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Projecting Banks' Net Interest Income: an Asset-Liability Approach, Applied to the Euro Area

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ABSTRACT

In a context of volatile interest rates, the impact of monetary policy decisions on banks' net interest income is a key question for financial stability, since changes in profitability may affect their capacity to absorb losses and to accumulate capital through retained earnings. This paper presents an ALM-like model developed to project the evolution of the aggregate balance sheet and the interest income and expense of a banking sector under various scenarios. Based on balance sheet structure data, the model simulates the expiration of maturing instruments and the progressive accumulation of new issuances. Using conservation laws valid at the aggregate level, the model provides a consistent accounting-based framework, where bank reserve holdings depend on central bank actions, and the volume of customer deposits results from net payments between the banking sector and the rest of the economy. A combination of financial data sources makes it possible to build a simplified balance sheet of the aggregate euro area banking sector, on which the model can be run. Its total net interest income turns out to be, on the whole, positively sensitive to changes in interest rates. The model can also quantify sensitivities to other factors, such as central bank operations on securities or changes in the cost structure of customer deposits. Back-testing results on 2016–23 confirm the model's ability to account for observed interest margins.

Keywords: Interest Rates; Banking Sector; Net Interest Income; Monetary Policies; Asset-liability Projection Model.

JEL classification: G21, E43, E44, E47, E58.

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NON-TECHNICAL SUMMARY

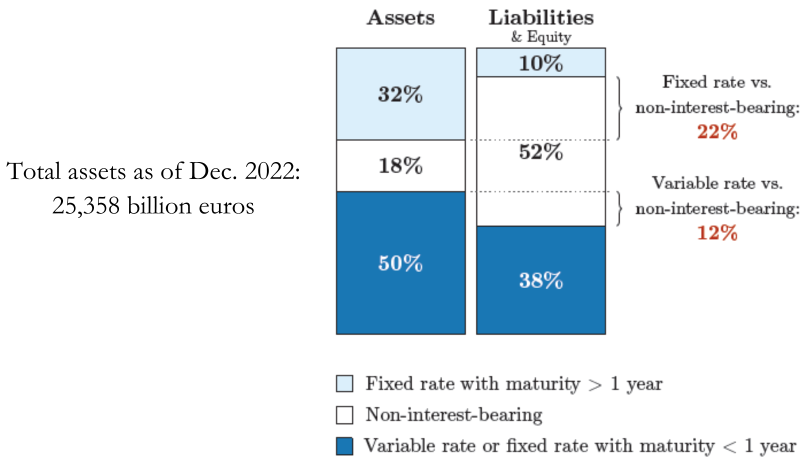
Banks' profitability relies prominently on their ability to collect more interests on their assets than they pay on their liabilities. The resulting net interest income (NII) is a key factor of financial stability, since it drives the capacity of banks to absorb losses and to accumulate capital through retained earnings. In this paper, we present an ALM-like model designed to project the banking sector's aggregate NII under different financial and monetary policy scenarios, using the available information on the structure of assets and liabilities (interest rate regime, maturities etc.).

The distinctiveness of MAP (*Modèle Actif-Passif*) lies in its "mechanistic" nature and its flexibility in terms of admissible scenarios. The model applies simple accounting principles to project the aggregate banks' balance sheet, leveraging the near-closed-system nature of the banking sector as a whole. It explicitly projects the expiration of existing assets and liabilities and their replacement with new ones, as well as the financial commitments between credit institutions and central banks. Scenarios are set externally and can be used to investigate the extent to which an extensive variety of circumstances may affect banks' NII: changes in interest rates, in the volume of loans, in monetary policy decisions or in the cost structure of banks' deposits.

The paper includes an application of the model to the aggregated euro area banking sector, whose total balance sheet amounts to more than 25 trillion euros at end-2022 and generates around 300 billion euros in annual net interest income. The scope covers the largest banking institutions (*Significant institutions* as determined by the EU supervisors), which together account for approximately 85% of the euro area's total banking assets. The results provide a view on the trends in euro area banking sector quarterly profits over a 5-year horizon.

