Optimal Domestic (and External) Sovereign Default

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[1]The views expressed here do not necessarily reflect those of the FRB Philadelphia or The Federal Reserve System.

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5. Largely a "forgotten" story in macroeconomics literature

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- Facts in line with R-R facts: Debt ratios rose sharply together with spreads in the last 5 years. Fig. Debt/Spreads
- Caveat: Eurozone is not a fiscal union

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- Quantitative analysis via time series simulations (long-run, default events, business cycle correlations, sensitivity)
- Study model's mechanism in RME functions and perform sensitivity analysis

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- If equilibria with debt exist, do they feature dynamics in which default risk and default events are observed?
- Can the model account for key facts of debt-crisis dynamics (debt ratios, rising spreads, low default prob., foreign v. domestic debt)?

Main Findings

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Lit. Review

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Overview Model

- Introduce endogenous public debt and default in a model of heterogeneous agents, incomplete markets, and public debt with aggregate risk
- Agents face idiosyncratic income shocks y, agg. gov. exp. shocks g, and save in non-contingent, pari-passu gov. bonds with a no-borrowing constraint
- Utilitarian government pays for g, B and lump sum transfers τ with income taxes τ^y and by issuing debt B' at price q
- Public debt sold to both foreign and domestic creditors
- Study Recursive Markov Equilibrium without commitment

Environment: Households

- Unit measure of households with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad u(c_t) = c_t^{1-\sigma}/(1-\sigma)$$

where $\beta \in (0,1)$ and c_t is individual consumption.

- Agents receive income $y_t \in \mathcal{Y} = \{\underline{y}, \dots, \overline{y}\}$. Income is iid across households, and persistent with transition $\pi(y_{t+1}, y_t)$.

$$\log(y_{t+1}) = (1-\rho_y)\mu_y + \rho_y \log(y_t) + u_t, \quad |\rho_y| < 1, \ u \sim N(0, \sigma_u)$$

Households (cont.)

- If the government does not default, the budget constraint is

$$c_t + q_t b_{t+1} = y_t(1-\tau^y) + b_t + \tau_t^{d=0}$$

- If the government defaults, the market for public debt closes and re-opens next period. The budget constraint is:

$$c_t = y_t(1-\tau^y) - \phi(g_t) + \tau_t^{d=1}$$

International Investors

- Pricing of gov. bonds is simplified by introducing risk-neutral competitive investors a'la Eaton-Gersovitz
- Expected profits:

$$\Omega_t = -q_t \hat{B}_{t+1} + \frac{(1-p_t)}{(1+\bar{r})} \hat{B}_{t+1}$$

- FOC yields arbitrage of expected risky return and international risk free rate $\overline{r}$.

Government

- Gov. expenditures follow exogenous Markov process $g_t \in \mathcal{G} \equiv \{\underline{g}, \ldots, \overline{g}\}$ with transition prob. matrix $F(g_{t+1}, g_t)$, independent of income shocks.

$$\log(g_{t+1}) = (1-\rho_g)\mu_g + \rho_g \log(g_t) + e_t, \quad |\rho_g| < 1, \ e \sim N(0, \sigma_e)$$

- If $d_t = 0$, the gov. budget constraint is:

$$\tau_t^{d=0} = \tau^y Y - B_t - g_t + q_t B_{t+1}$$

- If $d_t = 1$, the gov. budget constraint is:

$$\tau_t^{d=1} = \tau^y Y - g_t$$

Timing of Actions and Participation

1. Realizations of exogenous shocks y and g are observed.

2. Individual states $\{b, y\}$, wealth distribution $\Gamma_t(b, y)$ and aggregate states $\{B, g\}$ are known.

3. Income taxes are paid. Government chooses to default or not, $d_t \in \{0, 1\}$:

   - If $d_t = 0$, debt is repaid, new debt market opens, government sets supply of debt, lump-sum transfers satisfy GBC ($\tau_t = \tau^y Y - B_t - g_t + q_t B_{t+1}$), agents and foreign investors choose bond holdings with price q_t.

   - If $d_t = 1$, debt is not paid to all creditors, output cost $\phi(g)$, debt market does not open, transfers satisfy GBC ($\tau_t = \tau^y Y - g_t$).

4. Agents consume, period t ends.

Recursive Markov Competitive Eq. (given gov. policies)

Given $\Gamma_0(b,y)$, $d(B,g)$, $B'(B,g)$, and $\tau^d(B',B,g)$, a **Recursive Markov Equilibrium** (RMCE) is a value function, households' decision rules, bond price and transition function $H^d(\Gamma,B,g,g')$ such that:

1. Given prices and policies, the value function and saving decision rule solve the households' problem ▸ HH problem
2. The foreign investor's arbitrage condition holds ▸ Inv. problem
3. The distribution evolves according to $H^{d\in\{0,1\}}(\Gamma,B,g,g')$
4. The government budget constraint is satisfied period by period
5. The asset market clears: $\hat{B}' = B^{d'} - B'$
6. The aggregate resource constraint is satisfied ▸ aggregates

Government's Default Decision

$$\max_{d\in\{0,1\}} \left\{W^{d=0}(B,g), W^{d=1}(g)\right\}$$

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- Social Welfare Functions:

$$W^{d=0}(B,g) = \int_{\mathcal{Y}\times\mathcal{B}} V^{d=0}(b,y,B,g)d\omega(b,y),$$

$$W^{d=1}(g) = \int_{\mathcal{Y}\times\mathcal{B}} V^{d=1}(y,g)d\omega(b,y).$$

- Welfare weights are given by joint cdf.:

$$\omega(b,y) = \sum_{y_i\leq y} \pi^*(y_i)\left(1 - e^{-\frac{b}{\bar{\omega}}}\right)$$

Government's Debt Decision

- The value for each household of an alternative debt level $\tilde{B}'$

$$\tilde{V}(b,y,B,g,\tilde{B}') = \max_{\{c\geq 0, b'\geq 0\}} u(c) + \beta E_{(y',g')|(y,g)}[V(b',y',\tilde{B}',g')]$$

$$\text{s.t. } c + q(\tilde{B}',g)b' = b + y(1-\tau^y) + \tau(\tilde{B}',B,g)$$

- The optimal government policy is the solution to:

$$\max_{\tilde{B}'} \int_{\mathcal{Y}\times\mathcal{B}} \tilde{V}(b,y,B,g,\tilde{B}')d\omega(b,y).$$

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- A **Recursive Markov Equilibrium with Endogenous Policies** is an RMCE for which $B'(B, g)$ and $d(B, g)$ are the optimal debt and default decision rules.

