Strike while the Iron is Hot:

Optimal Monetary Policy with a Nonlinear Phillips Curve

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Motivation

▶ The recent inflation surge featured

    ▶ Increase in the frequency of price changes (Montag and Villar, 2023) US

    ▶ Increase in Phillips curve slope (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023) US

▶ Optimal monetary policy is mainly studied in models, in which the Phillips curve is linear and the frequency is held constant (Galí, 2008; Woodford, 2003)

▶ What does optimal monetary policy look like with a nonlinear Phillips curve and endogenous variation in frequency? How should CBs respond to a large inflation surge?

What do we do?

▶ We use the standard state-dependent pricing model of Golosov and Lucas (2007)

▶ Solve it nonlinearly using a new algorithm over the sequence space

▶ Positive analysis under a Taylor rule

     ▶ Trace the responses to shocks of different sizes

     ▶ Assess the nonlinearity of the Phillips curve

▶ Normative analysis: Ramsey optimal policy

     ▶ Optimal long-run inflation

     ▶ Trace the optimal responses to shocks

     ▶ Characterize the (nonlinear) targeting rule after *large* cost-push shocks

What do we find?

▶ In this model the Phillips curve is nonlinear: it gets steeper as frequency increases

▶ In response to small shocks, optimal monetary policy is similar to the one under Calvo

▶ In response to efficiency shocks, there is divine coincidence, as in Calvo

▶ Different responses to small and large cost-push shocks. Optimal policy leans aggressively against inflation when frequency rises: "it strikes while the iron is hot"

Literature

▶ Nonlinear Phillips curve (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023)

    ▶ Microfounded by state-dependent price setting

      (Golosov and Lucas, 2007; Gertler and Leahy, 2008; Auclert et al., 2022)

    ▶ In the presence of large aggregate shocks (Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020;

      Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024)

▶ Optimal policy in a menu cost economy

    ▶ Optimal inflation target (Burstein and Hellwig, 2008; Adam and Weber, 2019; Blanco, 2021)

    ▶ Small shocks, large shocks, optimal nonlinear target rule (comp. Galí, 2008; Woodford, 2003)

    ▶ Focus on aggregate shocks (unlike Caratelli and Halperin, 2023, who study *sectoral* shocks)

Overview of the model

▶ Heterogeneous-firm DSGE model with fixed costs of price-adjustment

▶ Households: consume a Dixit-Stiglitz basket of goods, and work (HH)

▶ Firms: produce differentiated goods using labor only and are subject to aggregate TFP shocks and idiosyncratic "quality" shocks. They have market power and set prices optimally subject to a fixed cost (Golosov and Lucas, 2007) (Firms).

▶ Monetary policy: either follows Taylor rule or set optimally to maximize household welfare under commitment (Policy)

## Model: Intuitive summary

▶ Each period, firms choose whether to reset
their price and, if so, what new price to set

▶ The firm's optimality conditions define the
reset price and the inaction region (s,S)

▶ Given the idiosyncratic shock, this
endogenously determines the price distribution

▶ Let $p_t(j) \equiv \log\left(P_t(j)/(A_t(j)P_t)\right)$ be the
quality-adjusted log relative price

▶ Let $x_t(j) \equiv p_t(j) - p_t^*(j)$ be the price gap

Distribution of Price Gaps in Steady State
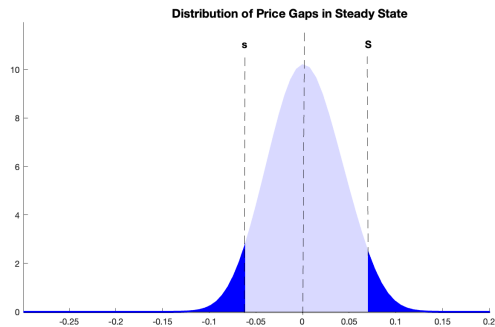
Model under large shock

- ▶ Large aggregate shock: shifts the distribution of price gaps for all firms
- ▶ Limited impact on the (s,S) bands
- ▶ Pushes a large fraction of firms outside of the inaction region
- ▶ Large increase in the frequency of price changes and hence additional flexibility of the aggregate price level (on top of "selection")

