Behavioral Sticky Prices*

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Abstract

We study a model where households make decisions according to a dual-process framework widely used in cognitive psychology. System 1 uses effortless heuristics but is susceptible to biases and errors. System 2 uses mental effort to make more accurate decisions. Through their pricing behavior, monopolistic producers can influence whether households deploy Systems 1 or 2. The strategic use of this influence creates a new source of price inertia and provides a natural explanation for the "rockets and feathers" phenomenon: prices rise quickly when costs increase but fall slowly when costs fall. Our model implies that price stability is not optimal.

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1 Introduction

We study a model where households make decisions according to a dual-process framework widely used in the cognitive psychology literature to describe human decision making (see, e.g., Stanovich and West (2000)). System 1 uses heuristics to make quick decisions that require little or no effort but are prone to biases and systematic errors. System 2 uses mental effort to make slower, more deliberate decisions that are more accurate. Our paper builds on the elegant formulation of dual process reasoning proposed by Ilut and Valchev (2023).

In our model, households make errors in their purchase decisions because of cognitive costs. Monopolistic producers, for whom these errors result in high levels of demand relative to the rational optimum, have an incentive to keep their prices constant to discourage households from activating System 2 and reconsidering their purchasing decisions. This behavior generates a novel type of price inertia.

This form of inertia is consistent with the "sticky winners" phenomenon documented by Ilut et al. (2020): firms that receive a high demand realization are less likely to change their prices.

Our model offers a natural explanation for a puzzling empirical regularity documented by Karrenbrock (1991), Neumark and Sharpe (1992), and Peltzman (2000) known as "rockets and feathers": prices increase rapidly when costs rise but decrease slowly when costs fall. This phenomenon arises naturally in our model from the strategic interaction between monopolistic producers and households.[1]

When costs rise significantly, all firms increase prices to avoid losses, so costs and

[1]This paper is related to the literature on price stickiness due to information frictions. Mankiw and Reis (2002), Maćkowiak and Wiederholt (2009), and Woodford (2009) place the information friction on the side of the firms (see also Ilut et al. (2020)).Matějka (2015) is an interesting example of strategic interaction between producers and households. In his model, monopolistic producers choose prices as a function of unit input costs. However, households cannot observe prices perfectly due to limited information-procession ability. As a result, it is optimal for monopolists to implement simple pricing policies where prices take only a few values. These policies make prices easier to observe for households, thereby reducing pricing uncertainty and increasing sales.

prices rise together. When costs fall, the firms that benefit from favorable demand have an incentive to keep their prices constant so that households do not reoptimize their purchase decisions. So, on average, prices decline by less than costs.

Price stability is generally optimal in cashless economies with sticky prices because it eliminates the relative price distortions produced by inflation (see Woodford (2003)). In our model, price stability is not optimal because of the strategic interaction between monopolists and boundedly rational households. When average inflation is zero, firms that receive favorable demand due to behavioral mistakes maintain their prices. The other firms increase or decrease their prices slightly to try to obtain a more favorable demand. As a result, sizeable behavioral mistakes become ingrained, and households choose a significantly inefficient consumption bundle. It is generally optimal to deviate from zero inflation to reduce this inefficiency.

We now discuss three observations consistent with the importance of System 1 in consumer behavior. The first is "shrinkflation," a situation where manufacturers reduce product sizes while keeping prices constant. The UK Office for National Statistics (2019) found 206 instances between September 2015 and June 2017 where products were downsized, yet their prices remained largely unchanged. Budianto (2024) documents that 35 percent of the products included in the U.K. consumer price index between 2012 and 2023 have suffered changes in quantity.

This practice suggests that some manufacturers are prepared to incur considerable expenses to keep prices stable, presumably to avoid triggering a re-optimization of household purchasing decisions.[2]

The second phenomenon is the increasing adoption of subscription-based business models, such as streaming or software-as-a-service, and the tendency for sub-

[2]President Biden deemed shrinkflation important enough to merit discussion in a February 2024 Super Bowl video broadcast. The president noted that "sports drinks bottles are smaller, a bag of chips has fewer chips, but they're still charging us just as much [...] ice cream cartons have shrunk in size but not in price. [...] Some companies are trying to pull a fast one by shrinking the products little by little and hoping you won't notice."

scription prices to remain stable. This stability can be interpreted as a tactic producers use to dissuade households from engaging System 2 and reassessing the value of their subscriptions.[3]

Amazon Prime subscription prices are remarkably sticky. Initially offered at an annual rate of $79 in 2011, the fee has only been adjusted a few times: to $99 in 2014, $119 in 2018, and $139 in 2022. These adjustments were often accompanied by enhancements in service offerings, including the introduction of Amazon Prime Day, which served to justify the higher fees.

Netflix provides a case study of both price stability and shrinkflation. The standard subscription price remained at $7.99 from November 2010 until May 2014. At that point, the price was increased to $8.99, but only for new subscribers. Existing subscribers were grandfathered in at the $7.99 rate for an additional two years. Concurrently, Netflix rolled out a new basic plan priced at $7.99, which offered only standard-definition video on a single screen, a downgrade from the two high-definition screens available under the regular plan. The price for this basic plan remained unchanged until 2019.

The third observation consistent with the elements of our model is that convenient prices that are slightly below a round number (e.g., $9.99 instead of $10) are widely used (Kashyap (1995) and Blinder et al. (1998)), and less likely to change than other prices (Levy et al. (2011) and Ater and Gerlitz (2017)). This practice can be interpreted as a way to exploit System 1 thinking, creating the perception that the price is lower than its actual value.

Our paper is organized as follows. Section 2 describes our model. Section 3 shows that our model is consistent with the rockets and feathers phenomenon. Section 4 discusses optimal fiscal and monetary policy. Section 5 summarizes our findings.

[3]See Della Vigna and Malmendier (2006) for evidence that consumers often fail to assess the value of subscription services rationally.

2 Model

In this section, we describe the household problem, the monopolistic producers' problem, the government's fiscal and monetary policy, and the economy's equilibrium.

2.1 Household problem

There is a representative household that maximizes its utility,

$$U = \frac{C^{1-\sigma}-1}{1-\sigma} - \frac{N^{1+\eta}}{1+\eta} - \int_0^1 \mathcal{I}_i di, \;\; \sigma, \eta > 0,$$

The variable N denotes the labor supply and $\mathcal{I}_i$ is the cognitive cost of using System 2 to choose how much of good i to buy. We discuss this cost in more detail below. Consumption, C, is a composite of differentiated goods, c_i,

$$C = \left(\int_0^1 c_i^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \;\; \theta > 1. \tag{1}$$

The household maximizes utility subject to the budget constraint,

$$\int_0^1 P_i c_i di \leq WN + \int_0^1 \Pi_i di - \mathcal{T}, \tag{2}$$

where P_i is the nominal price of good i, W is the nominal wage, Π_i is the nominal profits of firm i, and $\mathcal{T}$ is nominal lump-sum taxes.

The representative household observes ω, the vector with the relevant state variables,

$$\omega = \left[W, P_i, \int_0^1 \Pi_i di - \mathcal{T} \right].$$

Fully rational solution The familiar solution to this maximization problem is

$$c_i^* (\omega) = \left(\frac{P_i}{P} \right)^{-\theta} C^* (\omega),$$

$$\left[C^*\left(\omega\right)\right]^{\sigma}\left[N^*\left(\omega\right)\right]^{\eta}=w,, \tag{3}$$

$$C^*\left(\omega\right)=wN^*\left(\omega\right)+\frac{\int_0^1\Pi_i di-\mathcal{T}}{P}, \tag{4}$$

where the superscript * denotes the optimal value of different variables.

The price of aggregate consumption, P, is given by

$$P=\left(\int_0^1 P_i^{1-\theta}di\right)^{\frac{1}{1-\theta}}. \tag{5}$$

The variable w denotes the real wage rate,

$$w\equiv\frac{W}{P}.$$

Bounded rationality solution Now, consider the household problem with bounded rationality. Throughout, we use the formulation of dual process reasoning proposed by Ilut and Valchev (2023).

Households observe the vector of relevant state variables, ω, but cannot solve for the optimal values of $c_i^*(\omega)$ and $N^*(\omega)$. They have prior beliefs about $x_i^*(\omega)$–the optimal level of $ln\left[c_i^*(\omega)\right]$–and can use costly signals to update these beliefs. Once households choose the values of c_i, the labor demand, N, is chosen to satisfy the budget constraint.

Period $t=0$ In order for System 1 to be well defined at $t=1$, we need to consider a pre-period, $t=0$, in which households observe prices and make purchase decisions.[4] The household is uncertain about the optimal log demand for good i, as a function of P_i, so it treats this demand as a random variable. At the beginning of each period, households have a normally distributed prior about log demand $\mathcal{N}\left(x_{i,t-1}\left(P_i\right),\sigma_{i,t-1}\left(P_i,P_i'\right)\right)$. The household observes $P_{i,t}$ and chooses $\sigma^2_{\epsilon,i,t}$–

[4]For simplicity, we omit time subscripts whenever the interpretation is clear.

the variance of $s_{i,t}$, i.e. the noisy signal about $x^*\left(P_{i,t}\right)$. This signal is given by

$$s_{i,t} = x^*\left(P_{i,t}\right) + \sigma_{\epsilon,i,t}\epsilon_{i,t},$$

where $\epsilon_{i,t}$ follows a standard normal distribution.

Once the value of $s_{i,t}$ is realized, the household chooses its log-demand $\widetilde{x}_{i,t}\left(P_{i,t}\right)$ to minimize the expected value of the mean squared error:

$$\int_{-\infty}^{\infty}\left[z - \widetilde{x}_{i,t}\left(P_{i,t}\right)\right]^2 g_i\left(z\right) dz,$$

where $g_i(z)$ is the posterior distribution for the rational log demand for good i.

The solution to this problem is to set $\widetilde{x}_{i,t}$ equal to the mean of the posterior distribution

$$\widetilde{x}_{i,t} = x_{i,t}\left(P_{i,t}\right).$$

The cognitive cost, $\mathcal{I}_{i,t}$, associated with the choice of $\sigma^2_{\epsilon,i,t}$ takes a form familiar from the rational inattention literature (see, e.g., Maćkowiak et al. (2023))

$$\mathcal{I}_{i,t} = \kappa \ln\left[\frac{\sigma^2_{i,t-1}\left(P_{i,t}\right)}{\sigma^2_{i,t}\left(P_{i,t}\right)}\right],\ \ \kappa > 0,$$

where $\mathcal{I}_{i,t}$ is proportional to the average entropy reduction produced by the signal. The optimal variance of the signal is the one that results in the following variance of the posterior:

$$\sigma^2_{i,t}\left(P_{i,t}\right) = \begin{cases} \kappa, & \text{if } \sigma^2_{i,t-1}\left(P_{i,t}\right) > \kappa, \\ \sigma^2_{i,t-1}\left(P_{i,t}\right), & \text{if } \sigma^2_{i,t-1}\left(P_{i,t}\right) \leq \kappa. \end{cases}$$

We make two assumptions. First, $\sigma^2_{i,-1}\left(P_i\right) = \sigma^2_c > \kappa$, for all P_i and all i, so households draw a signal in the pre-period for all realizations of the initial price, $P_{i,0}$. Second, $\sigma_{i,-1}\left(P_i, P'_i\right) = 0$ for all $P_i \neq P'_i$, so households believe that $x^*_i\left(P_i\right)$ is uninformative about $x^*_i\left(P'_i\right)$. This independence assumption is important to keep System 1 simple. When the covariance $\sigma_{i,-1}\left(P_i, P'_i\right)$ is not zero, using System 1 requires the household to solve a complex inference problem that combines signals obtained for different prices.