Eq. Implications I: Demand for Bonds

- Assuming differentiability, FOC with respect to b':

$$u'(c) \leq \beta E_{(y',g')|(y,g)} \left[(1 - d(B',g')) \frac{u'(c')}{q(B',g)} \right]$$

with equality if $b' > 0$

  - Larger default set reduces the expected marginal benefit of b'
  - Higher default prob. lowers b', except for high enough (b, y), who demand more bonds at higher risk premia
  - Even if $d' = 0$, marginal benefit affected by future default risk (reduces bond demand for most (b, y))

Eq. Implications II: Public Debt for Liquidity

- Using $\tilde{b} = (b - B)$, agent's and gov. budget constraint imply:

$$\begin{aligned} c &= y + \tilde{b} - q(B', g)\tilde{b}' - \tau^y(y - Y) - g \\ \tilde{b}' &\geq -B' \end{aligned}$$

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- Debt redistributes resources
  - Repaying B favors the wealthy (agents with $\tilde{b} > 0$)
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- Income tax insures against idiosyncratic shocks

Eq. Implications III: Default Incentives

- Consumption differences in repayment v. default states:

$$\Delta c \equiv c^{d=0} - c^{d=1} = \tilde{b} - q(B', g)\tilde{b}' + \phi(g)$$

  - The two first terms in RHS reflect distributional effects of B and B'

Eq. Implications III: Default Incentives

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  - The two first terms in RHS reflect distributional effects of B and B'

- Larger mass with $\tilde{b} < 0$ and low q (high default risk), imply more agents with $\Delta c < 0$ and higher default incentives

- Larger mass with $\tilde{b}' < 0$ reduces fraction of agents with $\Delta c < 0$: "static" default incentives decrease as fraction of future net borrowers increases

Two Simple Examples

1. **Distributional Incentives** details fig
   - One period model with given fraction of rich and poor
   - Gov. always default as second best policy to attain efficient consumption dispersion unless rich weight more in the SWF than their actual share of wealth
   - Extended to two period model with uncertainty and optimal choice of debt/default (D'Erasmo and Mendoza (2015))

2. **Social Value of Debt** details
   - What is the welfare cost of "surprise" default in an economy with full commitment
   - Welfare costs: 1.35% for B/Y up to 5%
   - Social value of debt and agents in favor in repayment decrease monotonically with B/Y

Quantitative Analysis

Calibration - Spain

Parameter		Value	Target
Risk-Free Rate (%)	$\bar{r}$	2.07	Real return german bonds
Risk Aversion	σ	1.00	Standard value
Autocorrel. Income	ρ_y	0.85	Guvenen (2009)
Std Dev Error	σ_u	0.25	Spain wage data
Avg. Income	μ_y	0.75	GDP net of fixed capital investment
Autocorrel. G	ρ_g	0.88	Autocorrel. government consumption
Std Dev Error	σ_e	0.02	Std. Dev. government consumption
Avg. Gov. Consumption	μ_g	0.18	Avg. G/Y Spain
Proportional Income Tax	τ^y	0.35	Marginal labor income tax
Discount Factor	β	0.885	Avg. ratio domestic debt Spain
Welfare Weights	ω	0.051	Avg spread Spain (vs Germany)
Default Cost	ϕ_1	0.603	Avg. Debt to GDP Spain (maturity adjusted)

▸ Default Cost and Maturity Adjustment

Moments (%)	Model	Data
Avg. Ratio Domestic Debt	74.31	74.43
Avg. Spread Spain	0.94	0.94
Avg. Debt to GDP Spain (maturity adjusted)	5.88	5.56

Time-Series Dynamics: Long Run and Pre-Crisis

TABLE: Long-run and Pre-Crisis Moments: Data v. Model

	Data		Model	
Moment (%)	Avg.	Peak Crisis	Average	Prior Default
Gov. Debt B	5.43*	7.43	5.88	7.95
Domestic Debt B^d	4.04	4.85	4.29	4.84
Foreign Debt $\widehat{B}$	1.39	2.58	1.59	3.11
Ratio B^d/B	74.34*	65.28	74.31	60.94
Tax Revenues $\tau^y Y$	25.24	24.85	26.60	26.60
Gov. Expenditure g	18.12*	20.50	18.13	18.18
Transfers τ	7.04	7.06	8.35	8.73
Spread	0.94*	4.35	0.94	7.22

Note: * identifies moments used as calibration targets.

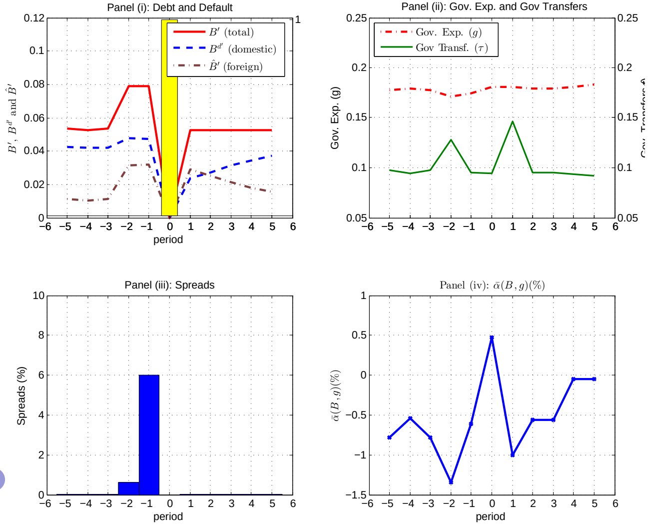
Time-Series Dynamics: Cyclical Properties

TABLE: Cyclical Moments: Data v. Model

	Standard Deviation		Correl$(x, hhdi)$		Correl$(x, g/GDP)$	
Variable x	Data	Model	Data	Model	Data	Model
Consumption	0.85	0.84	0.43	0.97	-0.32	-0.76
Trade Balance/GDP	0.63	0.55	-0.31	-0.82	0.15	0.08
Spreads	1.04	2.46	-0.44	-0.004	-0.22	-0.23
Gov. Debt / GDP	1.58	1.23	-0.18	-0.07	0.06	-0.07
Dom. Debt / GDP	1.68	0.32	-0.32	-0.34	-0.10	-0.22

