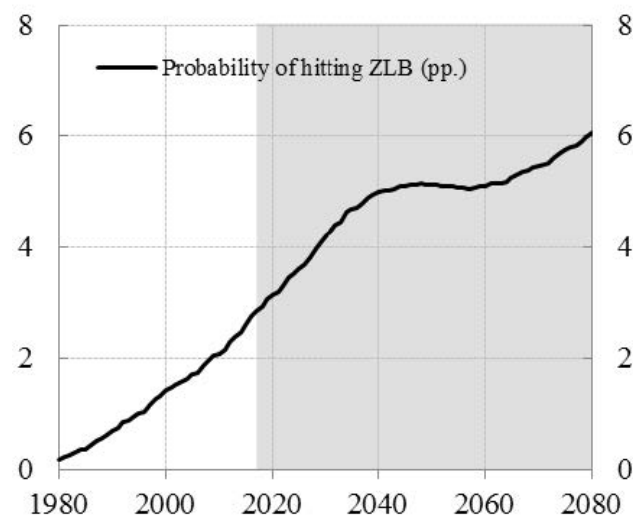
3.3.1 r^* and the risk of hitting the lower bound on nominal rates of interest

Demographic factors suggest that r^* is more likely to fall than to rise in the medium term, affecting the likelihood of the conventional interest rate instrument becoming constrained by the effective lower bound. Bielecki *et al.* (2018) show that (positing an unchanged inflation target) the risk of the nominal interest rate dropping below zero will increase from just 2% p.a. at the onset of the financial crisis to more than 4% p.a. in year 2030, and is projected to increase further (see Chart 17).

²² The massive differences in error bands arise from measuring different sources of uncertainty. The two polar cases are the small error bands by Gerali and Neri (2017), reflecting only parameter uncertainty, and the very large ones by Brand and Mazelis (2018) comprising uncertainty about parameter estimates, shocks, and states.

Chart 17

Projected influence of decrease in r^* on the probability of hitting the ZLB in Bielecki *et al.* (2018)



The lower bound constraint is at the heart of the demand-side view of the secular stagnation debate. The *ex-ante* desire to save relative to the one to invest may be so strong as to push the natural rate below the lower bound, so that equilibrium can only be regained by a fall in aggregate income, as posited by Summers (2016). Alternatively, the secular decline in yields may reflect the negative financial cycle in the aftermath of the financial crisis (Borio, 2014). The supply-side camp in the debate has pointed to the slowdown in productivity and unfavourable demographic developments (Gordon, 2017; Goodhart, Pardeshi and Pradhan, 2015). These arguments, however, are not mutually exclusive as demand side factors can be very closely linked to supply side factors. For example, a chronic weakness in demand leading to hysteresis would reduce the productive capacity of the economy (Blanchard and Summers, 1986; Blanchard, Cerutti and Summers, 2015).

3.3.2 Changes in monetary policy strategies versus changes in policy instruments

While concerns about the lower bound on nominal rates previously appeared to be a topic of mere academic curiosity, today there is an extensive debate about profound changes in monetary policy frameworks and instruments. Two broad options for safeguarding the effectiveness of monetary policy in a low r^* environment have been distinguished: changes in monetary policy strategies or use of unconventional policy instruments for instances when the effective lower bound on nominal interest rates become binding.

Proposal for raising inflation targets (advocated by Ball, 2014; Blanchard, Dell’Ariccia and Mauro, 2010; Williams, 2016; Dorich, Labelle, Lepetyuk, Mendes *et al.*, 2018) rest on the notion that – abstracting from higher costs of inflation – if credible, a

concomitant increase in inflation expectations affords central banks more leeway to engineer higher negative real rates than what otherwise would have been the case. Besides prominent suggestions to increase inflation targets, proponents have also advocated price-level or nominal GDP targeting. This latter option may be in either permanent or temporary guise (Coibion, Gorodnichenko and Wieland, 2012 and Bernanke, 2017).

Taking into consideration welfare costs from higher inflation and assuming ineffectiveness of monetary policy at the zero lower bound, using an estimated New Keynesian DSGE for the US and the euro area Andrade, Galí, Bihan and Matheron (2018) explore the welfare-optimal relationship between changes in r^* and inflation targets. Based on a second-order approximation of the household utility function, they perform counterfactual simulations using different target rates of inflation. Taking into account uncertainty about the parameters of the model, they estimate that the optimal inflation target rate (for r^* equal to 2.8% – the posterior mean of the estimate in their sample) is 2.2%. At this target level of inflation, the nominal interest rate will be constrained at the zero lower bound roughly 10% of the time. As r^* decreases, the optimal inflation target rises but less than one-for-one. For example, if r^* falls from 2.8% to 1.8%, the optimal inflation target rate rises, but by less than that, from 2.2% to 3.1%. At this target rate, the zero lower bound will be binding around 11% of the time.

It is an intriguing trait of this debate that the potency of non-standard measures introduced by major central banks to overcome the lower-bound is not discussed prominently.²³ Such measures have included credit easing and quantitative easing measures, such as asset purchases, forward guidance and negative interest rates on reserves. Negative nominal interest rates can enhance the scope for a central bank to engineer negative real interest rates, if warranted, affecting a wide range of return measures and asset prices relevant for the transmission of monetary policy – in particular if combined with other unconventional tools.²⁴

Changes in the central bank's tool-kit may, however, also be considered to become more permanent. Arguments have been advanced in favour of central banks running large balance sheets permanently (for an overview see Bernanke, 2016). Specifically, maintaining a permanent asset portfolio may create room for manoeuvre on short-term interest rates.