Distribution of Price Gaps after Large Shock
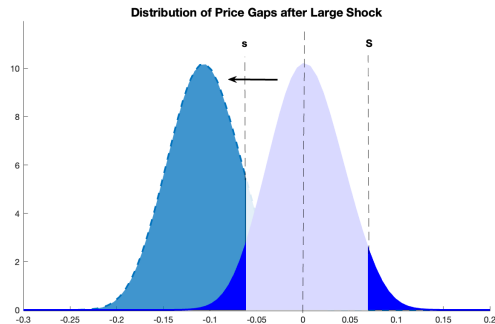
Model: Distortions

► Monopolistic competition and nominal frictions imply three distortions:

  ► Inefficient markup fluctuations
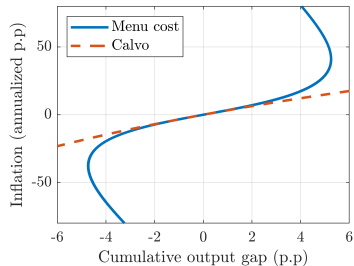
  ► Price dispersion

  ► Price adjustment (menu) costs

Calibration

		Households	
β	$0.96^{1/12}$	Discount rate	Golosov and Lucas (2007)
ϵ	7	Elasticity of substitution	Golosov and Lucas (2007)
γ	1	Risk aversion parameter	Midrigan (2011)
υ	1	Utility weight on labor	Set to yield $w = C$

		Price setting	
η	3.6%	Menu cost	Set to match 8.7% of frequency
σ	2.4%	Std dev of quality shocks	and 8.5% size in Nakamura and Steinsson (2008)

		Monetary policy	
ϕ_π	1.5	Inflation coefficient in Taylor rule	Taylor (1993)
ϕ_y	0.5/12	Output gap coefficient in Taylor rule	Taylor (1993)
ρ_i	$0.75^{1/3}$	Smoothing coefficient	

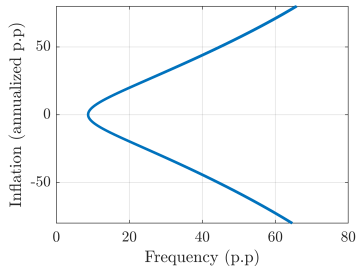|		Shocks	
ρ_A	$0.95^{1/3}$	Persistence of the TFP shock	Smets and Wouters (2007)
ρ_τ	$0.9^{1/3}$	Persistence of the cost-push shock	Smets and Wouters (2007)

Nonlinearity of the Phillips Curve at realistic frequency (20%) `US`
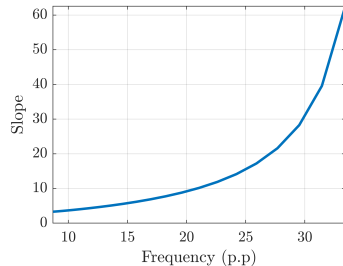
Consider the model under a Taylor rule `Robustness`

| Nonlinear Phillips Curve (PC)	Frequency	PC slope

Normative results: Computation

▶ Challenges

    ▶ Price change distribution and firms' value function are infinite-dimensional objects

    ▶ In the Ramsey problem, we need derivatives w.r.t. both

▶ New algorithm, inspired by González et al. (2024)

    ▶ Approximate distribution and value functions by piece-wise linear interpolation on grid

    ▶ Endogenous grid points: (s,S) bands and the optimal reset price

    ▶ Solve in the sequence space using Dynare (Adjemian et al. (2023))

Optimal long-run inflation rate

▶ The Ramsey steady-state inflation rate is slightly above zero: $\pi^* = 0.25\%$

  ▶ Close to the inflation rate that minimizes the steady-state frequency of price changes

▶ Why not exactly zero as in Calvo (1983)?