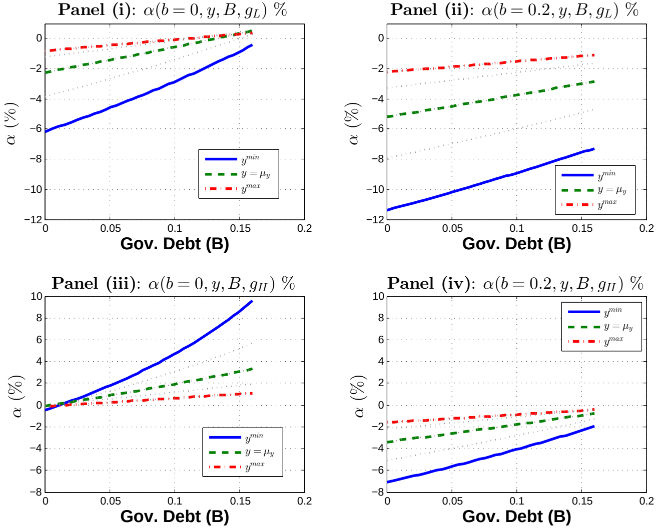
Note: $hhdi$ denotes household disposable income. In the model, $hhdi = Y + \tau + \tau^y Y$ and $TB = Y - C - g$.

Time-Series Dynamics: Event Analysis



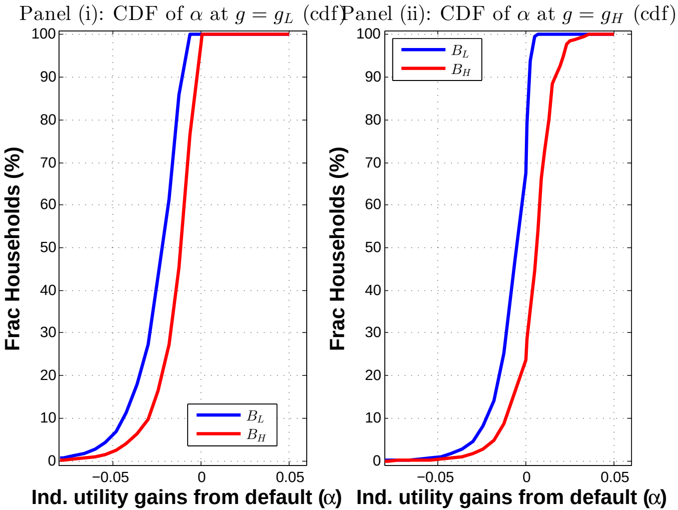
Highlights

Individual Gains from Default as a Function of B



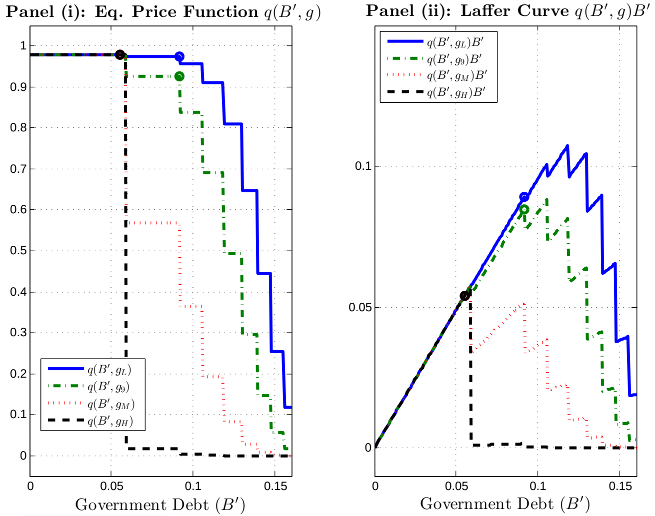
Highlights | Fig. α as function of g | Agg. Welfare Gains $\overline{\alpha}$

Social Distribution of α (for different B and g)



Highlights

Bond Prices & Debt Laffer Curves



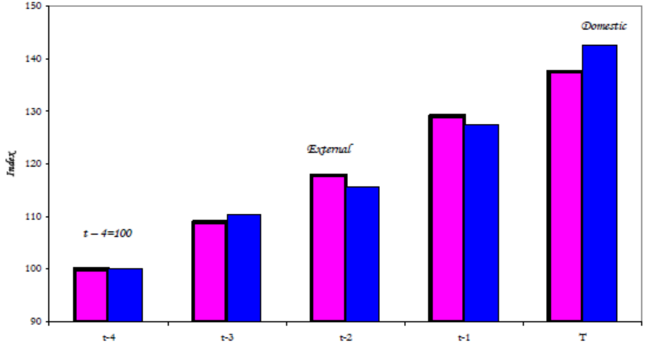
▸ Highlights ▸ Default Decision Rule

Conclusions

- Tradeoff between distributional incentives to default and social value of debt for self-insurance, liquidity provision and risk-sharing supports RME with debt exposed to default risk
- A rich feedback mechanism links debt issuance and default choices, government bond prices, the agent's optimal plans and the dynamics of the distribution of bonds across agents
- Results largely consistent with the data: sensitivity
  - Rapidly rising spreads at high debt ratios in periods leading to a default (rising dist. incentives, falling social value)
  - Long-run and pre-default averages are consistent with data counterparts, at low default frequency and with spreads of up to 700 basis points
  - Model also consistent with key cyclical moments observed in the data (e.g. correlation of g/GDP and spreads)

The Forgotten History of Domestic Defaults

Figure 5. The Runup in Domestic and External Debt on the Eve of Default, Average Default Episode: 1800-2006



Sources: See Data Appendices I and II in Reinhart and Rogoff (2008).