²³ For a detailed discussion of this debate see Jaubertie and Shimi (2016).

²⁴ For evidence on the effectiveness of unconventional policies, in particular, asset purchases see: for the euro area Altavilla, Carboni and Motto (2015); for the US Gagnon, Raskin, Remache, Sack *et al.* (2011), Chen, Cúrdia and Ferrero (2012); for the UK Kapetanios, Mumtaz, Stevens and Theodoridis (2012), Pesaran and Smith (2016); other important references include Rabanal and Quint (2017), Sheedy (2017), Rajan (2017).

3.3.3 Depressed r^* , central bank mandates, and the role of other public policies

A comprehensive appraisal of changes in inflation targets relative to unconventional policies in ensuring the effectiveness of monetary policy in a low r^* environment is beyond the scope of this report. But raising inflation targets to levels that are no longer consistent with price stability creates important welfare losses and risks to credibility from higher inflation. As also noted by Bernanke (2014), these negative effects would be incurred permanently. Permanently raising inflation targets in response to adverse economic conditions is not only difficult to reconcile with price stability mandates as stipulated in central bank law in a timeless way. It may also not be a credible or effective policy choice in a situation when, because of constraints on policy instruments, inflation expectations risk becoming unmoored towards too low levels.

These economic, reputational, and legal reservations about raising inflation targets need to be considered in the light of the experience by central banks that unconventional measures appear to have been effective. These measures have been controversial in some quarters and there has been lingering trepidation about side effects of negative interest rates and asset purchases. Yet, a prevalence of adverse side effects, e.g. of negative interest rates on bank profitability, has not been identified²⁵ and there is no broad-based evidence of banks' risk-taking in response to profitability pressures (ECB, 2018, p. 68).

Structural forces depressing r^* cannot be affected by monetary policy, but structural reforms supporting productivity growth and investment can do so. Of course, productivity growth is not solely a function of public policy and there are technological and resource constraints affecting growth. Yet structural policies can boost productivity in various ways, as highlighted in the report by Masuch *et al.* (2018).²⁶

A particularly relevant area where structural policies have long-lasting effects on r^* relates to demographics. The demographic trends described in section 1 cannot be considered independently of decisions about ages for pension eligibility, the generosity of pensions and how they are funded. Decreases in r^* due to ageing can be mitigated by changes in the retirement age and the pension system replacement rates (see e.g. Krueger and Ludwig, 2007; Sudo and Takizuka, 2018). Public policies that encourage labour force participation or human capital accumulation can boost investment rates and sustain the productivity of older age cohorts.

²⁵ Altavilla, Boucinha and Peydró (2017) illustrate that bank profitability is intrinsically linked to macroeconomic performance. Adverse effects on net interest margins possibly caused by unconventional policies are largely offset by the positive impact on intermediation activity, credit quality, and capital gains from the increase in the value of the securities held by banks.

²⁶ Masuch *et al.* (2018) illustrate how product market reforms can strengthen competition amongst firms which increases the incentives to innovate and invest in human and physical capital; institutional reforms increasing the efficient and impartial functioning of the judiciary and of public administration restrain rent-seeking activities; reforms which enhance labour mobility and reduce skill mismatches through training and education tend to support the diffusion of technology and the growth of more innovative and productive firms; finally, in the euro area completing Banking Union will not only enhance potential growth through more efficient allocation of financial resources, but also attenuate the scarcity of safe assets and thereby lift the equilibrium level of the safe rate.

Globally, the natural rate can be affected by governments' decisions to save. The massive increase in foreign exchange reserves and the desire of oil producers to transform their below ground assets into financial assets has significantly reduced r^* (see Bean, Broda, Itō Kroszner, 2015, for an extended discussion).

With a view to ensure that central banks carry out their price stability mandates in an independent way, an open debate needs to continue on how to ensure the effectiveness of policy frameworks under challenging economic and financial circumstances. The economic and financial factors depressing equilibrium real rates are a case in point: the ongoing demographic transition, flight to safety, global saving-investment imbalances, and low productivity growth all contribute to this challenge. At the same time, it needs to be recognised that these factors cannot be affected by central banks. A return to higher natural rates would have to come from pension reform (i.e. increasing retirement age in combination with measures to sustain human capital of ageing populations), a reversal in the pricing of risk, a boost in productivity, or a combination of these factors.

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Model appendix

A Demographic trends in OLG models and inequality

A.1 Bielecki *et al.* (2018)

Bielecki *et al.* (2018) calibrate a New-Keynesian OLG with investment adjustment costs, using age cohorts 20-99, and modelling the full life-cycle at annual frequency. The model is based on demographic forecast by the European Commission (EUROPOP 2013).

The first-order condition for household j affected by the time- and age-dependent mortality risk ω is:

$$1 = \beta(1 - \omega_{j,t})\mathbb{E}_t \left[\frac{c_{j,t}}{c_{j+1,t+1}} \frac{R_{t+1}^a}{\pi_{t+1}} \right]$$

The expected rate of return on asset portfolio equals the expected rate of return on capital:

$$\mathbb{E}_t[R_{t+1}^a] = \mathbb{E}_t \left[\frac{R_{t+1}^k + (1 - \delta)Q_{t+1}}{Q_t} \right]$$

Changes in fertility affect population growth rate n which in turn affects per capita capital accumulation, which matters for the determination of the return on capital.