  ▶ Asymmetry of the profit function leads to asymmetric (s,S) bands: a negative price gap is less desirable than a positive price gap of the same size

  ▶ At zero inflation, more mass around the lower (s) band than around the higher (S) band

  ▶ Slightly positive inflation raises p^* and pushes the mass of firms to the right inside (s,S)

  ▶ This leads to lower frequency and lower price-adjustment costs

Steady-state price distribution (at zero inflation)

Log-relative price deviation from optimal (x)
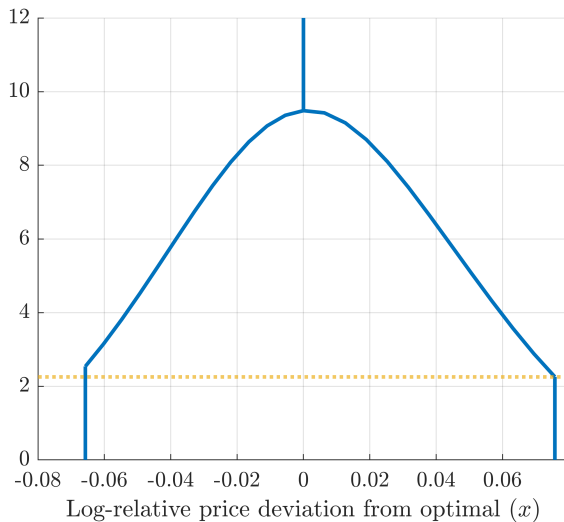
Optimal response to cost-push shocks

▶ In linearized Calvo (1983), optimal policy is a flexible inflation targeting rule

$$\pi_t = -\frac{1}{\epsilon}\Delta \tilde{y}_t$$

▶ Slope $-1/\epsilon$ is independent of the frequency of repricing or the slope of the PC

  ▶ An increase in frequency raises the slope of the Phillips curve κ

  ▶ But it also raises the relative weight of the *output-gap* in welfare, $\lambda = \kappa/\epsilon$

  ▶ Why? Because more price-flexibility implies that inflation is less costly.

▶ For small cost-push shocks, the slope of the targeting rule in Golosov and Lucas (2007) is also $-1/\epsilon$!

Nonlinear targeting rule

▶ Globally, the target rule is nonlinear `Robust`

▶ After large shocks, the planner stabilizes inflation more relative to the output gap

▶ Why? Stabilizing inflation is cheaper due to the lower sacrifice ratio (higher freq.)

    ▶ Similar results with quadratic objective

    ▶ The nonlinearity of the targeting rule is due to the nonlinear Phillips curve
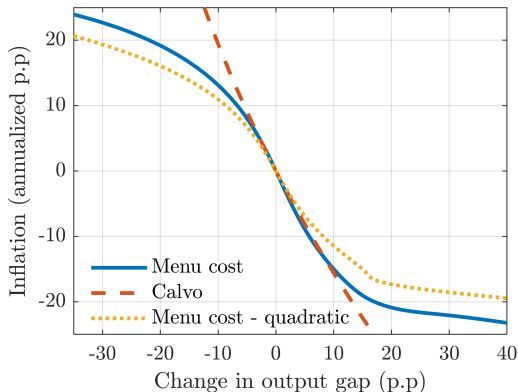
Nonlinear targeting rule for the real interest rate
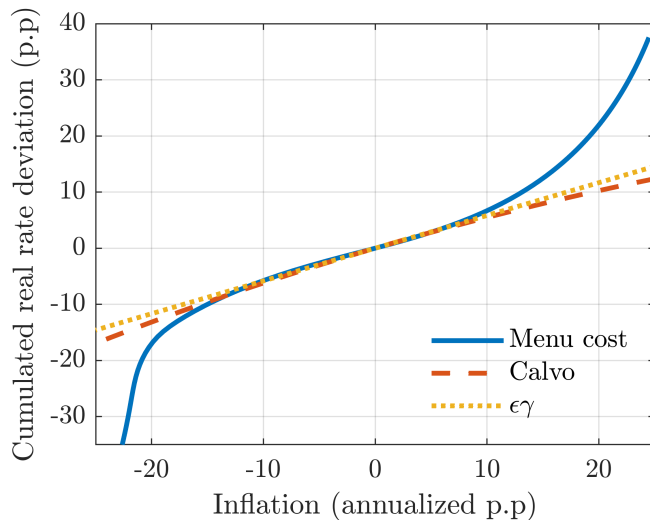
## Optimal responses to efficiency shocks: "divine coincidence"

▶ In the standard NK model with Calvo pricing: divine coincidence holds after TFP and other shocks affecting the efficient allocation

▶ Optimal policy fully stabilizes inflation and closes the output gap

▶ We show analytically, that, after a TFP shock, divine coincidence holds also in the menu cost model: inflation is fully stabilized at steady state and the output gap is closed

Conclusion

We study optimal policy in a state-dependent framework with a nonlinear Phillips curve

▶ Optimal long-run inflation is near zero (slightly positive)

▶ Divine coincidence holds for efficiency shocks

▶ For small cost shocks the optimal response is similar to Calvo (1983):

   the lower welfare weight on inflation offsets the higher slope of the Phillips curve

▶ For large cost shocks, CB leans aggressively against inflation: strike while the iron is hot!

