Extreme spectral risk measures: an application to futures clearinghouse margin requirements

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Outline

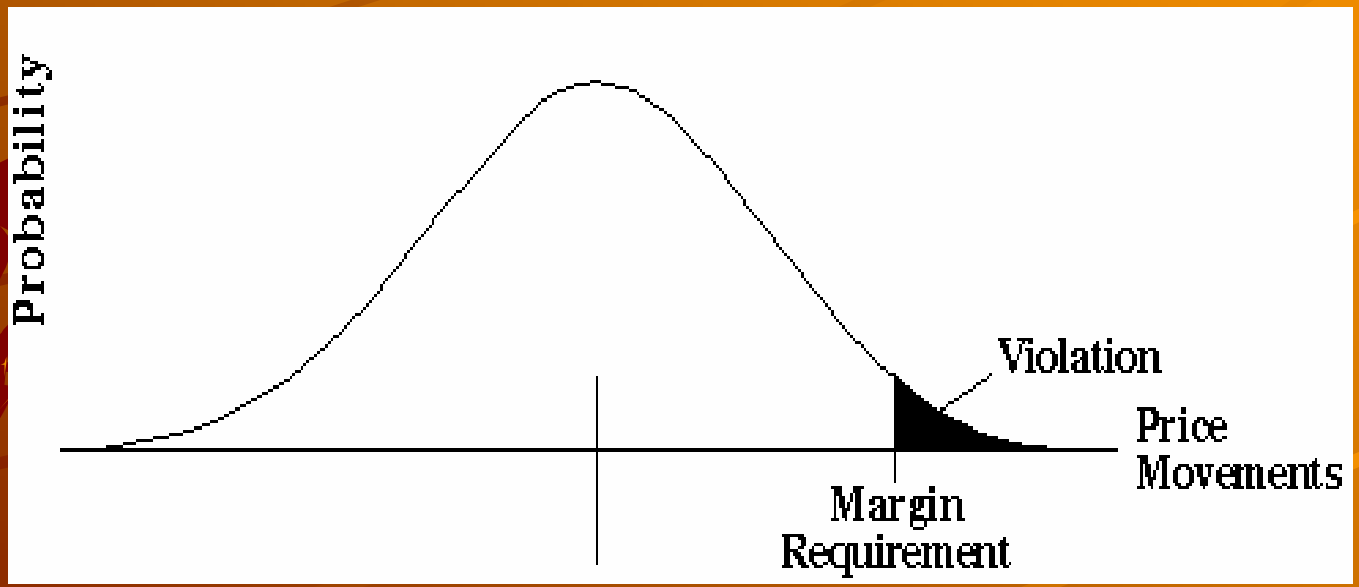
- Margin setting
- Risk measures
- Risk measures and margin setting
- Properties of SRMs
- Extreme risk and margin setting
- Data and preliminary analysis
- VaR and ES based margins
- Exponential SRMs based Margins
- Conclusions and future work