Data sources for Chart 2

Data for Old-age dependency ratio:

- 1982-2017: Eurostat, Population: Structure indicators [demo_pjanind], Old dependency ratio 1st variant (population 65 and over to population 15 to 64 years), euro area (18 countries), European Union (before the accession of Croatia) and European Union (current composition)
- 2018-2080: Eurostat, EUROPOP15, Baseline projections: demographic balances and indicators [proj_15ndbims], Old dependency ratio 1st variant (population 65 and over to population 15 to 64 years), European Union (current composition)

Data for Life expectancy at birth:

- 1960-2001: World Bank, World Development Indicators, Life expectancy at birth (years), euro area
- 2002-2015: Eurostat, Life expectancy by age and sex [demo_mlexpec], Age: Less than 1 year, euro area (19 countries)

- 2016-2080: Eurostat, EUROPOP15, Life expectancy by age and sex [proj_15nalexp], Age: Less than 1 year, constructed for euro area (19 countries) by weighting with Population on 1st January by age, sex and type of projection [proj_15npms], Age: Less than 1 year, Projection: Baseline projections

Data for Fertility rates:

- 1960-1999: World Bank, World Development Indicators, Fertility rate, total (births per woman), euro area
- 2000-2015: Eurostat, Fertility rates by age [demo frate], Total, euro area (19 countries)
- 2016-2080: Eurostat, EUROPOP15, Age specific fertility rates [proj_15naasfr], constructed for euro area (19 countries) by averaging

Time series for Life expectancy at birth and Fertility rates were subsequently smoothed with HP-filter with $\lambda = 6.25$

A.2 Papetti (2018)

Papetti (2018)'s OLG model is approximated with an aggregate representation (cf. Jones, 2018) and embodies perfect foresight transition with exogenous demographic change for EA12 (medium variant projections by UN (2017)) as unique driver. For age cohorts 15-100 the full life cycle at annual frequency is captured, featuring exogenous labour participation (ages 15-64) with age-varying productivity. r^* is the gross real return on savings that (by no arbitrage and no frictions) equals the gross net real return on capital that in the model, where t corresponds to 1 year, can be found to be:

$$1 + r_{t+1}^k - \delta = \frac{(L_{t+1}^g)^\sigma}{\underbrace{\beta \zeta_{t+1}^g (N_{t+1}^g)^\sigma}_{\text{exogenous}}} \left[\frac{\tilde{Y}_{t+1}(1 - \tilde{s}_{t+1})}{\tilde{Y}_t(1 - \tilde{s}_t)} \right]^\sigma$$

where $\tilde{Y}_t \equiv Y_t/L_t = \left[\psi (\tilde{K}_{t-1})^{\frac{\rho-1}{\rho}} + (1 - \psi) \right]^{\frac{\rho}{\rho-1}}$ denotes aggregate output per unit of labor efficiency $L_t = \sum_{j=15}^{64} h_j N_{j,t}$; h_j age-varying labor productivity; $N_{j,t}$ number of people of age j in year t ; $\tilde{K}_{t-1} \equiv K_{t-1}/L_t$ capital per unit of labor efficiency; $\tilde{s}_{t+1} = (\tilde{K}_t L_{t+1}^g - (1 - \delta)\tilde{K}_{t-1})/\tilde{Y}_t$ saving rate; $L_{t+1}^g = L_{t+1}/L_t$ growth of labor force in efficiency units; $N_{t+1}^g = N_{t+1}/N_t$ growth of population $N_t = \sum_{j=15}^{100} N_{t,j}$; $\zeta_{t+1}^g = \zeta_{t+1}/\zeta_t$ growth of (average across ages in each year) unconditional survival probability $\zeta_t = \sum_{j=15}^{100} \pi_{t,j} (N_{t,j}/N_t)$, $\pi_{t,j}$ such that $N_{t+j,j} = \pi_{t+j,j} N_{t,0}$; β representative consumer's discount factor; σ inverse intertemporal elasticity of substitution; ψ bias towards capital in the production function; δ depreciation rate of capital. In the baseline calibration: $\delta = .0906, \beta = .9822, \sigma = 2.5, \psi = .3, \rho = .5$.

A.3 Rannenberg (2018)

In Rannenberg (2018), rich households (indexed with S) derive utility from consumption $C_{S,t}$, and their stocks of safe real financial assets $b_{S,t}$ (consisting of bank deposits and government bonds), physical capital and housing, as well as disutility from supplying labor. The assumption that the rich derive utility from physical capital and real financial assets on top of housing and consumption is referred to as preferences over wealth. It is rationalised by appealing to a so-called “capitalist spirit” which says that the rich derive utility from accumulating wealth in various forms due to the sense of prestige and power it provides. Several authors have argued that “capitalist spirit” preferences over wealth are necessary to explain the saving behaviour of rich households in US data, specifically their wealth to income ratio and their positive marginal propensity to save out of an increase in their permanent income (see Kumhof *et al.*, 2015; Francis, 2009; Dynan *et al.*, 2004; Carroll, 2000).

Rich households’ first order condition with respect to real financial assets is given by

$$C_{S,t}^{-\sigma_S} = \beta \frac{R_t}{\Pi_{t+1}} E_t C_{S,t+1}^{-\sigma_S} + \phi_B (b_{S,t})^{-\sigma_B} \quad (9)$$

where Π_t and R_t denote inflation and the nominal interest rate. For $\phi_B > 0$, the household has preferences over wealth. To understand the effect of these preferences on saving behavior, note that for $\phi_B \geq 0$,

$$R_t \leq \frac{1}{E_t \left\{ \frac{\beta \Lambda_{S,t+1}}{\Lambda_{S,t} \Pi_{t+1}} \right\}} \equiv DIS_t \quad (10)$$

i.e. the households’ individual discount rate DIS_t , which the household applies to future nominal income streams, may be smaller than the nominal interest rate R_t . The reason why (10) can be an equilibrium is that for $\phi_B > 0$ the household derives benefits from saving over and above the future consumption opportunity saving entails.