The posterior mean for $x_{i,0}(P_{i,0})$ is

$$x_{i,0}(P_{i,0}) = x_{i,-1}(P_{i,0}) + \alpha\left[x^*(P_{i,0}) + \sigma_\epsilon \epsilon_{i,0} - x_{i,-1}(P_{i,0})\right],$$

where

$$\alpha \equiv 1 - \frac{\kappa}{\sigma_c^2},$$

$$\sigma_\epsilon \equiv \sqrt{\frac{\kappa}{\alpha}}.$$

The household does not draw signals for the optimal policy associated with prices not observed in the pre-period. For these prices, the posterior distribution is equal to the prior,

$$x_{i,0}(P_i) = x_{i,-1}(P_i), \quad P_i \neq P_{i,0}.$$

Period $t = 1$ Given the signals drawn in the pre-period, the prior variance at $t = 1$ is

$$\sigma_{i,0}^2(P_i) = \begin{cases} \sigma_c^2, & \text{if } P_i \neq P_{i,0}, \\ \kappa, & \text{if } P_i = P_{i,0}. \end{cases}$$

If $P_{i,1} = P_{i,0}$, households find themselves in a familiar situation and rely on System 1 to make decisions. If $P_{i,1} \neq P_{i,0}$, the situation is unfamiliar, and the household activates System 2:

$$x_{i,1}(P_{i,1}) = \begin{cases} x_{i,0}(P_{i,0}), & \text{if } P_{i,1} = P_{i,0}, \\ x_{i,0}(P_{i,1}) + \alpha\left[x^*(P_{i,1}) + \sigma_\epsilon \epsilon_{i,1} - x_{i,0}(P_{i,1})\right], & \text{if } P_{i,1} \neq P_{i,0}. \end{cases}$$

As in Ilut and Valchev (2023), we assume that the mean of the prior in the pre-period coincides with the fully rational value of log demand to ensure that our results are not generated by ex-ante biases, $x_{i,-1}(P_i) = x^*(P_i)$. Under this assumption, we obtain

$$x_{i,1}(P_{i,1}) = \begin{cases} x^*(P_{i,0}) + \alpha\sigma_\epsilon\epsilon_{i,0}, & \text{if } P_{i,1} = P_{i,0}, \\ x^*(P_{i,1}) + \alpha\sigma_\epsilon\epsilon_{i,1}, & \text{if } P_{i,1} \neq P_{i,0}. \end{cases}$$

Defining $\gamma \equiv \alpha\sigma_\epsilon$ and $p_i \equiv P_i/P$, we can write the demand for good i as

$$c_i = e^{\gamma\tilde{\epsilon}_i} c_i^* (\omega) = e^{\gamma\tilde{\epsilon}_i} p_i^{-\theta} C^* (\omega), \tag{6}$$

where

$$\tilde{\epsilon}_i \equiv \begin{cases} \epsilon_{i,0}, & \text{if } P_{i,1} = P_{i,0}, \\ \epsilon_{i,1}, & \text{if } P_{i,1} \neq P_{i,0}. \end{cases}$$

We can think of these households as following the satisficing approach proposed by Simon (1956). They are using a solution to their maximization problem that might not be the global optimum but is satisfactory.

2.2 Firms' problem

The producers of the differentiated goods are monopolistically competitive and are not subject to behavioral biases. Firm i produces y_i units of good i using labor (n_i) according to the production function

$$y_i = An_i. \tag{7}$$

The government provides a labor subsidy at rate τ, which we discuss further below. The firm makes pricing decisions in period 1, before observing the current demand shock, $\epsilon_{i,1}$.

Suppose that $P_{i,0} = P_0$ for all i. If the firm does not change the price, its relative price is $1/\pi$, where $\pi \equiv P/P_0$. The resulting profits are

$$e^{\gamma\epsilon_{i,0}} \left[\left(\frac{1}{\pi} \right) - (1 - \tau) \frac{w}{A} \right] \left(\frac{1}{\pi} \right)^{-\theta} C^* (\omega).$$

If the firm decides to change its price to relative price p_i, then the realized profits depend on the demand shock in period one. The firm's expected profit is

$$\mathbb{E} \left[e^{\gamma\epsilon} \right] \left[p_i - (1 - \tau) \frac{w}{A} \right] p_i^{-\theta} C^* (\omega),$$

where ϵ is a standard normal random variable. The optimal reset price is

$$p^* = \frac{\theta}{\theta - 1} (1 - \tau) \frac{w}{A}, \tag{8}$$

so maximal expected profits given a price change are

$$\mathbb{E}\left[e^{\gamma\epsilon}\right] \frac{1}{\theta} \left[\left(\frac{\theta}{\theta - 1}\right) (1 - \tau) \frac{w}{A}\right]^{1-\theta} C^* (\omega) .$$

There is a demand shock, ℓ, such that whenever $\epsilon_{i,0} \geq \ell$ the firm chooses to keep its price constant. The firm's optimal pricing policy is

$$p_i = \begin{cases} p^*, & \text{if } \epsilon_{i,0} < \ell, \\ \frac{P_0}{P} \equiv \frac{1}{\pi}, & \text{if } \epsilon_{i,0} \geq \ell, \end{cases} \tag{9}$$

and the value of ℓ is given by

$$\ell = \begin{cases} \frac{1}{2}\gamma + \frac{1}{\gamma} \ln \frac{\frac{1}{\theta}\left[\left(\frac{\theta}{\theta-1}\right)(1-\tau)\frac{w}{A}\right]^{1-\theta}}{\left[\left(\frac{P_0}{P}\right)-(1-\tau)\frac{w}{A}\right]\left(\frac{P_0}{P}\right)^{-\theta}}, & \text{if } \frac{1}{\pi} > (1-\tau)\frac{w}{A}, \\ \infty, & \text{if } \frac{1}{\pi} \leq (1-\tau)\frac{w}{A}. \end{cases} \tag{10}$$

If the System 1 demand is sufficiently high, the firm prefers keeping its nominal price constant (and its relative price equal to $\frac{1}{\pi}$) to changing prices and triggering System 2.[5]

Let

$$\chi \equiv 1 - \Phi(\ell) \tag{11}$$

denote the fraction of firms that change prices. Using equation (8) to substitute $(1-\tau)\frac{w}{A}$ in (10), as well as equations (9), (11) and (5), it is possible to show that p^* and χ are functions of π only that satisfy:[6]

$$1 = \chi(\pi)\left(\frac{1}{\pi}\right)^{1-\theta} + [1 - \chi(\pi)]\,[p^*(\pi)]^{1-\theta} . \tag{12}$$

[5] When the pre-period price is equal to the optimal reset price but $\epsilon_i < \ell$, the firm changes the price by an infinitesimal amount to get a new demand draw.

[6] In Lemmas 1 below and 4 in the Appendix, it is shown that these functions are well-defined.

This expression resembles the one for Calvo (1983) pricing with one important difference. Here, the probability of not changing prices, $\chi(\pi)$, is endogenous.

The following lemma characterizes a key property of $p^*(\pi)$.

Lemma 1. *For all* $\pi \geq \frac{\theta}{\theta-1}$, $p^*(\pi) = 1$ *and* $\chi(\pi) = 0$.
See the Appendix for proof.

This lemma states that whenever inflation is higher than $\theta/(\theta-1)$, all firms wish to reset their price. The reason is that otherwise, they would have negative profits.

When there is deflation, firms have an incentive to lower their price to sell a higher quantity. But there are firms with a demand shock, $\epsilon_{i,0}$, that is high enough to induce them to keep their nominal price constant, even for high levels of deflation.

It follows that ℓ can be written as

$$\ell(\pi) = \begin{cases} \frac{1}{2}\gamma + \frac{1}{\gamma}\ln \frac{\frac{1}{\theta}[p^*(\pi)]^{1-\theta}}{\left[\left(\frac{1}{\pi}\right)-\left(\frac{\theta-1}{\theta}\right)p^*(\pi)\right]\left(\frac{1}{\pi}\right)^{-\theta}}, & \text{if } \pi < \frac{\theta}{\theta-1}, \\ \infty, & \text{if } \pi \geq \frac{\theta}{\theta-1}. \end{cases} \tag{13}$$

2.3 Government

The government uses monetary policy to control nominal expenditure. It also implements a uniform *ad valorem* subsidy on labor costs at a rate τ, which is financed with lump-sum taxes,

$$\frac{\mathcal{T}}{P} = \tau w N. \tag{14}$$

We consider a simple form of monetary policy where the growth rate of money, μ targets nominal expenditure under full rationality,

$$\mu = \pi \frac{C^*(\omega)}{C_0}, \tag{15}$$

where C_0 is normalized to 1. An alternative policy is to target realized nominal expenditure. One drawback of this alternative is that the resulting equilibrium might not be unique.

In the Appendix, we also consider a dynamic version of the model in which the monetary authority follows a Taylor rule. We show that the resulting equilibrium is locally unique and that our key results are robust to this alternative formulation.

2.4 Equilibrium

We define the equilibrium as follows.

Definition 1. *An equilibrium is a set of prices, (p^*, π, w), allocations, $(c_i, y_i, n_i, \Pi_i, N)$, and policies, $(\tau_n, \mathcal{T})$, such that, given productivity A and monetary policy μ, the following are satisfied:*

1. *Given ω, c_i satisfies equation (6) and N is chosen to satisfy (2) with equality, where $C^*(\omega)$ and $N^*(\omega)$ solve equations (3) (4), and P satisfies (5).*

2. *Given (A, τ, w, π),*
$$\Pi_i \equiv \left(P_i - (1-\tau)\frac{W}{A}\right)c_i, \tag{16}$$
firms produce y_i units of output according to (7), and set prices according to (9).

3. *Policies $(\tau_n, \mathcal{T})$ are set to satisfy (14), and (15) holds.*

4. *The consumption and labor market clear:*
$$y_i = c_i, \tag{17}$$
$$\int_0^1 n_i di = N. \tag{18}$$

Using equations (1), (6) and (9), we can write the following expression for aggregate consumption:
$$C = \Delta_u(\pi)\, C^*(\omega), \tag{19}$$

where

$$\Delta_u(\pi) = \Big\{ \chi(\pi)\,\mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)\epsilon} \mid \epsilon \geq \ell(\pi)\right]\left(\frac{1}{\pi}\right)^{1-\theta} + \\ [1-\chi(\pi)]\,\mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)\epsilon}\right][p^*(\pi)]^{1-\theta}\Big\}^{\frac{\theta}{\theta-1}},$$

is a utility distortion arising from bounded rationality and price dispersion.

