The Coherence Side of Rationality

Theory and evidence from firm plans *

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Abstract

We study the forecasting heuristics MBA textbooks propose to make firms' plans. Using Duke-Survey data, we document the prevalence and heterogeneity of heuristics use among top US executives. We propose a theory of optimal multidimensional forecasting; evaluate heuristics in terms of forecast coherence; develop tests to distinguish coherence from accuracy. In our normative benchmark, technology parameters coherently link output-inputs forecasts. Our positive model rationalizes some heuristics as second-best responses to noisy signals, yielding a pecking order of heuristics. Consistent with our predictions, firm performance is negatively associated with incoherence and incoherent heuristics use. About one-half of CFOs make incoherent forecasts.

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I Introduction

Coherence—or "the consistency of the elements of the person's judgment" (Hammond (2007), p. xvi)—is one of the two standards of rationality together with accuracy.[1] In the context of multidimensional forecasting, coherence requires forecasts of individual variables to incorporate the connections among those variables; whereas accuracy requires individual forecasts not to differ systematically from realizations ex post. While forecast accuracy has been widely studied,[2] forecast coherence has received less attention.

This paper studies forecast coherence in a corporate setting, by combining novel theory and data on 'firm plans.' These plans consist of detailed internal forecasts over multiple balance-sheet variables, which outline the allocation of resources within the firm. When preparing such plans, Chief Financial Officers (CFOs) start from an output target by forecasting sale revenues ('top line' forecast), and then proceed to forecast other items (e.g., input costs). This is a challenging multidimensional forecasting problem in a high-stake setting, requiring CFOs to draw on their knowledge of the firm's production possibility, which provides a natural coherence benchmark for rational firm plans.[3]

Internal planning and forecasting underlie all resource allocation and investment decisions inside the firm and is still not well understood (Graham, 2022). MBA textbooks and case studies offer a series of 'rules of thumb' (RoT) to help CFOs make internal forecasts (e.g., Ruback (2004), Welch (2017), Koller et al. (2020)). These rules range from forecasting each item on its own, to anchoring each forecast to the output target, to multivariate rules. These rules produce very different forecasts from one another and differ in their implicit concern for coherence. To the best of our knowledge, these rules have never been evaluated theoretically or empirically. Our paper provides a first evaluation.

Our paper makes two additional contributions. First, it develops a series of

[1] Coherence can be traced back at least to Aristotle (Fogelin, 2003). Tversky and Kahneman (1981), p. 453, write, "there is general agreement that rational choices should satisfy some elementary requirements of consistency and coherence." See also Sen (1993), Becker (1996), Posner (2014).

[2] E.g., Tversky and Kahneman (1974); see Benjamin (2019) for a recent review.

[3] At least in static or stable environments. Dynamic disruptions due to aggregate shocks or technological innovation may require adaptation and, thus, incoherence relative to ex ante relationships and plans. The implications of dynamics and disruptions for coherence deserve careful analysis. But given the current lack of a conceptual framework to study forecast coherence, in this paper we start by laying the foundation of such a framework under stability.

approaches to empirically assess forecast coherence, spanning theory-based restrictions and formal statistical tests, some based on forecasts ('ex ante' approaches) and others on forecast errors ('ex post' approaches).[4] Second, two of our ex post approaches based on forecast errors—a regression test and an individual-level test—enable us to disentangle (in)coherence from (in)accuracy and provide further evidence consistent with our theory.

Exploiting unique data on simultaneous forecasts of multiple balance-sheet items from the Duke Survey of CFOs of large- and mid-size US corporations (Ben-David et al. (2013), Graham (2022)), linked with realizations from Compustat, we present the first empirical evidence on the prevalence and heterogeneity of RoT use for internal planning. Notably, we show that a continuous measure of ex ante incoherence, using the most sophisticated RoT ((R5) below) as an intuitive coherence benchmark, correlates negatively with ex post corporate performance.[5] Similarly, firms using RoTs whose forecasts are most distant from those implied by the sophisticated benchmark have lowest performance on average. Albeit descriptive, this evidence establishes the empirical relevance of forecast (in)coherence.[6]

To interpret this evidence, we develop a theory of optimal multidimensional forecasting in firm plans and propose a novel mechanism by which incoherence may arise: the use of suboptimal forecasting rules, which may induce resource misallocation within the firm. Since our model nests the RoTs from the managerial literature, we can use the model to establish under what conditions, if any, each rule can be first-best optimal under full information or second-best optimal under imperfect information. Our model yields a partial ordering of the existing rules along firm performance, and testable predictions relating forecast incoherence, RoT use, and firm outcomes, matching our evidence.

We analyze the rules of Welch (2017)'s taxonomy: (R1) plain growth, (R2) proportion of sales, (R3) economies of scale, (R4) industry based, and (R5) disaggregated. These rules vary in their concern for coherence and degree of sophistication. Conceptually, rules (R2)-(R5) reflect some concern for coherence, as they link the forecast of each input's

[4]For any given forecaster and variable, we define the forecast error as realization minus forecast.

[5](R5) uses multivariate regression to incorporate the relationship between the item being forecasted and other balance-sheet items.

[6]To make a step toward causality, we additionally perform an event study, examining corporate performance and other outcomes around the date when the CFO takes office and confirm that performance decreases in the years following the start of CFOs whose forecasts display larger incoherence.

growth to the forecast of output's growth (R2-R4), and also of other inputs' growth (R5); whereas (R1) prescribes forecasting each input's growth rate by projecting the input's past growth rates into the future, implicitly disregarding the input's relationship with the output and other inputs. Operationally, rules (R1)-(R4) are straightforward to implement, involving a mere sample average (R1) or univariate mean linear regressions (R2-R4); whereas (R5) is more sophisticated, requiring multivariate regression.

In our theory, a profit-maximizing firm combines two inputs (K, L) to produce output according to a standard technology.[7] The CFO issues a vector of forecasts over the growth rates of output and inputs, given knowledge of the firm's technology. Each forecast is made by minimizing expected inaccuracy under square loss, subject to a coherence constraint provided by the firm's production function. In the normative version of our model with complete information, we establish that the elements of the optimal forecast vector are coherently linked by the firm's technology parameters, rationalizing a version of (R5) as first-best optimal.[8] In the positive version of the model, we study the implications of imperfect information about (the values of) inputs and output.

Among the five rules, (R1) is reminiscent of "narrow bracketing," (R5) of "broad bracketing," and (R2)-(R4) appear to lie in between the two extremes of narrow and broad bracketing.[9] Starting from this observation and building on Lian (2021)'s model of "narrow thinking" in consumption choice, we model the forecaster's problem as an incomplete-information, common-interest game among multiple selves. Each self is in charge of forecasting one item, while observing noisy signals about other items. By creating intra-personal frictions in coordinating different selves forecasting different variables, imperfect information can, thus, generate incoherence.

[7]In the Online Appendix, we show our results extend naturally to $n > 2$ inputs. Moreover, since our data comes in the form of forecasts over input expenditures and sale revenues, we are unable to study forecast (in)coherence and (in)accuracy for prices and quantities separately. Given this data limitation, the model assumes that the firm is a price taker in both input and output markets. However, as we further discuss below, some of our ex post restrictions and tests are robust to the specifics of market structure as, under some conditions, they get differenced away when computing forecast errors.

[8]Being linear, (R5) can only be rationalized as first-best optimal under linear production functions. Alternatively, as discussed below, it should be viewed as a linear approximation of the optimal rule.

[9]"Narrow bracketing" refers to the behavior of decision makers who, when confronted with multiple related choices (e.g., consumption bundles), make each choice in isolation instead of jointly (hence, "broad bracketing"). By disregarding how budget constraint and utility function tie choice variables together, narrow bracketers obtain lower utility than broad bracketers (e.g., Thaler (1985), Read et al. (1999)).

We establish that when a forecasting self observes informative signals about the output and other input(s), a version of (R5) emerges as optimal also in a second-best sense. The 'narrow bracketing' rule (R1) emerges as second-best optimal in the limit in which the forecasting self observes infinitely noisy signals about both output and other input(s). The "economies of scale" rule (R3), projecting the univariate relationship between the input being forecasted and the output, emerges as second-best optimal when only the output's signal is informative. (R2), a special case of (R3) assigning to the input the same growth rate planned for the output, is generally inferior to (R3).

In our model, "narrow thinking" generates incoherent plans via the use of suboptimal forecasting rules. Incoherent plans, in turn, tend to induce suboptimal resource allocations to inputs and, hence, lower profits. Our model's predictions line up well with our empirical results: (R5) is best, (R3)-(R4) are intermediate, (R1)-(R2) are worst.

In our data, about 48% of CFOs give forecasts that are closest to those implied by (R1) or (R2).[10] (R2) can be expressed as a mean regression of each input's growth on the output's growth with a zero intercept and a unit slope, which is why we relabel it 'sales anchoring' rule. Despite its simplicity and popularity, it is easy to see (R2) is problematic by comparing it with the still simple but less popular (R3). (R3) can be expressed as the same mean regression, but with the intercept and slope actually estimated using Compustat data. The estimated slope is multiplied by the targeted sale revenues' growth and added to the estimated intercept. In Compustat data, while the slope is close to one, the intercept is positive, large, and significant. Intuitively, capital expenditures not only drive output growth but also provide maintenance to depreciated capital, which is captured by the intercept. By neglecting the latter, (R2) systematically projects a much smaller rate of increase in capital expenditures relative to sale revenues than warranted by the data. Importantly, this applies to all industries.[11]

In sum, viewing the use of (R2) and (R1) as a first plausible metric of incoherence,

[10] In particular, 8% are closest to (R1), 40% to (R2), and 27% of the latter are exactly equal to that implied by (R2). About 11% of CFOs give forecasts closest to (R3), 27% to (R4), and 15% to (R5). The popularity of (R2) is consistent with managerial and consulting teachings and with the observations that this rule is very simple to implement, potentially intuitive, and seemingly incorporates a coherence concern by exactly anchoring each input's forecast to the output's forecast.

[11] As further discussed below, (R1) too yields forecasts of capital expenditures' growth that are systematically too low relative to the planned sale revenues' growth.

we infer that nearly half of the CFOs in our sample make incoherent forecasts.

We then ask what we can learn by relying more directly on our theory's assumptions about the firm's environment and technology. We start with a general empirical formulation of a production function, whose main requirement is linearity in parameters (after relevant variable transformations),[12] and we use our assumptions that the CFO knows the firm's technology and there are no aggregate shocks or technological innovation. Then, ex ante coherent forecasts of inputs and output are cross-sectionally linked in the same way as their realizations. Subtracting forecasts from realizations on both sides of the equation differences away any firm-level heterogeneity known to (or predictable by) the CFO at the time of forecast, but unknown to the econometrician. That is, some industries have short horizons, others need long-term plans, and in others there may complicated lead-lag relationships, but as long as CFOs know their firm's technology and such technology can be expressed in a linear-in-parameter form, computing forecast errors (FE) will difference away all such heterogeneity and yield a simple relationship linking the FEs of output and inputs, with the inputs' loadings as weights.

Economic theory provides two intuitive restrictions on contemporaneous output-input FEs coherent forecasts should satisfy ex post. First, under free disposal, the loadings of each input's FE should be (weakly) above zero. Second, under no increasing returns of individual inputs, the loadings of each input's FE should be (weakly) below one. The two shaded areas in Figure 1 depict these coherence regions; Section V works out the details. Figure 2 plots the joint distribution of the contemporaneous FEs of output and capital in our data. By overlaying Figure 2 on Figure 1, we find that 52% of CFOs violate one or both restrictions, a result in the same ballpark as the 48% of CFOs using incoherent RoTs. We find similar results in all ten industries, consistent with the idea that contemporaneous FEs are informative about coherence irrespective of the lead-lag relationship or the short- vs long-term outlook of different industries.

The output-input relationship among FEs further suggests simple regression tests to assess coherence ex post and distinguish it from accuracy. Intuitively, accuracy requires

[12] Variables may enter the regression in levels, growth rates, logs, or other transformations that preserve linearity in parameters. Inputs and state variables are allowed to affect output with firm-specific lags and in a time-varying, firm-specific manner. The details are presented in Section V.

the FEs of each variable to be zero on average, whereas coherence requires the output-input relationship among FEs to mirror that for realizations. In our data, we reject forecast coherence on average and forecast accuracy for capital expenditures, but we do not reject forecast accuracy for sale revenues.

As a last step, we fully use the structure of our theoretical framework, including functional-form assumptions on the firm's technology and distributional assumptions on the process for prices, and derive formal statistical tests of forecast coherence and forecast accuracy at the individual level, some based on forecasts and some based on FEs. Focusing on the latter, whose implementation is feasible and more credible with our data, we reject the null of coherence at the 95% confidence level for about 56% of CFOs in our sample.

In sum, while relying on assumptions of varying number and strength, all our results point to about one half of CFOs providing incoherent forecasts of simultaneous variables.

At the individual level, forecast coherence requires the FEs of output and inputs to be 'sufficiently close to one another,' while forecast accuracy requires the FEs of individual variables to be 'sufficiently close to zero.' Figure 3 provides a graphical representation of this intuition in the output FE-capital FE space for a hypothetical firm with a capital loading of 1/3. In our data, allowing the loading of capital to vary by industry, we find that 31% of CFOs are coherent and accurate, about 44% are incoherent and inaccurate, 12% are incoherent but accurate, and 13% are inaccurate but coherent.

Distinguishing coherence from accuracy is important, as under some conditions coherence can be assessed ex ante, and ex ante coherence may aid accuracy ex post.[13] By abstracting away from unanticipated aggregate shocks and technological innovation, our framework rules out the existence of any conceptual or de facto wedge between ex ante and ex post coherence. This comes with two main advantages for our analysis. First, by nesting together coherence and accuracy within a single framework, our analysis clarifies the conceptual distinction between the two within a stable setting, and provides intuitive restrictions and statistical tests based on contemporaneous input-output FEs for disentangling (in)coherence and (in)accuracy empirically. Second, in our empirical analysis we can focus on cross-sectional variation in forecast coherence and its relation to

[13]Of course, ex post accuracy on all variables would necessarily imply coherence.

firms' outcomes. Accordingly, we examine a sample of large, mature firms over a quiet period with no major shocks or disruptions.

In our data CFOs are reasonably accurate in forecasting output, reflecting their incentives (Graham, 2022), while they are much less accurate in forecasting capital expenditures, likely reflecting that their forecast of capital expenditures' growth is incoherently linked to that of sale revenues' growth.[14]

These tests also help rule out alternative interpretations of our findings. A first issue is whether incoherence might reflect the CFO's differential control over different variables. CFOs have ultimate authority on capital expenditures, while sales depend also on external factors such as consumer behavior and product market competition. Yet, we find that CFOs are more accurate in forecasting sale revenues than capital expenditures.

Second, we show that incoherence is distinct from other CFO's traits such as overconfidence and optimism (e.g., Ben-David et al. (2013) and Guenzel and Malmendier (2020)'s review). Controlling for the latter, we find that incoherence is associated with lower investment spending, while the opposite applies for overconfidence and optimism.

A final issue is whether incoherence might be an optimal response to firm-level idiosyncratic shocks. Upon being hit by such a shock, a CFO might need to invest more than originally planned to achieve its sales target, consistent with the average positive capital expenditures FE and the average zero sales FE that we observe in the data. This explanation is unlikely to hold systematically in our data, for two reasons. First, our results are relatively stable across subsamples over time and across industries, consistent with the systematic use of heuristics and inconsistent with some subsamples—but not others—being hit by idiosyncratic shocks. Second, if firms are optimally responding to idiosyncratic shocks, we should not observe a systematic negative relationship between incoherence—and incoherent heuristics' use—and firm performance, as we do.

Studying coherence is challenging, as it requires referring to "the entire web of beliefs held by the individual" (Tversky and Kahneman (1974), p. 1130) and "to something external to choice behavior (such as objectives, values, or norms)" (Sen (1993), p. 495). Our corporate planning setting is naturally amenable to study forecast coherence, for

[14]For sale revenues, we reject the null of forecast accuracy at the 95% confidence level for 27% of CFOs. For capital expenditures, we reject the null of forecast accuracy for 48% of CFOs.

two reasons. First, executives of large mature firms should have a good sense of their firm's technology and agree on the firm's objectives. Second, these executives should be optimizing agents, or at least they should be seeking to achieve certain pre-defined goals.[15]

Our novel theory and data on firm plans enable us to evaluate the forecasting heuristics taught in managerial and consulting textbooks, and used by top executives, in terms of their implied forecast (in)coherence and associated firm outcomes. We show that not all heuristics are the same: two prominently used ones give rise to incoherent forecasts and are associated to lower performance, whereas two others work quite well.

Our evidence on incoherence in firm plans provides a possible microfoundation for understanding resource allocation within the firm (e.g., see Maksimovic and Phillips (2002, 2013), Hoberg and Phillips (2023)). Prior literature on managerial beliefs has focused on forecast accuracy of individual variables (e.g., Ben-David et al. (2013), Graham (2022)). Prior work on behavioral bracketing has not studied a firm production setting (e.g., Thaler (1985), Rabin and Weizsäcker (2009), Lian (2021)). Prior literature in behavioral corporate finance has focused on other psychological traits of top managers such as optimism and overconfidence (e.g., Baker et al. (2007), Guenzel and Malmendier (2020)). We show that incoherence differs from both optimism or overconfidence. We discuss the paper's contributions to the literature in greater detail in Section VI.

II Theoretical Framework

When preparing firm plans, CFOs typically start from output by forecasting sale revenues for a number of years, and then proceed to forecast the other balance-sheet variables, including capital and labor expenditures (Graham, 2022). To help CFOs with this multidimensional forecasting problem, managerial textbooks and case studies propose simple methods, surveyed in the following taxonomy by Welch (2017) (p. 593-594):

(R1) A **plain growth** forecast, separately projecting past growth rates of each item; henceforth 'narrow bracketing' rule. Welch (2017) implements this rule by

[15] Prior research has been criticized for its focus on universal, domain-free benchmarks of coherence such as modus ponens and tollens in propositional logic, or Bayes' rule in probability theory, since "norms of coherence are crucial in the service of an organism's goals" (Arkes et al. (2016), p. 31).

computing for each item the average of the two most recent annual growth rates.

(R2) A pure **proportion of sales** forecast, forecasting each item as a fixed proportion of sale revenues; henceforth 'sales anchoring' rule. Welch (2017) implements this rule by assigning to each item the same growth rate as that forecasted for sales. Thus, this rule can be expressed as a mean regression of each item's growth on sales' growth with a zero intercept and a unit slope.