Linearising equation (9) yields

$$\hat{C}_{S,t} = -\theta \frac{1}{\sigma_S} [\hat{R}_t - E_t \hat{\Pi}_{t+1}] + \theta E_t \hat{C}_{S,t+1} + (1 - \theta) \frac{1}{\sigma_S} \sigma_B \hat{b}_{S,t} \quad (11)$$

where a hat on top of a variable denotes the percentage deviation of that variable from the non-stochastic steady state, with $\theta = \beta \frac{R}{\Pi}$, i.e. the product of the steady-state household discount factor and the real interest rate. For $\phi_B > 0$, we have $\theta < 1$.

Iterating (11) until period $t + n$ yields

$$\hat{C}_{S,t} = E_t \left\{ \sum_{i=0}^n \theta^i \frac{1}{\sigma_S} [-\theta (\hat{R}_{t+i} - \hat{\Pi}_{t+1+i}) + (1 - \theta) \sigma_B \hat{b}_{S,t+i}] \right\} + \theta^{n+1} E_t \hat{C}_{S,t+n+1} \quad (12)$$

θ may be interpreted as the equilibrium weight the household attaches to period $t + 1$ consumption, i.e. the net effect of utility discounting and the (steady state) market real interest rate.

Now consider an increase in a household's permanent income, and assume for simplicity that the household saves only in the form of financial assets. In the absence of preferences over wealth ($\theta = 1 \Leftrightarrow \phi_B = 0$), an increase in the households permanent income -and thus consumption at some point in the future- will simply have a one-for-one effect on current consumption. By contrast, with $\theta < 1$, the higher future consumption does not have a one-for-one effect on current consumption. The anticipated decline in the future marginal utility of consumption simply matters less for current consumption. Indeed, it can be shown that with linear preferences over wealth ($\theta < 1$ and $\sigma_B = 0$), the household's MPS out of permanent income changes will equal one. For $\sigma_B > 0$, $0 < MPS < 1$, as the accumulation of save assets $\hat{b}_{S,t}$ lowers the marginal utility of saving relative to consumption. Allowing the household to save in the form of other asset classes does not change this result as long as the household has preferences over these asset classes as well.

At the aggregate level, a redistribution of income from other sectors to rich households will thus tend to lower the interest rate, unless another sector in the economy increases its borrowing sufficiently.

B Econometrics approaches

B.1 BVAR estimation of Jarocinski (2017)

Jarocinski (2017) uses a Bayesian VAR with a steady-state prior estimated with rolling regression windows to trace the time-varying steady state for real rates. The VAR includes the real GDP growth, GDP deflator inflation and several interest rates: overnight (EONIA), 3-month Euribor, 1-year Euribor, lending rate to NFCs and the 10-year government bond yield.

The real natural rate is approximated as the five-year ahead forecast of the real interest rate. The rationale behind this approach is that, in the long run, shocks and rigidities disappear, so the forecast converges to the natural rate.

Estimation is performed using a mean-adjusted BVAR (Villani, 2009) to forecast inflation and the nominal interest. The VAR is written in deviation of the *steady state* Ψ :

$$Y_t - \Psi = \sum_{k=1}^p \Phi_k (Y_{t-k} - \Psi) + u_t, \quad u_t \sim N(0, \Sigma)$$

with Minnesota prior for $\Phi_k: \Phi \sim N(\Phi_0, \Omega_0)$, a Normal diffuse (non-informative) prior for $\Sigma: \Sigma \propto |\Sigma|^{-(N+1)/2}$ and a steady state prior for $\Psi: \Psi \sim N(\Psi_0, \Lambda_0)$.

B.2 Variants of Laubach and Williams (2003)

B.2.1 Brand and Mazelis (2018)

Brand and Mazelis (2018) estimate the natural rate of interest for the US and the euro area in a closed, semi-structural model comprising a Taylor rule with nominal interest rate smoothing:

$$i_t = c_i i_{t-1} + (1 - c_i)(r_t^* + \pi_t^* + c_p(\pi_t - \pi_t^*) + c_y \tilde{y}_t + \epsilon_t^i)$$

with the nominal interest rate i_t , the natural rate r_t^* , observed inflation π_t , the inflation objective π_t^* , and the natural output gap \tilde{y}_t (defined as observed output \tilde{y}_t less potential output \tilde{y}_t^*). Interest rate smoothing c_i , Taylor coefficients p_π and p_y are parameters. The stochastic term ϵ_t^i captures transitory deviations from the rule.

The Phillips curve is specified in non-accelerationist form:

$$\pi_t = b_\pi \pi_{t-1} + b_y \tilde{y}_{t-1} + \epsilon_t^\pi \text{ with } 0 < b_\pi < 1$$

with a stochastic term ϵ_t^π and parameters b_π and b_y .

The nominal rate of interest and inflation are related to the observed real rate of interest r_t via the Fisher relation:

$$r_t = i_t - E_t\{\pi_{t+1}\}$$

Whereby $E_t\{\pi_{t+1}\}$ is approximated by a lagged averaging term, as in Laubach and Williams (2003).