CPI and frequency of price changes in the US, Montag and Villar (2023)
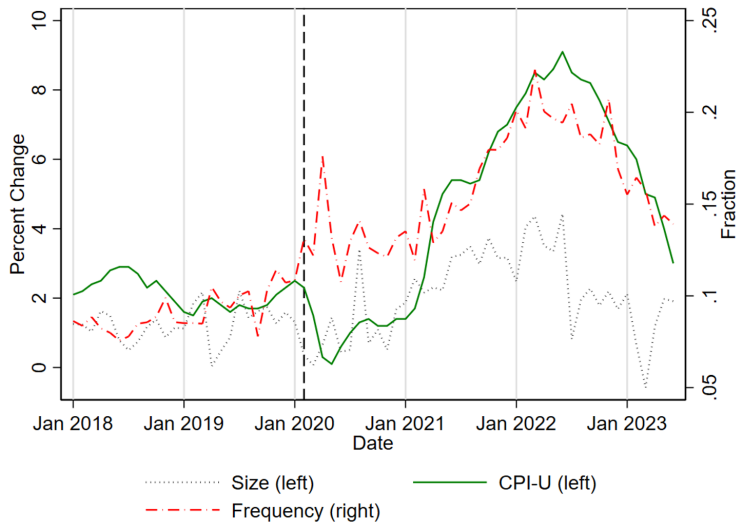
Phillips correlation across US cities, Cerrato and Gitti (2023)
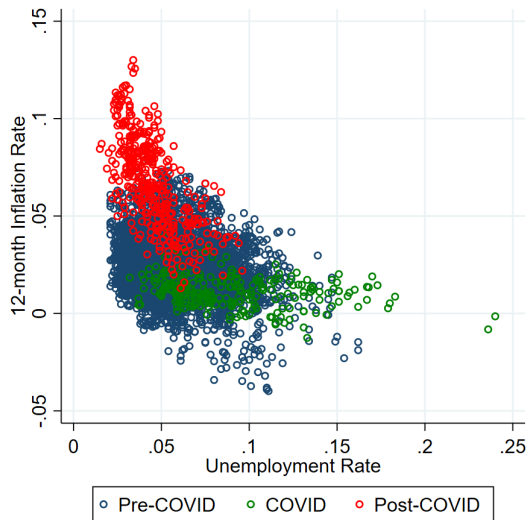
Households

▶ A representative household consumes (C_t), supplies labor hours (N_t) and saves in
one-period nominal bonds (B_t).

▶ The household's problem is:

$$\max_{C_t, N_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C)_t - \nu N_t$$

$$\text{s.t. } P_t C_t + B_t + T_t = R_{t-1} B_{t-1} + W_t N_t + D_t,$$

where P_t is the price level, R_t is the gross nominal interest rate, W_t is the nominal wage,
T_t are lump sum transfers and D_t are profits

Consumption and labor

▶ Aggregate consumption C_t and the price level are defined as:

$$C_t = \left\{ \int \left[A_t(i) C_t(i) \right]^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}, \quad P_t = \left[\int_0^1 \left(\frac{P_t(i)}{A_t(i)} \right)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

where $A_t(i)$ is product quality, ϵ is the elasticity of substitution.