Margin setting

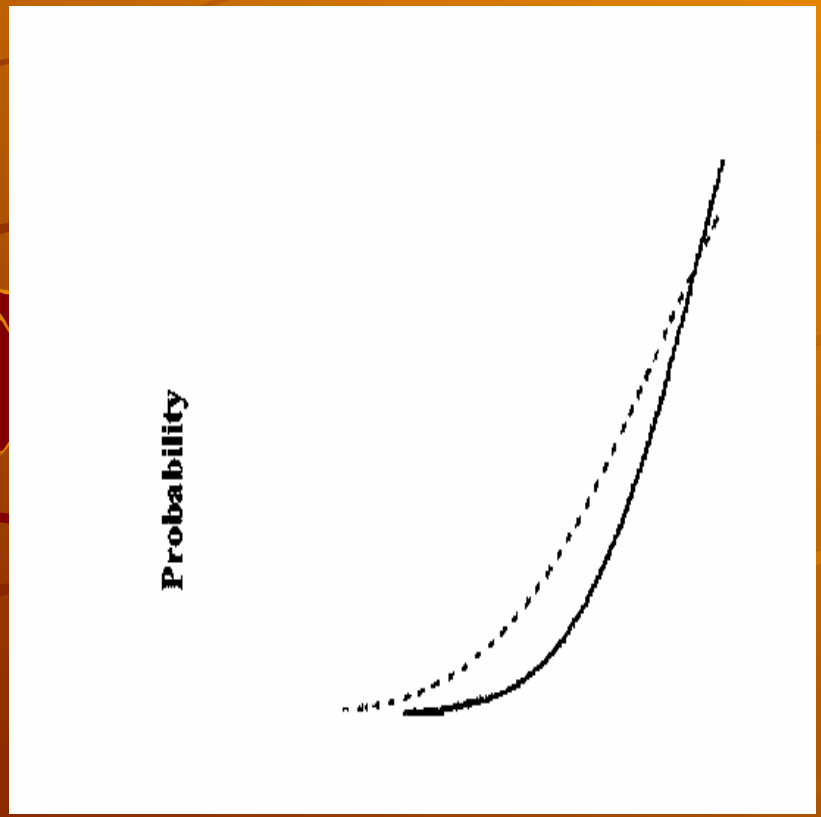
- Clearinghouses (CH) act as counterparty to trade
- CH manage counterparty risk by setting margins
- Literature has focused on statistical models for setting margins
- Examples of models employed include Extreme value theory, gaussian, historical distribution, conditional distributions etc
- Application of statistical models is to estimate VaR (probability of default/quantile or loss)
- PAPER examines properties and estimates of potential candidate measures for setting margins

Statistical models determine default probability

Margin Requirements for a Short Position and a Distribution of Price Changes



Left Tail of Fat-tailed and Normal Distribution



Risk Measures

- Value at Risk (VaR) – quantile of loss distribution

$$VaR_\alpha = q_\alpha$$

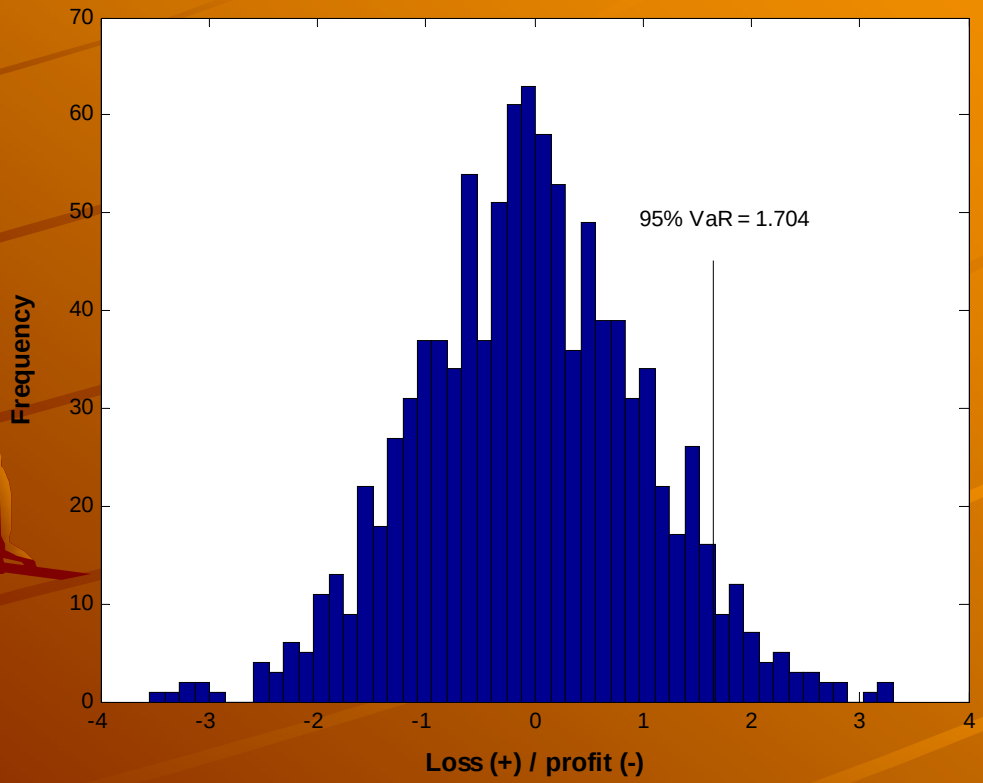
- Expected Shortfall (ES) – average of losses beyond VaR