The IS curve is approximated by the process

$$\tilde{y}_t = a_{y,1} \tilde{y}_{t-1} + a_{y,2} \tilde{y}_{t-2} + \frac{a_r}{2} (\tilde{r}_{t-1} + \tilde{r}_{t-2}) + \epsilon_t^{\tilde{y}}$$

with the real rate gap defined as $\tilde{r}_t \equiv r_t - r_t^*$. Potential output follows a random walk:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_t^{y^*}$$

where $\epsilon_t^{y^*}$ captures permanent shocks to the level of potential output, while the stochastic drift

$$g_t = g_{t-1} + \epsilon_t^g$$

features a permanent innovation to the period-on-period growth rate of potential output ϵ_t^g .

The law of motion for the natural rate is then given by

$$r_t^* = 4g_t + z_t$$

if we use quarterly data and interest rates are annualised. In addition to the trend growth rate of the natural rate g_t , any other non-growth determinants are captured by z_t , which follows a random walk:

$$z_t = z_{t-1} + \epsilon_t^z$$

Brand and Mazelis (2018) estimate the model both for the US and the euro area (EA), in case of the euro area with data from 1971Q1 to 2017Q3 from the ECB's Area-Wide Model, including real GDP, the 3-month short-term nominal interest rate, and the headline consumer price inflation (in terms of changes of the CPI or HICP index on a year earlier).

The introduction of a Taylor rule necessitates a corresponding time series for the inflation objective π_t^* . In the absence of an official objective, proxies are constructed: For the EA in the 1970s, average inflation is assumed to be an inflation objective. Towards the late 1970s, inflation decelerated. This coincides with the time of the negotiations to establish the European Monetary System (EMS), which went into effect in 1979. Inflation stabilised at a lower level in the late 1980s already. The European Central Bank announced a quantitative definition of price stability to be consistent with consumer price inflation below 2% in 1998 and clarified in 2003 that, within this definition, it aims at inflation rates close to 2%. An inflation objective is constructed that is divided into these three historical episodes.

Brand and Mazelis (2018) use the MH algorithm as implemented the RISE toolbox to estimate the model, relying largely on flat priors, with the exception of the prior on the variance σ_g^2 . σ_g^2 requires a tight prior to eschew procyclicality in the g -component and as data are insufficiently informative of it. This specific prior draws on the corresponding estimates obtained from a univariate unobserved component model for output. Rejection sampling is used to control for the signs of b_y and b_π and to ensure stability in the IS and Phillips-curve equation.

B.2.2 Krustev (2018)

Krustev (2018) estimates a version for the US, augmenting the model with a financial cycle, and the labour market featuring a non-accelerationist Phillips curve. Importantly, the model is estimated using a stepwise Maximum Likelihood approach. The model links the neutral rate r_t^* – to *sustainable growth*, i.e. trend growth adjusted for financial imbalances and allows for *financial headwinds* in the form of an endogenous credit gap to affect r_t^* in the short-run.

Formally, the natural rate is given by

$$r_t^* = c g_t^{in} + z_t = \underbrace{c g_t + z_t}_{long-run} + \underbrace{c \gamma \Delta \tilde{c}_t}_{headwinds}$$

B.2.3 Kupkovic (2017)

Kupkovic (2017) uses a version for Slovakia that is slightly modified on the basis of Berger and Kempa (2014)'s model to account for the real effective exchange rate q_t convergence prior to the adoption of the Euro in 2009. The models allows the equilibrium real effective exchange rate q_t^* to grow over time by μ_t :

$$\begin{aligned} q_t^* &= q_{t-1}^* + \mu_{t-1} + \varepsilon_{t+1}^{q^*} \\ \mu_t &= \mu_{t-1} + \varepsilon_{t+1}^\mu \end{aligned} \tag{13}$$

GDP data have been obtained from the statistical office of the Slovak Republic for GDP; ECB, NBS data are used for 3 months interest rates EURIBOR (BRIBOR until 2008); EC data for the real effective exchange rate deflated by HICP; NBS Macro-database for core HICP inflation.

B.2.4 Geiger and Schupp (2017)

Geiger and Schupp (2017) estimate the specification by Laubach and Williams (2003) for Germany using stepwise ML-estimation.

B.2.5 Pedersen (2015)

Pedersen (2015) estimates an open economy version of Laubach and Williams (2003) for Denmark. The model is summarised by the following equations:

$$\begin{aligned} y_t &= y_t^* + \tilde{y}_t \\ r_t &= r_t^* + \tilde{r}_t \\ q_t &= q_t^* + \tilde{q}_t \\ y_t^* &= y_{t-1}^* + g_{t-1} + \varepsilon_{t+1}^{y^*} \\ g_t &= g_{t-1} + \varepsilon_{t+1}^g \\ r_t^* &= \gamma g_{t-1} + z_{t-1} \\ z_t &= z_{t-1} + \varepsilon_{t+1}^z \\ \tilde{y}_t &= \alpha_0^{xy} \tilde{y}_{t-1} + \alpha_1^{xy} \tilde{y}_{t-2} + \alpha_{xr}^{xy} \tilde{r}_t + \alpha_0^{xq} \tilde{q}_t + \alpha_1^{xq} \tilde{q}_{t-1} + \varepsilon_{t+1}^x \\ \pi_t &= \bar{\pi} + \beta_\pi^\pi \pi_{t-1} + \kappa_x^\pi \tilde{y}_t + \beta_q^\pi \Delta q_{t-1} + \varepsilon_{t+1}^\pi \\ q_t^* &= q_{t-1}^* + \varepsilon_{t+1}^q \\ \tilde{q}_t &= \delta_0^{xq} \tilde{q}_{t-1} + \delta_1^{xq} \tilde{q}_{t-2} + \varepsilon_{t+1}^{xq} \\ \tilde{r}_t &= c \tilde{q}_{t-1} + k_{t+1} \end{aligned}$$

$$\kappa_t = \rho_\kappa \kappa_t + \varepsilon_{t+1}^\kappa$$

The acronyms are as follows: Inflation, π_t , output, y_t , the real interest rate, r_t , the real effective exchange rate, q_t . Superscript * denotes equilibrium measures. An upward movement in q_t is equal to an appreciation of the Danish real exchange rate. The model further consists of a sequence of stochastic shocks: The trend growth rate, g_t , transitory shocks, z_t , capturing risk premia in the UIP relation, κ_t , shocks to the output gap, $\varepsilon_{t+1}^{y^*}$, shocks to the real natural exchange rate, ε_{t+1}^q , shocks to the real exchange rate gap, ε_{t+1}^{xq} . The rest are constant parameters. Data have been obtained from the statistical office of Denmark and from the Danmarks Nationalbank.