(R3) An **economies-of-scale** forecast, positing for each item a fixed component and a variable component, the latter itself a proportion of sales' growth. Welch (2017) implements this rule by estimating a best linear predictor under square loss of each item's growth on contemporaneous sale revenue's growth using Compustat data. The estimated regression intercept is the fixed component and the estimated slope multiplied by the forecasted sale revenues' growth is the variable component.

(R4) An **industry-based** forecast, drawing on information from other firms in the same industry. Welch (2017) implements this rule exactly as (R3), but using only data on firms in the same industry as the firm under consideration.

(R5) A **disaggregated** forecast, recognizing that each item may comove not only with sales but also with other items; henceforth 'sophisticated' rule. Welch (2017) implements this rule by expanding the specification of (R3) to include additional contemporaneous items and using all Compustat data.

There is no consensus on which of the above rules, if any, constitutes best practice. In their best-selling guide to valuation, Koller et al. (2020) advocate (R2) writing, "*net Property, Plant and Equipment should be forecast as a percentage of revenues*" (p. 286).[16] Similarly, Harvard Business School case studies typically provide case solutions in which the forecast of capital expenditures' growth equals the forecast of sale revenues' growth, and in general suggest (R1), (R2), and (R4) (e.g., Luehrman and Heilprin (2009) and Stafford and Heilprin (2011)).[17] This literature lacks a formal framework designed to

[16] In turn, Koller et al. (2020) indicate that depreciation should be forecasted as a proportion of net Property, Plant, and Equipment (PPE), and that capital expenditures should be calculated by summing the projected increase in net PPE to depreciation (p. 286-287). As a result of this chain of calculations, capital expenditures is forecasted as a percent of revenues, and thus the forecast of capital expenditures growth should equal the forecast of sale revenues growth, as in (R2).

[17] Advocates of other rules include Ruback (2004), advocating (R2) and (R3); Titman and Martin (2016), advocating (R3); and Holthausen and Zmijewski (2020), describing a method akin to (R5).

evaluate these methods and offer guidance as to whether these methods are equivalent and, if not, which of them performs best under what conditions.

In this section, we address this gap. In Subsection *A.*, we present a normative framework of first-best rational forecasts under complete information. In Subsection *B.*, we present a positive framework of second-best optimal forecasts when CFOs observe noisy signals about the firm's inputs and output. Our frameworks nest the above RoTs and derive conditions under which some of them emerge as first- and/or second-best optimal.

A. A Benchmark Model of Optimal Corporate Forecasts

Consider a firm aiming to maximize its profits, $\Psi = p_y y - p_1 x_1 - p_2 x_2$, where y is output, x_1 and x_2 are input quantities (K, L),[18] p_1 and p_2 are the input prices, and p_y is the output price, which we normalize to 1.[19] Output is produced by a technology in the general class of CES production functions (Moysan and Senouci, 2016),

$$y = f(x_1, x_2) = \left(\frac{a}{a+b} x_1^{\xi} + \frac{b}{a+b} x_2^{\xi} \right)^{\frac{a+b}{\xi}}, \tag{1}$$

where $\nu \equiv a + b > 0$ are parameters governing the returns to scale (constant for $\nu = 1$, increasing for $\nu > 1$, and decreasing for $\nu < 1$), and the elasticity of substitution between x_1 and x_2 is $\chi = \frac{1}{1-\xi}$.[20] We assume factor-augmenting productivities are constant over time and normalize them to one.[21] We also assume that the technological relationship is stable over time and not subject to aggregate shocks. Moreover, we assume for now that input prices are i.i.d. log-normally distributed: $\{\pi_{i,t}\}_{t \geq 1} \sim \mathcal{N}(0, \sigma_i^2)$, with $corr(\pi_1, \pi_2) =$

[18] This is for illustration and wlog. We extend our theory to n inputs in the Online Appendix.

[19] We maintain basic "producer theory" assumptions: (i) firms are described by exogenously given technologies to convert inputs into output; (ii) firms are "competitive" producers, taking input and output prices as given and choosing a production plan to maximize profits. These further imply cost minimization. We denote the cost function by $Z(p_1, p_2, y)$ and its value at solution, y^*, by Z^*. These assumptions are for illustration and wlog, because all that matters for our results is that (1) the industry's competitive structure is known to the CFOs (but not necessarily to the econometrician), and (2) the industry structure is stable over the forecast horizon.

[20] This formulation nests a number of widely used specifications. For $\chi \to +\infty$, the inputs are perfect substitutes and the production function is linear; for $\chi \to 0$, there is no substitution and the production function is Leontieff; for $\chi = 1$, we have a Cobb-Douglas. The empirical literature suggests as plausible a range $\chi \in (0.5, 1]$ (e.g., Berndt (1976), Oberfield and Raval (2021)), implying $\xi \in (-1, 0]$.

[21] A TFP shock would be isomorphic to input price shocks in the same direction.

$\rho_{1,2}$, where $\pi_i = \log p_i$ for $i = 1, 2$.[22]

At time t, a forecaster issues a vector of forecasts, $\mathbf{F}_t = (Fy_{t+1},\, Fx_{1,t+1},\, Fx_{2,t+1})$, over the elements of $\mathbf{x}_{t+1} = (y_{t+1},\, x_{1,t+1},\, x_{2,t+1})$, by minimizing square loss,

$$\left(\min_{Fy_t} \mathbb{E}\left[(y_{t+1} - Fy_t)^2 \,|\Omega_t \right], \quad \min_{Fx_{1,t}} \mathbb{E}\left[(x_{1,t+1} - Fx_{1,t})^2 \,|\Omega_t \right], \quad \min_{Fx_{2,t}} \mathbb{E}\left[(x_{2,t+1} - Fx_{2,t})^2 \,|\Omega_t \right] \right),$$

where Ω_t denotes the information set at t, which includes knowledge of the production function in (1). In words, the forecaster minimizes expected inaccuracy subject to a coherence constraint embedded in Ω_t. At solution, $Fn_t^* = \mathbb{E}[n_{t+1}|\Omega_t] \equiv \mathbb{E}_t[n_{t+1}]$ for $n \in \{y, x_1, x_2\}$). Hence, this formulation nests rules (R1)-(R5).

A.1 Optimal Forecasts

Proposition 1 (Inequality). *Forecast coherence requires that the forecasts of output and inputs, $\mathbb{E}_t[y_{t+1}]$, $\mathbb{E}_t[x_{1,t+1}]$, and $\mathbb{E}_t[x_{2,t+1}]$, satisfy an inequality, whose direction depends on whether the CES production function is concave or convex. For $\xi \leq 1$ and $a + b \leq 1$, the CES function is concave and forecast coherence requires*

$$\mathbb{E}_t[y_{t+1}] \leq f\left(\mathbb{E}_t[x_{1,t+1}], \mathbb{E}_t[x_{2,t+1}]\right) = \left(\frac{a}{a+b} \mathbb{E}_t[x_{1,t+1}]^{\xi} + \frac{b}{a+b} \mathbb{E}_t[x_{2,t+1}]^{\xi} \right)^{\frac{a+b}{\xi}}. \qquad (2)$$

For $\xi \geq 1$ and $a + b \geq 1$, the CES function is convex and the inequality swaps sign.

All Proofs are in the Web Appendix.

Proposition 1 provides a first restriction, in the form of an inequality condition, ex ante coherent forecasts of output and inputs should satisfy. This inequality can be implemented empirically. We provide an illustration in the Web Appendix. Relative to some of the restrictions and test statistics involving forecast errors we derive in Section V, the inequality condition of Proposition 1 has two appealing features. First, it applies to a fairly general class of production functions. Second, it requires data on forecasts only. However, a credible empirical implementation of this inequality would require firm-specific estimates of the technology parameters our data does not allow.

[22] These assumptions are empirically plausible, as we implement most of our analysis over 2001-2007 at the peak of the 'great moderation,' when aggregate volatility was not a concern and prices were stable.

In general, the CES is a non-linear function of the inputs; whereas the rules of thumb (R1)-(R5) are linear. As one of our objectives is to assess whether we can rationalize any of these rules, at least in a second-best sense, we now consider the case of $\xi \to 0$, corresponding to a Cobb-Douglas technology, $y = x_1^a \cdot x_2^b$.

Corollary 1 (Cobb-Douglas). *In the limit case in which $\xi \to 0$,*

$$\mathbb{E}_t \log [y_{t+1}] = a \cdot \mathbb{E}_t \log [x_{1,t+1}] + b \cdot \mathbb{E}_t \log [x_{2,t+1}] \, .$$

Similarly,

$$\mathbb{E}_t \log \left[\frac{y_{t+1}}{y_t} \right] = a \cdot \mathbb{E}_t \log \left[\frac{x_{1,t+1}}{x_{1,t}} \right] + b \cdot \mathbb{E}_t \log \left[\frac{x_{2,t+1}}{x_{2,t}} \right] .$$

Because the Cobb Douglas is linear in logarithms, the coherence requirement holds with equality, both for forecasts expressed in levels and in growth rates.

A.2 Optimal Forecasts and Rules of Thumb

We now consider the forecasting problem in the case a and b are unknown to the forecaster.

Proposition 2 (Unknown parameters). *If parameters a and b are unknown, a forecaster can estimate them using a linear projection operator, with the forecasted variables in logarithms.*

Corollary 2 (Rationalizing R5). *In a multivariate linear projection, $\mathbb{E}_t \log [x_{1,t+1}] = \alpha + \beta_1 \cdot \mathbb{E}_t \log [y_{t+1}] + \beta_2 \cdot \mathbb{E}_t \log [x_{2,t+1}]$, the parameters are*

$$\alpha = \mu_1 - \frac{1}{a}\mu_y + \frac{b}{a}\mu_2 = 0, \qquad \beta_1 = \frac{1}{a}, \qquad \beta_2 = -\frac{b}{a},$$

where $\mathbb{E}\log [y_{t+1}]=\mu_y$ and $\mathbb{E}\log [x_{i,t+1}]=\mu_i$, for $i = 1, 2$, are the unconditional means. The same result obtains with the variables expressed in growth rates.

Corollary 2 rationalizes forecasts based on a version of (R5) as first-best optimal. However, the managerial education literature defines (R5), as well as (R1)-(R4), as linear functions of growth rates (not in logarithms). Thus, our analysis implies that such linear rules will only be correct up to a first-order Taylor approximation.

Corollary 3 (Incoherent R3). *In a univariate linear projection, $\mathbb{E}_t \log [x_{1,t+1}] = \alpha + \beta \cdot$*

$\mathbb{E}_t \log [y_{t+1}]$, *the parameters are*

$$\alpha = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_1^2}\mu_y, \qquad \beta = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_1^2}.$$

In a univariate linear projection with the variables in growth rates, $\mathbb{E}_t \log \left[\frac{x_{1,t+1}}{x_{1,t}}\right] = \alpha + \beta \cdot \mathbb{E}_t \log \left[\frac{y_{t+1}}{y_t}\right]$*, the parameters are*

$$\alpha = 0, \qquad \beta = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}.$$

Corollary 3 shows that, in general, (R3) yields different forecasts from (R5). Hence, in general, (R3) yields incoherent forecasts.

Corollary 4 (Rationalizing R3). *If* $\rho_{1,2} = 1$ *and* $\sigma_1^2 = \sigma_2^2 = \sigma_{1,2} = \sigma^2$*, then for a linear regression in growth rates,* $\mathbb{E}_t \left[\frac{y_{t+1}}{y_t}\right] = \alpha + \beta \cdot \mathbb{E}_t \left[\frac{x_{i,t+1}}{x_{i,t}}\right] + e_{i,t+1}$*, with* $i = 1, 2$*, we have* $\alpha > 0 \iff 0 < \beta < 1 \iff \nu < 1$*. The same is true for i.i.d. shocks, setting* $\gamma_i = 0 \ \forall i$*.*

Corollary 4 shows that (R3) can be rationalized as optimal only in the very special case in which input prices are perfectly correlated and, thus, there is no added benefit of using a multivariate rule over a univariate one. Corollary 4 further reveals that rule (R2), which amounts to setting $\mathbb{E}_t \left[\frac{x_{i,t+1}}{x_{i,t}}\right] = \mathbb{E}_t \left[\frac{y_{t+1}}{y_t}\right]$, is optimal when $\alpha = 0$ and $\beta = 1$, which occurs under constant returns to scale ($\nu = 1$). Then, (R2) and (R3) coincide with each other. If returns to scale are not constant, (R2) is inferior to (R3). More generally, when $\rho_{1,2} \in (-1, 1)$ and $\sigma_1^2 \neq \sigma_2^2$, both (R3) and (R2) yield incoherent forecasts and the forecaster would do better by using information on the output and all inputs.

Finally, (R1) amounts to setting the forecast of $x_{i,t+1}$ equal to the average of k past growth rates, $\log F_{i,t}^{R1} = \frac{1}{k}\sum_{j=1}^{k} \log \frac{x_{i,t+1-j}}{x_{i,t-j}}$ (e.g., Welch (2017) uses $k = 2$). We establish:

Corollary 5 (Expected Loss Under R1). *Under (R1) and* $k \to +\infty$, $\mathbb{E}_t \left[L_{t+1}^{R1}\right] = \mathbb{E}_t \left[L_{t+1}^{o}\right] + \left[(1-\gamma_i)\,\pi_{i,t}\right]^2 > \mathbb{E}_t \left[L_{t+1}^{o}\right]$ *for* $\gamma_i < 1$*, where* $\mathbb{E}_t \left[L_{t+1}^{R1}\right]$ *and* $\mathbb{E}_t \left[L_{t+1}^{o}\right]$ *denote the expected losses under (R1) and the optimal forecast, respectively.*

Corollary 5 implies that (R1) is optimal if and only if one uses a lot of data ($k \to +\infty$) and the input price follows a random walk ($\gamma_i = 1$), otherwise it is strictly inferior.

To sum up, to produce optimal forecasts a forecaster should be a broad bracketer, that is, s/he should take the structure of the firm's problem into account and use data on the output and all inputs. Under narrow bracketing, the forecaster ignores the problem's structure and, when forecasting x_i, disregards data on $x_{\neg i}$ and/or y.

In reality, narrow bracketing may be a second-best optimal response to imperfect information. Moreover, CFOs may issue forecasts between the extremes of broad and narrow bracketing, for instance, as they may be better informed about certain inputs relative to others. Next, we present a framework capturing these possibilities.

B. A Model of Bracketing in Corporate Forecasts

Building on Lian (2021), we now introduce noisy signals and recast the multidimensional forecasting problem as multiple selves playing an incomplete-information, common-interest game. With two inputs, capital and labor, the CFO "capital self" makes forecasts of capital expenditures' growth by observing imprecise signals of output and labor growth. Conversely, the CFO "labor self" makes forecasts of labor expenditures' growth by observing imprecise signals of output and capital growth. In equilibrium, each self does not perfectly know other selves' signals (states of mind) and, thus, makes forecasts with imperfect knowledge of other selves' forecasts. In this sense, narrow thinking reflects intra-personal frictions in coordinating forecasts of different variables.[23]

Consider a CFO's self making a forecast about the first input, $F \log x_1$, by

$$\min_{F \log x_1} \mathbb{E}\left[(\log x_1 - F \log x_1)^2 \,|\Omega\right].$$

The forecaster observes two noisy signals, one about the output, η_y, and one about the second input, η_2.[24] Specifically, $\eta_y = \log y + \epsilon_y$ and $\eta_2 = \log x_2 + \epsilon_2$, where $\epsilon_y \sim \mathcal{N}\left(\mu_y, s_y^2\right)$, $\epsilon_2 \sim \mathcal{N}\left(\mu_2, s_2^2\right)$, and $y = x_1^a \cdot x_2^b$.

Proposition 3 (Optimal forecasts). *The optimal forecast of input x_1 given signals η_y*

[23] In contrast to other models where multiple selves have conflicting interests (e.g., Piccione and Rubinstein (1997), Benabou and Tirole (2002)), in Lian (2021)'s model and ours multiple selves have common interests. Yet, because different selves do not share information with one another, they have difficulty in coordinating their decisions (forecasts).

[24] This is wlog, as inputs can be relabelled $(i, \neg i)$. Also, by stationarity we drop time subscripts.

and η_2 *is*

$$\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y\left(\eta_y - \mu_y\right) + \beta_2\left(\eta_2 - \mu_2\right),$$

where

$$\beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2 + s_2^2}}, \beta_2 = \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)}.$$

Proposition 3 shows that, given the signals η_y and η_2, the optimal forecast of the first input is provided by a linear projection of (the deviation of the signals from the prior means of) the output and second input. The intercept is the prior mean of the first input and the slope coefficients are functions of the fundamental uncertainty and signal precisions. Proposition 3 rationalizes a version of (R5) also as second-best optimal and clarifies that the accuracy of the linear projection depends on the signals' precision.

Next, we examine a number of special cases.

Corollary 6 (Narrow Bracketing). *When* s_y^2, $s_2^2 \to +\infty$, *the optimal forecast is* $\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1$.

When all signals are infinitely noisy, the optimal forecast projects the prior mean of the input being forecasted into the future. Thus, Corollary 6 provides conditions under which (R1) is second-best optimal.

Corollary 7 (Univariate Projections). *When* $s_2^2 \to +\infty$ *and* $0 < s_y^2 < +\infty$, *the optimal forecast is* $\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y\left(\eta_y - \mu_y\right)$, *where* $\beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}$.

When the signal of the second input is infinitely noisy and the signal of the output is noisy but informative, the optimal forecast is a univariate linear projection of (the deviation from the prior mean of) the output, where the intercept is still the prior mean of the first input and the slope is a function of the fundamental uncertainty and the precision of the output's signal. Thus, Corollary 7 provides conditions under which (R3) is second-best optimal. Because nothing necessarily implies $\mu_1 = 0$ and $\beta_y = 1$, (R2) is generally inferior to (R3) even in a second-best world.

The ranking between (R3) and (R4) is ambiguous. (R4) may be better, as a and b may be industry specific. However, (R4) uses a smaller sample, which may hurt the performance of the linear projection. We will evaluate these possibilities empirically.

Finally, we consider the case in which both signals are infinitely precise.

Corollary 8 (Precise Signals). *When* $s_y^2, s_2^2 \to 0$, *the optimal forecast is* $\mathbb{E}[\log x_1 | \eta_y, \eta_2] = \frac{1}{a}(\eta_y - b\eta_2)$.

As incoherence among forecasts—and, hence, in firm plans—induces suboptimal resource allocations to inputs, our theory predicts that firm profits will decrease as the deviation of the actual forecasts from the optimal ones increases. Furthermore, our theory yields a partial ordering of the managerial RoTs, (R5) $\succeq$ (R4)-(R3) $\succeq$ (R2)-(R1), and proposes a mechanism through which incoherence may arise: the use of (R1) and (R2). In the next sections, we introduce the data and evaluate empirically these predictions.