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_p dp$$

- Spectral Risk Measures (SRM) – risk measure related to user's risk aversion function

$$M_\phi = \int_0^1 \phi(p) q_p dp$$

Histogram of profits and losses



Risk measures and margin setting

- All risk measures have key parameter – confidence levels for VaR and ES, degree of risk aversion for SRM
- Confidence levels set in arbitrary fashion but degree of risk aversion can be obtained by user of risk measure
- CH would select CARA based on their risk appetite
- Risk measures react in similar way to key parameters eg. If CARA increases SRM increases
- VaR assumes user is risk lover whereas ES assumes user is risk neutral
- SRM assumes user is risk averse

Risk measures and margin setting

- VaR able to measure default probability associated with margin
- ES able to measure default probability associated with margin
- VaR not coherent whilst ES and SRMs are coherent
- VaR not subbadditive – investor would break up margin accounts to get reduction in margin requirement
- ES tells CH of loss that they should expect conditional that VaR is exceeded

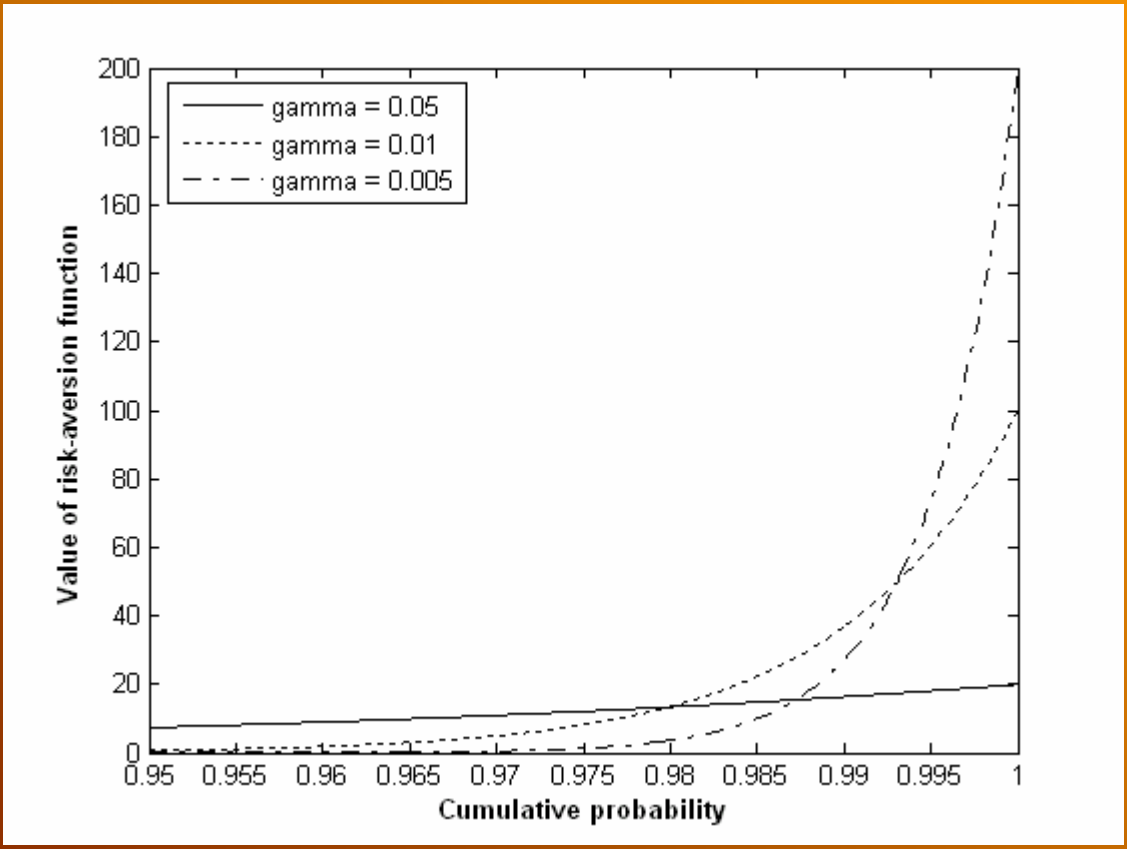
Properties of SRMs

- SRMs are coherent
- SRMs are not based on confidence interval
- SRMs are based on risk aversion function
- User of SRMs decide on their risk aversion function
- Potential risk aversion function is exponential $\phi_\gamma(p) = \frac{e^{-p/\gamma}}{\gamma(1-e^{-1/\gamma})}$
- Exponential SRM $M_\phi = \int_0^1 \phi(p) q_p dp = \int_0^1 \frac{e^{-(1-p)/\gamma}}{\gamma(1-e^{-1/\gamma})} q_p dp$

Properties of SRMs

- *Non-negativity:* weights non-negative
- *Normalization*: weights sum to 1
- *Weakly increasing*: Weights attached to higher losses at least weights attached to lower losses
- For high weights associated with high losses, expect higher risk aversion associated with these higher weights
- Weights should rise faster as *p* rises further