The balance sheet analysis performed to apply the model provides an analytical decomposition of the balance sheet by type of interest rate and maturity, taking into account the partial sensitivity of customer deposits to interest rates. At the euro area aggregate level, this structural decomposition indicates a positive net holding of interest-paying products as of end-2022: the surplus of long fixed-rate assets (22%) and the surplus of short and variable-rate assets (12%) are both effectively funded by non-interest-bearing liabilities (Figure 1). This configuration creates a positive sensitivity to higher interest rates for euro area.

Figure 1. Estimated balance sheet structure of the euro area aggregate banking sector, breakdown by interest rate and maturity as of Dec. 2022



Indeed, in an example scenario of rising interest rates, the model projects a rising net interest income for the euro area banking sector over the next five years, in line with the balance sheet structure. Furthermore, the MAP's analytical approach provides a detailed composition of the projected net interest income, allowing a decomposition of the projected interest fluctuations in four categories of operations, i.e. (i) central banks, (ii) other banks (iii) clients and (iv) securities.

The reliability of the MAP projections can be assessed by a back-testing method using past starting points with observed data, both on overall NII and balance sheet, and on each component. The back-testing exercise realised on the euro area banking sector model for the period 2016–23 gives robust results. While the model is adapted to the present situation of eurozone banks, its modular nature makes it easy to modify, either to fit a different banking sector, or to accommodate potential future changes in the financial system (e.g. issuance of central bank digital currencies).

Projection de la marge d'intérêt des banques : une approche actif-passif, appliquée à la zone euro.

RÉSUMÉ

Dans un contexte de variation des taux d'intérêt, l'incidence des décisions de politique monétaire sur la marge d'intérêt des banques est une question clé pour la stabilité financière, dans la mesure où les fluctuations des intérêts nets dégagés par les banques peuvent affecter leur capacité à absorber des pertes et à augmenter leurs capitaux propres via les bénéfices non distribués. Cet article présente un modèle actif-passif développé pour projeter l'évolution du bilan agrégé ainsi que les produits et les charges d'intérêt d'un secteur bancaire selon divers scénarios. Sur la base des données de structure du bilan, le modèle simule l'expiration des instruments arrivant à échéance et l'émission progressive de nouveaux produits. Dans le respect de la cohérence comptable, le modèle utilise des règles de conservation valables au niveau agrégé : les réserves bancaires dépendent des actions de la banque centrale et le volume des dépôts des clients résulte des paiements nets entre le secteur bancaire et le reste de l'économie. Le croisement de données financières permet de construire une version simplifiée du bilan du secteur bancaire agrégé de la zone euro et d'y appliquer le modèle. Dans ce cadre, les revenus nets d'intérêts des banques s'avèrent, dans l'ensemble, positivement sensibles aux variations des taux d'intérêt. Le modèle permet également de quantifier les sensibilités à d'autres facteurs, tels que les opérations des banques centrales sur les titres ou les changements dans la structure de coût des dépôts des clients. Le rétrocontrôle réalisé sur la période 2016-2023 atteste de la capacité du modèle à rendre compte des marges d'intérêt observées.

Mots-clés : taux d'intérêt ; système bancaire ; marge d'intérêt ; politique monétaire ; modèle actif-passif.

1 Introduction

The profitability of the banking sector has important implications for financial stability, as profits influence banks' retained earnings and capital positions, and therefore their capacity to finance the real economy. In the new inflationary context, the rapid rise of interest rates is driving significant changes in banks' interest incomes and expenses, given the wide structural differences between their assets and liabilities arising from their maturity-transformation activity: the sign and amplitude of the overall effect of these changes on their net interest income (NII) is an important question for financial stability.

In this paper, we present an approach to project the impact of interest rates fluctuations on the net interest income by taking into account the balance sheet structure, and particularly the respective maturities of assets and liabilities and the breakdown between fixed- and variable-rate interests on both sides of the balance sheet. The asset-liability model developed at Banque de France, called *Modèle Actif-Passif* (MAP), is an operational tool designed to project the components of the aggregate net interest income, i.e. the interest income and the interest expense, of the banking sector of an economy under any five-year scenario. The net interest income is a key indicator of European banks' profits since they earn around 60% of their net income from the margin between interest-bearing assets and liabilities.