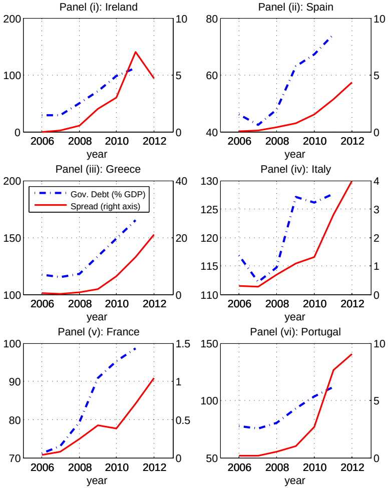
Return

Euro Area Fiscal and Debt Situation 2011

Moment in (%)	Gov. Debt	Gov. Debt Held by Residents	Gov. Exp.	Gov. Rev.	Primary Balance	Sov. Spreads
France	62.73	43.34	24.48	50.60	-2.51	0.71
Germany	51.49	45.04	19.27	44.50	1.69	0.00
Greece	133.10	26.73	17.38	42.40	-2.43	13.14
Ireland	64.97	14.43	18.38	34.90	-9.85	6.99
Italy	100.22	61.72	20.42	46.20	1.22	2.81
Portugal	75.84	33.64	20.05	45.00	-0.29	7.63
Spain	45.60	64.19	20.95	35.70	-7.04	2.83
Avg.	76.28	41.30	20.13	42.76	-2.74	4.87
Median	64.97	43.34	20.05	44.50	-2.43	2.83
GDP (w. avg)	66.49	49.18	21.02	44.99	-1.06	1.80

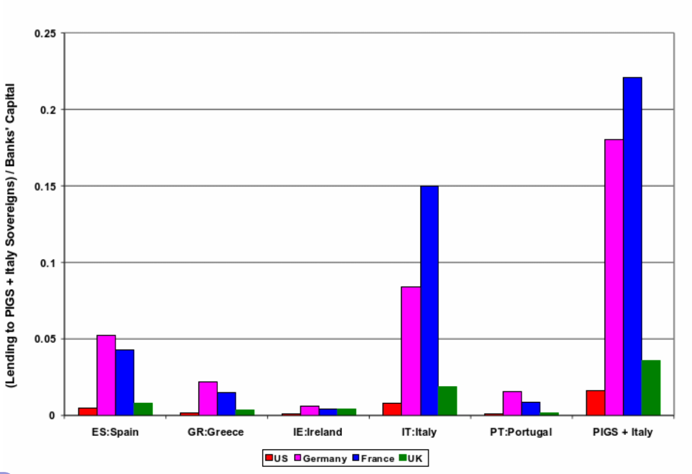
Return Definitions

Euro Area Evolution Debt and Spreads



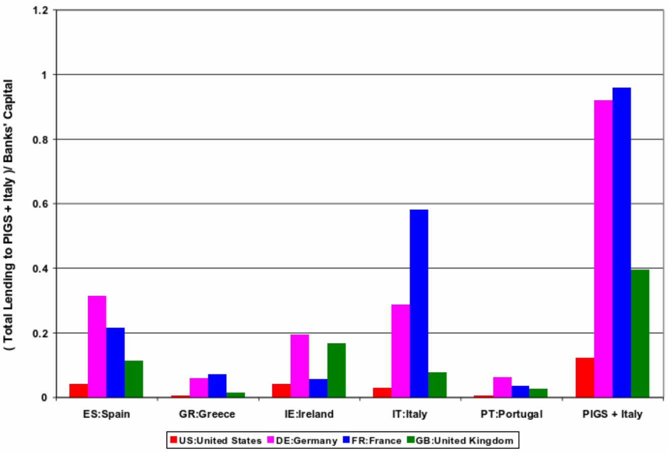
Return

Banks' Exposure to Sov. Risk (2011.q2)



Return

Banks' Exposure to Agg. Credit Risk (2011.q2)



Return

Definitions

- Reinhart and Rogoff (2008):
  - Domestic Public debt is issued under home legal jurisdiction.
  - In most countries, is has been denominated in local currency and held mainly by residents.
- Kumhof and Tamer (2005):
  - BIS aggregates comprehensive data on individual securities from market sources. The definition is very conservative.
  - Classifies as domestic security: issues by residents, target at resident investors in domestic currency.

Return

Related Literature

1. Incomplete Markets - Role of Debt:
   - Het. Agents: Aiyagari & McGrattan (98); Azzimonti, de Francisco and Quadrini (14); Heathcote (05); Floden (01); Bhandari, Evans Golosov and Sargent (16);
   - Rep. Agent: Aiyagari et al. (02); Presno and Pouzo (14);
2. External Default: Arellano (08); Aguiar and Gopinath (06); Cuadra, Sanchez & Sapriza (08); Dias, Richmond & Wright (12); Sosa Padilla (14); Du and Schreger (16)
3. Interaction with Domestic Agentes: Guembel & Sussman (09); Broner, Martin & Ventura (10); Gennaioli, Martin & Rossi (14); Aguiar and Amador (14); Mengus (14)
4. Het. Agents - Default: Dovis, Golosov and Shourideh (16); Aguiar, Amador, Farhi and Gopinath (15)

Return

Recursive Individual Agent's Problem

- Beginning-of-period value, before d is chosen:

$$V(b, y, B, g) = (1 - d(B, g))V^{d=0}(b, y, B, g) + d(B, g)V^{d=1}(y, g)$$

Recursive Individual Agent's Problem

- Beginning-of-period value, before d is chosen:

$$V(b, y, B, g) = (1 - d(B, g))V^{d=0}(b, y, B, g) + d(B, g)V^{d=1}(y, g)$$

- If $d = 0$, the agent's payoff is:

$$V^{d=0}(b, y, B, g) = \max_{\{c \geq 0, b' \geq 0\}} u(c) + \beta E_{y', g'|y,g}[V(b', y', B', g')]$$

s.t. $$c + q(B'(B, g), g)b' = b + y(1 - \tau^y) + \tau^{d=0}(B'(B, g), B, g)$$

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s.t. $$c + q(B'(B,g),g)b' = b + y(1-\tau^y) + \tau^{d=0}(B'(B,g),B,g)$$

- If $d=1$, the agents's payoff is:

$$V^{d=1}(y,g) = u(y(1-\tau^y) - g + \tau^y Y - \phi(g)) + \beta E_{y',g'|y,g}\left[V^{d=0}(0,y',0,g')\right]$$

return

Recursive Problem of International Investors

- Arbitrage condition for bond prices:

$$q(B', g) = \frac{(1 - p(B', g))}{(1 + \bar{r})},$$

where $p(B', g))$ is the default probability given by

$$p(B', g) = \sum_{g'} d(B', g')F(g', g).$$

- If supply of debt is short of domestic demand, agents buy bonds abroad at risk-free price