III Data

We use two main sources of data, one on CFO expectations and one on firm realizations. CFO expectations come from the Duke Survey, run by John Graham and Campbell Harvey. The study surveys 2,000 to 3,000 CFOs quarterly, asking their views about the US economy and corporate policies, as well as their expectations of future firm performance and operational plans. The typical response rate is 5% to 8%, with most responses arriving within a couple of days. Since the end of the 1990s, the survey has consistently asked respondents their expectations of the future twelve-month growth of key firm variables, including revenues, capital expenditures, employment, and earnings.[25]

Our data comprises 72 quarterly surveys conducted between March 2001 and December 2018. We observe corporate forecasts as a single number per variable, which we interpret as the CFO's expected value, corresponding to the firm's base case scenario.[26]

Forecasts are elicited for all variables jointly as follows:

Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, −2%, etc.) [Leave blank if not applicable] Revenues: _____; Capital spending: _____; R&D spending: _____; Technology spending: _____; Prices of your product: _____; Earnings: _____; Cash on balance sheet: _____; Number of domestic full-time employees: _____; Wage: _____; Dividends: _____. Advertising: _____. Share repurchases: _____.

Figure A1 of the Web Appendix shows a screenshot from the survey. It is worth noting

[25] Historical surveys and aggregated responses can be accessed at https://cfosurvey.fuqua.duke.edu/.

[26] For many firms, this is the only scenario leading to fleshed-out forecasts in internal planning. Some firms consider also a downside scenario to plan for contingencies and an upside scenario to lay out stretch goals. However, the latter do not usually lead to fleshed-out forecasts (see Graham (2022), p. 1997).

that by eliciting all forecasts jointly, the question's format might aid coherence relative to alternative formats eliciting individual forecasts in separate questions.

Firm realizations come from Compustat, which extracts the information from the Security and Exchange Commission (SEC)-required public filing of financial statements. Compustat covers all publicly traded firms across all sectors of the US economy since 1955. We exclude firms with negative assets and we winsorize at the 1% level.

Table A1 of the Web Appendix shows that, on average, Duke firms are larger in sales and assets, more profitable, and hoard more cash than Compustat firms, but are otherwise similar in terms of market-to-book ratio, investment, and leverage. These patterns concur with prior work using the Duke data (e.g., Ben-David et al. (2013)).

When matching Duke and Compustat by firm ID, we face four sources of attrition: (1) due to privacy restrictions, not all Duke respondents report their firm ID; (2) not all Duke respondents give forecasts about all variables; (3) some of the variables whose forecasts are elicited in the Duke Survey do not have a precise counterpart in Compustat, namely, technology spending, outsourced employees, health spending, productivity, product prices, and share repurchases; and (4) some of the variables with a counterpart in Compustat do not have full coverage. Chiefly, wages are missing for about 90% of Compustat firms. R&D and advertising expenditures are also missing for many Compustat firms.[27]

Table A2 of the Web Appendix reports summary statistics on twelve-month ahead growth forecasts (Panel A), followed by the corresponding realizations (Panel B), in the matched Duke-Compustat sample.[28] Comparing the two panels shows that while CFOs are on average slightly optimistic about sale revenues (output), the empirical medians of forecasts and realizations are quite close to each other, consistent with Graham (2022)'s observation that CFOs care about getting revenues forecasts right. Conversely, CFOs' forecast of capital expenditures are systematically too low, with the distribution of

[27]Points (1) and (2) raise the possibility of selection. However, under the assumption that missing forecasts reflect lack of knowledge, our respondents are likely positively selected from those more likely to provide complete and high quality answers. Points (3) and (4) imply that our analysis of FEs needs to be limited to items with full coverage in both Duke and Compustat. Finally, the matched Duke-Compustat sample mostly refers to the first part of the sample period, until 2011Q4. This is not a problem for us, as we conduct most of our regression analysis in the pre-financial crisis period.

[28]For some firm-year pairs there might be multiple CFO forecasts, as we match Duke and Compustat by firm ID. Table A3 of the Web Appendix reports the same statistics in the full Compustat population.

forecasts shifted to the left relative to the distribution of realizations.

IV Empirical Analysis

A. Rules of Thumb Indicators and Ex Ante Incoherence

We now use our data to study CFOs' use of heuristics in producing forecasts for internal planning. In the main text, we focus on forecasts of sale revenues (output) and capital expenditures (capital input), as they have a clear mapping with theory and we observe their realizations. In the Web Appendix, we report evidence on other items.

We consider the five RoTs described in Section II and implement each of them following Welch (2017). Table 1 reports the results.

The first row of the table implements (R1). The estimated coefficient is -0.089, precisely estimated, indicating mean reversion in capital expenditures (hence, CapEx) growth. That is, under (R1), a firm experiencing high CapEx growth in the past two years should set a low one-year ahead forecast of CapEx growth, regardless of any goal for contemporaneous sale revenues' growth, or other inputs' growth patterns.

The second row implements (R3) by estimating the univariate regression $\text{CapEx Growth} = \alpha + \beta \cdot \text{Sales Growth} + \varepsilon$. We find $\widehat{\alpha}$ =0.217 and $\widehat{\beta}$ =1.055, precisely estimated. Thus, a firm wishing to achieve a 5% sales growth rate should under (R3) forecast a CapEx growth rate of 1.055 x 5% + 21.7% = 26.7%. Importantly, these estimates imply that (R2), which assumes α =0 and β =1, provides much lower CapEx growth forecasts than (R3), which uses actual data. Indeed, the same firm wishing to achieve a 5% sales growth rate should under (R2) forecast a CapEx growth rate of only 5%. The discrepancy between the two rules is driven by the intercept: equal to zero according to (R2), positive and large according to (R3).[29]

The subsequent ten rows implement (R4) by estimating the same regression

[29] The estimated slope of 1.055 under (R3) is statistically indistinguishable from the value of 1 assumed by (R2). Relative to (R3), (R2) leads to underpredicting the growth rates of other inputs as well, although by a smaller difference. For example, a firm aiming at a 5% sales growth should under (R3) forecast a 12.7% advertising growth and a 7.7% wage growth.

separately for each of ten industries, defined by one-digit SIC codes.[30] In all ten industries the intercept is statistically significantly larger than 0, ranging from 0.163 to 0.402. Furthermore, the slope of sales ranges from 0.859 to 2.050. These results confirm that in all industries using (R2) implies setting a (much) lower CapEx growth forecast relative to sales growth than observed in the data.

Finally, the last row reports results from a multivariate regression designed to describe the implementation of a version of (R5). Welch (2017) does not make an explicit recommendation on which regressors to include, ultimately illustrating (R5) with a 'kitchen sink' specification including all Compustat variables. Our model indicates that the optimal forecasting rule for input j should be a multivariate linear regression including the firm's output and the remaining $n-1$ inputs. Regrettably, as discussed in Section III above, data limitations imply we need to restrict our attention to variables for which we observe both the CFO forecast in Duke and corresponding realization in Compustat, with enough coverage on both to allow for sufficient statistical power. In the main body of the paper, we focus on the following specification:

$$\text{CapEx Growth} = \alpha + \beta \cdot \text{Sales Growth} + \theta \cdot \text{Earnings Growth} + \varepsilon.$$

Earnings corresponds in our model to $f(x_1, ..., x_n - 1, K) - \sum_{j=1}^{n-1} p_j x_j$, and as such contains information on the $n-1$ inputs besides CapEx. Furthermore, the Earnings ('bottom line') forecast is widely reported by CFOs, allowing for a sizable sample.

We now try to infer the rule used by each CFO in two steps. First, we compute five values, each equal to the orthogonal distance between the CFO's forecast and that implied by each RoT. Second, we compute the minimum among the five values. For each CFO, i, we refer to the distance-minimizing RoT, $\tau^i \in \{1, ..., 5\}$, as to the CFO's 'type.'[31]

Table 2 shows that, among the 396 CFOs for whom we observe the identity and the

[30] Corresponding to the following sectors: Agriculture, forestry, and fishing; Mining; Construction; Manufacturing; Transportation, communications, and public utilities; Wholesale trade; Retail trade; Finance, insurance, real estate; Services; and Public administration.

[31] To focus on coherence and minimize confounding with accuracy, we compute the distance relative to the sales forecast, as CFOs are more accurate at forecasting sales than any other item. In the Web Appendix, we present a number of robustness checks. Table A5 finds similar results when implementing (R5) using advertising instead of earnings. Tables A6 and A7 report the distributions of RoTs, when the distance is measured relative to earnings and CapEx, respectively.

joint forecasts and realizations of all variables, a plurality of about 40% makes a forecast that is closest to that implied the sales anchoring rule, (R2), 27% of whom use exactly (R2). Another 7.6% of CFOs are closest to the narrow bracketing rule, (R1). Thus, about 48% of CFOs in our sample appear to use an incoherent RoT. With regards to the remaining rules, we find that about 11% of CFOs make forecasts closest to (R3), 27% to (R4), and 15% to (R5). These results underscore the large heterogeneity in forecasting rules used by CFOs, reflecting the twin facts that making coherent forecasts is challenging and there is no clear consensus on what rule(s) should be recommended in what case(s).

In Table 2, the average minimum distance of 0.033 is reasonably small, while masking substantial heterogeneity as revealed by a standard deviation of 0.059. Small discrepancies between actual and minimum-distance forecasts are not concerning, as they likely reflect rounding or truncation in CFOs' reports and/or small differences in rule implementation between CFOs and us (e.g., the use of a different number of past observations for (R1) or of a different sample size for (R3)-(R5)). Larger discrepancies raise the possibility that the CFOs used a different method altogether. To investigate this potential issue, we compute the fraction of CFOs for whom the distance between their actual forecast and the minimum-distance one falls within a pre-specified interval. We find that the minimum-distance forecast falls within a $+/-0.05$ interval around the actual forecast for more than 80% of CFOs and within a $+/-0.025$ interval for close to 60% of CFOs. By way of comparison, the standard deviation of sales growth forecasts is 0.271 (see Panel A of Table A2). Hence, the widths of the $+/-0.05$ and $+/-0.025$ intervals are about 1/3 and 1/6, respectively, of that standard deviation. These figures suggest that our classification is plausible and CFOs most likely use the rule we assign to them.

To account for the possibility that some CFOs use a different method, we consider an alternative categorization in which we assign a unique rule among (R1)-(R5) only to those CFOs for whom the minimum-distance forecast falls within a $+/-0.05$ interval around their forecast and we combine the remaining 18.7% CFOs in a residual category, (R6). Below, we use this alternative categorization in robustness analyses.

For each CFO, we now compute our ex ante measure of incoherence at t, as the orthogonal distance between the three-dimensional vector of the CFO's forecasts at t

of the sales growth, CapEx growth, and an additional item at $t+1$, and the (R5)'s hyperplane. We implement (R5) via a multivariate regression of sales growth ($y_{i,t}$) on CapEx growth ($x_{1i,t}$) and an additional regressor ($x_{2i,t}$),

$$y_{i,t} = \beta_0 + \beta_1 x_{1i,t} + \beta_2 x_{2i,t} + \varepsilon_{i,t}, \tag{3}$$

where i indexes interchangeably CFOs and firms. Specifically, we define

$$\text{Incoherence}_{i,t} = \frac{\left| F_{i,t}\left[y_{i,t+1}\right] - \widehat{\beta}_1 F_{i,t}\left[x_{1i,t+1}\right] - \widehat{\beta}_2 F_{i,t}\left[x_{2i,t+1}\right] - \widehat{\beta}_0 \right|}{\sqrt{1^2 + \widehat{\beta}_1^{\,2} + \widehat{\beta}_2^{\,2}}}, \tag{4}$$

where $\widehat{\beta}_0$, $\widehat{\beta}_1$, $\widehat{\beta}_2$ are the estimated coefficients of (3).

When we use earnings growth as our $x_{2i,t}$ (i.e., net income, as in the bottom row of Table 1), we end up with 396 CFOs in our final sample. When we use advertising growth (as in column 8 of Table A4 in the Web Appendix), we obtain similar results, although based on 130 CFOs only. Using wages or R&D as a proxy for $x_{2i,t}$ results in even smaller samples of 50 CFOs or less; similarly, using more than two regressors to implement (R5) results in yet fewer observations. Although when doing these latter exercises we find broadly similar patterns, sample sizes are too small to allow for an informative comparison. However, rather than the specific variable used as our $x_{2i,t}$, what turns out to be important is moving from a univariate regression of sales growth on capital expenditures growth to a multivariate regression including at least an additional regressor. That is, it is important that forecasters take into account the contemporaneous relationship among multiple items.

Our model predicts a pecking order of rules of thumb: (R5) is best (see Corollary 2 and Proposition 1); (R3) and (R4) are intermediate (see Corollaries 4 and 7); (R1) and (R2) are worst (see Corollaries 5 and 6). We evaluate these predictions by regressing the ex ante incoherence measure on binary indicators for the CFO type. Columns 1-4 of Table 3 present estimates of univariate regressions including one dummy at the time, while column 5 presents the full specification where the (R5) group is used as reference. Consistent with our model's predictions, the estimates of column 5 show that (R1) CFOs have the largest incoherence relative to (R5) CFOs, followed by (R2) CFOs. The average differences in incoherence of (R3) and (R4) CFOs relative to (R5) CFOs are much smaller in magnitude and not statistically significant.

Robustness analyses yield similar results. When using advertising instead of earnings to implement (R5), we still find that (R1) is most distant from (R5), followed by (R2), and that both differences are strongly statistically significant (see Table A8 of the Web Appendix). When using the alternative categorization (R1)-(R6) in Table A9, we find as expected that (R6) is the most distant from (R5) and, more remarkably, that the relative ranking of (R1)-(R4) is unchanged. As in the baseline case, only (R1) and (R2) are statistically significantly different from (R5).

Table 4 reports sample characteristics and investigates heterogeneity in ex ante incoherence by CFO and firm characteristics. In Panel A, 45% of CFOs have an MBA and 9% are women. On average, CFOs are 50.4 years old and have been on the job for 4.3 years. Average firm size is 2.5 billion USD of sales and 64% of firms pay a dividend. These figures are in line with those in Ben-David et al. (2013).

Panel B shows that incoherence is lowest at the intermediate age range, 41-50, suggesting that experience may help CFOs form more coherent forecasts and that coherence might decline at older ages. Incoherence is unrelated to optimism or miscalibration, consistent with the idea from psychology that incoherence and overconfidence are different constructs. Having an MBA does not correlate with incoherence, likely reflecting that, on the one hand, some rules of thumb are quite simple and CFOs may come up with them on their own and, on the other hand, there is no consensus in MBA textbooks on which rule is best; although in practice, as shown in Table 3, different rules perform differently in terms of ex ante coherence. Finally, CFOs of larger firms and firms paying dividends display lower incoherence, although the statistical significance is marginal. These results suggest that in stable and more predictable environments it might be easier to make (more) coherent forecasts.[32]

[32]CFO tenure does not correlate with incoherence. This, combined with the fact that longer tenured CFOs delegate less (Graham et al., 2015), suggests that incoherence is unrelated to delegation of authority.

B. Incoherence, Rules of Thumb, and Firm Performance

We now examine our model's main prediction that corporate performance should decrease with managerial incoherence, by estimating the regression,

$$\text{ROA}_{i,j,t} = \alpha + \lambda_j + \delta_t + \beta \cdot \text{Incoherence}_{i,j,t} + \theta \cdot X_{i,j,t} + \varepsilon_{i,j,t},$$

where i indexes CFO-firm pairs, j indexes industries, t indexes time, the dependent variable $\text{ROA}_{i,j,t}$ is the percent return on the firm's assets, λ_j are industry fixed effects, δ_t are survey fixed effects, and $X_{i,j}$ includes firm-level regressors (firm size, market-to-book, and dividends) and CFO-level regressors (miscalibration and optimism, measured at both short- and long-term horizons). Based on our model, we hypothesize $\beta < 0$.

We conduct the estimation over 2001-2007 to avoid the effect of the financial crisis, an aggregate shock which impacted managerial beliefs (Boutros et al., 2020), and to focus on the peak of the 'great moderation,' when aggregate volatility was not a concern. We compute bootstrap standard errors and cluster them at the firm level. Given the lack of exogenous variation in incoherence, the results should be interpreted as correlational.

Table 5 shows the results. In column 1, a 1-standard deviation increase in incoherence (0.079 from Table 3) is associated with a 3-percent lower ROA, significant at the 5% level. Results are stable when we further condition on CFO miscalibration and optimism (columns 2 and 3), industry and survey fixed effects (column 4), and firm-level regressors (columns 5-8). Thus, while sample size changes across columns reflecting availability of regressors, the estimated negative association between CFO incoherence and firm performance remains quite stable in magnitude and statistically significant.

Next, we investigate the extent to which this result reflects the use of different RoTs. To do so, we regress firm performance on CFO-type indicators and other regressors as in the previous analysis. Firms with (R5) CFOs act as a reference group. Table 6 reports our results. Consistent with our model's ranking of the five rules, we find that firms whose CFOs use the narrow-bracketing rule (R1) have a 5%-to-6% lower ROA relative to those whose CFO uses (R5). These are very large differences in economic terms. We also find that firms whose CFOs use the sales-anchoring rule (R2) or the economies-of-scale rule

(R3) have a 2%-3% lower ROA on average than (R5) firms, whereas the performance of firms whose CFOs use (R4) is indistinguishable from that of (R5) firms.

In a robustness analysis using the (R1)-(R6) categorization, we find that firms whose CFOs are classified as (R6) have a 4% lower average ROA and results for (R1)-(R4) firms are very similar to those presented in Table 6. Therefore, CFOs whose forecasts are quite distant from those implied by all the five rules we study are unlikely to use a more sophisticated or 'better' approach, further supporting our focus on Welch (2017)'s taxonomy and our version of (R5) as a benchmark rule. In sum and consistent with our model, firm performance is negatively associated with incoherence and is on average lowest for firms whose CFOs use rules (R1) and (R2).

C. Incoherence, Rules of Thumb, and Corporate Policies

Given the workings of (R1) and (R2) and our empirical results so far, we conjecture that these rules may lead to a suboptimal mix of inputs and, thus, lower firm performance, by inducing a lower planned investment spending growth than needed to achieve the targeted sale revenues growth. In Table 7, we investigate this potential mechanism by estimating

$$\mathrm{Y}_{i,j,t} = \alpha + \lambda_j + \delta_t + \beta \cdot \mathrm{RoT}_{i,j,t} + \theta \cdot X_{i,j,t} + \varepsilon_{i,j,t},$$

where the indexes i, j, and t are defined as in the earlier regression for ROA; $\mathrm{RoT}_{i,j,t}$ is a vector of indicators for (R1)-(R4); and $\mathrm{Y}_{i,j,t}$ is CapEx over assets (columns (1)-(3)), or long-term book debt over assets (columns (4)-(6)).