Compared to standard approaches used in academic research or by banking supervisors, the distinctiveness of this ALM-like model lies in its "mechanistic" nature and its flexibility in terms of admissible scenarios.

First, it is based on a consistent accounting view of the bank balance sheet dynamics, with explicit modelling of the expiration of existing assets and liabilities and their replacement with new ones. Leveraging the near-closed-system nature of the aggregated banking sector, it tracks the mechanical effects of each type of banking operation across the balance sheet, ensuring for instance that the amounts of loans and deposits evolve consistently with each other.[1] The analytical approach, which provides a detailed composition of the projected net interest income, can be used to track the origins of the projected interest fluctuations.

Second, the MAP can be used to investigate the extent to which an extensive variety of circumstances may affect banks' NII, unconstrained by macro-economical relationships. Scenarios can independently specify, not only the evolution of interest rates and of the volume of loans, but also changes in monetary policy decisions, which are channelled to banks by an explicit model of the link between central bank asset holdings and commercial bank reserve holdings. Scenarios can also incorporate changes in the cost structure of banks' deposits, a key driver of banks' net interest income (Grodzicki et al., 2023).

This paper relates to both theoretical and empirical literature on interest rate risk management and its implication for banks' profits.

A substantial body of the literature developed after the Great Financial Crisis is dedicated

[1] Such relationships do not generally hold at the individual level: when a new loan is extended by a bank, the corresponding deposit creation frequently moves out of its balance sheet as the borrower uses it for a purchase. At the aggregate level of all banks in an economy, however, the created deposit can only circulate within the balance sheet until it is used to pay a bank.

to models projecting the evolution of banks' balance sheet and income statement items under various scenarios in order to ultimately estimate the financial strength of banks at an individual level. Developed mainly by central banks, they use or combine structural, semi structural and/or empirical approaches to evaluate the evolution of net revenues under different scenarios.[2] Our approach is different in that it applies a simpler framework based on accounting and financial rules to the last known state of the aggregated banking system in order to project the balance sheet and the net interest income over a five-year horizon. We employ the theoretical principles developed by McLeay et al. (2014) showing that the evolution of bank balance sheets at the aggregate level is mainly asset-driven. Conceptually, our methodology is closer to the one employed by Carpenter et al. (2015) projecting the evolution of Federal Reserve's balance sheet and income statements under various scenarios by taking into account quantitative easing policies.

The topic dealing with explaining the drivers of banks' realised interest margins is largely documented in the literature (Demirgüç-Kunt and Huizinga (1999), Borio et al. (2017), Claessens et al. (2018), Altavilla et al. (2018)). Some studies use bank stock prices to build forward-looking indicators to quantify the sensitivity of banks' financial position to market rates fluctuations (Ampudia and Van den Heuvel (2022), English et al. (2018)). For example, the economic value of equity (EVE) uses market valuations to evaluate of the banking-book interest rate risk for supervision monitoring purposes (BCBS (2019), EBA (2022), ECB (2022), Claußena and Plattea (2023)). It is argued in Drechsler et al. (2021) that banks do not take on interest rate risk despite having a large maturity mismatch, as they use their deposit market power to match the sensitivity of their interest income and expense to changes in central banks' funding rates and to stabilise their NII. However, Begenau and Stafford (2022) claim that stable NII does not imply near zero interest rate risk according to standard risk measures.

The present paper adopts a financial perspective to project the net interest income of a banking sector at an aggregate level. It leaves aside the question of the validity of individual interest rate risk indicators, as well as any idiosyncratic risk such as liquidity-related vulnerabilities. It is designed to be used as a macro monitoring tool, rather than from a micro perspective. To the best of our knowledge, this is the first study to present a model that projects the evolution of the aggregate balance sheet and net interest income of the banks of a monetary zone.