return

Definition (RME): Aggregates

- Aggregate Consumption is

$$C = \int_{\mathcal{Y}\times\mathcal{B}} c \;\; d\Gamma(b, y),$$

- Aggregate income is

$$Y = \int_{\mathcal{Y}\times\mathcal{B}} y \; d\Gamma(b, y),$$

- The domestic asset demand is

$$B^{d'} = \int_{\mathcal{Y}\times\mathcal{B}} b' \; d\Gamma(b, y)).$$

- The aggregate resource constraint in the no default periods is

$$C + g = Y + \hat{B} - q(B', g)\hat{B}',$$

and in the default period is

$$C + g = Y - \phi(g).$$

Return

Simple Example I: Distributional Incentives

- One-period economy where gov. has issued B.
- Same y for all agents, default can cost a fraction ϕ of y
- Exogenous wealth distribution :
  - Fraction γ holds $b^L = B - \epsilon$
  - Fraction $(1-\gamma)$ holds $b^H = \frac{B-\gamma b^L}{1-\gamma} = B + \frac{\gamma}{1-\gamma}\epsilon$
  - $\epsilon \in [0, B]$ is exogenous demand for gov. bonds
- Government solves: $\max_{d\in\{0,1\}} \left\{W^{d=0}(B,g), W^{d=1}(g)\right\}$,

$$W^{d=0}(B,g) = \omega u(y - g + b^L - B) + (1-\omega)u(y - g + b^H - B)$$

$$W^{d=1}(g) = u(y(1-\phi) - g)$$

return

Distributional Incentives to Default

- Efficient consumption dispersion chosen by planner satisfies:

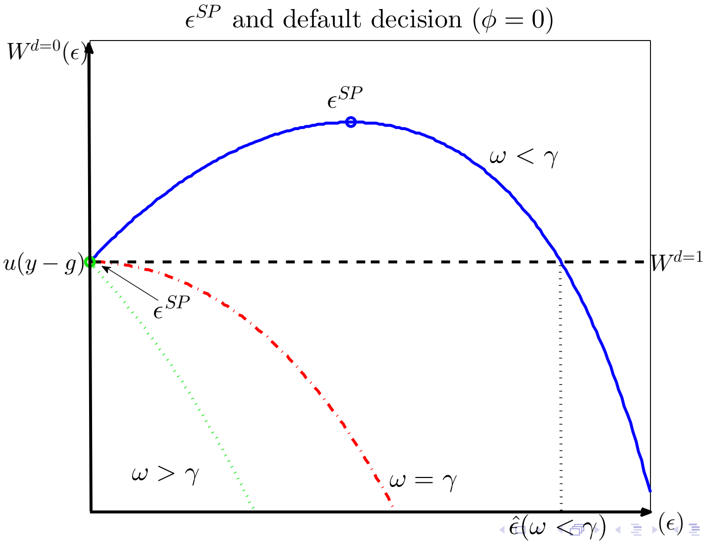
$$\frac{u'\left(y-g+\frac{\gamma}{1-\gamma}\epsilon^{SP}\right)}{u'\left(y-g-\epsilon^{SP}\right)}=\left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right).$$

- If $\phi = 0$:
  - $\omega \geq \gamma \Rightarrow$ default is always optimal for any $\epsilon > 0$
  - $\omega < \gamma \Rightarrow \exists\, \widehat{\epsilon} > 0$: if $\epsilon < \widehat{\epsilon}$ repayment is optimal

- If $\phi > 0$:
  - For any $\{\omega, \gamma\} \Rightarrow \exists\, \widehat{\epsilon} > 0$: if $\epsilon < \widehat{\epsilon}$ repayment is optimal
  - Repayment range widens as $\gamma - \omega$ or ϕ increase (i.e. tolerance for dispersion is akin to default costs)

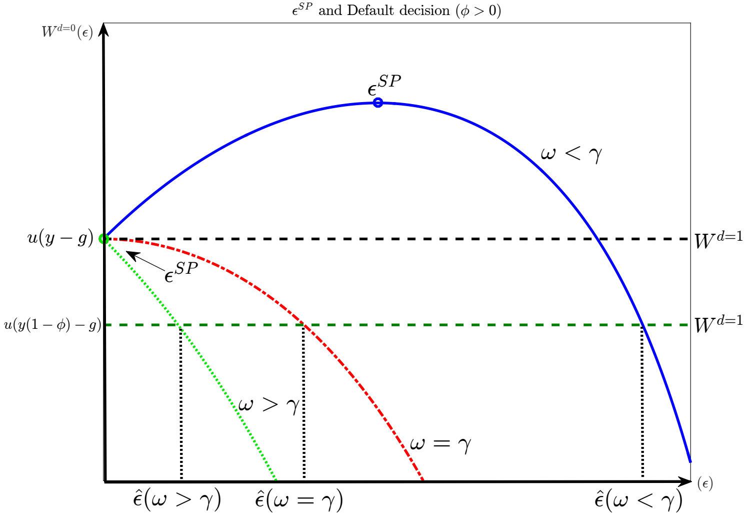
return

Distributional Mechanism *(given B)* return



Distributional Mechanism (given B)

return | fig $\phi = 0$



Simple Ex. II: Social Value of Debt

- Compare an economy with government committed to repay with one experiencing a once-and-for-all unanticipated default
- In both cases $\bar{q} = 1/(1+\bar{r})$ (gov. committed/default is a surprise)
- Compensating variation in consumption for each agent:

$$\alpha(b, y, B, g) = \left[\frac{V^{d=1}(y, g)}{V^{c}(b, y, B, g)}\right]^{\frac{1}{1-\sigma}} - 1$$

- Social value of public debt:

$$\bar{\alpha}(B, g) = \int \alpha(b, y, B, g) d\omega(b, y)$$

return

Social Value of Debt (cont.)

return

B/GDP	B^d/GDP	τ/GDP	$\bar{\alpha}(B, \mu_g)$	$\bar{\alpha}(B, \underline{g})$	$\bar{\alpha}(B, \overline{g})$	hh's $\alpha > 0$
5.0	4.5	32.4	-1.35	-2.49	-0.94	12.4
10.0	4.5	30.8	-0.66	-1.82	-0.23	49.3
15.0	4.5	29.0	0.05	-1.14	0.51	79.5
20.0	4.5	26.6	0.77	-0.44	1.26	94.2

Note: All moments are in percentage.