In columns 1 and 2, use of (R1) and of (R2) is associated with 1.3%-1.6% lower CapEx relative to (R5). The difference is large in economic terms and for (R2) also statistically significant. In columns 4 and 6, (R1) and (R2) are associated with 4% and 9% higher leverage, respectively, relative to (R5). Again, for (R2) the difference is statistically significant. These results continue to hold after conditioning for miscalibration and optimism. This makes sense, as we also confirm Ben-David et al. (2013)'s prior finding that miscalibration and optimism are positively (rather than negatively) associated with investment spending, underscoring that miscalibration and optimism are distinct constructs from incoherence. In sum, use of (R1) and (R2) is associated with lower investment and higher leverage, suggesting that forecast incoherence in internal planning may imply suboptimal investment and financing policies.

D. Change in Firm Behavior when CFOs Take Office

The negative association between performance and incoherence may reflect different possibilities. Higher incoherence may lead to lower investment and performance. Or, lower investment may induce CFOs to make incoherent forecasts, for example, to forecast too high a sales growth. Finally, more incoherent CFOs might be selected by—or self-select into—firms with lower investment spending and worse performance.

To make a step toward causality and shed light on its direction, we study how firm performance, investment, and leverage evolve in the years around a CFO's hiring. We extract the dates when CFOs join firms from Execucomp and Boardex data and supplement this information by hand-collecting data from 10-K filings. A CFO is considered to take office in a firm when he or she first signs the financial reports. We match performance, investment, leverage, and characteristics from Compustat for the year of taking office. We use as dependent variables the within-firm differences in ROA, investment, and leverage between the two years after and prior the CFO's taking office.

Table 8 presents the results. Performance declines following the appointment of incoherent CFOs, especially those using (R1). (R1) is also associated with a 2.2% lower investment intensity relative to an average of 4.5 percentage points.

While not unambiguously causal, these results are consistent with the cross-sectional ones as well as with the predictions of our model.

V Statistical Tests of Coherence Vis-à-Vis Accuracy

So far, we have focused on an ex ante notion of coherence and have made limited use of the assumptions of Section II. Next, we combine forecasts and realizations and show how forecast errors can help disentangle (in)coherence from (in)accuracy ex post. To do so, we make greater use of the assumptions of Section II. This enables us to derive novel tests of coherence and shed light on potential alternative interpretations of our findings.

A. Why Forecast Errors

We begin by considering an empirical formulation of a generic production function,

$$y_{t+1}^{f} = \alpha + \alpha^{f} + \sum_{i=1}^{n} \beta_{i}^{f} x_{i,t+1}^{f} + \sum_{i=1}^{n} \sum_{s=0}^{t} \delta_{i,t-s}^{f} x_{i,t-s}^{f} + \sum_{j=1}^{m} \sum_{s=0}^{t} \gamma_{j,t-s}^{f} z_{j,t-s}^{f} + \varepsilon_{t+1}^{f}, \quad (5)$$

where f indexes firms, i inputs, t time, s lags, and j any relevant state variable (e.g., inventory, cash, etc.). The loadings of inputs and state variables, $\delta_{i,t-s}^{f}$ and $\gamma_{j,t-s}^{f}$, may vary across firms and over time, allowing for a wide array of lead-lag relationships. For example, firms in short-term industries will load more on inputs and state variables in recent years, whereas firms in long-term industries will load more on inputs and state variables in more distant years. The variables could be in levels, growth rates, logarithms, or any other transformation that preserves linearity in parameters.

Coherent forecasts of firm-specific variables should be cross-sectionally related as are their realizations in equation (5), that is,

$$\mathbb{E}_{t}\left[y_{t+1}^{f}\right] = \alpha + \alpha^{f} + \sum_{i=1}^{n} \beta_{i}^{f} \mathbb{E}_{t}\left[x_{i,t+1}^{f}\right] + \sum_{i=1}^{n} \sum_{s=0}^{t} \delta_{i,t-s}^{f} x_{i,t-s}^{f} + \sum_{j=1}^{m} \sum_{s=0}^{t} \gamma_{j,t-s}^{f} z_{j,t-s}^{f}. \quad (6)$$

Subtracting (6) from (5) *within firms* yields

$$\mathrm{FE}_{t}\left[y_{t+1}^{f}\right] \equiv y_{t+1}^{f} - \mathbb{E}_{t}\left[y_{t+1}^{f}\right] = \sum_{i=1}^{n} \beta_{i}^{f} \mathrm{FE}_{t}\left[x_{i,t+1}^{f}\right] + \varepsilon_{t+1}^{f}, \quad (7)$$

further implying that the FEs associated with coherent forecasts of output and inputs should also be cross-sectionally linked by the technology parameters on contemporaneous inputs, β_{i}^{f}. Differently from the forecasts equation (6), the FEs equation (7) does not depend on any firm-level heterogeneity known to, or predictable by, the CFO at the time of forecast—but likely unobserved to the econometrician—as such heterogeneity gets differenced away when subtracting the forecast from the realization. This result holds for any production function admitting a separable representation with respect to its inputs, possibly after log-linearization, as in (5).

This discussion suggests two intuitive restrictions on FEs of contemporaneous inputs and output. First, as long as inputs contribute positively to output, that is, input loadings are positive, one would expect FEs of output and each input to be positively associated. Second, as long as no individual input has increasing returns, that is, input loadings are not larger than one, one would expect forecast errors to lie below the 45 degree line. Together, these restrictions imply the shaded areas in Figure 1.

Figure 2 plots the joint distribution of FEs of Sales and CapEx.[33] While the regression slope is positive as expected, 42% of CFOs have FEs of Sales and CapEx of opposite sign, violating the first restriction of positive loadings. An additional 10.4% of FE pairs imply an input loading larger than one, thus violating the second restriction. Therefore, more than 50% of CFOs report contemporaneous FEs that violate intuitive coherence bounds. We find similar results in all ten industries, consistent with the idea that examining contemporaneous FEs is informative about coherence irrespective of the lead-lag relationships or the short- vs- long-term outlooks of different industries. We also find similar results for additional FE pairs, including sales and earnings.

Building on the intuition introduced in this subsection, we now derive more formal statistical tests of coherence vis-à-vis accuracy. We present regression tests in Subsection V.*B.* and individual-level tests in Subsection V.*C.*

B. *Regression Tests*

Besides suggesting coherence restrictions, equation (7) provides an intuition about the connection between forecast accuracy and forecast coherence, and how to distinguish between the two. Specifically, forecast accuracy requires that for each variable being forecasted the FEs are on average zero, Avg $\text{FE}_t\left[y^f_{t+1}\right] = 0$ and Avg $\text{FE}_t\left[x^f_{i,t+1}\right] = 0$ for each input i. Forecast coherence across variables requires that the FEs of contemporaneous inputs and output are "close to one another" in the sense of (7).

[33] In our model, the first input is the stock of physical capital, K, typically measured by the balance-sheet item Property, Plant, and Equipment; whereas, in our empirical implementation, we use balance-sheet item CapEx, I. In the Web Appendix, we show that the FE of CapEx growth equals the FE of capital growth under general conditions. The intuition is again that computing FEs differences away any heterogeneity known or predictable at the time of forecast.

These observations suggest regression-based tests of accuracy and coherence. Accuracy can be assessed by testing for each variable separately that the mean of the FEs is zero.[34] Coherence can be tested by regressing FEs of output on FEs of inputs, as per (7). Under rationality, one expects the slope of the FE of each input to equal the loadings of that input in (5).

C. Individual-Level Tests

We assume an AR(1) process for the log of input prices, that is, $\pi_{i,t+1} = \gamma_i \pi_{i,t} + \epsilon_{i,t+1}$, with $0 < \gamma_i < 1$ ($\gamma_i = 0$ denotes the i.i.d. case), where the error terms are i.i.d., normally distributed, and uncorrelated, $\{\epsilon_{1,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_1^2)$, $\{\epsilon_{2,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_2^2)$, and $\{\epsilon_{1,t}\}_{t\geq 1} \perp \{\epsilon_{2,t}\}_{t\geq 1}$. This assumption provides a way to capture different conditions and uncertainty firms face, which cannot be directly assessed from the point forecasts observed in our data. We further assume $\xi \to 0$.

Proposition 4 (Individual-Level Test Statistics). *If $\xi \to 0$ and $\rho_{1,2} = 0$, under the null hypothesis of coherent forecasts, the following holds:*

$$\text{C1-stat} \equiv \frac{\frac{\mathbb{E}_t \log y_{t+1} - a \mathbb{E}_t \log x_{1,t+1}}{b} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1) \tag{8}$$

and

$$\text{C2-stat} \equiv \frac{\text{FE}_t \log y_{t+1} - a\text{FE}_t \log x_{1,t+1}}{\sigma_2 b} \sim \mathcal{N}(0,1), \tag{9}$$

where $\text{FE}_t \log y_{t+1} = \log y_{t+1} - \mathbb{E}_t \log y_{t+1}$ *and* $\text{FE}_t \log x_{1,t+1} = \log x_{1,t+1} - \mathbb{E}_t \log x_{1,t+1}$.

The two statistics derived in Proposition 4 have intuitive interpretations. Under the null of coherence, the forecasts of output and one input cannot be "too far" from each other in the sense of (8). Similarly, the FEs of output and one input cannot be "too far" from each other in the sense of (9).[35] Proposition 4 is written for the 2-input case, as it is

[34]It can also be tested by regressing realizations on forecasts for each variable. Under rationality, one expects a unit slope and, under no aggregate shocks, a zero intercept. There is a long tradition in economics of rational expectations tests of this type (e.g., Muth (1961), Lovell (1986), Pesaran and Weale (2006), D'Haultfoeuille et al. (2021), Crossley et al. (2024), and Giustinelli and Shapiro (2024)).

[35]Relative to Subsection V.*A*., Proposition 4 clarifies that output-input FEs of opposite sign do not necessarily imply incoherence, as long as they are sufficiently close to each other. On the other hand, FEs of the same sign can reflect incoherence, if they are sufficiently far from each other.

the one we implement empirically below. In the Web Appendix, we also provide a proof for the n-input case. Relative to (7), whose implementation requires observing FEs on all n inputs, implementation of Proposition 4 only requires observing FEs for $n-1$ inputs. This is an implication of the $\xi \to 0$ assumption.

Under the assumptions of Proposition 4, forecast accuracy for a generic variable x, corresponding to the output or any of the input, can be tested using $\frac{\mathrm{FE}_t \log x_{t+1}}{\sigma_x} \sim \mathcal{N}(0,1)$. Implementation of this test along with the test in (9) enables us to empirically distinguish forecast accuracy from forecast coherence.

Figure 3 illustrates the theoretical connection between the concepts of forecast accuracy and forecast coherence for a firm whose input has a loading of 1/3. There are four conceptual cases, corresponding to the four areas of the figure. In the first area, the forecasts are both accurate and coherent, as both $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1}$ and $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y}$ are close to zero and also close to each other. In the second area, the forecasts are inaccurate but coherent, as $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1}$, $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y}$, or both are statistically different from zero but quite close to each other. In the third area, the forecasts are accurate but incoherent, as both $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1}$ and $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y}$ are close to zero but sufficiently apart from each other. In the fourth area, the forecasts are both inaccurate and incoherent, as $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1}$, $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y}$, or both are statistically different from zero and also far apart from each other.

D. Test Implementation and Results

Regression tests. To implement the regression tests from subsection V.B., we need $\mathrm{FE}_t \log x_{1,t+1}$ and $\mathrm{FE}_t \log y_{t+1}$, which are not directly observed in our data as the forecasts were not elicited in logs. To address this issue, we use the following transformation:

$$\mathbb{E}_t \log x_{t+1} = \log \mathbb{E}_t x_{t+1} - \frac{1}{2} V_t \log x_{t+1}, \tag{10}$$

written for a generic variable x and where $V_t \log x_{t+1}$ is the conditional variance of $\log x$. Therefore, $V_t \log x_{i,t+1} = \sigma_i^2 = (1-\gamma_i^2)\, V \log x_{i,t+1}$ for $i = 1, 2$, where $V \log x_{i,t+1}$ is the unconditional variance and γ_i is the coefficient of an AR(1) regression of $\log x_{i,t}$. The conditional variance of the output is then $V_t \log y_{t+1} = a^2\sigma_1^2 + b^2\sigma_2^2$. We compute a and b

using the universe of industries from the Bureau of Economic Analysis.

Table 9 reports the results. To test for accuracy, we regress $\text{FE}_t \log x_{1,t+1}$ on a constant (column 1) and $\text{FE}_t \log y_{t+1}$ on a constant (column 2). In column 1, the constant is statistically different from zero at the 10% level; whereas in column 2 the constant is not statistically different from zero. Hence, CFO forecasts are on average inaccurate about CapEx growth and accurate about sales revenue growth. The latter is consistent with Graham (2022)'s observation that top executives care the most about sales forecasts.

To test for coherence, we first estimate the regression,

$$\text{FE}_t \log y_{t+1} = \alpha + \beta_1 \cdot \text{FE}_t \log x_{1,t+1} + \beta_2 \cdot \text{FE}_t \log x_{2,t+1} + \varepsilon_{t+1}, \tag{11}$$

based on equation (7), whereby under forecast coherence one would expect α=0, β_1=a, and β_2=b. In column 3, $\widehat{\alpha}$ =0.46, statistically different from 0 at the 5% level. $\widehat{\beta_1}$ =0.113, marginally significant at the 10% level and significantly smaller than the capital share of 0.4 at all significance levels. And $\widehat{\beta_2}$ =0.023, not statistically different from zero and thus smaller than the labor share. Hence, we reject the null of coherence.

The coherence test in column 3 requires FEs on wages, implying that we could perform it on 51 observations only. To address this limitation, we take advantage of Proposition 4, which suggests a coherence test based on (7) using FEs on $n-1$ inputs only. In column 4, we implement the regression test without wages. We now find $\widehat{\alpha}$, not statistically different from zero, and $\widehat{\beta_1}$ =0.135, precisely estimated and significantly smaller than the capital share of 0.4. In fact, we reject the null for any estimate of the capital share of 0.3 or above. When we allow the capital share to vary by industry, we reject the null of coherence for 7 out of 13 industries, representing 86% of the total observations (not shown). Furthermore, we find $\widehat{\beta_2}$ to be not statistically different from zero. For completeness, in column 5 we estimate (7) without CapEx and find the coefficient on wages to be statistically insignificant.

In sum, CFO forecasts are on average accurate for sales, inaccurate for CapEx, and incoherent across the two. These results indicate that accuracy and coherence are not only distinct concepts in theory but also in our data. We now dig deeper in this direction by implementing our individual-level tests, which aim to disentangle (in)coherence and (in)accuracy at the individual level.

Individual-level tests. We test for coherence using the C2-stat, as it does not require FEs on the second input (wages) or other firm-level characteristics hard to observe (Z^*). For each CFO, we observe four items (two forecasts and two realizations) and we estimate three parameters (a, b, and σ_2) using aggregate industry data from the Bureau of Economic Analysis and Bureau of Labor Statistics. As a result, the C2-stat is distributed as a Student t with $N - K = 4 - 3 = 1$ degree of freedom.

Table 10 presents the results. In Panel A, we reject the null of coherence at the 95% confidence level for 55.7% of CFOs in our sample.[36] This result parallels our previous findings that 48% of CFOs use incoherent rules of thumb, (R1) and (R2), and that 52% of CFOs violate the intuitive restrictions on FEs discussed in Subsection V.A.

Figures in Panel A further confirm that CFO forecasts are fairly accurate on output and fairly inaccurate on capital. Specifically, we reject the null of forecast accuracy at the 95% confidence level for 27.2% of CFOs with respect to sales revenue and for 47.9% of CFOs with respect to CapEx. When considering output and capital input forecasts together, we reject the null of accuracy for 57% of CFOs in our sample.

These results rule out an alternative interpretation of our earlier findings, that is, that CFOs wield control over sales and capital to a different extent. While CFOs have ultimate authority over their firm's capital expenditures, their firm's sales depend also on external factors, including customer behavior and product market competition. Thus, one might expect CFOs to be more accurate about CapEx than sales. This is the opposite of what we find. CFOs are much less accurate in forecasting CapEx, likely reflecting the fact that their CapEx forecast is incoherently linked to their planned sales.

Panel B of Table 10 assesses coherence and accuracy together. It shows that 31.1% of CFOs in our sample are coherent and accurate, 13.2% are coherent but inaccurate, 12.0% are accurate but incoherent, and 43.7% are incoherent and inaccurate (all at the

[36] At the 99% confidence level, we reject coherence for 7.7% of CFOs. This difference reflects the distribution of the Student t with 1 degree of freedom.

95% confidence level).[37] The summary statistics of the cross-sectional distribution of the calculated C2-stat and of the FEs on output and capital input are shown in Panel C.

One concern with these results is the extent to which the computed C2-stat, as well as the accuracy statistics, are sensitive to the uncertainty coming from estimating the parameters a, b, and σ_2. To address this concern, we perform a non-parametric bootstrap procedure, as follows. For each CFO, we generate 1,000 bootstrap repetitions of the C2-stat.[38] Using these 1,000 replications, we compute the fraction of cases out of 1,000 for which we reject the null of coherence at the 95% and 99% confidence levels. Hence, for each CFO and confidence level, the computed statistic is a number between 0 and 1, where 0 means that the null of coherence was never rejected across all 1,000 repetitions and 1 means that the null of coherence was rejected for all 1,000 repetitions.

In Figure A2, we plot the value of this statistic (on the y-axis) against its empirical cumulative distribution function across CFOs (on the x-axis). The top graph refers to the calculation of the statistic at the 95% confidence level. The null of coherence is rejected in all bootstrap repetitions for about 40% of CFOs, in none for about 15% of CFOs, and in strictly more than none and less than all repetitions for the remaining 45% of CFOs. Consistent with the results in Table 10, the proportion of CFOs for whom the null of coherence is rejected more than half of the times is approximately 55%.[39]

VI Related Literature

Our evidence on incoherence in firm plans provides a possible microfoundation for understanding resource allocation within the firm (e.g., Maksimovic and Phillips (2002, 2013), Hoberg and Phillips (2023)). We discuss here our contributions to the study of

[37] The figures at the 99% confidence level are 89.4%, 2.9%, 3.4%, and 4.3%, respectively.