Figure : Exponential Risk-Aversion Functions



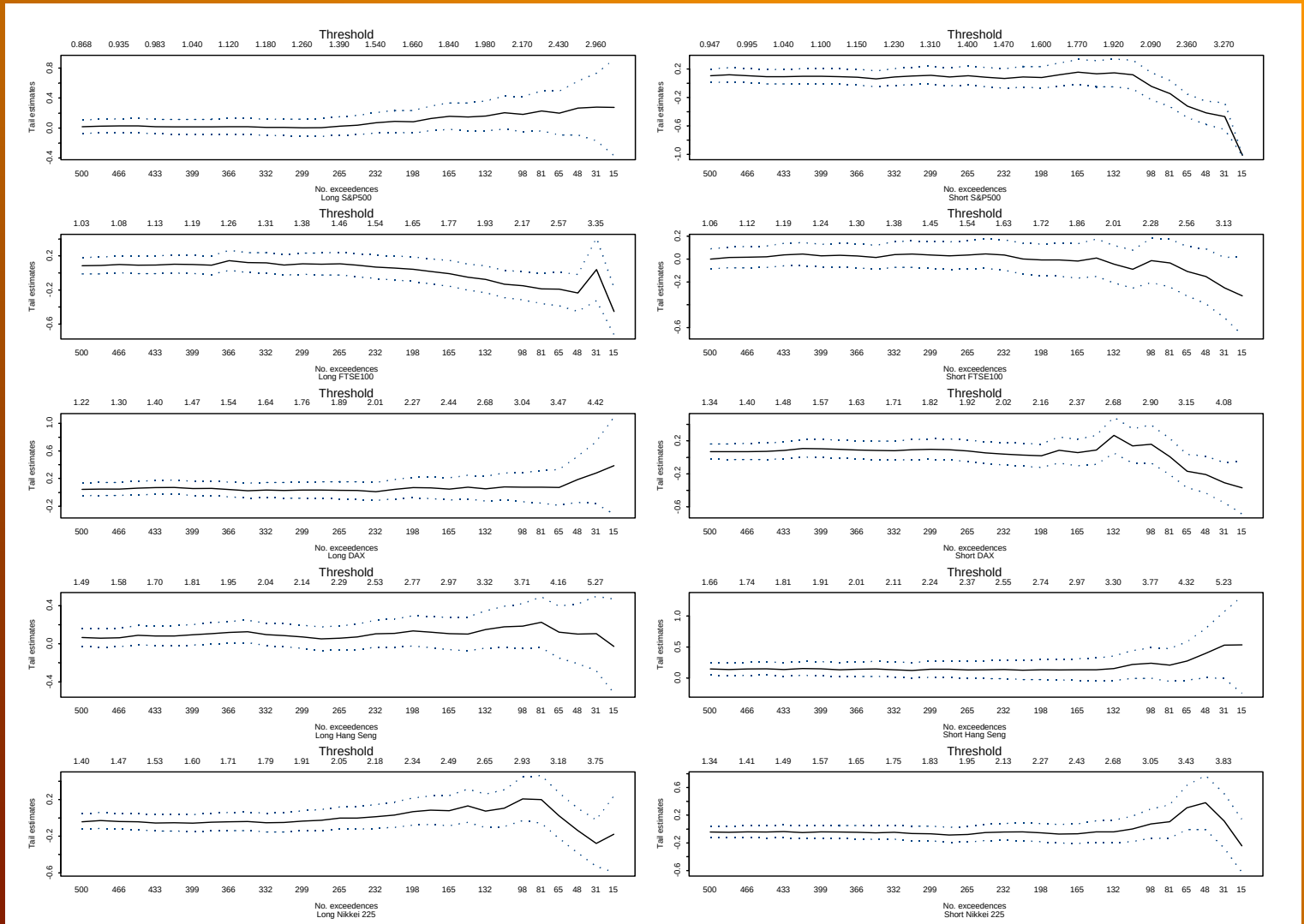
Extreme risk and margin setting

- Use Peaks over Threshold (GPD) approach
- Model realisations of random variable over high threshold
- Shape parameter indicates tail property with literature supporting fat-tailed property
- GPD parameters incorporated into VaR engine to give risk measures

Data and Preliminary Analysis

- Use heavily traded (eg. S&P500, FTSE100, DAX, Hang Seng, Nikkei 225) futures between 1/1/91 – 31/12/03)
- QQ plots indicate fat-tails
- QQ plots show tail threshold values
- Tail index plots confirm tail threshold values
- GPD tail parameters reasonable in terms of literature
- Fit of Exceedences to GPDs good

Figure : Tail Index Plots as Functions of Numbers of Exceedances



Comparison of alternative risk based margins

- Use high confidence intervals to reflect CH concern with large losses and potential defaults
- ES larger than VaRs and ES similar to SRM:

  eg. SRMs with CARA of 100 similar to ES with 0.99 confidence level and VaR with 0.995 confidence level
- Risk measures increase for increasing confidence intervals/risk aversion
- All measures are reasonably symmetric margins for long and short positions
- ES more precise than VaR and reasonably similar to SRM

Figure : Generalised Pareto VaRs of Futures Positions at Extreme Confidence Levels