- Social value of debt (i.e. cost of a surprise default) is large and monotonically decreasing in B/GDP

Social Value of Debt (cont.)

return

B/GDP	B^d/GDP	τ/GDP	$\bar{\alpha}(B,\mu_g)$	$\bar{\alpha}(B,\underline{g})$	$\bar{\alpha}(B,\overline{g})$	hh's $\alpha > 0$
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- Social value of debt (i.e. cost of a surprise default) is large and monotonically decreasing in B/GDP
- Estimates are significantly larger than those in Aiyagari & McGrattan (98) (which find a max. value of 0.1 percent)

Social Value of Debt (cont.) ▸ return

B/GDP	B^d/GDP	τ/GDP	$\bar{\alpha}(B,\mu_g)$	$\bar{\alpha}(B,\underline{g})$	$\bar{\alpha}(B,\overline{g})$	hh's $\alpha > 0$
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- Social value of debt (i.e. cost of a surprise default) is large and monotonically decreasing in B/GDP
- Estimates are significantly larger than those in Aiyagari & McGrattan (98) (which find a max. value of 0.1 percent)
- Higher debt ratios reduce transfers and the extent to which the government can redistribute

Default Cost and Maturity Adjustment

- Default Cost Function:

$$\phi(g) = \phi_1 \max\{0, (\mu_g - g)^{1/2}\}.$$

- Maturity Adjustment:
  - Bonds issued in year t promise to pay one unit in year $t+1$ and $(1-\delta)^{s-1}$ units in year $t+s$ for $s>1$
  - Duration can be written as: $D = \frac{1+r^*}{r^*+\delta}$
  - If we let $\overline{B}$ denote the value of total outstanding debt and B represents the maturity adjusted (one period) stock of debt, B can be written as

$$B = \frac{\overline{B}}{D}$$

Return

Time Series Dynamics: Event Analysis

- Debt accelerates just before default with foreign and domestic holdings rising but the former rising faster
- A lower value of g weakens the incentives to default and allows the government to increase B and τ (resulting in a reduction in $\bar{\alpha}$)
- Higher debt results in higher spreads that spike when g rises
- The increase in g strengthen default incentives resulting in a sharp increase in $\bar{\alpha}$ causing a "sudden" default
- The sudden default and the surge in spreads (both occurring with unchanged debt) may look as if equilibrium multiplicity is the culprit but this is not the case

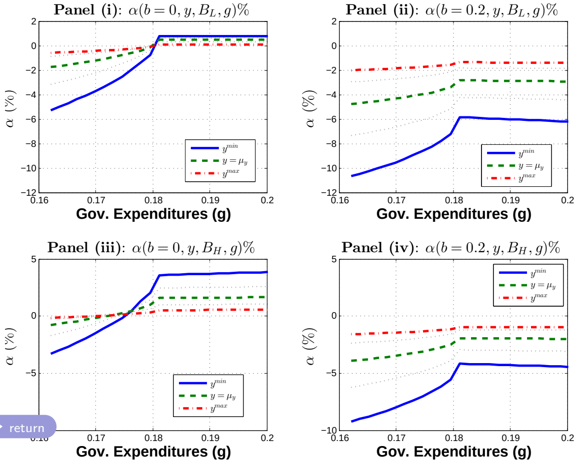
Return

Gains of default across (B, y)

- Gains from default differ sharply for the non-debt holders (low b) and debt holders (high b)
  - Non-debt holders receive the same lump-sum transfers and pay same taxes that debt-holders but do not suffer wealth losses from a default
- Gains are non-monotonic in income
  - Low wealth high income agents value repayment because they would like to start building a buffer
  - High wealth, low income agents value repayment more because they would like to use their buffer stock
- Default gains are convex in government debt: non-debt holders value increasingly more redistribution of resources in their favor when a larger B is defaulted on

Return

Individual Gains from Default as a Function of g



Highlights return

Gains of default across (g, y)

- Individual default gains are increasing and convex in g for $g < \mu_g$
    - Default risk increases with g
    - Exogenous default cost falls as g rises
- Response of default gains to increases in g is weaker for high−income agents

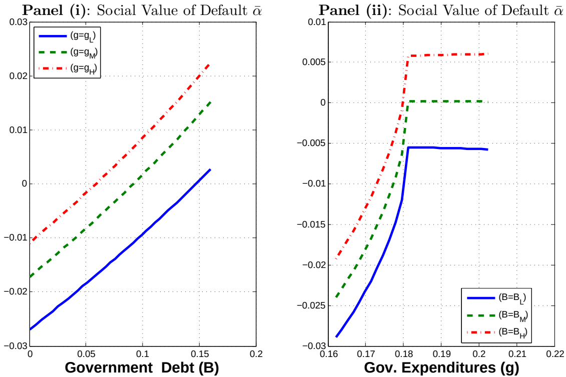
Return

Welfare Gain of Default and Tax Differential

- The social value of default rises with B with the same convex pattern identified in the individual gains of default
- Social gains from default rise much faster at at $g \leq \mu_g$
- Social gains yield smaller numbers than individual gains because they reflect government's aggregation

Return

Social Gain of Default



$$\overline{\alpha}(B,g) = \int_{\mathcal{B}\times\mathcal{Y}} \alpha(b,y,B,g)d\omega(b,y)$$

Highlights return

Social Distribution of α (for different B and g)

- Welfare weights $\omega(b, y)$ are exogenous but the social distribution of gains from default across agents varies endogenously with the aggregate states (B, g).
- The non-linear, non-monotonic responses of the individual α's to changes in B and g imply that the α's move in different directions across (b, y) pairs when (B, g) changes.
- The social distribution of default gains shifts to the right as B rises, and a larger fraction of agents are assessed as benefiting from a default.

Return

Bond Prices & Debt Laffer Curves

- Price function has similar shape that those observed in EG models
- For debt that carries default risk, prices are lower at higher g because the probability of default is increasing in g
- For low g (and long-run B), debt is sold at the risk free price and below the maximum of the Laffer curve.
- For average or high g the government chooses B' to maximize resources.
- On the equilibrium path, we also observe B' choices that are interior and carry default risk ($g = g_9$)

Return

Sensitivity I: Government Welfare Weights

Highlights

Moment (%)	benchmark	$\bar{\omega}=0.051$ $z=0.025$	$\bar{\omega}=0.0435$ $z=0$
Long Run Averages			
Gov. Debt B	5.88	4.22	4.56
Dom. Debt B^d	4.29	3.84	4.16
Default Frequency	0.93	1.00	0.53
Spreads	0.94	1.01	0.54
Transf τ	8.35	8.39	8.38
Frac. Hh's $b=0$	68.74	69.15	67.41
$\bar{\alpha}(B,g)$	-0.341	-0.306	-0.483
Averages Prior Default			
Gov. Debt B	7.95	6.00	6.12
Dom. Debt B^d	4.84	4.76	4.66
Spreads	7.22	6.84	4.56
Def. Th. $\hat{b}(\mu_y)$	0.073	0.051	0.051
%. Favor Repay (1-$\omega(\tilde{b}(\mu_y),\mu_y)$)	23.45	21.99	29.98
% Favor Repay (1-$\gamma(\tilde{b}(\mu_y),\mu_y)$)	3.68	4.16	4.07