[38] Specifically, for each of the 15 BEA industries and pair of consecutive years between 1987 and 2018, we resample observations with replacement 1,000 times (aka bootstrap replications). At each replication, we obtain an estimate of σ_1, based on the residual sum of squares (RSS) of the regression of total capital compensation on its lag, and an estimate of σ_2, based on the RSS of the regression of total labor compensation on its lag. We use cluster bootstrap with 6 clusters, corresponding to the following year windows: 1988-1992, 1993-1997, 1998-2002, 2003-2007, 2008-2012, and 2013-2018. We additionally generate bootstrap estimates of a and b, as the capital and labor shares of total factor compensation. Endowed with bootstrap estimates of a, b, σ_1^2, and σ_2^2, we derive corresponding estimates for σ_y^2, for the FEs of the log of output (y) and of the log of capital input (x_1), and thus for the C2-stat.

[39] The bottom graph refers to the calculation of the statistic at the 99% confidence level.

survey expectations of firms and to behavioral corporate finance and economics.

Survey Expectations of Firms. Our paper is related to the recent and growing literature studying top executives' beliefs and forecasts about their firm's variables or the macroeconomy (Ben-David et al. (2013), Boutros et al. (2020), Campello et al. (2010), Campello et al. (2011, 2012)).[40] This literature typically studies the accuracy of managerial forecasts, one variable at a time. Bloom et al. (2021) show that forecasting firms' own variables is even harder than forecasting the aggregate economy. Gennaioli et al. (2016) show that both investment plans and actual investment are explained by CFOs' expectations of earnings growth. Graham (2022) documents that the sales revenue growth forecast is most important for firms' plans and consequences. We confirm that firms make on average accurate forecasts about sales, but we also show that expectations of other variables—most notably, capital expenditures—are much less accurate, and we link this to incoherence. We add to this literature by documenting wide heterogeneity in the methods top managers use to make forecasts of multiple related balance-sheet variables, some of which lead to incoherence.

Behavioral Research on Bracketing. Experimental and behavioral research in economics and finance has shown that decision makers often narrowly bracket and make interrelated decisions in isolation (e.g., Tversky and Kahneman (1981), Read et al. (1999), Rabin and Weizsäcker (2009), Ellis and Freeman (2020)). Thaler (1985, 2018) and Heath and Soll (1996) have proposed that individuals hold a mental account for each decision, thus failing to take into account the interdependence among choice variables implied by their budget constraint and utility function.

Economic theories of narrow bracketing and mental accounting include Barberis et al. (2006), Rabin and Weizsäcker (2009), Hastings and Shapiro (2013, 2018), and Lian (2021).[41] Our theory is closest to Lian (2021)'s, which endogenizes bracketing by modeling a narrow thinker who makes consumption decisions about individual goods with imperfect knowledge of the others and thus faces difficulties at coordinating different decisions.

[40] See also Bachmann and Bayer (2013, 2014), Born et al. (2023), and Candia et al. (2023).

[41] These models study monetary gambles, stock market participation, and consumption decisions, and show that narrow bracketing can lead to suboptimal choices in these domains.

We add to this literature by developing the first model of bracketing in a firm setting and by providing field evidence of narrow bracketing in a sample of CFOs dealing with internal planning. Similar to Lian (2021), there are no explicit mental budgets, which avoids the need to take a stand on where those come from, and agents make decisions (forecasts) based on different non-nested information,[42] which differentiates Lian (2021) and us from rational inattention models (e.g., Sims (2003), Mackowiak and Wiederholt (2009), Matějka and McKay (2015), Kőszegi and Matějka (2020)).[43] Unlike Lian (2021), we study a production setting where the interconnection among different decisions come from the production technology. Our model delivers novel predictions about corporate forecasts and performance, which we then test in the data.

Coherence and Accuracy Requirements of Rationality. The psychology literature has long recognized that rationality of judgements involves both accuracy ('correspondence') and coherence ('consistency'), and has typically maintained that the two are separate properties (e.g., Hammond (1996, 2007), Osherson et al. (1994), Gigerenzer et al. (1999), Gigerenzer and Gaissmaier (2011), Lee and Zhang (2012), Arkes et al. (2016)). This literature, however, has struggled to provide a formal framework or direct evidence to assess this claim, and has instead focused on predicting systematic inaccuracy from specific violations of statistics and probability laws.[44]

The coherence principle (deFinetti (1937)) states that subjective probabilities must conform to the logical and probability axioms posited in Kolmogorov (1933). In a series of famous experiments, Tversky and Kahneman (1971, 1974, 1983) document systematic misconceptions of logic, statistics, and probability theory, including the law of large numbers, the conjunction rule, the law of total probability, and Bayes' theorem. Since

[42] In the sense of Blackwell, neither input i's signal is more informative than $\neg i$'s signal, nor vice versa.

[43] Rational inattention models also use noisy signals, but different decisions are based on the same nested information. Relatedly, but using a deterministically imperfect perception of fundamentals rather than noisy signals, Gabaix (2014, 2019) develop a sparsity model in which an agent's multiple decisions are based on the same imprecise perception of the fundamentals.

[44] Philosophy and legal scholars have often seen incoherence in the context of moral and legal systems as concerning (e.g., Raz (1994), Sunstein et al. (2002), Fogelin (2003), Posner (2014)). A theoretical literature in philosophy develops axiomatic definitions of coherence, whereby two propositions are coherent if they are correlated according to some suitably defined notion of correlation (e.g., Thagard (2000), Schippers (2014), and references therein). In actual data, however, multiple forecasts may be correlated for reasons other than coherence. Furthermore, none of these papers disentangle coherence from accuracy.

then, a number of theories and experiments have shown how such misconceptions predict systematic inaccuracy in specific prediction tasks (e.g., Wright et al. (1994), Rabin (2002), Benjamin et al. (2016), Zhu et al. (2022)). Tversky and Kahneman (1974) have famously labeled this research program "heuristics and biases," whereby the use of heuristics generates systematic and predictable forecast errors (see also Thaler (2018)). Some scholars have pointed out how some of these results can also be cast in terms of the coherence-accuracy framework. For instance, Hammond (1996), Tentori et al. (2013), Arkes et al. (2016), Jönsson and Shogenji (2019), and others discuss how Tversky and Kahneman (1983)'s conjunction fallacy can be understood as a violation of coherence with respect to probability laws. In this literature, documenting a form of incoherence in one domain typically serves the purpose of predicting future systematic inaccuracy in another domain, without aiming at disentangling coherence from accuracy.

Our main contribution to this literature is to provide a formal framework, in which coherence and accuracy are defined with respect to the same forecasting task and which allows us to jointly assess forecast accuracy and forecast coherence. We show that to do so one needs both theory and data. In terms of theory, an economic model provides a benchmark against which to judge coherence. This is similar to the use of probability theory for assessing coherence of probabilistic judgments, but crucially economic theory allows us to nest coherence and accuracy in a setting with optimizing agents. In terms of data, observing both forecasts and realizations—and, thus, FEs—allows us to jointly assess accuracy and coherence, and also to disentangle the two. Accuracy is assessed by testing whether FEs of each variable are 'sufficiently' close to zero. Coherence is assessed by testing whether FEs of different variables (inputs and output in our setting) are 'sufficiently' close to one another. In both cases, the extent of 'sufficiently' is pinned down by economic theory. In our data, we find that 12% of individuals are incoherent but accurate, and 13% of individuals are inaccurate but coherent.

Disentangling coherence and accuracy is important, since under some conditions coherence can be assessed ex ante. The intuition is similar to that of the debiasing program in psychology (e.g., Fischhoff (1982)), with the key difference that in our framework one can use economic theory and regression analysis to determine the ex ante optimal rule.

Behavioral Corporate Finance. Baker et al. (2007) organize the behavioral corporate finance literature under the two themes of irrational investors and irrational managers (Guenzel and Malmendier (2020) discuss also other biased parties). The more developed irrational-investors literature studies how rational managers respond to irrational fluctuations in capital market prices. Our study belongs to the less developed and growing irrational-managers literature, which assumes that, while markets are arbitrage free, managerial behavior can be influenced by psychological biases. So far, this stream of research has focused mostly on how psychological traits such as optimism, overconfidence, or preference for skewness can distort managerial expectations about the future and thus distort investment and financing decisions (see Malmendier and Tate (2005, 2008), Landier and Thesmar (2009), Gervais et al. (2011), Schneider and Spalt (2014)). We study incoherence, a novel managerial trait, in the context of firm plans, and controlling directly for measures of optimism and overconfidence (Ben-David et al. (2013)) we show that incoherence differs from optimism or overconfidence both in theory and in the data.[45]

VII Conclusion

We develop a theory of forecast coherence in firm planning, which yields a benchmark of first-best coherent forecasts. In a positive version of our model, incoherence arises from 'narrow thinking' and operates via the use of forecasting heuristics. Our model rationalizes as second-best optimal responses to noisy signals on inputs and output some—although not all—of the 'rules of thumb' proposed by managerial textbooks to help top financial executives make forecasts of multiple balance-sheet variables for internal planning. Our model additionally provides a partial ranking of these rules of thumb as well as predictions on the relationships between forecast incoherence and rule-of-thumb use and between each of the latter and firm performance.

Using Duke-Compustat linked data, we document wide heterogeneity in CFOs' rule-

[45] Krüger et al. (2015) and Dessaint et al. (2021) show that incorrectly using a company-wide cost of capital for non-core projects or a CAPM-based beta can distort investment or M&A activity, resulting in negative wealth effects for investors. Unlike these papers, we examine managerial rules of thumb in firm plans rather than in capital budgeting, and we provide theory and evidence showing how some rules of thumb may be second-best responses to imperfect information.

of-thumb use. Consistent with our model's predictions, firm performance is negatively associated with forecast incoherence and is on average lowest among firms whose CFOs use 'narrow bracketing' and 'sales anchoring' rules (R1-R2).

Making increasing use of our model's assumptions, we then introduce theory-informed restrictions, regression tests, and individual-level tests of coherence based on FEs. While relying on different setups and assumptions, all our empirical results point to about one half of CFOs providing incoherent forecasts of their firm's input and output growth.

We see these results as reflecting a lack of consensus among managerial textbooks and case studies and a lack of theory and evidence to distinguish among different forecasting rules. We also believe that these results suggest two main takeaways. First, simple, intuitive, and much advertised heuristics, such as 'sales anchoring' (R2) and 'narrow bracketing' (R1), perform poorly and should not be part of future managers' toolkit.[46] Second, and more generally, while much prior research has been cast in terms of whether heuristics are unambiguously good or bad, we show that heuristics are not necessarily all the same. Some may be helpful, others harmful. Heuristics should be evaluated with respect to their proposed goals. It typically requires both theory and data to do so.

[46] We do not address here the broader question of what the best possible method may be to provide forecasts over balance-sheet variables, beyond those proposed by managerial textbooks. Given the nonlinear nature of the firm's problem, combining flexible prediction methods (nonparametric regressions, machine learning algorithms, etc.) with big data sets (if available) will likely yield superior forecasts.

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Figure 1: **Coherence Restrictions on Contemporaneous Forecast Errors**

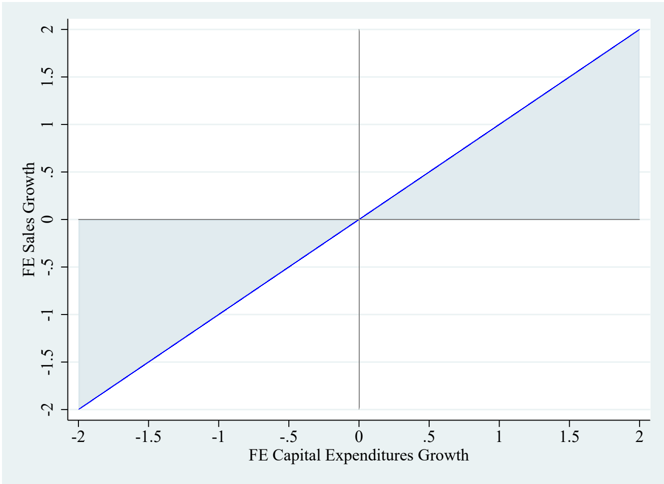

Figure 2: **Contemporaneous Forecasts Errors of Output and Capital**

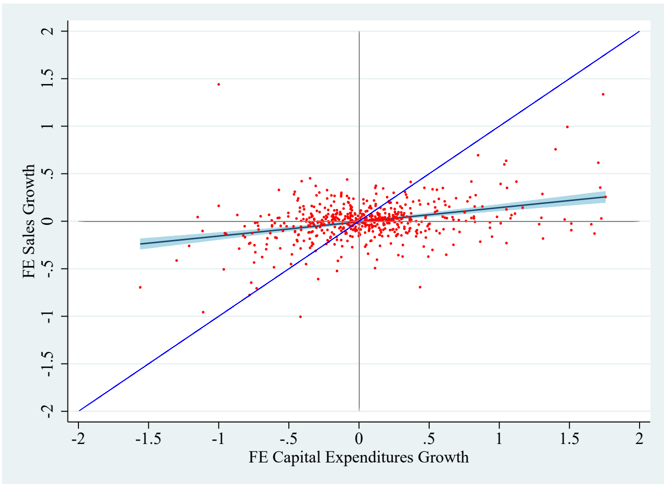

Figure 3: **(In)Coherence and (In)Accuracy Areas**

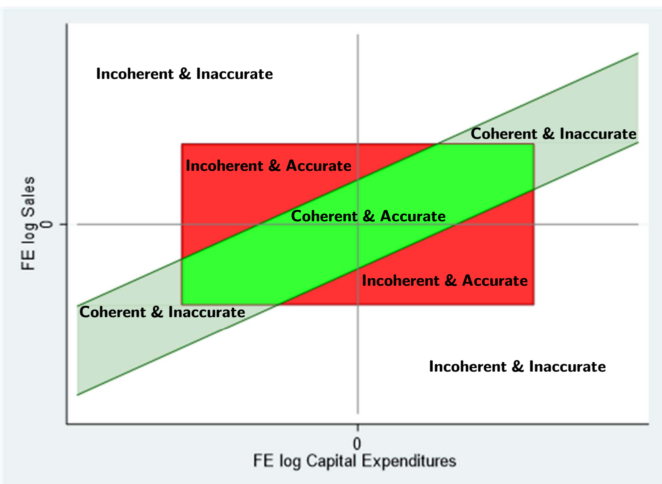

Table 1: **Cross-Sectional Regressions for Rules of Thumb using Compustat Data**

	Intercept	Slope 1	Slope 2	R^2	N Obs
Rule 1 ('narrow bracketing')	0.316***	-0.089***		0.004	74,413
	(0.041)	(0.016)			
Rule 3 ('economies of scale')	0.217***	1.055***		0.081	100,441
	(0.025)	(0.036)			
Rule 4 ('industry based')					
SIC 0	0.330***	2.050***		0.097	358
	(0.156)	(0.344)			
SIC 1	0.243***	0.950***		0.115	8,983
	(0.045)	(0.072)			
SIC 2	0.243***	0.859***		0.050	14,777
	(0.026)	(0.054)			
SIC 3	0.186***	1.188***		0.104	24,852
	(0.024)	(0.058)			
SIC 4	0.180***	0.925***		0.064	14,398
	(0.022)	(0.091)			
SIC 5	0.163***	1.281***		0.081	10,266
	(0.027)	(0.121)			
SIC 6	0.402***	0.963***		0.036	7,477
	(0.041)	(0.062)			
SIC 7	0.202***	1.162***		0.105	14,673
	(0.039)	(0.090)			
SIC 8	0.198***	1.216***		0.088	3,911
	(0.023)	(0.128)			
SIC 9	0.222***	1.288***		0.123	746
	(0.058)	(0.182)			
Rule 5	0.217***	1.042***	0.018***	0.082	100,040
	(0.025)	(0.036)	(0.004)		

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.

Table 2: **Minimum Distance of Sales Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.033	0.058	0.030	0.019	0.031	0.043
Std. Dev.	0.059	0.100	0.064	0.017	0.035	0.069
Frac. Zeros	0.106	0.000	0.268	0.000	0.000	0.000
P10	0.000	0.008	0.000	0.005	0.002	0.003
P25	0.007	0.015	0.000	0.006	0.007	0.008
P50	0.019	0.028	0.014	0.010	0.023	0.023
P75	0.036	0.064	0.035	0.028	0.048	0.043
P90	0.071	0.114	0.071	0.048	0.072	0.089
N of Observations	396	30	157	43	107	59
Fraction	1.000	0.076	0.396	0.109	0.270	0.149

Notes: Cross-sectional analysis with 396 CFOs.

Table 3: **Incoherence and Rules of Thumb: Distance from Optimal Forecast**

	(1)	(2)	(3)	(4)	(5)
Rule 1 ('narrow bracketing')	0.081***				0.104***
	(0.014)				(0.016)
Rule 2 ('sales anchoring')		0.039***			0.053***
		(0.008)			(0.011)
Rule 3 ('economies of scale')			-0.055***		-0.020
			(0.012)		(0.014)
Rule 4 ('industry based')				-0.027***	0.010
				(0.009)	(0.012)
Constant	0.066***	0.057***	0.079***	0.080***	0.043***
	(0.004)	(0.005)	(0.004)	(0.005)	(0.009)
R^2	0.071	0.057	0.045	0.023	0.175
N observations	396	396	396	396	396

Summary Statistics of the dependent variable	
Mean	0.073
Std. Dev.	0.079
P10	0.012
Median	0.059
P90	0.139

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table 4: **Incoherence and CFO & Firm Characteristics**

Panel A – Summary statistics

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
CFO has MBA	0.452	0.498	0.000	0.000	1.000	396
Age	50.43	6.937	42.00	50.00	60.00	396
Tenure	4.273	4.099	0.000	3.000	9.000	396
Gender	0.088	0.284	0.000	0.000	0.000	396
Miscalibration ST	0.035	0.920	-1.166	0.329	0.985	360
Optimism ST	0.052	0.981	-0.918	-0.077	1.285	373
Miscalibration LT	0.039	0.979	-1.095	0.262	0.917	362
Optimism LT	0.033	1.088	-1.008	-0.078	1.077	374
Firm size	7.829	2.296	4.898	7.805	10.61	396
Market-to-Book	1.685	0.900	0.998	1.393	2.731	364
Dividends	0.636	0.482	0.000	1.000	1.000	396

Panel B – Incoherence and CFO characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
CFO has MBA	0.005					0.007
	(0.009)					(0.011)
Tenure > Median	0.008					0.006
	(0.008)					(0.011)
Age 40-		-0.011				-0.025
		(0.022)				(0.029)
Age 41-50		-0.027*				-0.038**
		(0.016)				(0.019)
Age 51-60		-0.024				-0.030*
		(0.017)				(0.017)
Gender		0.002				0.005
		(0.010)				(0.012)
Miscalibration ST			-0.012			-0.014
			(0.008)			(0.010)
Optimism ST			-0.012			-0.010
			(0.007)			(0.009)
Miscalibration LT				-0.005		-0.002
				(0.004)		(0.006)
Optimism LT				0.001		0.005
				(0.004)		(0.007)
Firm size					-0.006*	-0.005*
					(0.003)	(0.003)
Market-to-Book					0.011	0.014
					(0.013)	(0.014)
Dividends					-0.015	-0.020
					(0.012)	(0.013)
Constant	0.043*	0.078***	0.052*	0.046**	0.137**	0.159***
	(0.022)	(0.025)	(0.027)	(0.021)	(0.036)	(0.049)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Survey FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.111	0.112	0.154	0.126	0.162	0.227
N of Observations	396	396	360	362	364	332

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.