Equations (6), (7), (17), (18) and (19) imply that

$$C = \frac{\Delta_u(\pi)}{\Delta_c(\pi)} AN, \tag{20}$$

where

$$\Delta_c(\pi) = \chi(\pi)\,\mathbb{E}\left[e^{\gamma\epsilon} \mid \epsilon \geq \ell(\pi)\right]\left(\frac{1}{\pi}\right)^{-\theta} + [1-\chi(\pi)]\,\mathbb{E}\left[e^{\gamma\epsilon}\right][p^*(\pi)]^{-\theta}$$

is a production distortion arising from both bounded rationality and price dispersion. The following Lemma shows that the first-best allocation is not attainable due to cognitive costs.

Lemma 2. *For any π, $\Delta_u(\pi) < \Delta_c(\pi)$.*

Proof. The result follows from the repeated application of Jensen's inequality. □

Using the property,

$$\mathbb{E}\left[e^{a\epsilon} \mid \epsilon \geq \underline{\epsilon}\right] = \mathbb{E}\left[e^{a\epsilon}\right]\frac{1-\Phi(\underline{\epsilon}-a)}{1-\Phi(\underline{\epsilon})},$$

and defining δ_u and $\delta(\pi)$ as

$$\delta_u(\pi) \equiv 1 - \Phi\left(\ell(\pi) - \gamma\left(\frac{\theta-1}{\theta}\right)\right), \tag{21}$$

$$\delta(\pi) \equiv 1 - \Phi(\ell(\pi) - \gamma), \tag{22}$$

we obtain,

$$\mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)\epsilon} \mid \epsilon \geq \ell(\pi)\right] = \mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)\epsilon}\right]\frac{\delta_u(\pi)}{\chi(\pi)}.$$

$$\mathbb{E}\left[e^{\gamma\epsilon} \mid \epsilon \geq \ell\left(\pi\right)\right] = \mathbb{E}\left[e^{\gamma\epsilon}\right]\frac{\delta\left(\pi\right)}{\chi\left(\pi\right)}.$$

The distortions $\Delta_u\left(\pi\right)$ and $\Delta_c\left(\pi\right)$ can be simplified as

$$\Delta_u\left(\pi\right) = \left\{\mathbb{E}\left[e^{\gamma\left(\frac{\theta-1}{\theta}\right)\epsilon}\right]\right\}^{\frac{\theta}{\theta-1}}\left\{\delta_u\left(\pi\right)\pi^{\theta-1} + \left[1-\chi\left(\pi\right)\right]\left[p^*\left(\pi\right)\right]^{1-\theta}\right\}^{\frac{\theta}{\theta-1}}, \tag{23}$$

and

$$\Delta_c\left(\pi\right) = \mathbb{E}\left[e^{\gamma\epsilon}\right]\left\{\delta\left(\pi\right)\pi^{\theta} + \left[1-\chi\left(\pi\right)\right]\left[p^*\left(\pi\right)\right]^{-\theta}\right\}. \tag{24}$$

The government's budget constraint, (14), and the definition of nominal profits, (16), imply that

$$\frac{\int_0^1 \Pi_i di - \mathcal{T}}{P} = \int_0^1 \left(p_i - \frac{w}{A}\right)c_i di.$$

Using equation (8) to substitute w and the boundedly rational demands (6) to substitute c_i, we obtain

$$\frac{\int_0^1 \Pi_i di - \mathcal{T}}{P} = \left[1-\vartheta\left(\pi\right)\right]C^*\left(\omega\right), \tag{25}$$

where

$$1-\vartheta\left(\pi\right) \equiv \int_0^1 \left[p_i - \left(\frac{\theta-1}{\theta}\right)\frac{1}{1-\tau}p^*\right]p_i^{-\theta}di.$$

Using equation (25) to substitute profits net of taxes in (4), and using (3) to substitute $N^*\left(\omega\right)$, we obtain an expression for $C^*\left(\omega\right)$,

$$C^*\left(\omega\right) = \left\{\frac{\left[\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{1-\tau}\right)p^*\left(\pi\right)\right]^{1+\eta}}{\left[\vartheta\left(\pi\right)\right]^{\eta}}\right\}^{\frac{1}{\sigma+\eta}} A^{\frac{1+\eta}{\sigma+\eta}} \tag{26}$$

Equation (26) describes the aggregate consumption that a fully rational household would choose. Using (19) and (15), we obtain the equations

$$C\left(\pi\right) = \Delta_u\left(\pi\right)\left\{\frac{\left[\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{1-\tau}\right)p^*\left(\pi\right)\right]^{1+\eta}}{\left[\vartheta\left(\pi\right)\right]^{\eta}}\right\}^{\frac{1}{\sigma+\eta}} A^{\frac{1+\eta}{\sigma+\eta}} \tag{27}$$

and

$$\mu = \pi \frac{C(\pi)}{\Delta_u(\pi)}. \tag{28}$$

Together with equation (12) and the definitions (11), (13), and (21)-(24), these equations characterize the equilibrium aggregate consumption $C(\pi)$ and inflation π.

3 Rockets and Feathers

We now study the impact of cost shocks and show that our model is consistent with the rockets and feathers phenomenon: prices rise quickly when costs increase but fall slowly when costs fall.

We establish our results by doing comparative statics for two productivity levels: $A = 1 + v$ and $A = 1/(1 + v)$, where $v > 0$. Log inflation responds symmetrically to cost shocks in the economy with fully rational households since $\pi^f = 1/C_f$.

To study the response of our economy, we set $1 - \tau = (\theta - 1)/\theta$ and the growth rate of nominal expenditure, μ, to one.

A cost increase (a productivity fall from $A = 1$ to $A = 1/(1 + v)$) generates inflation, while a cost decrease (a productivity rise from $A = 1$ to $A = 1 + v$) creates deflation. To compare the response of prices to these two types of shocks, we plot in Figure 1 the absolute value of the logarithm of gross inflation as a function of the magnitude of the shocks, v. The orange and blue lines correspond to a cost increase and decrease, respectively. In a fully rational model, these two lines coincide. In absolute value, inflation's response is the same for positive and negative cost shocks.

This symmetry is preserved in our model for infinitesimal cost shocks. However, for larger cost shocks, prices respond more to cost increases than declines. When costs rise significantly in our model, all firms increase prices to avoid losses, so costs and prices rise together. When costs fall, the firms that benefit from favorable demand have an incentive to keep their prices constant so that households do not reoptimize their purchase decisions. So, on average, prices decline by less than costs.

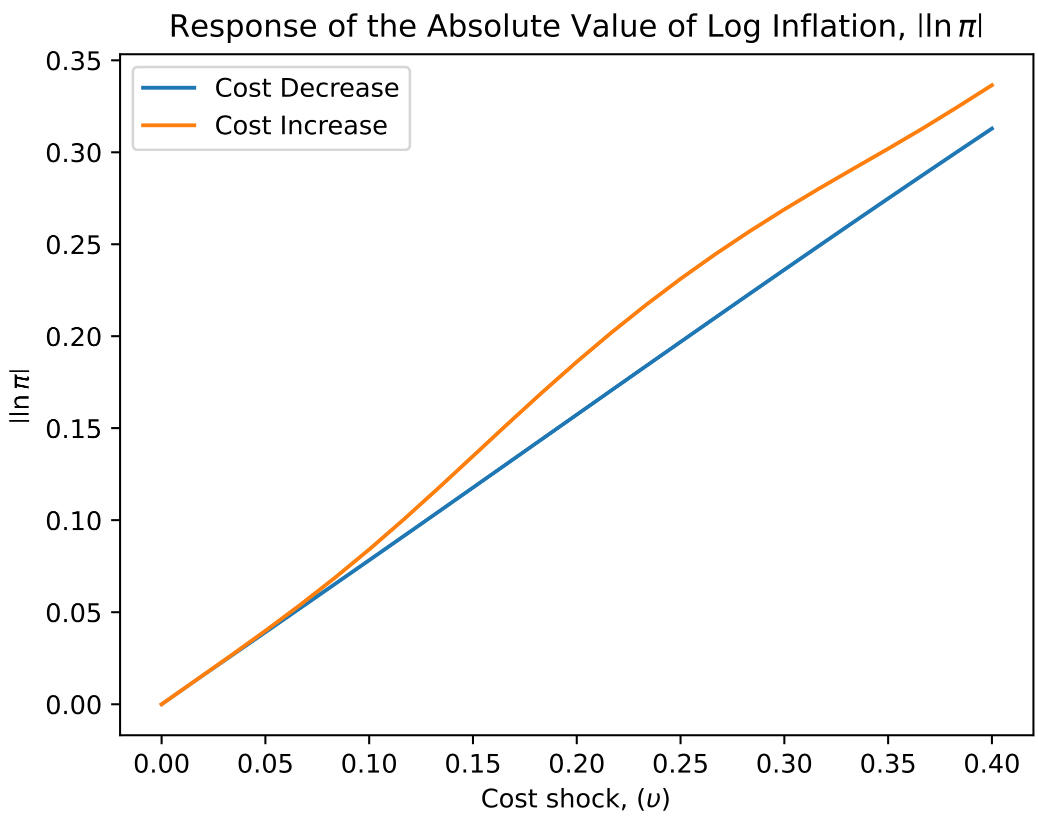

Figure 1: The impact of cost shocks on the absolute value of the logarithm of inflation

For cost shocks higher than 43 percent, all firms change their prices. However, for cost shocks lower than -43 percent, some firms do not lower their prices. As the absolute value of the cost shock increases, the orange and blue lines in Figure 1 eventually converge.

The following proposition shows the main theoretical result for a configuration of parameters that makes the equilibrium analytically tractable.

Proposition 1. *Suppose $\sigma = 1$ and $\eta = 0$. Let $\pi^*(A)$ be the equilibrium level of inflation associated with productivity A. There is $\overline{v}$ such that if $v \geq \overline{v}$,*

$$-\ln\left[\pi^*(1+v)\right] < \ln\left[\pi^*\left(\frac{1}{1+v}\right)\right].$$

See the Appendix for proof.

This proposition implies that for large enough shocks, the percentage response of inflation is higher than the percentage response of deflation to cost shocks with the same absolute value.

4 Optimal Monetary Policy

We now characterize the optimal values for the labor subsidy rate, τ, and the growth rate of nominal expenditure, π. We assume that the government has the ability to implement any desired inflation level.

Social welfare is given by

$$\mathcal{W}(\tau,\pi) = \frac{\left[C(\tau,\pi)\right]^{1-\sigma}-1}{1-\sigma} - \frac{\left[N(\tau,\pi)\right]^{1+\eta}}{1+\eta} - \kappa\left[1-\chi(\pi)\right]\ln\left(\frac{\sigma_c^2}{\kappa}\right),$$

where, given an inflation level π and a labor subsidy τ, the equilibrium allocations must satisfy

$$C(\tau,\pi) = \Delta_u(\pi)\left\{\frac{\left[\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{1-\tau}\right)p^*(\pi)\right]^{1+\eta}}{\left[\vartheta(\tau,\pi)\right]^{\eta}}\right\}^{\frac{1}{\sigma+\eta}} A^{\frac{1+\eta}{\sigma+\eta}}, \tag{29}$$

and

$$C(\tau,\pi) = \frac{\Delta_u(\pi)}{\Delta_c(\pi)} AN(\tau,\pi). \tag{30}$$

We first choose τ given an inflation rate π. Since the fraction of sticky firms $\chi(\pi)$ does not depend on τ, the problem can be written as

$$\max \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\eta}}{1+\eta} \text{ s.t. } C \leq \frac{\Delta_u(\pi)}{\Delta_c(\pi)} AN.$$

Notice that the restriction is equivalent to (30), written in terms of the allocations, C and N. Given the solution for the optimal C and N, condition (29) determines the optimal level of τ.