Note: Benchmark model parameters are $\bar{\omega}=0.051$, $z=0$

$$\omega(b,y)=\sum_{y_i\leq y}\pi^*(y_i)\left(1-e^{-\frac{(b+z)}{\bar{\omega}}}\right)$$

return

Sensitivity II: Preferences and Income Process

Highlights

Moment (%)	bench.	β 0.85	β 0.90	σ 0.5	σ 2	σ_u 0.200	σ_u 0.300
Long Run Averages							
Gov. Debt B	5.88	5.96	6.32	5.06	6.80	6.28	6.40
Dom. Debt B^d	4.29	1.16	6.24	0.02	6.82	1.22	6.39
Foreign Debt $\hat{B}$	1.59	4.80	0.08	5.04	-0.02	5.06	0.01
Def. Freq.	0.93	1.02	0.27	19.58	0.25	0.29	0.49
Spreads	0.94	1.027	0.266	24.340	0.249	0.296	0.490
Transf τ	8.35	8.35	8.35	9.20	8.34	8.34	8.34
Frac. Hh's $b = 0$	68.74	91.66	63.49	98.96	22.25	93.27	61.19
$\bar{\alpha}(B, g)$	-0.341	-0.506	-0.305	-0.646	-0.448	-0.320	-0.323
Averages Prior Default							
Gov. Debt B	7.95	7.99	8.47	6.31	8.72	8.17	8.46
Dom. Debt B^d	4.84	1.27	8.34	0.03	8.72	1.32	8.42
Foreign Debt $\hat{B}$	3.11	6.72	0.13	6.28	0.00	6.85	0.04
Spreads	7.22	7.03	3.76	43.49	3.72	3.59	4.69

Note: Benchmark model parameters are $\beta = 0.885$, $\sigma = 1$ and $\sigma_u = 0.25$ return

Sensitivity III: Default Cost

Highlights

		ϕ_1		ψ		$\hat{g}$	
Moment (%)	bench.	0.35	0.75	0.35	0.75	0.176	0.186
Long Run Avg							
Gov. Debt B	5.88	5.59	6.04	7.23	5.37	5.36	7.17
Dom. Debt B^d	4.29	4.30	4.31	4.35	4.29	4.29	4.32
Foreign Debt $\hat{B}$	1.59	1.29	1.73	2.88	1.08	1.07	2.85
Def. Freq.	0.93	0.49	0.95	2.89	0.13	0.14	1.68
Spreads	0.94	0.494	0.955	2.976	0.135	0.137	1.706
Transf τ	8.34	8.36	8.35	8.33	8.36	8.36	8.35
Frac. Hh's $b = 0$	68.74	68.78	68.71	65.51	68.87	68.79	69.05
$\bar{\alpha}(B, g)$	-0.341	-0.230	-0.449	-0.668	-0.136	-0.174	-0.520
Avg Prior Default							
Gov. Debt B	7.95	6.92	8.48	11.76	5.96	7.67	8.06
Dom. Debt B^d	4.84	4.66	4.90	5.48	4.42	4.82	4.66
Foreign Debt $\hat{B}$	3.11	2.26	3.57	6.28	1.54	2.85	3.40
Spreads	7.22	4.64	7.19	15.42	1.59	5.87	6.12

Note: Benchmark model parameters are $\phi_1 = 0.572$, $\hat{g} = \mu_g = 0.182$ and $\psi = 1/2$.

$$\phi(g) = \phi_1 \max\{0, (\hat{g} - g)^{\psi}\}$$

return

Sensitivity IV: Proportional Income Taxes

		τ^y	
Moment (%)	benchmark	0.20	0.45
Long Run Averages			
Gov. Debt B	5.88	6.40	6.34
Dom. Debt B^d	4.29	6.42	2.36
Foreign Debt $\hat{B}$	1.59	-0.02	3.98
Def. Freq.	0.93	0.49	0.52
Spreads	0.94	0.49	0.52
Transf τ	8.35	8.34	8.34
Frac. Hh's $b = 0$	68.74	59.81	85.87
$\bar{\alpha}(B, g)$	-0.3408	-0.3480	-0.3041
Averages Prior Default			
Gov. Debt B	7.95	8.45	8.06
Dom. Debt B^d	4.84	8.43	2.60
Foreign Debt $\hat{B}$	3.11	0.01	5.47
Spreads	7.22	4.71	4.56

Note: Benchmark model parameters are $\tau^y = 0.35$.

return

Sensitivity I: Government Welfare Weights

- Increasing z for given $\bar{\omega}$:
  - Weights of agents at $b = 0$ increases considerably (0 vs 38.62 percent)
  - The default threshold and the fraction that benefit from repayment drop
  - These changes reflect stronger incentives to default and less desire to issue debt

- Decreasing $\bar{\omega}$ for given z:
  - Stronger incentives to default put an additional constraint on government borrowing
  - Incentives to use debt for redistribution decrease: lower average debt and spreads

Return

Sensitivity II: Preferences and Income Process

- Observed changes in B^d are standard: increasing incentives for self-insurance by rising β, σ or σ_u increases domestic holdings
- Higher β, σ or σ_u also allows the government to issue higher levels of debt: default incentives decrease (lower spreads)
- The benefit of defaulting as a mechanism for redistribution that cannot happen via self-insurance decreases
- The scenario with lower β results in higher debt levels and spreads: similar mechanism to external debt literature

Return

Sensitivity III: Income Taxes and Default Cost

- As the cost of default increases (higher ϕ_1, lower ψ or higher $\hat{g}$) the government is able to borrow more
- Everything else equal the default probability decreases; however, the higher level of debt results in higher spreads
- Higher spreads induce a higher domestic demand for government bonds
- The average welfare cost of default increases