Table 5: **Incoherence and Corporate Performance (Return on Assets)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Incoherence	-0.377**	-0.378**	-0.360**	-0.396**	-0.399**	-0.386**	-0.317*	-0.307*
	(0.157)	(0.179)	(0.162)	(0.162)	(0.186)	(0.169)	(0.192)	(0.181)
Miscalibration ST		0.003			0.001		-0.001	
		(0.005)			(0.005)		(0.004)	
Optimism ST		0.000			0.000		0.001	
		(0.006)			(0.006)		(0.005)	
Miscalibration LT			0.004			0.002		0.001
			(0.005)			(0.005)		(0.005)
Optimism LT			0.008			0.007		0.009
			(0.006)			(0.006)		(0.006)
Firm size							0.009***	0.009**
							(0.003)	(0.003)
Market-to-Book							0.028**	0.027*
							(0.014)	(0.015)
Dividends							0.022*	0.023*
							(0.012)	(0.013)
Constant	0.069***	0.069***	0.068***	0.054***	0.056***	0.057***	-0.131***	-0.123***
	(0.011)	(0.011)	(0.011)	(0.014)	(0.020)	(0.019)	(0.047)	(0.0471)
Industry FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Survey FE	No	No	No	Yes	Yes	Yes	Yes	Yes
R^2	0.046	0.042	0.047	0.071	0.064	0.068	0.177	0.185
N of CFOs	311	282	284	311	282	284	263	265
N of Firms	277	252	254	277	252	254	235	237
N of Observations	468	423	428	468	423	428	396	401

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Bootstrap standard errors clustered at the firm level are reported in parenthesis.

Table 6: **Rules of Thumb and Corporate Performance (Return on Assets)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Rule 1 ('narrow bracketing')	-0.057**	-0.061**	-0.059**	-0.051**	-0.059**	-0.055**	-0.053**	-0.051**
	(0.022)	(0.025)	(0.024)	(0.023)	(0.025)	(0.025)	(0.026)	(0.025)
Rule 2 ('sales anchoring')	-0.026*	-0.027*	-0.023	-0.023	-0.028*	-0.024	-0.034	-0.031
	(0.0138)	(0.015)	(0.015)	(0.015)	(0.017)	(0.016)	(0.021)	(0.019)
Rule 3 ('economies of scale')	-0.031*	-0.036*	-0.034*	-0.027	-0.037*	-0.034	-0.047**	-0.045**
	(0.017)	(0.019)	(0.019)	(0.019)	(0.020)	(0.021)	(0.023)	(0.022)
Rule 4 ('industry based')	-0.012	-0.010	-0.010	-0.008	-0.008	-0.007	-0.012	-0.011
	(0.012)	(0.014)	(0.014)	(0.013)	(0.014)	(0.015)	(0.015)	(0.015)
Miscalibration ST		0.001			-0.001		-0.002	
		(0.005)			(0.005)		(0.004)	
Optimism ST		0.001			0.000		0.001	
		(0.006)			(0.005)		(0.005)	
Miscalibration LT			0.003			0.002		0.001
			(0.006)			(0.005)		(0.004)
Optimism LT			0.007			0.006		0.008
			(0.006)			(0.006)		(0.005)
Firm size							0.010***	0.009***
							(0.004)	(0.004)
Market-to-Book							0.028**	0.028*
							(0.014)	(0.015)
Dividends							0.029**	0.030**
							(0.013)	(0.014)
Constant	0.065***	0.066***	0.064***	0.040***	0.045**	0.046*	-0.147***	-0.137***
	(0.011)	(0.012)	(0.013)	(0.015)	(0.019)	(0.028)	(0.046)	(0.050)
Industry FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Survey FE	No	No	No	Yes	Yes	Yes	Yes	Yes
R^2	0.014	0.014	0.019	0.033	0.031	0.034	0.165	0.170
N of CFOs	311	282	284	311	282	284	263	265
N of Firms	277	252	254	277	252	254	235	237
N of Observations	468	423	428	468	423	428	396	401

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Bootstrap standard errors clustered at the firm level are reported in parenthesis.

Table 7: **Rules of Thumb and Corporate Policies**

	Investment			**Leverage**		
	(1)	(2)	(3)	(4)	(5)	(6)
Rule 1 ('narrow bracketing')	-0.016	-0.014	-0.015	0.055	0.041	0.047
	(0.011)	(0.011)	(0.012)	(0.092)	(0.101)	(0.092)
Rule 2 ('sales anchoring')	-0.013**	-0.015**	-0.012	0.093*	0.098	0.092*
	(0.006)	(0.007)	(0.008)	(0.053)	(0.060)	(0.053)
Rule 3 ('economies of scale')	-0.007	-0.011	-0.010	-0.023	-0.015	-0.027
	(0.008)	(0.010)	(0.010)	(0.073)	(0.091)	(0.084)
Rule 4 ('industry based')	-0.003	-0.003	-0.003	-0.004	0.005	0.001
	(0.007)	(0.008)	(0.008)	(0.045)	(0.050)	(0.046)
Miscalibration ST		0.001			0.012	
		(0.003)			(0.024)	
Optimism ST		0.002			-0.006	
		(0.003)			(0.019)	
Miscalibration LT			0.002			0.013
			(0.002)			(0.018)
Optimism LT			0.004*			-0.010
			(0.002)			(0.017)
Firm size	-0.002	-0.002	-0.002	0.012	0.011	0.012
	(0.002)	(0.002)	(0.002)	(0.017)	(0.019)	(0.018)
Market-to-Book	0.006**	0.006*	0.006*	-0.082***	-0.081***	-0.081***
	(0.003)	(0.003)	(0.003)	(0.020)	(0.021)	(0.022)
Dividends	0.000	0.004	0.004	0.037	0.044	0.043
	(0.009)	(0.008)	(0.008)	(0.072)	(0.076)	(0.080)
Constant	0.044*	0.043*	0.050**	0.568**	0.666***	0.620**
	(0.025)	(0.023)	(0.023)	(0.249)	(0.228)	(0.249)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Survey FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.210	0.223	0.230	0.069	0.062	0.066
N of Observations	437	397	402	437	397	402

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Bootstrap standard errors clustered at the firm level are reported in parenthesis.

Table 8: **Change in Performance and Corporate Policies when New CFOs Take Office**

	Change in ROA		**Change in Investment**		**Change in Leverage**	
	(1)	(2)	(3)	(4)	(5)	(6)
Incoherence	-1.633*		-0.049		-0.047	
	(0.989)		(0.045)		(1.115)	
Rule 1 ('narrow bracketing')		-0.274		-0.022*		-0.011
		(0.213)		(0.012)		(0.231)
Rule 2 ('sales anchoring')		-0.000		-0.003		-0.201
		(0.036)		(0.008)		(0.199)
Rule 3 ('economies of scale')		-0.057		-0.008		-0.110
		(0.051)		(0.012)		(0.153)
Rule 4 ('industry based')		0.019		0.001		-0.070
		(0.048)		(0.009)		(0.118)
Firm characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.391	0.192	0.024	0.017	0.053	0.042
N of Observations	142	142	140	140	146	146

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Bootstrap standard errors clustered at the firm level are reported in parenthesis.

Table 9: **Regression Tests of Accuracy and Coherence**

	FE of Log CapEx Growth	FE of Log Sales Growth			
	(1)	(2)	(3)	(4)	(5)
FE of Log CapEx Growth			0.113*	0.135***	
			(0.063)	(0.032)	
FE of Log Wages Growth			0.023		0.019
			(0.309)		(0.321)
Constant	-0.042*	-0.009	0.046**	-0.004	0.033
	(0.025)	(0.009)	(0.023)	(0.009)	(0.022)
R^2	0.000	0.000	0.108	0.127	0.018
N observations	359	359	51	359	52

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.
Robust standard errors are reported in parenthesis.

Table 10: **The Coherence and Accuracy Sides of Rationality**

Panel A – Separate Assessment of Coherence and Accuracy (Percent of Rejections of Null)

Significance level α	Coherence Sales-CapEx	Accuracy Sales	Accuracy CapEx	Accuracy Both
	(1)	(2)	(3)	(4)
5%	55.7%	27.2%	47.9%	57.0%
1%	7.7%	1.8%	6.4%	7.1%

Panel B – Joint Assessment of Coherence and Accuracy

Significance level α	Coherent + Accurate	Coherent + Inaccurate	Incoherent + Accurate	Incoherent + Inaccurate
	(1)	(2)	(3)	(4)
5%	31.1%	13.2%	12.0%	43.7%
1%	89.4%	2.9%	3.4%	4.3%

Panel C – Test Statistics: Summary Statistics

	Mean	Std.Dev.	P05	Median	P95	N Obs.
C-statistic	-0.193	4.846	-8.335	-0.135	7.871	560
FE Sales	-0.538	19.07	-23.53	0.554	22.24	563
FE CapEx	-0.988	31.28	-54.18	1.186	41.20	560

Notes: In Panel B, Accuracy means both accurate; and inaccuracy means at least one inaccurate. Critical values are those of the t-student with one degree of freedom, +/-12.706 at the 5% and +/-63.657 at the 1%. Sales are the output. Capital Expenditures (CapEx) are input 1. Labor Expenditures are input 2 (unobserved).

Online Appendix With Supplementary Material for

The Coherence Side of Rationality
Rules of thumb, narrow bracketing, and managerial incoherence in corporate forecasts

Pamela Giustinelli and Stefano Rossi

A Proofs

Proof of Proposition 1. Recall that for a concave function, f, it holds that $\mathbb{E}[f(x)] \leq f(\mathbb{E}[x])$. Assume $\xi \leq 1$ and start by assuming that $a+b=1$. The CES function f is homogeneous of degree one because, for a scalar λ, we have that

$$f(\lambda \mathbf{x}) = \left[a(\lambda x_1)^{\xi} + b(\lambda x_2)^{\xi}\right]^{\frac{1}{\xi}} = \lambda\left(a x_1^{\xi} + b x_2^{\xi}\right)^{\frac{1}{\xi}}.$$

Furthermore, note that f is also quasiconcave because it is a monotone transformation of a concave function. In fact,

$$f = g^{\frac{1}{\xi}},$$

and to see that g is concave, compute its Hessian, H_g,

$$H_g = \begin{bmatrix} \frac{\partial^2 g}{\partial x_1^2} & \frac{\partial^2 g}{\partial x_2 \partial x_1} \\ \frac{\partial^2 g}{\partial x_1 \partial x_2} & \frac{\partial^2 g}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} a\xi(1-\xi)x_1^{\xi-2} & 0 \\ 0 & b\xi(1-\xi)x_2^{\xi-2} \end{bmatrix}.$$

Since H_g is negative semi-definite, we can conclude that g is concave.

Now, let $a+b \leq 1$. We have that $f = \left(g^{\frac{1}{\xi}}\right)^{a+b}$, where g is concave, as shown above. Then, f is a concave increasing function of a concave function, from which we can conclude that f is concave, which proves the first part of the proposition. The second part of the proposition on convexity follows very similar arguments.
QED

Proof of Corollary 1. Here we prove the statement in growth rates (the one in levels follows similar steps). Given our assumptions, we have $y = x_1^a \cdot x_2^b$ and $Z^* \equiv p_1 x_1^* + p_2 x_2^*$. As a result, the optimal input quantities satisfy,

$$x_1^* = \frac{Z^*}{p_1}\frac{a}{a+b}, \qquad x_2^* = \frac{Z^*}{p_2}\frac{b}{a+b}.$$

Therefore, for $i = 1, 2$ we have

$$\begin{aligned} \log\left[\frac{x_{i,t+1}}{x_{i,t}}\right] &= \log\left[\frac{p_{i,t}}{p_{i,t+1}}\right] \\ \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \log\left[\frac{p_{1,t}}{p_{1,t+1}}\right] + b \cdot \log\left[\frac{p_{2,t}}{p_{2,t+1}}\right]. \end{aligned}$$

Putting these together, we obtain

$$\begin{aligned}\log\left[\frac{y_{t+1}}{y_t}\right] &= a\cdot\log\left[\frac{x_{1,t+1}}{x_{1,t}}\right]+b\cdot\log\left[\frac{x_{2,t+1}}{x_{2,t}}\right]\\ \mathbb{E}_t\log\left[\frac{y_{t+1}}{y_t}\right] &= a\cdot\mathbb{E}_t\log\left[\frac{x_{1,t+1}}{x_{1,t}}\right]+b\cdot\mathbb{E}_t\log\left[\frac{x_{2,t+1}}{x_{2,t}}\right].\end{aligned}$$

QED

Proof of Proposition 2. The Proof follows directly from the observation that in our setting the conditional expectation function is

$$\mathbb{E}\left[y|x_1,x_2\right]=\mathbb{E}\left[y\right]+\beta_1\left(x_1-\mathbb{E}\left[x_1\right]\right)+\beta_2\left(x_2-\mathbb{E}\left[x_2\right]\right),$$

where the parameters can be derived by the Frisch-Waugh-Lovell theorem, as

$$\beta_1=\frac{cov\left(y,x_1\right)-\left(\frac{cov(x_1,x_2)cov(y,x_2)}{var(x_2)}\right)}{var\left(x_1\right)-\frac{cov(x_1,x_2)^2}{var(x_2)}},\quad \beta_2=\frac{cov\left(y,x_2\right)-\left(\frac{cov(x_1,x_2)cov(y,x_1)}{var(x_1)}\right)}{var\left(x_2\right)-\frac{cov(x_1,x_2)^2}{var(x_1)}},$$

and where $var\left(x_1\right)$, $var\left(x_2\right)$, $cov\left(y,x_1\right)$, and $cov\left(y,x_2\right)$ are functions of parameters a and b. QED

Proof of Corollary 2. In levels,

$$\mathbb{E}\left[\log x_1|\log y,\log x_2\right]=\mu_1+\beta_y\left(\log y-\mu_y\right)+\beta_2\left(\log x_2-\mu_2\right),$$

where coefficients equal

$$\beta_y=\frac{cov\left(\log y,\log x_1\right)-\left(\frac{cov(\log y,\log x_2)cov(\log x_1,\log x_2)}{var(\log x_2)}\right)}{var\left(\log y\right)-\frac{cov(\log y,\log x_2)^2}{var(\log x_2)}},$$

$$\beta_2=\frac{cov\left(\log x_2,\log x_1\right)-\left(\frac{cov(\log y,\log x_2)cov(\log x_1,\log y)}{var(\log y)}\right)}{var\left(\log x_2\right)-\frac{cov(\log y,\log x_2)^2}{var(\log y)}}.$$

In detail, we have:
$var\left(\log x_2\right)=\sigma_2^2$,
$var\left(\log y\right)=a^2\sigma_1^2+b^2\sigma_2^2$,
$cov\left(\log x_2,\log x_1\right)=0$,
$cov\left(\log y,\log x_1\right)=cov\left(a\log x_1+b\log x_2,\log x_1\right)=a\sigma_1^2$,
$cov\left(\log y,\log x_2\right)=cov\left(a\log x_1+b\log x_2,\log x_2\right)=b\sigma_2^2$.