▶ Labor supply condition and Euler equation are given by:

$$W_t = \upsilon P_t C_t, \quad 1 = \mathbb{E}_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{R_t}{\Pi_{t+1}} \right]$$

Monopolistic producers

▶ Production of good i is given by $Y_t(i) = A_t \dfrac{N_t(i)}{A_t(i)}$, where quality follows a random walk

$$log(A_t(i)) = log(A_{t-1}(i)) + \varepsilon_t(i), \quad \varepsilon_t(i) \sim N(0, \sigma_t^2)$$

▶ Firms real profits

$$\frac{D_t(i)}{P_t} = \frac{P_t(i)}{P_t} Y_t(i) - (1 - \tau_t)\frac{W_t}{P_t} N_t(i)$$

▶ Firms face a fixed cost η to update prices

Quality-adjusted relative prices

- Let $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log relative price
- Real profit then is

$$\Pi(p_t(i), w_t, A_t) \equiv \frac{D_t(i)}{P_t} = C_t e^{p_t(i)(1-\epsilon)} - C_t(1-\tau_t)\frac{w_t}{A_t}e^{p_t(i)(-\epsilon)}$$

  where w_t is the real wage.
- When nominal price $P_t(i)$ stays constant, $p_t(i)$ evolves: $p_t(i) = p_{t-1}(i) - \sigma_t\varepsilon_t(i) - \pi_t$

Pricing decision

▶ Let $\lambda_t(p)$ be the price-adjustment probability

▶ Value function is

$$
\begin{aligned}
V_t(p) &= \Pi(p, w_t, A_t) \\
&+ \mathbb{E}_t\left[(1 - \lambda_{t+1}\left(p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1}\right))\Lambda_{t,t+1}V_{t+1}(p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1})\right] \\
&+ \mathbb{E}_t\left[\lambda_{t+1}\left(p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1}\right)\Lambda_{t,t+1}\left(\max_{p'}V_{t+1}\left(p'\right) - \eta w_{t+1}\right)\right].
\end{aligned}
$$

▶ The adjustment probability is

$$
\lambda_t(p) = I[\max_{p'}V_t\left(p'\right) - \eta w_t > V_t(p)]
$$

where $I[\cdot]$ is the indicator function.

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Monetary Policy

▶ The central bank either sets policy optimally, or follows a Taylor rule:

$$\log\left(R_t\right) = \rho_r \log\left(R_{t-1}\right) + (1 - \rho_r)\left[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^e)\right] + \varepsilon_{r,t} \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2)$$

▶ Shocks: employment subsidy (τ_t), TFP (A_t), volatility (σ_t)

$$\log\left(A_t\right) = \rho_A \log\left(A_{t-1}\right) + \varepsilon_{A,t} \quad \varepsilon_{A,t} \sim N(0, \sigma_A^2)$$

$$\tau_t - \tau = \rho_\tau(\tau_{t-1} - \tau) + \varepsilon_{\tau,t} \quad \varepsilon_{\tau,t} \sim N(0, \sigma_\tau^2),$$

$$\log\left(\sigma_t/\sigma\right) = \rho_\sigma \log\left(\sigma_{t-1}/\sigma\right) + \varepsilon_{\sigma,t} \quad \varepsilon_{\sigma,t} \sim N(0, \sigma_\sigma^2)$$

Aggregation and market clearing

▶ Aggregate price index

$$1 = \int e^{p(1-\epsilon)} g_t(p) \, dp,$$

▶ Labor market equilibrium

$$N_t = \frac{C_t}{A_t} \underbrace{\int e^{p(-\epsilon)} g_t(p) \, dp}_{\text{dispersion}} + \eta \underbrace{\int \lambda_t (p - \sigma_t \varepsilon - \pi_t) g_{t-1}(p) dp}_{\text{frequency}},$$

Law of motion of the price density

$$g_t(p) = \begin{cases} (1 - \lambda_t(p)) \int g_{t-1}(p + \sigma\varepsilon + \pi_t)d\xi(\varepsilon) & \text{if } p \neq p_t^*, \\ \\ (1 - \lambda_t(p_t^*)) \int g_{t-1}(p_t^* + \sigma\varepsilon + \pi_t)d\xi(\varepsilon) + \\ \quad \int_{\underline{p}}^{\overline{p}} \lambda_t(\tilde{p}) \left(\int g_{t-1}(\tilde{p} + \sigma\varepsilon + \pi_t)d\xi(\varepsilon) \right) d\tilde{p} & \text{if } p = p_t^*. \end{cases}$$