B.2.6 Bragoudakis (2018)

Bragoudakis (2018) presents an estimation of the natural or “equilibrium” real interest rate, r^* , for the Greek economy using a simplified version of Laubach and Williams (2003) methodology. Importantly, and different from other papers, this approach uses the bank lending rate, as it might be a more representative interest rate for the Greek economy as compared to 3 months Euribor. Specifically, it better captures the high country risk premia, like higher default risk, and other idiosyncratic risk of the current Greek economic crisis. In that sense, the empirical estimates provided in this note can be considered as a “natural bank lending rate”.

Bragoudakis (2018) uses the specification by Laubach and Williams (2003) and Belke and Klose (2017), featuring an accelerationist Phillips curve.

The following data are used: Real GDP, potential output, output gap, HICP, and interest rates have been collected for the Greek economy and cover the period from 1998q1 to 2017q4 owing to data availability. The data for GDP and HICP are taken from the Hellenic Statistical Authority (ELSTAT), while potential output and output gap data are based on estimations from Bank of Greece. The non-financial corporate (NFC) short term lending rate comes also from Bank of Greece database.

Finally, the model is estimated by maximum likelihood method using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm with Marquardt steps and the maximum iterations and convergence tolerance set at 10.000 and 0.001, respectively.

B.3 Multi-country model of Hledik and Vlcek (2018)

The multi-country model for the euro area is a structural model developed in Hledik and Vlcek (2018). It encompasses seven country blocks, the euro area wide block, and the rest of the world, proxied by the US economy. The euro area countries included are the largest one measured by their per-capita GDP. The set includes Austria (AT), Belgium (BE), Germany (GE), France (FR), Italy (IT), Netherlands (NL) and Spain (SP). In contrast to Fries *et al.* (2016), each national economy is modelled as a small open economy under a fixed exchange rate regime vis- a-vis its euro area peers. There are also trade links with the rest of the world, proxied by the US. The EUR/USD exchange rate is affected by both the ECB’s monetary policy and short-term

interest rates in the US. Euro-area wide measures are constructed using a weighted average of national counterparts, based on trade matrices. The model is partially calibrated (trade), but the rest of the parameters are estimated using a Bayesian approach.

Each country block consists of a Phillips curve (PC) and an output gap equation (IS). The PC is defined as

$$\pi_t^i = \alpha_0 \cdot \pi_{t+1}^i + (1 - \alpha_0) \cdot \pi_{t-1}^i + \alpha_1 \cdot (\alpha_2 \cdot \tilde{y}_t^i + (1 - \alpha_2) \cdot \tilde{q}_t^i) + \varepsilon_t^{\pi^i}$$

where π is inflation and \tilde{y} as the output gap and \tilde{q} as the country-specific effective real exchange rate are parts of the real marginal costs. The super index i stands for countries.

The IS curve, specified in the gap form, is defined as follows:

$$\tilde{y}_t^i = \beta_0 \cdot \tilde{y}_{t+1}^i + \beta_1 \cdot \tilde{y}_{t-1}^i - \beta_2 \cdot (\beta_3 \cdot \tilde{r}_{t-1}^i - (1 - \beta_3) \cdot \tilde{q}_t^i) + \beta_4 \cdot \tilde{y}_{t-1}^W + \varepsilon_t^{\tilde{y}^i}$$

where \tilde{y}_{t-1}^W is foreign demand and \tilde{r}_{t-1}^i is the country-specific real interest rate gap. It is computed as the difference between country-specific real interest rate and the natural rate of interest which is obviously also country-specific. The real interest rate is computed from the Fisher equation, using the ECB's nominal interest rate and the country's expected inflation rate. The natural rate of interest \tilde{r}_t^i for country i is given by:

$$\tilde{r}_t^i = \gamma \tilde{r}_{t-1}^i + (1 - \gamma) g^i + \varepsilon_t^{\tilde{r}^i}$$

where \tilde{q} is the real exchange rate gap, \tilde{y}^W is foreign demand gap, g is the estimated q-o-q change in potential output. The natural rates of interest follows, with some inertia, the q-o-q change in potential GDP.

In contrast to Laubach and Williams (2003) and (Fries *et al.*, 2016), the model is forward-looking and closed by a monetary policy rule. The ECB's sets the interest rate based on a forward-looking Taylor type monetary policy rule. Euro area wide measures are constructed using a weighted average of national counterparts using trade matrices:

$$\pi_t = \sum_{i=1}^7 \pi_t^i$$

Similar equations are used for the output gap. Each country trades with the rest of the world and the euro dollar exchange rate is floating.