Lemma 3. *Given π, the optimal consumption and labor allocations are*

$$C_{opt}(\pi) = \left[\frac{\Delta_u(\pi)}{\Delta_c(\pi)} A\right]^{\frac{1+\eta}{\sigma+\eta}},$$

$$N_{opt}(\pi) = \left[\frac{\Delta_u(\pi)}{\Delta_c(\pi)} A\right]^{\frac{1-\sigma}{\sigma+\eta}},$$

corresponding to the labor subsidy rate $\tau(\pi)$ implicitly defined by

$$\Delta_u(\pi) \left\{ \frac{\left[\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{1-\tau(\pi)}\right) p^*(\pi)\right]^{1+\eta}}{\left[\vartheta(\tau(\pi),\pi)\right]^{\eta}} \right\}^{\frac{1}{\sigma+\eta}} = \left[\frac{\Delta_u(\pi)}{\Delta_c(\pi)}\right]^{\frac{1+\eta}{\sigma+\eta}}.$$

We now discuss some properties of the optimal inflation rate, π.

Proposition 2 (With high cognitive costs, price stability is better than high inflation)**.** *Let $\mathcal{W}_s$ be the welfare level attained when gross inflation, $\pi \geq \frac{\theta}{\theta-1}$. Since $\kappa > 0$, there is a value $\overline{\sigma}_c^2$ such that when the household's prior uncertainty about the optimal consumption is higher than $\overline{\sigma}_c^2$ ($\sigma_c^2 \geq \overline{\sigma}_c^2$), price stability is better than high inflation, $\mathcal{W}(1) > \mathcal{W}_s$.*
See the Appendix for proof.

The intuition for this proposition is as follows. Recall that when households opt to gather information regarding the optimal consumption policy, they reduce their uncertainty to κ. When the prior uncertainty is high, this reduction involves significant cognitive effort that the households deem justified in times of high inflation. When inflation is zero, only a few firms adjust their pricing, so households incur low cognitive costs. Because of these low costs, social welfare is higher than when inflation is high.

Proposition 3 (Price stability is not optimal)**.** *There is a value of $\pi < 1$ such that $\mathcal{W}(\pi) > \mathcal{W}(1)$.*
See the Appendix for proof.

The intuition for this result is as follows. When average inflation is zero, firms experiencing high demand due to household decision errors do not change their prices. Other firms slightly increase or decrease their prices to draw a new demand shock. As a result, sizeable behavioral mistakes become ingrained, leading households to select a highly suboptimal consumption basket. Moving away from zero inflation mitigates this inefficiency by improving consumption choices.

Why is deflation locally better than inflation? The logic is as follows. Due to cognitive costs, households do not choose the optimal value of c_i. Instead, they consume an amount of good i that is proportional to the optimal value. The planner would like to reduce the consumption of goods supplied by firms that have sticky prices, since these firms received large demand shocks that drive consumption far away from the optimum. When inflation is positive, the relative price of the goods produced by firms with sticky prices falls, inducing households to consume more of these goods and exacerbating the impact of behavioral biases. In contrast, when inflation is negative, the relative price of the goods produced by firms with sticky prices rises. As a result, the consumption of these goods falls, mitigating the impact of behavioral biases.

To sharpen our intuition, it is helpful to compare the price distortions that emerge in our economy with those in a model with Calvo (1983) sticky prices. Consider a version of our model in which the threshold value of the shock above which firms keep their prices constant, ℓ, is fixed. We adopt this setup because we are interested in studying the behavior of inflation around zero, and locally, ℓ is constant. In this economy, the firms that change prices are the same for inflation and deflation rates with the same absolute value.

Figure 2 compares the production distortion, Δ_c, in this version of our model with that in an analogous model with Calvo pricing. In the Calvo economy, there is no selection–all firms have the same likelihood of changing prices. Consequently, as depicted in Figure 2, the production distortion reaches its lowest point when prices are stable ($\pi^p = 1$). In contrast, in the version of our economy with constant ℓ, the production distortions are minimized when the rate of inflation is negative ($\pi^p < 1$). The reason is the selection effect. Firms with large demand shocks do not change their prices, resulting in high production distortions under price stability.

5 Conclusion

This paper studies a model where households make decisions according to a dual process framework. This framework gives rise to a new kind of price rigidity that emerges from the interaction between consumers and monopolistic suppliers. There is a range of cost shocks for which some producers refrain from adjusting prices so that households do not reassess their purchasing decisions.

Our model explains the intriguing ”rockets and feathers” phenomenon: prices rise quickly when costs increase but fall slowly when costs fall. The model is also consistent with an important empirical regularity documented by Ilut et al. (2020): firms that receive a high demand realization are less likely to change their prices.

Unlike in other cashless economies with sticky prices, price stability is not opti-

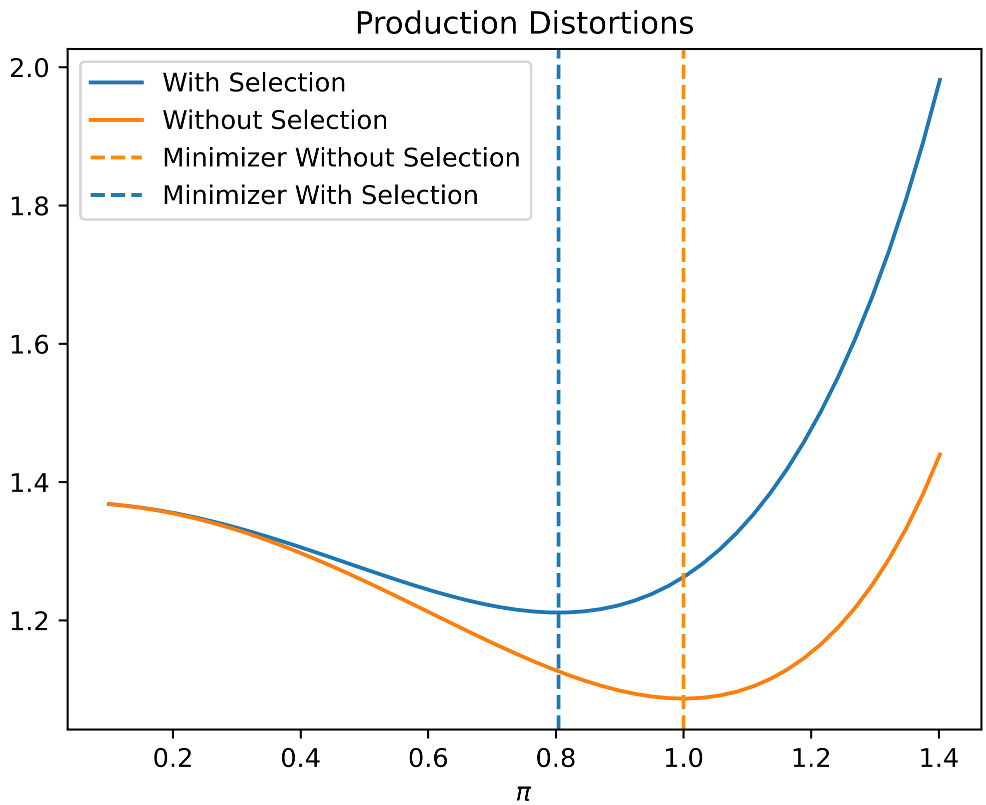

Figure 2: Production distortions as a function of the rate of inflation in a model with Calvo pricing and a version of our economy with ℓ constant

mal in our model.

We predict that the advent of Artificial Intelligence will make the strategic exploitation of the type of consumer behavioral biases present in our model more prevalent.

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6 Appendix

This appendix contains the proofs of our three propositions, a characterization of the properties of the price distribution generated by the model, as well as a derivation of the rockets and feathers result in an infinite-horizon model where policy is conducted through a Taylor rule.

6.1 Proof of Lemma 1

Proof. Let

$$v(p, p^*) \equiv \left[p - \left(\frac{\theta - 1}{\theta}\right) p^*\right] p^{-\theta},$$
$$v^* \equiv v(p^*, p^*),$$

and

$$\overline{v}(\pi, p^*) \equiv v\left(\frac{1}{\pi}, p^*\right).$$

Whenever $\overline{v}(\pi, p^*) > 0$, we can use equations (8) and (10) to write

$$\ell(\pi, p^*) \equiv \ln\left\{\mathbb{E}\left[e^{\gamma\epsilon}\right] \frac{v^*}{\overline{v}(\pi, p^*)}\right\}.$$

We first show that for all $\pi \geq \frac{\theta}{\theta-1}$, equation (12) implies that $\overline{v}(\pi, p^*) \leq 0$. Suppose that this property does not hold, i.e. $\pi \geq \frac{\theta}{\theta-1}$ but $\overline{v}(\pi, p^*) > 0$. Then

$$\overline{v}(\pi, p^*) > 0 \iff \left[\frac{1}{\pi} - \left(\frac{\theta - 1}{\theta}\right) p^*\right] \left(\frac{1}{\pi}\right)^{-\theta} > 0 \iff p^* < 1,$$

which implies that

$$(p^*)^{1-\theta} > 1.$$

From equation (12),

$$\pi^{\theta-1} = \frac{1 - (1 - \chi)(p^*)^{1-\theta}}{\chi}.$$

Therefore

$$\pi^{\theta-1} = \frac{1-(1-\chi)(p^*)^{1-\theta}}{\chi} < 1.$$

This inequality contradicts the initial assumption that $\pi \geq \frac{\theta}{\theta-1} > 1$. Therefore $\overline{v}(\pi, p^*) \leq 0$, and $\ell(\pi, p^*) = \infty$ for all $\pi \geq \frac{\theta}{\theta-1}$. In addition, $\chi = 0$, and equation (12) implies

$$p^* = 1.$$

□

6.2 Lemmas regarding $p^*(\pi)$

This section shows additional lemmas regarding $p^*(\pi)$. The following lemma shows that $p^*(\pi)$ is well-defined for $\pi < \frac{\theta}{\theta-1}$. Let

$$f(\pi, p^*) = \chi(\pi, p^*)\left(\frac{1}{\pi}\right)^{1-\theta} + [1-\chi(\pi, p^*)](p^*)^{1-\theta}$$

and

$$\chi(\pi, p^*) \equiv 1 - \Phi[\ell(\pi, p^*)],$$

where

$$\ell(\pi, p^*) \equiv \begin{cases} \frac{1}{\gamma}\ln\left\{\mathbb{E}[e^{\gamma\epsilon}]\frac{v(p^*,p^*)}{v\left(\frac{1}{\pi},p^*\right)}\right\}, & \text{if } v\left(\frac{1}{\pi}, p^*\right) > 0, \\ \infty, & \text{if } v\left(\frac{1}{\pi}, p^*\right) \leq 0. \end{cases}$$

We now show that the equation

$$f(\pi, p^*) = 1,$$

has a unique solution $p^*(\pi)$ for any $\pi < \frac{\theta}{\theta-1}$.