The Ramsey problem

$$\max_{\{g_t^c(\cdot),g_t^0,V_t(\cdot),C_t,w_t,p_t^*,s_t,S_t,\pi_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, \frac{C_t}{A_t}\left(\int e^{(x+p_t^*)(-\epsilon_t)} g_t^c(p)\,dx + g_t^0 e^{(p_t^*)(-\epsilon)}\right) + \eta g_t^0\right)$$

subject to

$$1 = \int e^{(x+p_t^*)(1-\epsilon)} g_t^c(x)\,dx + g_t^0 e^{(p_t^*)(1-\epsilon)},$$

$$V_t'(0) = \Pi_t'(x) + \frac{1}{\sigma}\Lambda_{t+1}\int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x')\frac{\partial\phi\left(\frac{x-x'-\pi_t^*}{\sigma}\right)}{\partial x}dx' + \Lambda_{t+1}\left(\phi\left(\frac{S_{t+1}-\pi_t^*}{\sigma}\right) - \phi\left(\frac{s_{t+1}-\pi_t^*}{\sigma}\right)\right)(V_{t+1}(0)-\eta w_{t+1}),$$

$$V_t(s_t) = V_t(0) - \eta w_t,$$

$$V_t(S_t) = V_t(0) - \eta w_t,$$

$$w_t = \upsilon C_t^\gamma,$$
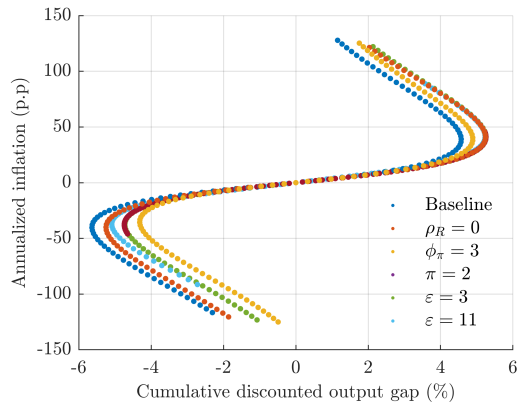
$$V_t(x) = \Pi(x,p_t^*,w_t,A_t) + \Lambda_{t,t+1}\frac{1}{\sigma}\int_{s_t}^{S_t}\left[V_{t+1}(x')\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right]dx' + \Lambda_{t,t+1}\left(1-\frac{1}{\sigma}\int_{s_t}^{S_t}\left[\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right]dx'\right)[(V_{t+1}(0)-\eta w_{t+1})],$$

$$g_t^c(x) = \frac{1}{\sigma}\int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1})\phi\left(\frac{x_{-1}-x-\pi_t^*}{\sigma}\right)dx_{-1} + g_{t-1}^0\phi\left(\frac{-x-\pi_t^*}{\sigma}\right),$$
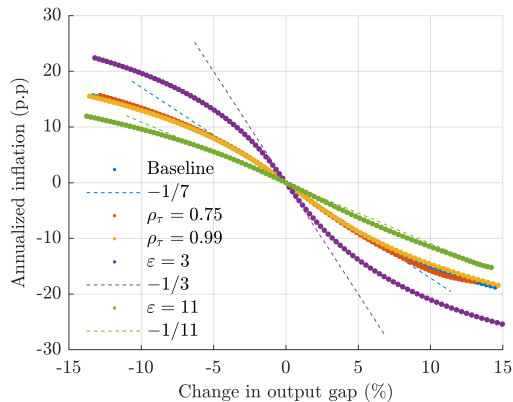
$$g_t^0 = 1 - \int_{s_t}^{S_t} g_t^c(x)dx.$$
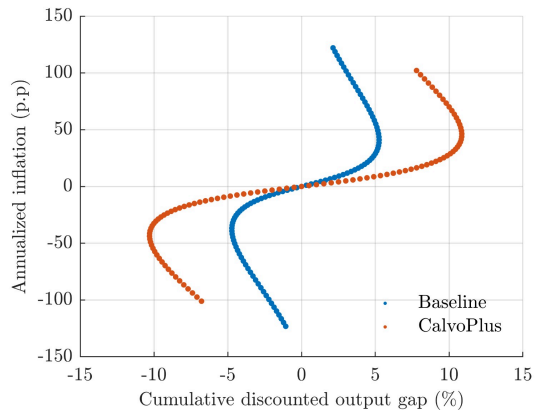
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Robustness

Nonlinear Phillips Curve

Target rule

CalvoPlus model

Nonlinear Phillips Curve

Target rule