The model is partially calibrated, using trade matrices. The rest of the parameters are estimated using Bayesian approach. Due to the structural nature of the model it is immune to Lucas critique. The model is used to identify the natural rate in a simultaneous system of equations, making the results more robust compared to other studies. The estimated country-specific real interest rate gaps (the difference between the real interest rates and the natural rates of interest) are consistent with the output gap estimates. Since the estimates also reflect the restriction arising from the Phillips curves, they are also consistent with the countries' inflation dynamics.

B.4 Local level model and Panel ECM in Fiorentini *et al.* (2018)

Fiorentini *et al.* (2018) estimate r^* using two different econometric approaches. In the first, Fiorentini *et al.* (2018) use a local level model which decomposes the observed real rate into a $I(1)$ component, labelled r^* , and an $I(0)$ component, which resembles the real rate gap. Since the natural real rate in this model is a simple random walk, conditional forecasts are simply the most recently observed value.

In the second part, Fiorentini *et al.* (2018) estimate a Panel error correction model (ECM) at annual frequency over the period 1899-2016. The unbalanced panel of advanced economies includes the following 17 countries: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States. The observed real rate serves as the dependent variable in the ECM, while some indicators about total factor productivity, demographics and risk serve as regressors.

Data on TFP growth comes from Penn World Tables (see Feenstra, Inklaar and Timmer, 2015) and de Vries and Erumban (2017); data on demographic composition come from the Human-Mortality-Database (2018). The spread between long-term and short-term interest rates is used as proxy for the term premium to measure the time-varying risk aversion of agents. Interest rates data come from the Jordà, Schularick and Taylor (2017) Macrohistory Database and from the OECD Main Economic Indicators database.

Exploiting projections on demographics, the authors are then able to forecast the real rate until 2025.

B.5 Details on Box 2: Return on capital and its determinants

The analysis in Box 2 uses a simplified version of the Caballero *et al.* (2017) framework, by adopting a Cobb-Douglas production function instead of a CES production function. The set of equations and reasoning is similar. The absence of arbitrage between physical capital and risk-free bonds requires:

$$r^{k,e} = \gamma(r^s + KRP + \delta - (1 - \delta)g^e) \quad (14)$$

where $r^{k,e}$ is the expected real rental rate of capital (expressed in units of consumption goods), r^s is the real risk free rate, KRP the capital risk premium, δ the depreciation rate, γ the price of investment goods relative to consumption goods, and g^e its expected growth rate. The capital risk premium, KRP , is the return in excess of the risk free rate that physical capital is expected to yield, while the last two terms capture the expected capital losses either due to depreciation of the physical capital, or to the decline in the relative price of investment goods over time.

The real average return to capital is defined as the ratio of real capital income to the stock of capital (expressed in unit of consumption goods), net of depreciation, with capital income being composed of rental income and profits:

$$APK = \frac{r^k K + Y \left(1 - \frac{1}{\mu}\right)}{\gamma K} - \delta$$

where μ is the average mark-up. Taking expectations and substituting $r^{k,e}$ gives:

$$APK^e = r^s + KRP + \frac{Y}{\gamma K} \left(1 - \frac{1}{\mu}\right) - (1 - \delta)g^e$$

According to this equation, an increase in the wedge between the expected return on capital and the risk free rate can be accounted for by an increase in the risk premium KRP , an increase in the average mark-up μ , or a slower rate of growth of the price of investment goods g^e . Profit maximisation requires:

$$w = \frac{MPL}{\mu}$$

where w is the real wage rate and MPL the real marginal product of labour. Further, assuming that production follows a Cobb-Douglas function with output elasticity to capital α , this implies:

$$s_l = \frac{(1 - \alpha)}{\mu}$$

where s_l is the labour share, measured as wL/Y . Since the labour share s_l can be observed, the above equation pins down an estimate of μ , which can then be substituted into the equation for average return on capital to quantify the decomposition of the wedge into its three components, namely the risk premium KRP , the contribution of profits $Y/\gamma K \left(1 - \frac{1}{\mu}\right)$, and expected capital losses due to the price of investment declining $(1 - \delta)g^e$.

B.6 Macro-finance model of Ajevskis (2018)

Ajevskis (2018) proposes a shadow rate macro-finance model of the term structure of interest rates. Yields are driven by the vector, $X_t = [X_t^{unobs}, X_t^{macro}]$ consisting of four factors: Two of which are unobservable factors, $X_t^{unobs} = [X_t^1, X_t^2]$, while the other two factors are observable and include the deviation of inflation and unemployment from their respective steady states, $X_t^{macro}[\tilde{\pi}_t, \tilde{u}_t]$. All factors follow a first-order vector autoregressive process under the real measure:

$$X_{t+1} = (1 - \Phi^{\mathbb{P}})\mu^{\mathbb{P}} + \Phi^{\mathbb{P}}X_t + \Sigma\varepsilon_{t+1}, \varepsilon_{t+1} \sim N(0, I_N)$$

where Σ is a triangular matrix. The matrix of market price of risk, Λ_t is given as an affine function of the factors

$$\Lambda_t = \lambda_0 + \lambda_1 X_t$$

The dynamics of the factors under the risk-neutral measure follow

$$X_{t+1}^1 = (1 - \Phi^{\mathbb{Q}})\mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}}X_t^1 + \Sigma\varepsilon_{t+1}, \sim N(0, I_N)$$

where $\mu^{\mathbb{Q}}$ and $\Phi^{\mathbb{Q}}$ satisfy $\mu^{\mathbb{Q}} = \mu^{\mathbb{P}} - \Sigma\lambda_0$, and $\Phi^{\mathbb{Q}} = \Phi^{\mathbb{P}} - \Sigma\lambda_1$, respectively.