Substituting yields

$$\begin{aligned}
\mathbb{E}\left[\log x_1 | \log y, \log x_2\right] &= \mu_1 + \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 - \frac{b^2\sigma_2^4}{\sigma_2^2}}\left(\log y - \mu_y\right) + \frac{-\left(\frac{b\sigma_2^2 a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\right)}{\sigma_2^2 - \frac{b^2\sigma_2^4}{a^2\sigma_1^2 + b^2\sigma_2^2}}\left(\log x_2 - \mu_2\right) \\
&= \mu_1 + \frac{1}{a}\left(\log y - \mu_y\right) - \frac{b\sigma_2^2 a\sigma_1^2}{\sigma_2^2\left(a^2\sigma_1^2 + b^2\sigma_2^2\right) - b^2\sigma_2^4}\left(\log x_2 - \mu_2\right) \\
&= \mu_1 + \frac{1}{a}\left(\log y - \mu_y\right) - \frac{b}{a}\left(\log x_2 - \mu_2\right).
\end{aligned}$$

where $\mu_1 - \frac{1}{a}\mu_y + \frac{b}{a}\mu_2 = 0$ follows by Corollary 1. Proving the statement in growth rates follows similar steps.
QED

Proof of Corollary 3. In levels,

$$\mathbb{E}\left[\log x_1 | \log y\right] = \mu_1 + \beta_y\left(\log y - \mu_y\right),$$

where coefficients equal

$$\alpha = \mu_1 - \beta_y\mu_y, \qquad \beta_y = \frac{cov\left(\log y, \log x_1\right)}{var\left(\log y\right)}.$$

We have:
$cov\left(\log x_2, \log x_1\right) = 0$,
$cov\left(\log y, \log x_1\right) = cov\left(a\log x_1 + b\log x_2, \log x_1\right) = a\sigma_1^2$,
$var\left(\log y\right) = a^2\sigma_1^2 + b^2\sigma_2^2$.
Substituting yields

$$\alpha = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\mu_y, \qquad \beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2},$$

and thus

$$\mathbb{E}\left[\log x_1 | \log y\right] = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\mu_y + \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\left(\log y - \mu_y\right).$$

Proving the statement in growth rates follows similar steps and intercept α is differenced away.
QED

Proof of Corollary 4. Consider the regression

$$\frac{y_{t+1}}{y_t} = \alpha + \beta\frac{x_{i,t+1}}{x_{i,t}} + e_{t+1}.$$

Denoting variable at optimum with superscripts *, we have

$$\begin{aligned}
\beta &= \frac{cov\left(\frac{y_{t+1}}{y_t}, \frac{x^*_{i,t+1}}{x^*_{i,t}}\right)}{var\left(\frac{x^*_{i,t+1}}{x^*_{i,t}}\right)} = \frac{\mathbb{E}\left[\frac{y_{t+1}}{y_t}\cdot\frac{x^*_{i,t+1}}{x^*_{i,t}}\right] - \mathbb{E}\left[\frac{y_{t+1}}{y_t}\right]\cdot\mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right]}{\mathbb{E}\left[\left(\frac{x^*_{i,t+1}}{x^*_{i,t}}\right)^2\right] - \left(\mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right]\right)^2}, \\
\alpha &= \mathbb{E}\left[\frac{y_{t+1}}{y_t}\right] - \beta\mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right].
\end{aligned}$$

Recall that

$$\left\{\begin{pmatrix}\pi_{1,t}\\ \pi_{2,t}\end{pmatrix}\right\} \overset{iid}{\sim} \mathcal{N}_2\left(\mathbf{0}, \begin{bmatrix}\sigma_1^2 & \sigma_{1,2}\\ \sigma_{1,2} & \sigma_2^2\end{bmatrix}\right).$$

Under the assumption that $\sigma_1^2 = \sigma_2^2 = \sigma^2$, for every t we have that the Pearson correlation coefficient $\rho_{1,2} = 1$, from which it follows that $\pi_{1,t} = \pi_{2,t} + c$ almost surely, where c is a constant.[47] As a result, the setting can be recast as one in which prices are constant and Z is stochastic because

$$\begin{aligned}\frac{x^*_{1,t+1}}{x^*_{1,t}} &= \frac{p_{1,t}}{p_{1,t+1}} \overset{\text{a.s.}}{=} \frac{p_{2,t}}{p_{2,t+1}} = \frac{x^*_{2,t+1}}{x^*_{2,t}}\\ \frac{y_{t+1}}{y_t} &= a\frac{p_{1,t}}{p_{1,t+1}} + b\frac{p_{2,t}}{p_{2,t+1}} \overset{\text{a.s.}}{=} (a+b)\frac{p_{1,t}}{p_{1,t+1}}\end{aligned}$$

and

$$\begin{aligned}&\left\{\frac{x^*_{i,t+1}}{x^*_{i,t}}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\sigma^2\right), \quad i = 1,2\\ &\left\{\frac{y_{t+1}}{y_t}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\left(a+b\right)^2\sigma^2\right).\end{aligned}$$

In fact, denote $z_t = \log(Z) \overset{\text{iid}}{\sim} \mathcal{N}\left(0,\sigma^2\right)$, and assume p_1 and p_2 constant for all t. We have

$$\begin{aligned}\frac{x^*_{1,t+1}}{x^*_{1,t}} &= \frac{x^*_{2,t+1}}{x^*_{2,t}} = e^{z_{t+1}-z_t}\\ \frac{y_{t+1}}{y_t} &= e^{(a+b)(z_{t+1}-z_t)}\end{aligned}$$

and

$$\begin{aligned}&\left\{\frac{x^*_{i,t+1}}{x^*_{i,t}}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\sigma^2\right), \quad i = 1,2\\ &\left\{\frac{y_{t+1}}{y_t}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\left(a+b\right)^2\sigma^2\right),\end{aligned}$$

as it was with stochastic prices. Therefore, for clarity from now on we drop the subscript i. Now, recalling that $\nu \equiv a+b$, that $cov\left(z_{t+1}, z_t\right) = \gamma\frac{\sigma^2}{1-\gamma^2}$, and that for any scalar, c, we have

$$\mathbb{E}\left[e^{c(z_{t+1}-z_t)}\right] = e^{\frac{1}{2}c^2 var(z_{t+1}-z_t)} = e^{\frac{1}{2}c^2\left(2\frac{\sigma^2}{1-\gamma^2}-2\gamma\frac{\sigma^2}{1-\gamma^2}\right)} = e^{c^2\left(\frac{\sigma^2}{1+\gamma}\right)},$$

[47]To see this, suppose that X, Y are two random variables such that $\rho(X,Y) = 1$. Let $V = X - \mathbb{E}[X]$ and $W = Y - \mathbb{E}[Y]$. We have $\mathbb{E}\left[(V-W)^2\right] = var(X) + var(Y) - 2cov(X,Y) = 0$, so that $V \overset{\text{a.s.}}{=} W$, from which the result $\pi_{1,t} = \pi_{2,t} + c$ follows.

we obtain the expressions

$$\begin{aligned}\beta &= \frac{e^{\left[(\upsilon+1)^2\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\left(\upsilon^2+1\right)\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(4\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(2\frac{\sigma^2}{1-\gamma}\right)}}=\frac{e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\upsilon^2\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(3\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(\frac{\sigma^2}{1-\gamma}\right)}}\\ \alpha &= \frac{e^{\left[\left(\upsilon^2+2\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(2\frac{\sigma^2}{1-\gamma}\right)}-1}.\end{aligned}$$

We can then directly verify the coimplications of the Corollary, that is,

$$\begin{aligned}\beta &< 1 \Longleftrightarrow e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\upsilon^2\frac{\sigma^2}{1-\gamma}\right]}<e^{\left(3\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(\frac{\sigma^2}{1-\gamma}\right)}\Longleftrightarrow \upsilon<1\\ \alpha &> 0 \Longleftrightarrow e^{\left[\left(\upsilon^2+2\right)\frac{\sigma^2}{1-\gamma}\right]}>e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}\Longleftrightarrow \upsilon<1,\end{aligned}$$

which also holds for i.i.d. shocks, that is, for $\gamma = 0$.
QED

Proof of Corollary 5. We have

$$\log\frac{x_{i,t+1}}{x_{i,t}}=\log\frac{1/p_{i,t+1}}{1/p_{i,t}}=\pi_{i,t}-\pi_{i,t+1}.$$

Denote $\log F_t^0$ the optimal forecast of log input $x_{i,t}$ growth. We have

$$\log F_t^o=\mathbb{E}_t\left[\log\frac{x_{i,t+1}}{x_{i,t}}\right]=(1-\gamma_i)\,\pi_{i,t}.$$

Under the optimal forecast, the forecast error will be minus the innovation of the log price shock,

$$\log\frac{x_{i,t+1}}{x_{i,t}}-\mathbb{E}_t\left[\log\frac{x_{i,t+1}}{x_{i,t}}\right]=-\epsilon_{i,t+1}|\Omega_t\ \sim\mathcal{N}\left(0,1\right),$$

so that the loss and the expected loss under the optimal forecast, L_{t+1}^o and $\mathbb{E}_t\left[L_{i,t+1}^o\right]$, are

$$\begin{aligned}L_{t+1}^o &= \epsilon_{i,t+1}^2=\sigma_i^2\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2\\ \mathbb{E}_t\left[L_{i,t+1}^o\right] &= \sigma_i^2\mathbb{E}_t\left[\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2\right]=\sigma_i^2,\end{aligned}$$

where the last equality follows from $\epsilon_{i,t+1}^2=\sigma_i^2=\sigma_i^2\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2$, and $\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2|\Omega_t\ \sim\chi^2$ with mean 1.

Under the narrow-bracketing rule (R1), $\log F_{i,t}^{R1}=\frac{1}{k}\sum_{j=1}^{k}\log\frac{x_{i,t+1-j}}{x_{i,t-j}}$, the forecast error in logs is

$$\log\frac{x_{i,t+1}}{x_{i,t}}-\log F_{i,t}^{R1}.$$

There are several possibilities. If $k=1$, $\log F_{i,t}^{R1}=\log\frac{x_{i,t}}{x_{i,t-1}}$, then the forecast error is $\log\frac{x_{i,t+1}}{x_{i,t}}-$

$\log \frac{x_{i,t}}{x_{i,t-1}} = -\left[\epsilon_{i,t+1} - (1-\gamma_i)\,\pi_{i,t} - (\pi_{i,t} - \pi_{i,t-1})\right]$, and

$$\mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \sigma_i^2\left(1 + \frac{\left[(1-\gamma_i)\,\pi_{i,t} + (\pi_{i,t} - \pi_{i,t-1})\right]^2}{\sigma_i^2}\right) = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t} + (\pi_{i,t} - \pi_{i,t-1})\right]^2.$$

For a general k, one obtains

$$\mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t} + \frac{1}{k}\sum_{j=1}^{k}\left(\pi_{i,t+1-j} - \pi_{i,t-j}\right)\right]^2.$$

For $k \to \infty$,

$$\lim_{k\to\infty} \mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t}\right]^2.$$

QED

Proof of Proposition 3. The Proposition is stated in the text for the case of $\rho_{1,2} = 0$. Here we prove the Proposition for the general case with correlated prices, i.e., for a generic value of $\rho_{1,2} \in [0.1]$. We have that

$$\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y\left(\eta_y - \mu_y\right) + \beta_2\left(\eta_2 - \mu_2\right),$$

where coefficients equal

$$\beta_y = \frac{cov\left(\eta_y, \log x_1\right) - \left(\frac{cov(\eta_y,\eta_2)cov(\log x_1,\eta_2)}{var(\eta_2)}\right)}{var\left(\eta_y\right) - \frac{cov(\eta_y,\eta_2)^2}{var(\eta_2)}},\quad \beta_2 = \frac{cov\left(\eta_2, \log x_1\right) - \left(\frac{cov(\eta_y,\eta_2)cov(\log x_1,\eta_y)}{var(\eta_y)}\right)}{var\left(\eta_2\right) - \frac{cov(\eta_y,\eta_2)^2}{var(\eta_y)}}.$$

We have:
$cov\left(\eta_y, \log x_1\right) = cov\left(a\log x_1 + b\log x_2 + \epsilon_y, \log x_1\right) = a\sigma_1^2 + b\rho_{1,2}$,
$cov\left(\eta_2, \log x_1\right) = \rho_{1,2}$,
$cov\left(\eta_y, \eta_2\right) = cov\left(a\log x_1 + b\log x_2 + \epsilon_y, \log x_2 + \epsilon_2\right) = b\sigma_2^2 + a\rho_{1,2}$,
$var\left(\eta_y\right) = a^2\sigma_1^2 + b^2\sigma_2^2 + \sigma_y^2 + 2ab\rho_{1,2}$,
$var\left(\eta_2\right) = \sigma_2^2 + s_2^2$.
Substituting yields

$$\beta_y = \frac{a\sigma_1^2 + b\rho_{1,2} - \frac{\rho_{1,2}\left(a\rho_{1,2} + b\sigma_2^2\right)}{\sigma_2^2 + s_2^2}}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 + 2ab\rho_{1,2} - \frac{\left(a\rho_{1,2} + b\sigma_2^2\right)^2}{\sigma_2^2 + s_2^2}}$$

$$\beta_2 = \frac{\rho_{1,2} - \frac{\left(a\rho_{1,2} + b\sigma_2^2\right)\left(a\sigma_1^2 + b\rho_{1,2}\right)}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 + 2ab\rho_{1,2}}}{\sigma_2^2 + s_2^2 - \frac{\left(a\rho_{1,2} + b\sigma_2^2\right)^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 + 2ab\rho_{1,2}}}$$

For $\rho_{1,2} = 0$, we obtain

$$\begin{aligned}
\beta_y &= \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}}, \\
\beta_2 &= \frac{-\frac{\left(a\rho_{1,2}+b\sigma_2^2\right)\left(a\sigma_1^2\right)}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}}{\sigma_2^2 + s_2^2 - \frac{b^2\sigma_2^4}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}} = -\frac{ab\sigma_1^2\sigma_2^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2} \times \frac{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}{\left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right) - b^2\sigma_2^4} \\
&= \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)}.
\end{aligned}$$

QED

Proof of Corollary 6.

$$\begin{aligned}
\lim_{s_y,s_2\to+\infty} \beta_y &= \lim_{s_y,s_2\to+\infty} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = 0, \\
\lim_{s_y,s_2\to+\infty} \beta_2 &= \lim_{s_y,s_2\to+\infty} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = 0.
\end{aligned}$$

QED

Proof of Corollary 7.

$$\begin{aligned}
\lim_{s_2\to+\infty} \beta_y &= \lim_{s_2\to+\infty} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}, \\
\lim_{s_2\to+\infty} \beta_2 &= \lim_{s_2\to+\infty} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = 0.
\end{aligned}$$

QED

Proof of Corollary 8.

$$\begin{aligned}
\lim_{s_y,s_2\to 0} \beta_y &= \lim_{s_y,s_2\to 0} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + -\frac{b^2\sigma_2^4}{\sigma_2^2}} = \frac{1}{a}, \\
\lim_{s_y,s_2\to 0} \beta_2 &= \lim_{s_y,s_2\to 0} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \sigma_2^2\left(a^2\sigma_1^2 + b^2\sigma_2^2\right)} = -\frac{b}{a}.
\end{aligned}$$

QED

Proof of Proposition 4. To derive the test statistic, note that we have $\mathbb{E}_t \log y_{t+1} = a\mathbb{E}_t \log x_{1,t+1} + b\mathbb{E}_t \log x_{2,t+1}$ and $\log x_{2,t+1} = \log \frac{b}{a+b} Z - \pi_{2,t+1}$, implying that the CFO forecast for input 2 is

$$\mathbb{E}_t \log x_{2,t+1} = \log \frac{b}{a+b} Z - \gamma_2 \pi_{2,t}. \tag{12}$$

Note that (12) depends on the technological parameters, a and b, and on Z, which we assume are known to the CFO at the time of the forecast and are stable over time. From (12) we obtain that

$$\frac{\mathbb{E}_t \log x_{2,t+1} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1).$$

We can then derive our test statistic C1-stat based on the joint forecasts of the first input and output by recalling that $\log y = a \log x_1 + b \log x_2$ as follows:

$$\text{C1-stat} \equiv \frac{\frac{\mathbb{E}_t \log y_{t+1} - a\mathbb{E}_t \log x_{1,t+1}}{b} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1),$$

where the distribution here is obtained under the null hypothesis of coherent forecasts.

To derive our C2-stat, we start by defining the forecast error of a generic variable x forecasted at t and realized at $t+1$ as the difference between the realization and the forecast, $\text{FE}_t x_{t+1} = x_{t+1} - \mathbb{E}_t x_{t+1}$. We then have that

$$\begin{aligned} \text{FE}_t \log x_{2,t+1} &= \log x_{2,t+1} - \mathbb{E}_t \log x_{2,t+1} \\ &= \log \frac{b}{a+b} Z - \pi_{2,t+1} - \mathbb{E}_t \left[\log \frac{b}{a+b} Z - \pi_{2,t+1} \right] \\ &= -\text{FE}_t \pi_{2,t+1} = -\epsilon_{2,t+1}. \end{aligned}$$

As a result, the forecast error of the log of the second input is the negative of the innovation of the second log-price process. It follows that

$$\frac{\text{FE}_t \log x_{2,t+1}}{\sigma_2} \sim \mathcal{N}(0,1).$$

Noting that $\text{FE}_t \log y_{t+1} = a\text{FE}_t \log x_{1,t+1} + b\text{FE}_t \log x_{2,t+1}$, we obtain our C2-stat,

$$\text{C2-stat} \equiv \frac{\text{FE}_t \log y_{t+1} - a\text{FE}_t \log x_{1,t+1}}{\sigma_2 b} \sim \mathcal{N}(0,1).$$

QED

Coherence Test Statistic: Multiple Inputs Case. Here we generalize our C2-stat to a multivariate case with N inputs. For this subsection, the production setting is

$$y = \prod_{i=1}^{N} x_i^{a_i}$$

$$\boldsymbol{p}'\boldsymbol{x} = Z,$$

where $\boldsymbol{p}$ and $\boldsymbol{x}$ are the column vectors of factor prices and quantities, respectively. As in the $N = 2$ case we have a linear relationship between the logs of inputs and output,

$$\log y = \sum_{i=1}^{N} a_i \log x_i,$$

where the same equation holds for the forecast errors. Analogously to the bivariate case, we have that $\text{FE}_t \log x_{1,t+1} = -\epsilon_{1,t+1}$ so that

$$\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1} \sim \mathcal{N}(0,1).$$

Then, using the linear relationship between the logs of inputs and output we obtain our generalized C2-stat,

$$\frac{\text{FE}_t \log y_{t+1} - \sum_{i=2}^{N} a_i \text{FE}_t \log x_{i,t+1}}{\sigma_1 a_1} \sim \mathcal{N}(0,1).$$

Proof of Equivalence of Forecast Error of Investment and Forecast Error of Capital. In our model, we take our input x_1 to be capital, that is, the stock of physical capital, K, typically measured by the balance sheet item Property, Plant, and Equipment, whereas in our empirical implementation we use balance sheet item capital expenditures, that is, I. Here we discuss the relation between investment growth (ΔI) and capital growth (ΔK), and we show that in general forecast errors of investment growth should equal the forecast errors of capital growth, $\text{FE}[\Delta I_{t+1}] = \text{FE}[\Delta K_{t+1}]$. To do so, denote end-of-period capital, K_{t+1}, and consider the law of motion:

$$K_{t+1} = K_t (1 - \delta) + I_{t+1}$$

where K_t is beginning-of-period capital, I_{t+1} denotes per-period capital expenditures, and δ is the depreciation rate. Rearranging:

$$I_{t+1} = \Delta K_{t+1} + \delta K_t$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. Therefore,

$$\Delta I_{t+1} = I_{t+1} - I_t = \Delta K_{t+1} + \delta K_t - \Delta K_t - \delta K_{t-1} = \Delta K_{t+1} - \Delta K_t + \delta \Delta K_t$$

and then

$$\Delta I_{t+1} = \Delta K_{t+1} - (1 - \delta) \Delta K_t$$

Similarly, in expectations it holds that:

$$\mathbb{E}[\Delta I_{t+1}] = \mathbb{E}[\Delta K_{t+1}] - (1 - \delta) \Delta K_t$$

Finally, subtracting expectations from realizations we obtain forecast errors, $\text{FE}[\Delta I_{t+1}] = \Delta I_{t+1} - \mathbb{E}[\Delta I_{t+1}]$, and $\text{FE}[\Delta K_{t+1}] = \Delta K_{t+1} - \mathbb{E}[\Delta K_{t+1}]$, respectively, which must satisfy the relationship:

$$\text{FE}[\Delta I_{t+1}] = \text{FE}[\Delta K_{t+1}]$$

That is, the forecast error of capital should equal the forecast error of investment. The intuition is the same of Section V.A., namely computing forecast errors differences away any heterogeneity known to, or predictable by, the firm at the time of the forecast.
QED

B Implementation of Inequality Condition

Inequality test. We implement the inequality restriction in Proposition 1, developed under a general CES function. We view this inequality as imposing on the data as little restriction as possible. We compute a and b using the universe of industries from the Bureau of Economic Analysis and find that $a + b \leq 1$. Furthermore, the elasticity of substitution between capital and labor in the US economy, denoted with χ, is typically found to be between 0.5 and 1 (e.g., see Berndt (1976) and Oberfield and Raval (2021)), where $\chi = 1$ defines the Cobb-Douglas production function. Therefore, the CES function is weakly concave and thus inequality (2) is the relevant one. We account for heterogeneity by allowing a and b to vary by industry and by presenting our results for three different values of the elasticity of substitution between capital and labor, $\chi = 0.5, 0.7, 0.9$. We implement our inequality restriction both in levels and in growth rates.[48]

Table A10 of the Web Appendix reports our results. Panel A shows that most CFOs' forecasts violate the inequality restriction of Proposition 1. In levels, almost all CFOs give joint forecasts of capital, labor, and output that jointly violate the inequality. However, as just discussed, results in levels should be seen with caution, as they refer to a much smaller sample given the limitations of Compustat data on wages. In growth rates, about 73% of CFOs' forecasts violate the inequality. These results are quite stable across different values of the elasticity of substitution between capital and labor. If anything, moving toward $\chi = 1$ (Cobb-Douglas) appears to give a slightly better shot at CFOs to give coherent forecasts, perhaps because much business teaching uses examples based on the Cobb-Douglas.