Lemma 4. *For $\pi < \frac{\theta}{\theta-1}$, $f(\pi, p^*) = 1$ has a unique solution for p^* that involves $\overline{v}(\pi, p^*) > 0$.*

Proof. For fixed $\pi < \frac{\theta}{\theta-1}$, we have that if

$$\frac{1}{\pi} \leq \left(\frac{\theta-1}{\theta}\right) p^* \iff p^* \geq \left(\frac{\theta}{\theta-1}\right) \frac{1}{\pi} > 1,$$

then $\overline{v}(\pi, p^*) \leq 0$. Therefore $\ell(\pi, p^*) = \infty$, $\chi(\pi, p^*) = 0$, and

$$\begin{aligned} f(\pi, p^*) &= \chi(\pi, p^*) \left(\frac{1}{\pi}\right)^{1-\theta} + [1 - \chi(\pi, p^*)] (p^*)^{1-\theta} \\ &= (p^*)^{1-\theta} < 1. \end{aligned}$$

On the other hand, as $p^* \to 0$,

$$v\left(\frac{1}{\pi}, p^*\right) \equiv \left[\frac{1}{\pi} - \left(\frac{\theta-1}{\theta}\right) p^*\right] \left(\frac{1}{\pi}\right)^{-\theta} \to \left(\frac{1}{\pi}\right)^{1-\theta},$$

and

$$v(p^*, p^*) = \frac{1}{\theta} \left[\left(\frac{\theta}{\theta-1}\right) p^*\right]^{1-\theta} \to \infty,$$

which means again that $\ell(\pi, p^*) \to \infty$, and therefore

$$f(\pi, p^*) \to \infty.$$

Since $\lim_{p^* \to 0} f(\pi, p^*) = \infty$ and $f(\pi, p^*) < 1$ for all $p^* \geq \frac{\theta}{\theta-1}\left(\frac{1}{\pi}\right)$, all that is left to be done is to show that $f(\pi, p^*)$ is strictly decreasing for $p^* \in \left(0, \left(\frac{\theta}{\theta-1}\right)\frac{1}{\pi}\right)$, in which case we are assured there is a unique solution in this region.

We have

$$e^{\gamma\ell(\pi,p^*)} = \mathbb{E}\left[e^{\gamma\epsilon}\right] \frac{v(p^*, p^*)}{v\left(\frac{1}{\pi}, p^*\right)} \iff \gamma\ell(\pi, p^*) = \ln\left(\mathbb{E}\left[e^{\gamma z}\right]\right) + \ln\left[v(p^*, p^*)\right] - \ln\left[v\left(\frac{1}{\pi}, p^*\right)\right].$$

and

$$\begin{aligned} \frac{dv(p^*, p^*)}{dp^*} &= (1-\theta) \frac{1}{\theta} \left[\left(\frac{\theta}{\theta-1}\right)\right]^{1-\theta} (p^*)^{1-\theta} \frac{1}{p^*} \\ &= -(\theta-1) v(p^*, p^*) \frac{1}{p^*}, \end{aligned}$$

and

$$v_{p^*}\left(\frac{1}{\pi}, p^*\right) = -\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{\pi}\right)^{-\theta},$$

so

$$\begin{aligned}\gamma \ell_{p^*}(\pi, p^*) &= -(\theta-1)\frac{1}{p^*} + \frac{\left(\frac{\theta-1}{\theta}\right)\left(\frac{1}{\pi}\right)^{-\theta}}{\left[\frac{1}{\pi} - \left(\frac{\theta-1}{\theta}\right)p^*\right]\left(\frac{1}{\pi}\right)^{-\theta}} \\ &= (\theta-1)\left[\frac{p^* - \frac{1}{\pi}}{\left[\frac{1}{\pi} - \left(\frac{\theta-1}{\theta}\right)p^*\right]p^*}\right].\end{aligned}$$

Therefore, we have

$$\begin{aligned}f_{p^*}(\pi, p^*) = \chi_{p^*}(\pi, p^*)\left(\frac{1}{\pi}\right)^{1-\theta} - \chi_{p^*}(\pi, p^*)(p^*)^{1-\theta} - (\theta-1)\left[1 - \chi(\pi, p^*)\right](p^*)^{-\theta} \\ = -\frac{\phi\left[\ell(\pi, p^*)\right]}{\gamma}(\theta-1)\left[\frac{p^* - \frac{1}{\pi}}{\left[\frac{1}{\pi} - \left(\frac{\theta-1}{\theta}\right)p^*\right]p^*}\right]\left[\left(\frac{1}{\pi}\right)^{1-\theta} - (p^*)^{1-\theta}\right] - \\ - (\theta-1)\left[1 - \chi(\pi, p^*)\right](p^*)^{-\theta}.\end{aligned}$$

But

$$p^* - \frac{1}{\pi},$$

and

$$\left(\frac{1}{\pi}\right)^{1-\theta} - (p^*)^{1-\theta},$$

have the same sign. Therefore $f_{p^*}(\pi, p^*) < 0$, which completes the proof. □

Lemma 5. *For $\pi < \frac{\theta}{\theta-1}$, the function $p^*(\pi)$ has elasticity*

$$\frac{p^{*\prime}(\pi)}{p^*(\pi)}\pi = \frac{\chi(\pi)\left(\frac{1}{\pi}\right)^{1-\theta} - \Omega(\pi)}{\Omega(\pi) + \left[1 - \chi(\pi)\right]\left[p^*(\pi)\right]^{1-\theta}},$$

where

$$\Omega(\pi) \equiv \frac{\phi\left[\ell(\pi)\right]}{\gamma}\frac{\left[p^*(\pi) - \frac{1}{\pi}\right]\left\{\left(\frac{1}{\pi}\right)^{1-\theta} - \left[p^*(\pi)\right]^{1-\theta}\right\}}{\frac{1}{\pi} - \left(\frac{\theta-1}{\theta}\right)p^*(\pi)} > 0,$$

and $\ell(\pi)$ has semi-elasticity

$$\ell'(\pi)\,\pi = \frac{1}{\gamma}(\theta-1)\left[\frac{p^*(\pi)-\frac{1}{\pi}}{\frac{1}{\pi}-\left(\frac{\theta-1}{\theta}\right)p^*(\pi)}\right]\left[\frac{1}{\Omega(\pi)+[1-\chi(\pi)]\,[p^*(\pi)]^{1-\theta}}\right].$$

Proof. The proof follows from the total differentiation of (12). □

Lemma 6. *The function p^* has the following properties:*

1. $p^*(1) = 1$;
2. *If* $\pi > 1$, $p^*(\pi) > 1$;
3. *If* $\pi < 1$, $p^*(\pi) < 1$.

Proof. These properties follow directly from equation (12). □

Lemma 7. *$\ell(\pi)$ is minimized at $\pi = 1$, with $\ell(1) = \frac{1}{2}\gamma$. Therefore, $\chi(\pi)$ and $\delta(\pi)$ are maximized at* 1.

Proof. From Lemma 5, $\text{sign}[\ell'(\pi)] = \text{sign}\left[p^*(\pi)-\frac{1}{\pi}\right]$. Therefore $\ell(\pi)$ is decreasing for $\pi < 1$, and increasing for $\pi > 1$, which implies that ℓ is minimized at $\pi = 1$. Equations (11) and (22) imply that χ and δ are decreasing in ℓ. It follows that $\chi(\pi)$ and $\delta(\pi)$ are maximized at $\pi = 1$. □

Lemma 8. *$p^{*\prime}(\pi)$ has exactly one maximum in $\left(1, \frac{\theta}{\theta-1}\right)$ and exactly one minimum in $(0,1)$.*

Proof. At any extremum, Lemma 5 implies that

$$\Omega(\pi) = \chi(\pi)\left(\frac{1}{\pi}\right)^{1-\theta}.$$

Now

$$\Omega(\pi) = \frac{\phi[\ell(\pi)]}{\gamma}\frac{\left[p^*(\pi)-\frac{1}{\pi}\right]\left\{\left(\frac{1}{\pi}\right)^{1-\theta}-[p^*(\pi)]^{1-\theta}\right\}}{\frac{1}{\pi}-\left(\frac{\theta-1}{\theta}\right)p^*(\pi)},$$

so

$$\frac{\phi\left[\ell\left(\pi\right)\right]}{\gamma}\frac{\left[p^{*}\left(\pi\right)-\frac{1}{\pi}\right]\left\{\left(\frac{1}{\pi}\right)^{1-\theta}-\left[p^{*}\left(\pi\right)\right]^{1-\theta}\right\}}{\frac{1}{\pi}-\left(\frac{\theta-1}{\theta}\right)p^{*}\left(\pi\right)}=\chi\left(\pi\right)\left(\frac{1}{\pi}\right)^{1-\theta}$$

$$\iff\frac{\mathbb{E}\left[\epsilon\mid\epsilon\geq\ell\left(\pi\right)\right]}{\gamma}=\frac{\frac{1}{\pi}-\left(\frac{\theta-1}{\theta}\right)p^{*}\left(\pi\right)}{\left[p^{*}\left(\pi\right)-\frac{1}{\pi}\right]\left\{\left(\frac{1}{\pi}\right)^{1-\theta}-\left[p^{*}\left(\pi\right)\right]^{1-\theta}\right\}}\left(\frac{1}{\pi}\right)^{1-\theta}. \tag{31}$$

Consider the function

$$g\left(a\right)=\mathbb{E}\left[\epsilon\mid\epsilon\geq a\right]=\frac{\phi\left(a\right)}{1-\Phi\left(a\right)}.$$

The first derivative is

$$\begin{aligned}g'\left(a\right)&=\frac{-a\phi\left(a\right)\left[1-\Phi\left(a\right)\right]-\phi\left(a\right)\left[-\phi\left(a\right)\right]}{\left[1-\Phi\left(a\right)\right]^{2}}\\&=\frac{-a\phi\left(a\right)}{1-\Phi\left(a\right)}+\mathbb{E}\left[\epsilon\mid\epsilon\geq a\right]^{2}\\&=-a\mathbb{E}\left[\epsilon\mid\epsilon\geq a\right]+\mathbb{E}\left[\epsilon\mid\epsilon\geq a\right]^{2}\\&=\left\{\mathbb{E}\left[\epsilon\mid\epsilon\geq a\right]-a\right\}\mathbb{E}\left[\epsilon\mid\epsilon\geq a\right]>0,\end{aligned}$$

so $g\left(a\right)$ is an increasing function.

Consider the region $\pi\in\left[1,\frac{\theta}{\theta-1}\right]$. As we have shown before, $\ell\left(\pi\right)$ is strictly increasing in this region. Therefore

$$\frac{\mathbb{E}\left[\epsilon\mid\epsilon\geq\ell\left(\pi\right)\right]}{\gamma}$$

is strictly increasing in π. Moreover,

$$\lim_{a\to\infty}\frac{\phi\left(a\right)}{1-\Phi\left(a\right)}=\lim_{a\to\infty}\frac{-a\phi\left(a\right)}{-\phi\left(a\right)}=\infty.$$

Now, let's look at

$$h\left(\pi\right)\equiv\frac{\frac{1}{\pi}-\left(\frac{\theta-1}{\theta}\right)p^{*}\left(\pi\right)}{\left[p^{*}\left(\pi\right)-\frac{1}{\pi}\right]\left\{\left(\frac{1}{\pi}\right)^{1-\theta}-\left[p^{*}\left(\pi\right)\right]^{1-\theta}\right\}}\left(\frac{1}{\pi}\right)^{1-\theta}.$$

As $\pi \to 1$, the numerator is clearly finite and positive, whereas the denominator goes to zero, so $\lim_{\pi \to 1^+} h(\pi) = \infty$. As $\pi \to \frac{\theta}{\theta-1}$, $h(\pi) \to 0$. it follows that there must be at least one value of π in this region that solves equation 31.