The shadow rate is given by

$$s_t = X_t^1 + X_t^2 + a_\pi \tilde{\pi}_t + a_u \tilde{u}_t$$

with the observed short term interest rate being a censored version of the shadow rate $i_t = \max(s_t, i_j^{LB})$, where $i_j^{LB}, j = 1, 2, \dots, k$, are lower bounds, k is a number of lower bounds. The model framework allows for time-varying lower bounds of interest rates. The effective lower bounds are assumed to equal the ECB's deposit facility when it is negative and zero otherwise.

Observable yields are represented by

$$y_t^n = g_n(X_t, \theta, i_j^{LB}) = -\frac{1}{n} \ln \left\{ E_t^{\mathbb{Q}} \exp \left(\sum_{i=0}^{n-1} \max(s_{t+i}, i_j^{LB}) \right) \right\} + \eta_t$$

where η_t is a vector of measurement errors and θ is a vector of parameters.

In the model, X_t^1 follows a unit root process. Since equilibrium nominal interest rate equal long-run nominal short-term rate which in turn equal the persistent latent factor, it follows that $i_t^* = X_t^1$.

Expected real rates are given by $E_t r_{t+\tau} = E_t \max(s_{t+\tau}, i_j^{LB}) - E_t \pi_{t+\tau}$. The natural rate is defined as $r_t^* = \frac{1}{60} \sum_{i=61}^{120} E_t r_{t+i}$. The model is estimated using the extended Kalman filter under the assumption that all yields are observed with measurement errors for yields having the same standard deviation σ and for inflation and unemployment being equal to zero.

Over the sample, from July 2005 to July 2017, the natural rate of interest has declined from 1.70% to 0%. For the same period the 3 and 10-years OIS rates have declined by 2.63% and 2.54%, respectively. The term premium for the 3 and 10-years OIS rates have decreased by 0.44% and 0.35%, respectively. Just a minor proportion of changes in yields can be explained by changes in the term premium.

The second unobservable factor, X_2 , which is less persistent than X_1 , is responsible for the effect of the ECB's Asset Purchase Programme on the yield curve. Indeed, this factor started to decline (increase in absolute value) significantly after January 2015. In this way, the X_1 component is constructed to flatten the yield curve mostly by changing the expectations of short-term interest rate.

C DSGE models

We present in-depth analysis of developments and drivers of r^* using two DSGE models for the euro area: Gerali and Neri (2017) and Haavio *et al.* (2017).

Gerali and Neri (2017) also include risk premium shocks in the set of "real" shocks, which are a short-cut for changes in households' preference for safe assets, as well as shocks to the efficiency of investment. They find that indeed the risk premium shocks accounted for the majority of the movements in the natural rate of interest in the euro area.

Haavio *et al.* (2017) build a model with both financial frictions as in Gertler and Karadi (2011) and shocks to households' preferences for riskless assets. Interestingly, the Euler equation is also affected by the rate of technological change. Their results also indicate that the developments in the natural rate of interest were dominated by the preference for risk-free assets.

Models in this class offer a view on the short-term behaviour of the natural rate of interest – different from approaches relying on slower moving concepts. In addition, they offer a decomposition of the sources of variation in the natural rate of interest, which (when interpreted with caution) offer guidance on whether the deviation of current rates from the underlying steady state level is likely to persist in the near future. Both models presented here highlight the role of imperfections and preference shocks in the financial sector in contributing to the post-crisis decline of the short-term natural rate of interest. This result is consistent with evidence of an increase in the preferences of economic agents towards higher demand for liquid and safe assets, i.e. what Krishnamurthy and Vissing-Jorgensen (2012) and Del Negro *et al.* (2017) refer to as the convenience yield.

C.1 Gerali and Neri (2017)

Gerali and Neri (2017) estimate a medium-scale DSGE model with a rich stochastic structure. The model is a closed economy New Keynesian model à la Smets and Wouters (2003), but abstracts from nominal wage rigidity and variable capacity utilization, with both stationary and non-stationary shocks. The estimation is done using Bayesian techniques. Importantly, the model retains the Laubach and Williams (2003) decomposition $r_t^* = \gamma g_t + z_t$, but adds additional structure. In steady state r^* is constant and equals $\frac{\gamma^C}{\beta}$. However, away from the steady state, r^* is influenced by all shocks affecting consumption growth and by the risk premium (θ): $r_t^* \cong E_t(\gamma_{t+1}^C) - \theta_t$ (Euler equation for safe bond). The model features a balanced growth path for consumption that is given by: $\gamma_t^C = \gamma_t^A \gamma_t^B (\gamma_t^{AK})^{\frac{\alpha}{1-\alpha}}$.

C.2 Haavio *et al.* (2017)

The model in Haavio *et al.* (2017) is also based on Smets and Wouters (2003), but extends it with unemployment (Galí, Smets and Wouters, 2012) and financial frictions (Gertler and Karadi, 2011). The model is estimated using Bayesian techniques and data for the euro area. Notably, two return measures (the 3 month EURIBOR and the spread between risky and riskless securities) are included as observables. Importantly, the household's Euler equation is affected by the taste for riskless assets ($\varphi_{risk,t+1}$) and technological change ($\gamma_{TFP}, \varphi_{TFP,t+1}$) and formally given by

$$1 = \beta E_t \left[e^{-\gamma_{TFP} - \varphi_{TFP,t+1} + \varphi_{risk,t+1}} \frac{\lambda_t^S + 1}{\lambda_t^S} \frac{1 + R_t}{1 + \pi_{t+1}} \right]$$

Ultimately, the natural rate turns out to be mostly driven by the risk shock (taste for riskless assets) and the TFP shock.

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