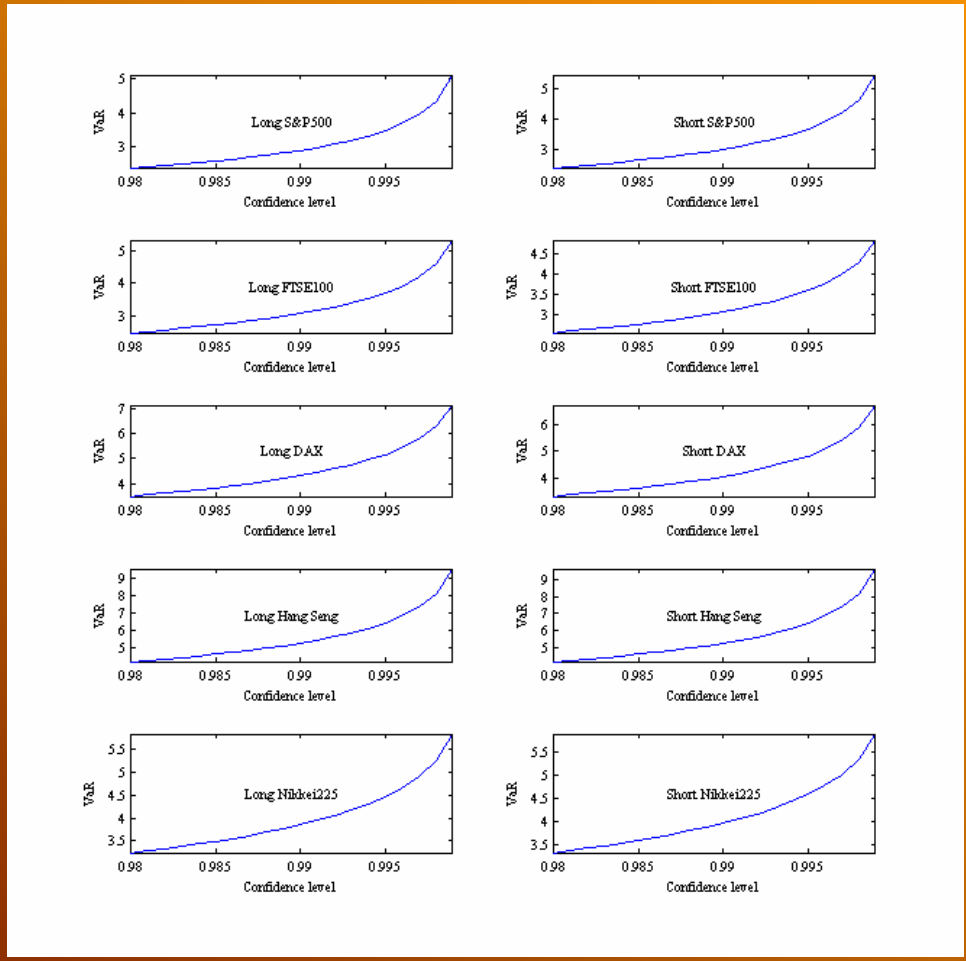


Figure : Generalised Pareto Expected Shortfalls of Futures Positions at Extreme Confidence Levels