Panel B reports summary statistics of the difference between the left-hand side and the right-hand side of the inequality of Proposition 1. Most CFOs forecast a growth of output that is larger than the output growth implied by feeding into a CES production function the CFOs' forecasts of capital and labor input growth.

[48]We observe CFO forecasts of growth rates, not of levels. Moreover, while we observe the CFO forecast of the growth rate of labor expenditures, $\mathbb{E}_t \frac{[x_{2,t+1}]}{[x_{2,t}]}$, for a large sample, in Compustat we observe few realizations of $x_{2,t+1}$. Therefore, when we compute $\mathbb{E}_t [x_{2,t+1}] = x_{2,t} \cdot \mathbb{E}_t \frac{[x_{2,t+1}]}{[x_{2,t}]}$ to implement the inequality restriction in levels, we end with much fewer observations in levels than in growth rates.

C Further Figures and Tables

Figure A1: **Survey Questions of Firm Forecasts**

4. Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, -2%, etc.) [Leave blank if not applicable]

% Prices of your products	% Technology spending
% Overtime	% Earnings
% Advertising/Marketing spending	% Revenues
% Number of employees	% Inventory
% Productivity (output per hour worked)	% M&A activity
% Wages/Salaries	% Capital spending
% Health care costs	% Dividends

Figure A2: **Bootstrap of Coherence Test Statistic C2-stat**

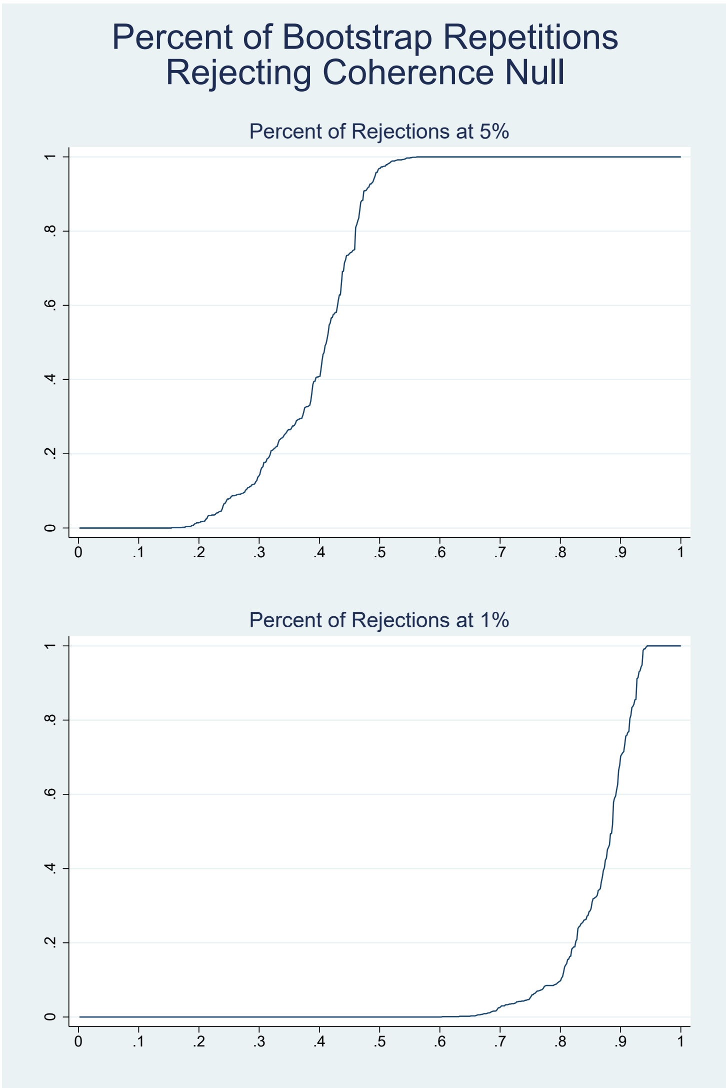

Table A1: **Summary Statistics**

Panel A – Matched Duke-Compustat sample

	Mean	Std. Dev.	P05	Median	P95	N Obs.
Market-to-book	1.845	1.629	0.875	1.402	4.157	15,929
ROA	0.025	0.235	-0.154	0.037	0.167	16,591
Sales	10,617.34	28,482.18	53.66	2,043.96	49,545.00	17,799
Log(sales)	7.591	2.093	4.022	7.629	10.813	17,757
Assets	37,698.73	187,568.6	74.12	2,894.43	113,960.0	17,799
Log(assets)	7.993	2.221	4.306	7.971	11.644	17,799
Book Leverage	0.411	1.755	0.000	0.371	0.910	17,733
Capital Expenditure	0.045	0.058	0.001	0.031	0.134	16,200
R & D	0.060	0.202	0.000	0.025	0.211	9,043
Cash Flow	0.302	13.075	-1.127	0.413	3.005	17,010
Cash	4.895	62.328	0.014	0.563	15.188	17,269
Advertising	0.025	0.043	0.000	0.009	0.098	6,729
Dividends	0.117	1.420	0.000	0.060	0.349	17,391
Dividends (0/1)	0.607	0.488	0.000	1.000	1.000	17,391

Panel B – Compustat data

	Mean	Std. Dev.	P05	Median	P95	N Obs.
Market-to-book	1.843	2.526	0.708	1.297	4.489	105,769
ROA	0.006	0.279	-0.273	0.021	0.187	123,155
Sales	3,521.58	15,220.67	17.992	315.011	14,687.00	127,307
Log(sales)	5.911	2.062	2.890	5.753	9.595	127,307
Assets	12,626.69	98,056.55	24.147	597.555	30,241.99	140,894
Log(assets)	6.529	2.170	3.184	6.393	10.317	140,894
Book Leverage	0.408	22.120	0.000	0.362	0.996	139,264
Capital Expenditure	0.063	0.159	0.000	0.032	0.212	108,909
R & D	0.075	0.135	0.000	0.027	0.290	52,955
Cash Flow	0.050	0.261	-0.222	0.061	0.256	118,905
Cash	0.192	0.396	0.002	0.081	0.685	111,475
Advertising	0.034	0.103	0.000	0.009	0.131	39,813
Dividends	0.144	7.286	0.000	0.000	0.557	122,340
Dividends (0/1)	0.431	0.495	0.000	0.000	1.000	122,340

Table A2: **CFO Growth Forecasts and Realizations of Selected Balance Items**

Panel A – CFO Growth Forecasts (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Expected Growth in Revenues and in Earnings						
Revenues	9.30	27.13	-5.00	5.00	20.00	14,490
Earnings	11.00	42.37	-10.00	5.00	30.00	25,472
Expected Growth in Capital-Related Expenditures						
Capital Expenditures	8.11	43.90	-15.00	3.00	25.00	25,305
R & D	4.51	21.65	0.00	0.00	15.00	8,325
Technology Spending	6.68	28.02	-5.00	3.00	20.00	22,404
Expected Growth in Labor-Related Costs						
Wages	3.90	12.41	0.00	3.00	7.00	27,472
Employees	3.95	30.16	-5.00	1.00	10.00	25,471
Outsourced Employees	3.74	21.19	0.00	0.00	10.00	10,990
Health Spending	8.59	11.65	1.00	8.00	15.00	25,064
Expected Growth in Productivity, Product Prices, and Advertising						
Productivity	3.91	9.38	0.00	3.00	10.00	18,197
Product Prices	2.08	8.22	-3.00	2.00	7.00	24,499
Advertising	4.75	21.83	-5.00	2.00	15.00	20,989
Expected Growth in Cash Holdings and Corporate Payout						
Cash	5.02	38.56	-20.00	0.00	20.00	16,876
Dividends	4.54	30.52	0.00	0.00	15.00	5,227
Share Repurchases	1.55	24.40	0.00	0.00	5.00	5,487

Panel B – Realizations, Matched Compustat-Duke Sample (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Actual Growth in Revenues and in Earnings						
Revenues	6.80	21.32	-13.56	5.23	27.25	14,549
Earnings	-10.36	307.02	-161.71	2.36	124.59	14,580
Actual Growth in Capital-Related Expenditures						
Capital Expenditures	15.87	67.07	-42.59	3.96	75.70	13,770
R & D	7.09	29.57	-19.85	4.27	33.33	6,456
Technology Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Actual Growth in Labor-Related Costs						
Wages	7.02	14.65	-7.23	5.35	22.10	2,836
Employees	2.98	16.95	-11.88	1.19	17.71	14,359
Outsourced Employees	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Health Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Actual Growth in Productivity, Product Prices, and Advertising						
Productivity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Product Prices	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Advertising	8.03	42.14	-26.76	2.79	38.46	5,735
Actual Growth in Cash Holdings and Corporate Payout						
Cash	35.42	132.66	-46.26	5.76	113.78	14,520
Dividends	12.68	52.88	-12.22	6.15	38.44	8,762
Share Repurchases	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table A3: **Growth Realizations of Selected Balance Items**

Realizations in Compustat (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Growth in Revenues and in Earnings						
Revenues	13.38	35.96	-16.64	6.89	46.24	105,866
Earnings	-16.21	432.06	-207.66	-3.92	174.42	105,841
Growth in Capital-Related Expenditures						
Capital Expenditures	35.71	132.99	-56.41	5.26	129.28	100,633
R & D	15.91	53.09	-25.24	6.75	57.64	40,715
Technology Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Growth in Labor-Related Costs						
Wages	10.57	24.22	-9.57	6.94	31.96	29,491
Employees	6.18	25.30	-14.29	2.07	29.17	107,435
Outsourced Employees	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Health Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Growth in Productivity, Product Prices, and Advertising						
Productivity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Product Prices	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Advertising	19.12	80.07	-37.35	4.39	71.33	34,251
Growth in Cash Holdings and Corporate Payout						
Cash	76.55	308.36	-57.50	5.23	184.62	103,833
Dividends	18.74	97.74	-56.43	5.42	60.56	54,841
Share Repurchases	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table A4: **Additional Cross-Sectional Regressions in Compustat Data for Rules of Thumb**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
CapEx Growth$_{t-1 \text{ to } t}$	-0.060***									
	(0.010)									
Avg. CapEx Growth$_{t-1 \text{ to } t}$		-0.089***								-0.089***
		(0.016)								(0.040)
Sales Growth$_{t-1 \text{ to } t}$			1.055***				1.042***	0.996***	0.652***	0.702***
			(0.036)				(0.036)	(0.045)	(0.060)	(0.178)
Wages Growth$_{t-1 \text{ to } t}$				1.036***					0.589***	0.380***
				(0.082)					(0.081)	(0.159)
Earnings Growth$_{t-1 \text{ to } t}$					0.053***		0.018***			-0.034
					(0.005)		(0.004)			(0.024)
Advertising Growth$_{t-1 \text{ to } t}$						0.317***		0.178***		0.263***
						(0.021)		(0.016)		(0.076)
Constant	0.327***	0.316***	0.217***	0.215***	0.355***	0.274***	0.217***	0.178***	0.192***	0.166***
	(0.042)	(0.041)	(0.025)	(0.026)	(0.039)	(0.034)	(0.025)	(0.027)	(0.023)	(0.034)
R^2	0.004	0.004	0.081	0.047	0.005	0.042	0.082	0.096	0.063	0.074
N observations	85,838	74,413	100,441	15,955	100,040	33,202	100,040	33,202	15,955	3,013

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table A5: **Minimum Distance of Sales Forecasts from Rules of Thumb: Robustness to Alternative Definition of Rule 5**

	All	R1	R2	R3	R4	R5
Mean	0.029	0.040	0.025	0.030	0.041	0.023
Std. Dev.	0.039	0.031	0.044	0.031	0.056	0.016
Frac. Zeros	0.146	0.000	0.365	0.000	0.000	0.000
P10	0.000	0.005	0.000	0.006	0.002	0.004
P25	0.005	0.017	0.000	0.009	0.003	0.009
P50	0.016	0.031	0.007	0.022	0.023	0.019
P75	0.035	0.064	0.032	0.040	0.065	0.031
P90	0.071	0.094	0.071	0.074	0.122	0.048
P95	0.106	0.094	0.106	0.094	0.222	0.049
N of Observations	130	9	52	30	18	21
Fraction	1.000	0.069	0.400	0.231	0.138	0.162

Notes: Cross-sectional analysis with 130 CFOs.

Table A6: **Minimum Distance of Earnings Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.026	0.045	0.021	0.028	0.032	0.031
Std. Dev.	0.033	0.054	0.028	0.046	0.035	0.030
Frac. Zeros	0.197	0.000	0.356	0.000	0.000	0.000
P10	0.000	0.002	0.000	0.002	0.005	0.004
P25	0.004	0.003	0.000	0.008	0.007	0.009
P50	0.014	0.027	0.014	0.010	0.017	0.015
P75	0.035	0.064	0.035	0.017	0.050	0.052
P90	0.068	0.101	0.057	0.099	0.071	0.073
P95	0.101	0.177	0.085	0.182	0.111	0.090
N of Observations	396	24	219	35	48	70
Fraction	1.000	0.061	0.553	0.088	0.121	0.177

Notes: Cross-sectional analysis with 396 CFOs.

Table A7: **Minimum Distance of CapEx Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.064	0.079	0.054	0.161	0.079	0.104
Std. Dev.	0.099	0.150	0.072	0.189	0.121	0.083
Frac. Zeros	0.106	0.000	0.146	0.000	0.000	0.000
P10	0.000	0.009	0.000	0.006	0.005	0.012
P25	0.014	0.022	0.014	0.008	0.029	0.039
P50	0.035	0.046	0.035	0.089	0.039	0.090
P75	0.071	0.076	0.071	0.321	0.062	0.157
P90	0.141	0.144	0.127	0.488	0.357	0.173
P95	0.220	0.264	0.163	0.508	0.390	0.293
N of Observations	396	58	288	14	25	11
Fraction	1.000	0.147	0.727	0.035	0.063	0.028

Notes: Cross-sectional analysis with 396 CFOs.

Table A8: **Incoherence and Rules of Thumb: Alternative Definition of R5**

	(1)	(2)	(3)	(4)	(5)
Rule 1 (‘narrow bracketing’)	0.099***				0.134***
	(0.022)				(0.025)
Rule 2 (‘sales anchoring’)		0.018			0.053***
		(0.012)			(0.016)
Rule 3 (‘economies of scale’)			-0.024*		0.023
			(0.014)		(0.018)
Rule 4 (‘industry based’)				0.003	0.044**
				(0.018)	(0.020)
Constant	0.058***	0.058***	0.071***	0.065***	0.023*
	(0.006)	(0.008)	(0.007)	(0.007)	(0.014)
R^2	0.126	0.009	0.014	-0.008	0.175
N observations	130	130	130	130	396
Summary Statistics of the dependent variable					
Mean		0.065	P10		0.012
Std. Dev.		0.069	Median		0.045
			P90		0.153

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table A9: **Incoherence, Rules of Thumb, and Corporate Performance: Robustness Using Alternative Definition of Rule 5**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Incoherence	-0.279*	-0.335**	-0.320**	-0.317**			
	(0.156)	(0.151)	(0.159)	(0.152)			
Rule 1 (‘narrow bracketing’)					-0.062**	-0.071*	-0.071*
					(0.031)	(0.042)	(0.042)
Rule 2 (‘sales anchoring’)					0.009	0.012	0.014
					(0.032)	(0.035)	(0.035)
Rule 3 (‘economies of scale’)					0.018	0.023	0.020
					(0.037)	(0.037)	(0.038)
Rule 4 (‘industry based’)					0.020	0.020	0.026
					(0.033)	(0.039)	(0.036)
Miscalibration ST		0.009				0.004	
		(0.005)				(0.007)	
Optimism ST		0.011				0.009	
		(0.011)				(0.011)	
Miscalibration LT			0.014				0.012
			(0.010)				(0.011)
Optimism LT			0.008				0.007
			(0.011)				(0.011)
Constant	0.051***	0.051***	0.048***	0.014	0.024	0.015	0.015
	(0.011)	(0.011)	(0.012)	(0.013)	(0.031)	(0.034)	(0.034)
Industry FE	No	No	No	Yes	No	No	No
Survey FE	No	No	No	Yes	No	No	No
R^2	0.055	0.075	0.073	0.140	0.017	0.020	0.026
N of Observations	136	122	121	136	136	122	121

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.

Table A10: **Violations of Coherence Inequality Restrictions**

Panel A – Inequality Test of Coherence

	$\chi = 0.5$	$\chi = 0.7$	$\chi = 0.9$
Inequality in Levels			
% Incoherent	100.00	100.00	99.07
% Coherent	0.00	0.00	0.93
% Total	100.00	100.00	100.00
N. Obs.	107	107	107
Inequality in Growth Rates			
% Incoherent	73.31	73.14	72.96
% Coherent	26.69	26.86	27.04
% Total	100.00	100.00	100.00
N. Obs.	577	577	577

Panel B – Summary stats of difference, LHS − RHS

	$\chi = 0.5$	$\chi = 0.7$	$\chi = 0.9$
Inequality in Levels			
Mean	15,419.59	15,201.16	15,033.78
Std. Dev.	28,574.83	28,233.56	27,990.89
Q10	213.3352	207.60	194.3495
Median	3,252.30	3228.92	3,205.96
Q90	39,737.28	39,505.75	39,360.82
N. Observations	107	107	107
Inequality in Growth Rates			
Mean	0.047	0.044	0.042
Std. Dev.	0.122	0.125	0.128
Q10	-0.061	-0.067	-0.067
Median	0.034	0.033	0.033
Q90	0.164	0.163	0.162
N. Observations	577	577	577

Notes: χ denotes the elasticity of substitution between capital and labor.