Suppose that we can show that at any solution for $\pi \in \left[1, \frac{\theta}{\theta-1}\right]$, $h'(\pi) < 0$. It would then follow that there is exactly one solution. Suppose there are two solutions. By continuity, there must be another solution in between. But then, the derivative has to be positive at that solution, which is a contradiction.

Note that $h(\pi)$ has to be strictly positive in $\left(1, \frac{\theta}{\theta-1}\right)$. So we can take logs and differentiate to obtain

$$\begin{aligned}
\frac{h'(\pi)}{h(\pi)} &= \frac{\left[\left(\frac{1}{\pi}\right)^{1-\theta}\right]'}{\left(\frac{1}{\pi}\right)^{1-\theta}} + \frac{-\frac{1}{\pi^2}}{\frac{1}{\pi} - \left(\frac{\theta-1}{\theta}\right) p^*(\pi)} - \frac{\frac{1}{\pi^2}}{p^*(\pi) - \frac{1}{\pi}} - \frac{\left[\left(\frac{1}{\pi}\right)^{1-\theta}\right]'}{\left(\frac{1}{\pi}\right)^{1-\theta} - \left[p^*(\pi)\right]^{1-\theta}} \\
&= -(\theta-1)\left(\frac{1}{\pi}\right)^{2-\theta} \left\{ \frac{\left[p^*(\pi)\right]^{1-\theta}}{\left(\frac{1}{\pi}\right)^{1-\theta}\left[\left(\frac{1}{\pi}\right)^{1-\theta} - \left[p^*(\pi)\right]^{1-\theta}\right]} \right\} - \\
&\quad - \frac{1}{\pi^2}\left[\frac{\frac{1}{\theta}p^*(\pi)}{\left[\frac{1}{\pi} - \left(\frac{\theta-1}{\theta}\right)p^*(\pi)\right]\left[p^*(\pi) - \frac{1}{\pi}\right]}\right].
\end{aligned}$$

which must be strictly negative in this region. The argument is exactly the same for $\pi \in \left(0, \frac{\theta}{\theta-1}\right)$.

We have shown that $p^{*\prime}(\pi)$ has exactly one extremum in $\left(1, \frac{\theta}{\theta-1}\right)$ and exactly one extremum in $(0, \overline{\pi})$.

Lemma 5 implies that $p^{*\prime}(1) > 0$. Since at $\pi = \frac{\theta}{\theta-1}$, $p^*(\pi) = 1$ and the derivative is continuous if there were a minimum in $\left(1, \frac{\theta}{\theta-1}\right)$ there would also have to be a maximum, contradicting the assumption that there is only one extremum. The same argument holds for $\pi \in (0, 1)$. □

6.3 Proof of Proposition 1

Proof. Set $\sigma = 1$ and $\eta = 0$. The equilibrium conditions become

$$C(\pi) = \Delta_u(\pi)\, p^*(\pi)\, A \tag{32}$$

and

$$1 = \pi \frac{C(\pi)}{\Delta_u(\pi)}. \tag{33}$$

Substituting $C(\pi)$ yields the equilibrium condition

$$\frac{1}{A} = e\left[\pi^*(A)\right], \tag{34}$$

where

$$e(\pi) \equiv \pi p^*(\pi).$$

It is evident that $e(\pi) \to 0$ as $\pi \to 0$ and $e(\pi) \to \infty$ as $\pi \to \infty$. Therefore, a solution to (34) exists. Moreover,

$$\frac{e'(\pi)}{e(\pi)}\pi = 1 + \frac{p^{*\prime}(\pi)}{p^*(\pi)}\pi = \frac{1}{\Omega(\pi) + [1-\chi(\pi)]\,[p^*(\pi)]^{1-\theta}},$$

where the last equality follows from (5). Therefore $e(\pi)$ is strictly increasing in π. It follows that the solution to (34) is unique, and that $\pi^{*\prime}(A) < 0$.

Consider a shock v such that

$$\pi^*\left(\frac{1}{1+v}\right) = \frac{\theta}{\theta-1}.$$

Substituting in (34) we get

$$1 + v = \frac{\theta}{\theta - 1}.$$

Now consider cost shocks $1 + v \geq \frac{\theta}{\theta-1}$. We want to show that $\pi^*(1+v) > \frac{1}{1+v}$. Since $e(\pi)$ is strictly increasing, we simply need to show that $e\left(\frac{1}{1+v}\right) < \frac{1}{1+v}$. Now

$$g\left(\frac{1}{1+v}\right) < \frac{1}{1+v} \iff p^*\left(\frac{1}{1+v}\right) < 1.$$

But since $1+v \geq \frac{\theta}{\theta-1}$, $\frac{1}{1+v} \leq 1$. By Lemma 6, $p^*\left(\frac{1}{1+v}\right) < 1$.

There are rockets and feathers when the increase in cost is such that all firms raise prices ($\frac{1}{1+v} < \frac{\theta-1}{\theta}$). The reason is that for a symmetric fall in costs, some firms with favorable demand still keep their prices constant. Since $e(\cdot)$ is continuous, $\pi^*(A)$ is also continuous. This property implies that even when the cost rise does not induce all firms to increase prices, there are values of $\overline{v}$ that produce rockets and feathers. □

6.4 Proof of Proposition 2

Proof. Let

$$\frac{\Delta_u(\pi)}{\Delta_c(\pi)} \equiv \zeta(\pi)$$

At any $\pi \geq \frac{\theta}{\theta-1}$, $\zeta(\pi) = \zeta_s$, a constant that is independent from π. Therefore $C(\pi) = C_s$ and $N(\pi) = N_s$ are also independent from inflation. We can write

$$C(1) = \left[\frac{\zeta(1)}{\zeta_s}\right]^{\frac{1+\eta}{\sigma+\eta}} C_s,$$

and

$$N(1) = \left[\frac{\zeta(1)}{\zeta_s}\right]^{\frac{1-\sigma}{\sigma+\eta}} N_s.$$

Substituting $C(1)$ and $N(1)$ in $\mathcal{W}(1)$ we get

$$\mathcal{W}(1) - \mathcal{W}_s = \left\{\left[\frac{\zeta(1)}{\zeta_s}\right]^{\frac{(1+\eta)(1-\sigma)}{\eta+\sigma}} - 1\right\}\left[\frac{C_s^{1-\sigma}-1}{1-\sigma} - \frac{N_s^{1+v}}{1+v}\right] + \frac{\left[\frac{\zeta(1)}{\zeta_s}\right]^{1-\sigma} - 1}{1-\sigma} + \kappa\chi(1)\ln\left(\frac{\sigma_c^2}{\kappa}\right).$$

As $\sigma_c^2 \to \infty$, the first two terms go to a finite number. The third term goes to infinity. Therefore there must be $\overline{\sigma}_c^2$ such that $\sigma_c^2 \geq \overline{\sigma}_c^2$ implies that $\mathcal{W}(1) - \mathcal{W}_s > 0$. □

6.5 Proof of Proposition 3

Proof. Let

$$\widetilde{A}(\pi) \equiv \zeta(\pi) A.$$

For any π,

$$\mathcal{W}'(\pi) = \widetilde{A}(\pi)^{\frac{(1+\eta)(1-\sigma)}{\sigma+\eta}-1} \widetilde{A}'(\pi) + \kappa \ln\left(\frac{\sigma_c^2}{\kappa}\right) \chi'(\pi).$$

At $\pi = 1$, $\ell'(\pi) = 0$, so $\chi'(1) = \delta_u'(1) = \delta'(1) = 0$. Since $\widetilde{A}(\pi) = A\zeta(\pi)$,

$$\mathcal{W}'(1) \propto \hat{\zeta}(1),$$

where $\hat{\zeta}(\pi) \equiv \frac{d\ln\zeta(\pi)}{d\ln(\pi)}$.

At $\pi = 1$,

$$\hat{\zeta}(1) = \theta\left[\frac{\delta_u(1) - \chi(1)}{\delta_u(1) + 1 - \chi(1)} - \frac{\delta(1) - \chi(1)}{\delta(1) + 1 - \chi(1)}\right].$$

Since $\delta_u = 1 - \Phi\left[\ell - \left(\frac{\theta-1}{\theta}\right)\gamma\right]$ and $\delta = 1 - \Phi(\ell - \gamma)$, $\delta_u(1) < \delta(1)$. Therefore $\hat{\zeta}(1) < 0$ and $\mathcal{W}'(1) < 0$. □

7 Price Distribution

We now describe the equilibrium relation between the optimal relative reset price, $p^*(\pi)$, and the inflation rate. For analytical convenience, we measure inflation with the price index for an economy with fully rational households, which we denote by π.

Figure 3 illustrates the properties described in lemmas 1, 6, and 8.

The intuition for the behavior of the reset price is as follows. When inflation is sufficiently high, nominal marginal costs are such that the profit margin at the old price is negative. As a result, all producers reset their prices, and therefore the relative reset price is equal to one.

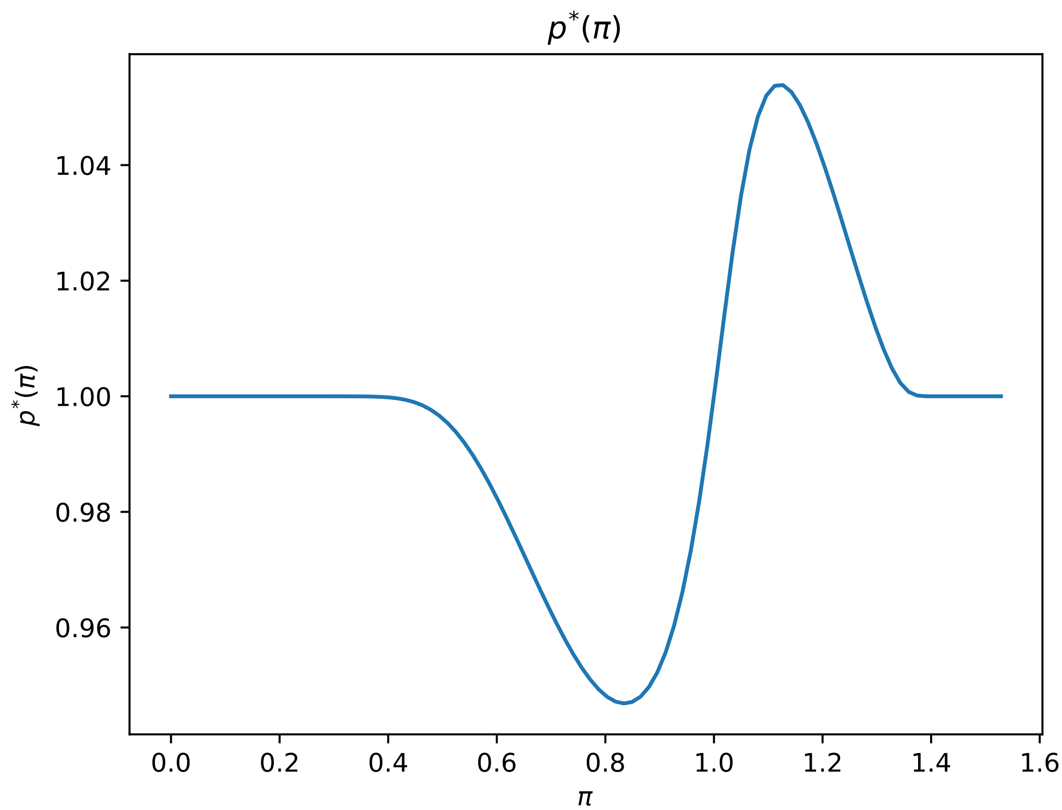

Figure 3: Reset relative price as a function of the inflation rate

As gross inflation goes to zero (and inflation goes to -100 percent), the real price charged by sticky firms, $\frac{P_0}{P} = \frac{1}{\pi}$ goes to infinity. In this case, almost no firm has a demand shock ϵ_i that makes it worthwhile to keep a real revenue near zero. In the limit, all producers reset their prices, again implying that the relative reset price must equal one.