Return

Default Decision

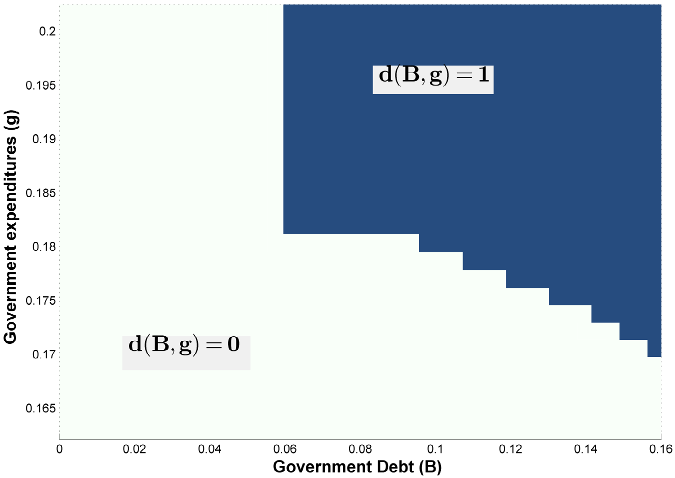
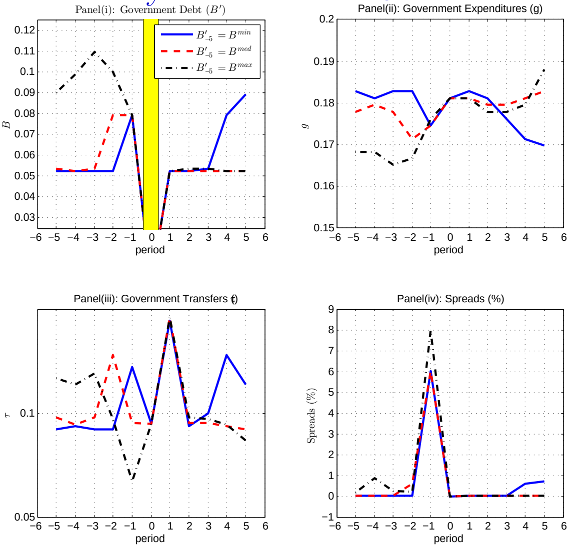
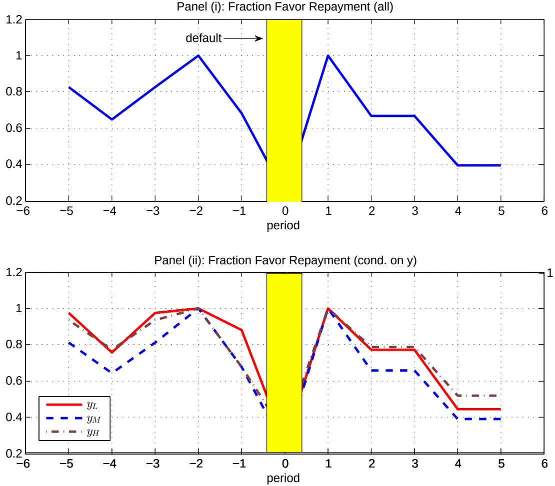


Figure q/Laffer

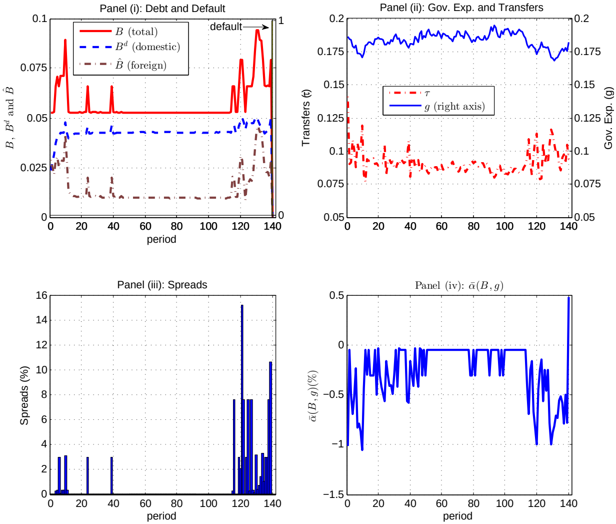
Default Event Analysis: Additional Cases



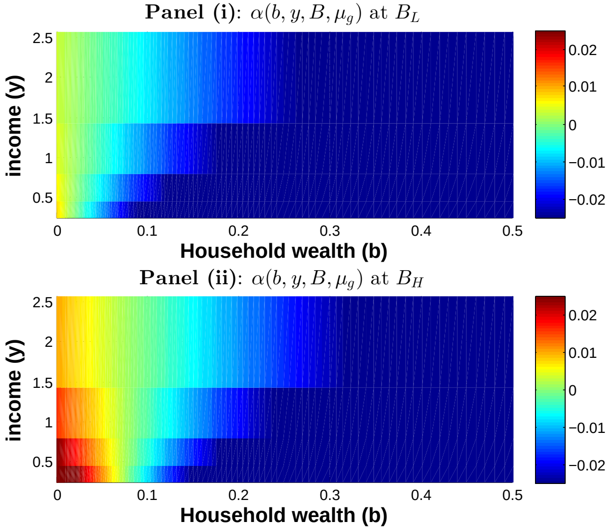
Preferences over Repayment



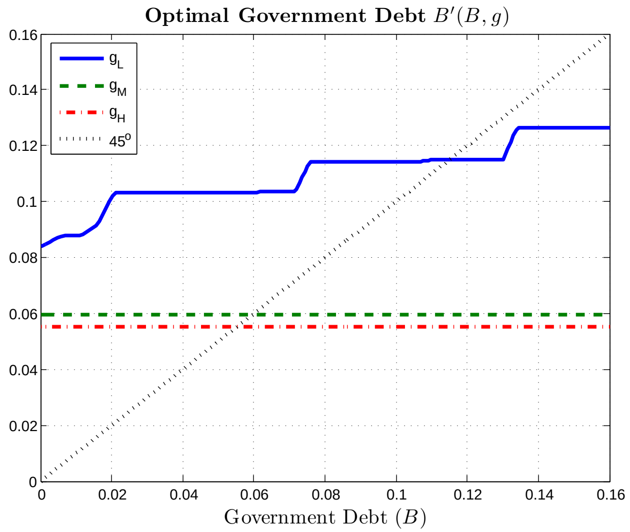
Time-Series Dynamics between Default Events



$\alpha(b, y, B, g)$ (for different B at $g = \mu_g$)



Optimal Debt $B'(B, g)$



"Average" Wealth Distribution $\bar{\Gamma}(b,y)$ and Welfare Weights $\omega(b,y)$