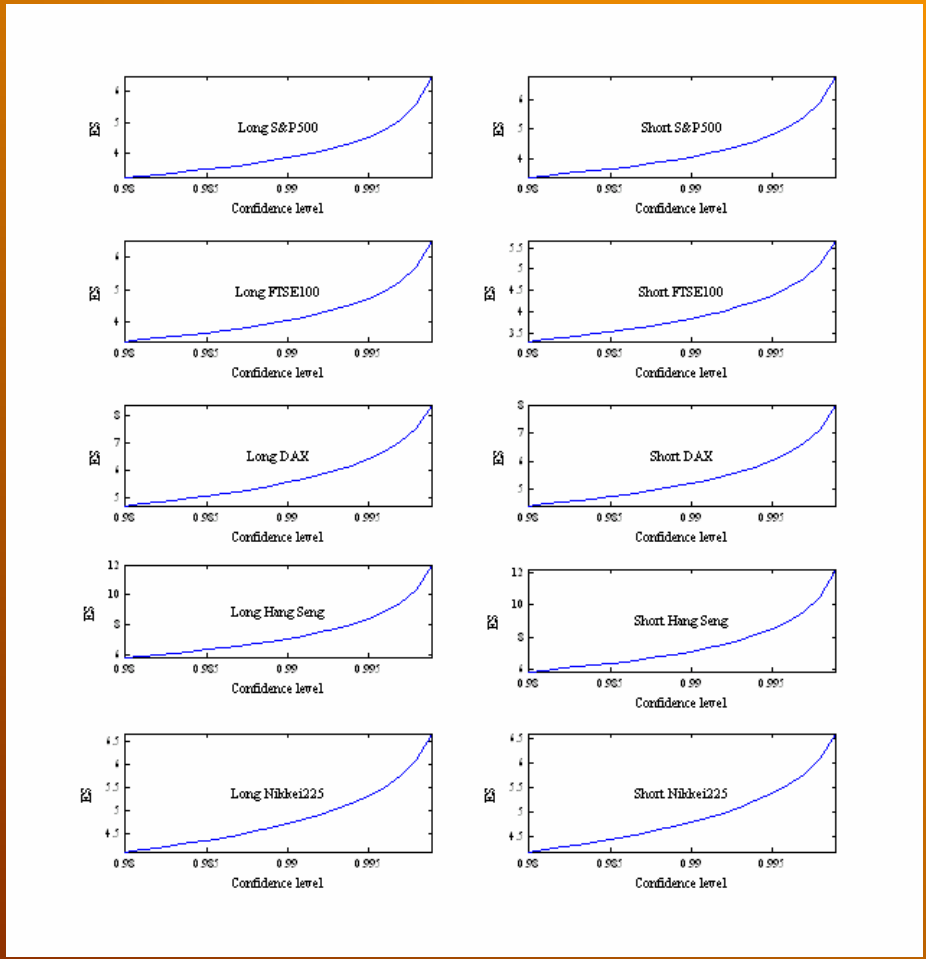
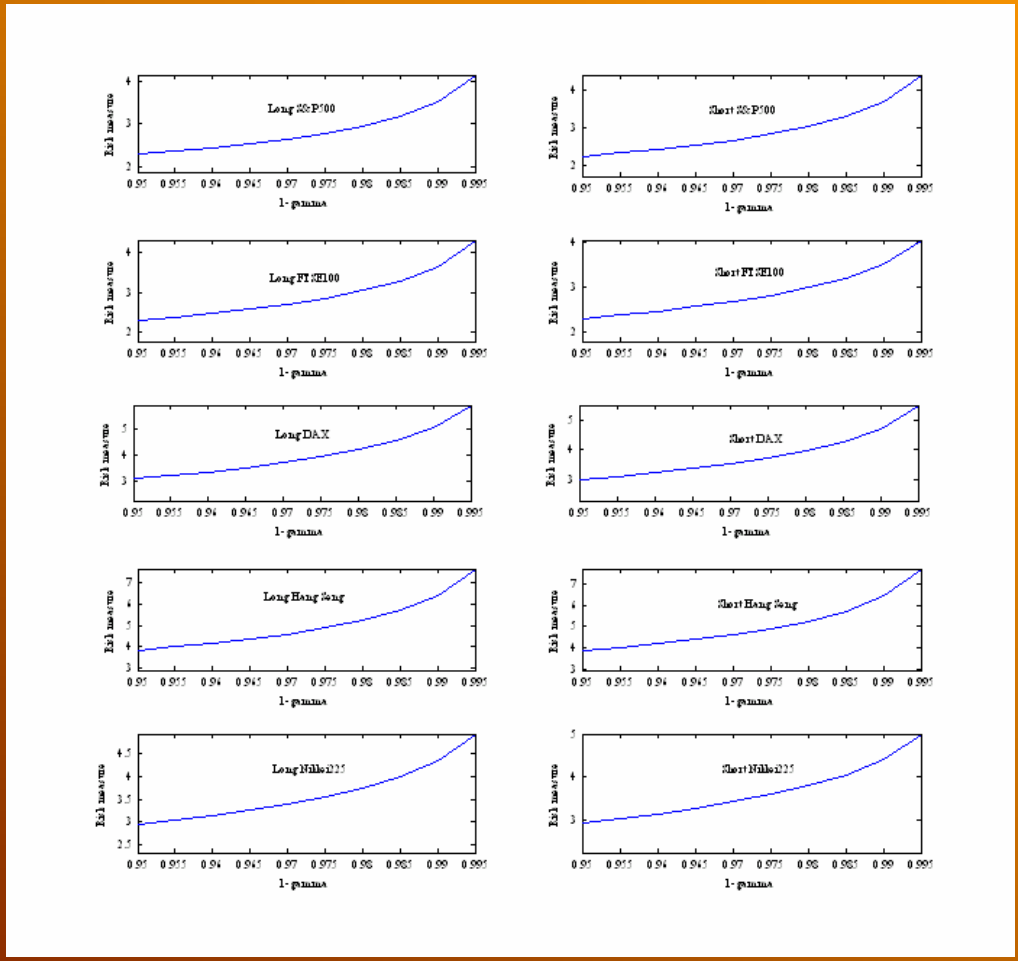


Figure : Spectral-Exponential Risk Measures of Futures Positions



Conclusions and future work

- CH impose margins to protect against extreme price movements
- Three risk measures outlined and estimated
- SRMs and ES coherent
- SRMs attractive by including user's risk aversion
- Extreme SRMs similar in magnitude and reasonably precise
- Future work will compare actual margins set by A CH with 3 risk measure estimates