When gross inflation is equal to one, the definition of P implies that the reset price is also equal to one regardless of the fraction of sticky firms:

$$1 = \chi(\pi)\,\pi^{\theta-1} + [1-\chi(\pi)]\,[p^*(\pi)]^{1-\theta}$$

which implies that

$$p^*(1) = 1.$$

In this case, the old price is equal to the nominal reset price, P^*, since $p^*(\pi) = 1$ implies that $P^* = P = P_0$. Therefore, firms with $\epsilon_{i,0} \geq \ell(1)$ keep their price, and firms with $\epsilon_{i,0} < \ell(1)$ change their price by an infinitesimal amount to induce the household to draw a new signal.

Figure 4 shows that the minimum demand shock that makes it worthwhile for firms to keep their price is minimized at $\pi = 1$. The old price maximizes the rational component of demand, so it takes a relatively small demand shock to induce firms to keep their price. This fact implies that the fraction of firms with sticky prices and high demand is large around $\pi = 1$.

We now explore the non-monotonicity of the reset price with respect to the rate of inflation implicit in lemma 6. This non-monotonicity reflects the interplay between the intensive and extensive margins of price adjustment. Using the definition of P,

$$1 = \chi(\pi)\,(\pi)^{\theta-1} + [1-\chi(\pi)]\,[p^*(\pi)]^{1-\theta},$$

we obtain another expression for the elasticity $\hat{p}^*(\pi) \equiv \frac{p^{*\prime}(\pi)}{p(\pi)}\pi$:

$$\hat{p}^*(\pi) = \overline{p}^*(\pi) + \varphi(\pi)\,\hat{\chi}(\pi),$$

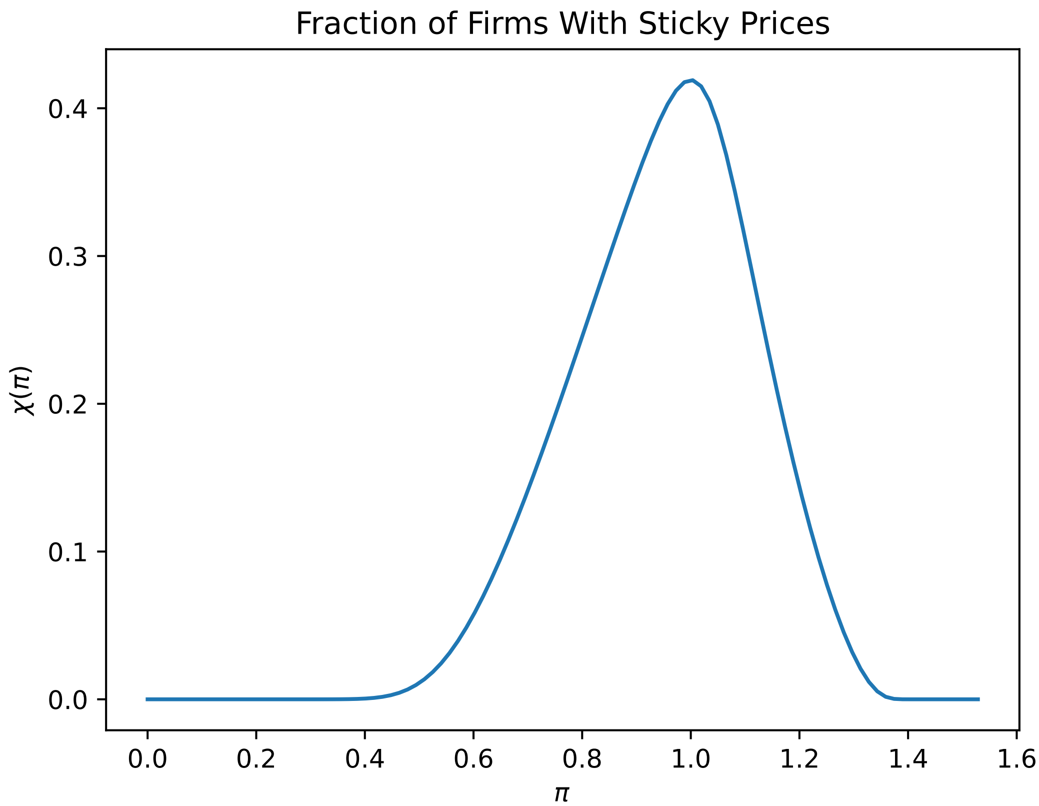

Figure 4: Fraction of firms with sticky prices as a function of the inflation rate

where

$$\overline{p}^*(\pi) \equiv \left[\frac{\chi(\pi)}{1-\chi(\pi)}\right]\left[\frac{\frac{1}{\pi}}{p^*(\pi)}\right]^{1-\theta} > 0,$$

$$\varphi(\pi) \equiv \left(\frac{1}{\theta-1}\right)\left[\frac{\chi(\pi)}{1-\chi(\pi)}\right]\left\{\left(\frac{1}{\pi}\right)^{1-\theta} - \left[p^*(\pi)\right]^{1-\theta}\right\},$$

and

$$\hat{\chi}(\pi) \equiv \frac{\chi'(\pi)}{\chi(\pi)}\pi.$$

It is easy to show that $\varphi(\pi) > 0$ when $\pi > 1$ and $\varphi(\pi) < 0$ when $\pi < 1$.

The first term of $\hat{f}(\pi)$, $\overline{f}(\pi)$, relates to the intensive margin of price adjustment, and the second term, $\varphi(\pi)\hat{\chi}(\pi)$, to the extensive margin.

Along the intensive margin, there is a positive relation between the relative reset price and inflation ($\overline{p}^*(\pi) > 0$). If inflation is high, sticky firms charge a low relative price. In equilibrium, flexible firms must charge a high relative price so that $\mathbb{E}_i\left[p_i^{1-\theta}\right] = 1$.

Along the extensive margin, there is a negative relation between the relative reset price and inflation ($\varphi(\pi)\hat{\chi}(\pi) < 0$). If inflation is high, the fraction of sticky firms is low ($\hat{\chi}(\pi) < 0$) because fewer demand shocks make keeping a low nominal price with high nominal marginal costs worthwhile. Flexible firms must charge a smaller relative price so that in equilibrium $\mathbb{E}_i\left[p_i^{1-\theta}\right] = 1$.

It turns out that there is a gross inflation level $\overline{\pi} > 1$ such that if $\pi > \overline{\pi}$, the effect of the extensive margin dominates and $\hat{p}^*(\pi) < 0$.

The dynamics of deflation are analogous to those of inflation. As inflation becomes more negative, the firms that change prices reduce these prices by more (the intensive margin). But, since more firms change prices (the extensive margin), prices do not have to fall by much to ensure that the harmonic mean of the relative prices is one. Again, there is an inflation level $\underline{\pi}$ such that if $\pi < \underline{\pi}$, $p^*(\pi) > p^*(\underline{\pi})$.

7.1 Taylor Rule

To analyze the response of inflation to the cost, v, under a Taylor rule, we consider a deterministic infinite-period version of the model. There is a pre-period, $t = 0$, with an exogenous price level P_0 and System 1 demands. After this pre-period, time is indexed by $t \geq 1$.

We assume that the household is fully rational from $t = 2$ onwards. In each period t, utility from consumption and labor is

$$U_t = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta},$$

where

$$C_t = \left(\int_0^1 c_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}.$$

In each period t, the household invests in nominal government bonds, D_{t+1}, at price $1/R_t$, where R_t is the gross nominal interest rate. The flow-of-funds constraint is

$$\int_0^1 P_{i,t} c_{i,t} di + \frac{D_{t+1}}{R_t} \leq W_t N_t + \int_0^1 \Pi_{i,t} di + D_t - \mathcal{T}_t.$$

As in the main text, production of good i is conducted by a monopolistically competitive firm with the following production function

$$y_{i,t} = A_t n_{i,t}.$$

The flow-of-funds constraint of the government is

$$D_t + \tau_{n,t} W_t N_t = \frac{D_{t+1}}{R_t} + \mathcal{T}_t.$$

The monetary authority sets interest rates according to the Taylor rule

$$R_t = \frac{1}{\beta} \pi_t^{\phi}, \ \ \phi > 1.$$

We define an equilibrium in this economy as follows.

Definition 2. *An equilibrium is a sequence of prices* $\{W_t, P_{i,t}\}_{t=1}^{\infty}$*, allocations,* $\{c_{i,t}, N_t, D_{t+1}, \Pi_{i,t}\}_{t=1}^{\infty}$*, and policies* $\{\tau_{n,t}, \mathcal{T}_t, R_t\}_{t=1}^{\infty}$ *such that, given* P_0 *and the productivity sequence* $\{A_t\}_{t=1}^{\infty}$*,*

1. *For* $t \geq 2$*,* $\{c_{i,t}, N_t, D_{t+1}\}_{t=2}^{\infty}$ *maximize*

$$\sum_{t=2}^{\infty} \beta^{t-2} U_t$$

subject to the flow-of-funds constraints and a transversality condition.

2. *For* $t = 1$*,*

$$c_{i,1} = e^{\gamma \widetilde{\epsilon}_{i,1}} p_{i,1}^{-\theta} C_1^*,$$

$$\widetilde{\epsilon}_{i,1} = \begin{cases} \epsilon_{i,0}, & \text{if } P_{i,1} = P_0 \\ \epsilon_{i,1}, & \text{if } P_{i,1} \neq P_0 \end{cases},$$

$$\epsilon_{i,t} \sim \mathcal{N}(0,1),$$

$$D_2 = D_2^*,$$

and N_1 *is chosen to satisfy the flow-of-funds constraint in period 1.* C_1^* *and* D_2^* *are the period-1 aggregate consumption and savings plans for period one that maximize*

$$\sum_{t=1}^{\infty} \beta^{t-1} U_t$$

subject to the flow-of-funds constraints and a transversality condition.

3. *In each period* t*, firms choose* $P_{i,t}$*,* $n_{i,t}$ *and* $y_{i,t}$ *to maximize expected profits* $\mathbb{E}_t[\Pi_{i,t}]$ *subject to* $y_{i,t} = A_t n_{i,t}$ *and* $y_{i,t} = c_{i,t}$*.*

4. *Policies satisfy the flow-of-funds constraints*

$$D_t + \tau_{n,t} W_t N_t = \frac{D_{t+1}}{R_t} + \mathcal{T}_t,$$

and the nominal interest rate satisfies

$$R_t = \frac{1}{\beta} \pi_t^{\phi},$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$*.*

5. *The labor market clears:* $\int_0^1 n_{i,t} di = N_t$.

Condition 1 states that the household is fully rational from period two onwards. Condition 2 states that in period one, consumption of the differentiated goods solve the bounded rationality problem in the main text, nominal government savings are rationally chosen, and labor is chosen to satisfy the budget constraint. Condition 3 incorporates the fact that because the household is rational from period two onwards, and prices are not rigid, the problem of the firm is static.

We consider the equilibrium associated with ad valorem subsidies satisfying

$$1 - \tau_{n,t} = \frac{\theta - 1}{\theta}, \ \ t \geq 1$$

and a productivity sequence

$$A_t = 1, \ \ t \geq 2.$$

Equilibrium For $t \geq 2$ From period two onwards, consumption and labor satisfy the conditions

$$c_{i,t} = p_{i,t}^{-\theta} C_t, \ \ t \geq 2,$$

$$C_t^{\sigma} N_t^{\eta} = w_t, \ \ t \geq 2,$$

$$\frac{1}{\beta} \left(\frac{C_{t+1}}{C_t} \right)^{\sigma} = R_t. \ \ t \geq 2.$$

The price level is given by

$$P_t^{1-\theta} = \int_0^1 P_{i,t}^{1-\theta} di.$$

Since the consumer is fully rational and $1 - \tau_{n,t} = \frac{\theta-1}{\theta}$, it follows that all firms set the nominal price to

$$P_{i,t} = \frac{W_t}{A_t},$$

which implies that $p_{i,t} \equiv P_{i,t}/P_t = 1$ and $w_t \equiv \frac{W_t}{P_t} = A_t = 1$. It follows that $c_{i,t} = C_t = N_t = 1$.

Since the government sets the nominal interest rate according to an interest rate rule with a zero inflation target, it is able to implement a locally unique solution with

$$\pi_t = 1, \;\; t \geq 2.$$

Firms' Problem At $t = 1$ The problem of the firms in period 1 is identical to that described in the main text. Therefore

$$p_{i,1} = \begin{cases} p_1^*, & \text{if } \epsilon_{i,0} \geq \ell \\ \frac{1}{\pi_1}, & \text{if } \epsilon_{i,0} < \ell \end{cases},$$

where

$$p_1^* \equiv \frac{w_1}{A_1}$$

and

$$\ell \equiv \begin{cases} \mathbb{E}\left[e^{\gamma\epsilon}\right] \frac{\frac{1}{\theta}\left(p_1^*\right)^{1-\theta}}{\left(\frac{1}{\pi_1} - \frac{\theta-1}{\theta}p_1^*\right)\left(\frac{1}{\pi_1}\right)^{-\theta}}, & \frac{1}{\pi_1} > \frac{\theta-1}{\theta}p_1^* \\ \infty, & \frac{1}{\pi_1} \leq \frac{\theta-1}{\theta}p_1^* \end{cases}.$$

Using

$$P_1^{1-\theta} = \int_0^1 P_{i,1}^{1-\theta} di$$

It follows that p_1^* can be implicitly defined as the same function of inflation used in the main text, i.e., $p_1^* = p_1^*(\pi_1)$.

Household's Problem at $t = 1$ As before, to obtain the boundedly rational demands in period 1, we need to characterize the rational plans in that period. From the point of view of period 1, the conditions that characterize the solution to the utility-maximization problem include

$$c_{i,t}^* = p_{i,1}^{-\theta} C_t^*,$$

$$\left(C_t^*\right)^\sigma \left(N_t^*\right)^\eta = w_t,$$

$$\frac{1}{\beta}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{\sigma} = \frac{R_t}{\pi_{t+1}},$$

$$\sum_{t=1}^{\infty} \mathcal{Q}_{1,t} P_t C_t^* = \sum_{t=1}^{\infty} \mathcal{Q}_{1,t} \left[W_t N_t^* + \int_0^1 \Pi_{i,t} di - \mathcal{T}_t\right] + D_1,$$

$$\mathcal{Q}_{k,t} \equiv \begin{cases} 1 & \text{, if } t = k \\ \prod_{\tau=k}^{t-1} \frac{1}{R_\tau} & \text{, if } t > k \end{cases},$$

It is convenient to also consider the equations

$$\sum_{t=2}^{\infty} \mathcal{Q}_{2,t} P_t C_t^* = \sum_{t=2}^{\infty} \mathcal{Q}_{2,t} \left[W_t N_t^* + \int_0^1 \Pi_{i,t} di - \mathcal{T}_t\right] + D_2^*$$

$$P_1 C_1^* + \frac{D_2^*}{R_1} = W_1 N_1^* + \int_0^1 \Pi_{i,1} di + D_1 - \mathcal{T}_1.$$

Plugging the equilibrium variables in the utility-maximization problem of period 1, we conclude that the rational plans in period 1 satisfy

$$C_t^* = C_2^*, \ \ t \geq 2,$$

$$N_t^* = N_2^*, \ \ t \geq 2,$$

$$\left(C_2^*\right)^{\sigma} \left(N_2^*\right)^{\eta} = 1.$$

Combining the intertemporal household and government budget constraints from period two onwards,

$$\begin{aligned} &\sum_{t=2}^{\infty} \mathcal{Q}_{2,t} P_t C_t^* = \sum_{t=2}^{\infty} \mathcal{Q}_{2,t} \left[W_t N_t^* + \int_0^1 \Pi_{i,t} di - \mathcal{T}_t\right] + D_2^* \\ \iff &\sum_{t=2}^{\infty} \mathcal{Q}_{2,t} P_t C_t^* = \sum_{t=2}^{\infty} \mathcal{Q}_{2,t} \left[W_t N_t^* + \int_0^1 \Pi_{i,t} di - \tau_{n,t} W_t N_t\right] - D_2 + D_2^* \\ \iff &\frac{1}{1-\beta} C_2^* = \frac{1}{1-\beta} N_2^* + \frac{D_2^* - D_2}{P_2}. \end{aligned}$$

From condition 2 in the equilibrium definition, $D_2 = D_2^*$ and therefore

$$C_2^* = N_2^*.$$

Combining with the intratemporal condition for consumption and labor in period 2,

$$C_2^* = N_2^* = 1.$$

Using the intertemporal budget constraint for period 1,

$$\begin{aligned}
\sum_{t=1}^{\infty} \mathcal{Q}_{1,t} P_t C_t^* &= \sum_{t=1}^{\infty} \mathcal{Q}_{1,t} \left[W_t N_t^* + \int_0^1 \Pi_{i,t} di - \mathcal{T}_t \right] + D_1 \\
\Longleftrightarrow P_1 C_1^* + \frac{1}{R_1} \frac{1}{1-\beta} P_1 C_2^* &= W_1 N_1^* + \frac{1}{R_1} \frac{1}{1-\beta} P_1 N_2^* + \int_0^1 \left(P_{i,1} c_{i,1} - W_1 n_{i,1} \right) di \\
\Longleftrightarrow C_1^* &= w_1 N_1^* + \int_0^1 \left(p_{i,1} - \frac{w_1}{A_1} \right) c_{i,1} di.
\end{aligned}$$

Since

$$c_{i,1} = e^{\gamma \widetilde{\epsilon}_{i,1}} p_{i,1}^{-\theta} C_1^*,$$

We obtain the same expression for rational consumption in period 1:

$$C_1^* (\pi_1) = \left\{ \frac{\left[A_1 p_1^* (\pi_1) \right]^{1+\eta}}{\left[\vartheta (\pi_1) \right]^{\eta}} \right\}^{\frac{1}{\sigma+\eta}}.$$

Equilibrium Conditions in Period 1 Using the fact that $C_2^* = 1$, the Euler equation in period 1 implies

$$\frac{1}{\beta} \left[\frac{1}{C_1^* (\pi_1)} \right]^{\sigma} = R_1.$$

Combining with the Taylor rule, we obtain the two equilibrium conditions

$$C_1^* (\pi_1) = \left\{ \frac{\left[A_1 p_1^* (\pi_1) \right]^{1+\eta}}{\left[\vartheta (\pi_1) \right]^{\eta}} \right\}^{\frac{1}{\sigma+\eta}}, \tag{35}$$

and

$$C_1^* (\pi_1) = \pi_1^{-\frac{\phi}{\sigma}}. \tag{36}$$

Rockets and Feathers Consider again productivity levels

$$A_{1,H} = 1 + v$$

and

$$A_{1,L} = \frac{1}{1+v},$$

with $v > 0$. Let $\pi_{1,L}(v)$ be the equilibrium inflation associated with an increase in costs of size v, and $\pi_{1,H}(v)$ the equilibrium inflation associated with a decrease in costs also of size v. We can now show an analogous rockets and feathers proposition.

Proposition 4 (Rockets and Feathers With Taylor Rule)**.** *Suppose $\sigma = 1$ and $\eta = 0$. If $\phi > 1$, there is $\overline{v}$ such that if $v \geq \overline{v}$,*

$$|\ln \pi_{1,L}(v)| > |\ln \pi_{1,H}(v)|.$$

Proof. With $\sigma = 1$ and $\eta = 1$, the equilibrium conditions (35) and (36) become

$$C_1^*(\pi_1) = A_1 p_1^*(\pi_1)$$

and

$$C_1^*(\pi_1) = \pi_1^{-\phi}.$$

Combining the two, the equilibrium condition for inflation is

$$\frac{1}{A_1} = p_1^*(\pi_1)\,\pi_1^{\phi}.$$

Let $e_{\text{Taylor}}(\pi_1) \equiv p_1^*(\pi_1)\,\pi_1^{\phi}$. If $\phi > 1$, it is still true that e_{Taylor} is strictly increasing in inflation, since

$$e_{\text{Taylor}}(\pi_1) = e(\pi_1)\,\pi_1^{\phi-1}.$$

As in proposition 1, $e(\pi_1)$ is strictly increasing. When $\phi > 1$, so is $e_{\text{Taylor}}(\pi_1)$. Therefore, the equilibrium is locally unique, and inflation decreases in A_1.

As before, there is a value of v such that $\pi_{1,L} = \frac{\theta}{\theta-1}$ and $p_1^* \left(\pi_{1,L}\right) = 1$:

$$1 + v = \left(\frac{\theta}{\theta - 1}\right)^{\phi}.$$

To show that for this v, $\left|\ln \pi_{1,L}(v)\right| > \left|\ln \pi_{1,H}(v)\right|$, we need only to show that

$$e_{\text{Taylor}}\left(\frac{\theta - 1}{\theta}\right) < \left(\frac{\theta - 1}{\theta}\right)^{\phi}.$$

Substituting,

$$p_1^* \left(\pi_{1,H}(v)\right) \left(\frac{\theta - 1}{\theta}\right)^{\phi} < \left(\frac{\theta - 1}{\theta}\right)^{\phi} \iff p_1^* \left(\pi_{1,H}(v)\right) < 1.$$

This inequality holds, since $\frac{\theta-1}{\theta} < 1$, in the region where $p_1^* < 1$. This statement completes the proof. □