Though the MAP has originally been designed to be used on the euro monetary area, its underlying principles are flexible and may be conformed to any banking sector, subject to some adaptations to the local structure of the financial system, and in particular to the operational framework of monetary policy. In order to illustrate how the model operates, we present in this paper an application of the MAP to the euro area banking sector.[3]

For the euro area banking sector starting at end-2022, the model projects a rising net interest income over 2023–27 in a context of higher interest rates, in line with a balance sheet structure displaying a positive net holding of interest-paying products, both with fixed and variable

[2] For example, the Bank of England stress tests model of Burrows et al. (2012), the European Central Bank's frameworks developed by Dees et al. (2017) and Budnik et al. (2020), Covas et al. (2014) model, the Banque de France model by Camara et al. (2015), the Dodd-Frank Act Stress Test (Federal Reserve Board, 2021).

[3] For internal Banque de France purposes, another implementation of the model exists for the aggregated French banking sector.

interest rates. This favourable outcome, which is robust against moderately adverse scenarios such as a shift of depositors towards more interest-paying products, is consistent with the symmetrical finding that a low interest rate environment hampers the ability of the banking sector to make profits from net interest income, cf. Claessens et al. (2018); Altavilla et al. (2018).

The model can be back-tested by comparing projections from past starting points with observed data, both on overall NII and balance sheet, and on their individual components. Deviations point to potential needs for model improvements. The back-testing realised on the euro area banking sector model for the period 2016–23 gives robust results.

The rest of the paper is organised as follows. Section 2 presents the general principles and main methodological aspects of the model. Section 3 describes the modelling of maturing products, forming the largest part of the banks' assets. Section 4 presents the modelling of the reciprocal financial commitments between credit institutions and central banks. Section 5 is dedicated to the evolution of non-maturing items, involving flows with maturing and monetary items, and the computation of the financial statements. Section 6 illustrates the application of the model to the euro area banking sector, describing the stating point for the projection and the main results: projected NII in an example scenario, sensitivity computations, and back-testing results. The last section concludes.

2 General modelling principles

2.1 Object

The MAP projects a simplified representation of the aggregate financial statements of the banking sector, with a quarterly time step corresponding to the publication frequency of banks' financial statements. It combines two sets of accounts, illustrated in figure 1:

- The **balance sheet** represents the state of what banks own and owe at a point in time. Banks' assets are mostly composed of loans and purchased securities, while the main liabilities are customer deposits and issued securities.
- The **income statement** represents the profits and losses accumulated by the banks during a given quarter. Banks' income statements include interest income and interest expense, whose difference equals the net interest income (NII), the main quantity of interest in output of the model. For completeness, the income statement also includes credit losses, and a wide variety of other sources of income and expense, which, for the purposes of the MAP, are treated as a single net cost, since its net value is generally negative.

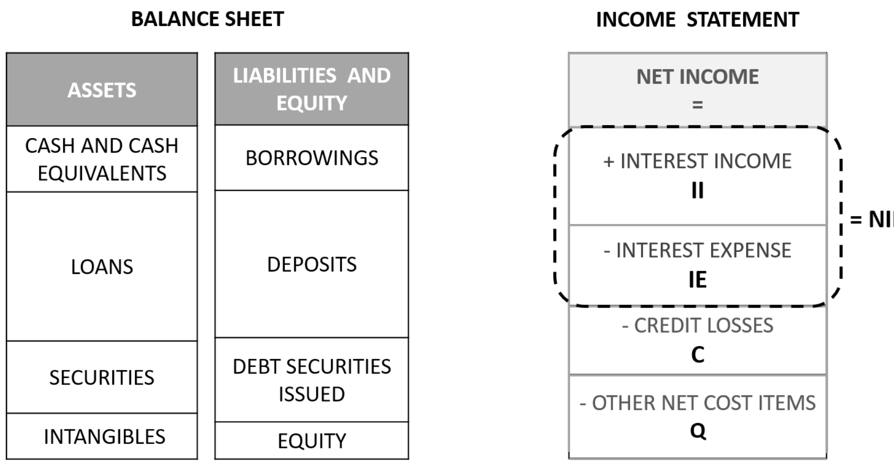


Figure 1: Simplified balance sheet and income statement

2.2 Principles

The MAP model was built with two overarching principles in mind.

First, it fully respects the principles of **double-entry accounting**. The modelled financial statements are projected using accounting entries that systematically match a credit and a debit. This does not only ensure that the accounting identity (Assets = Liabilities + Equity) is respected throughout the simulation: it also tracks the mechanical effects of banking operations across the balance sheet, e.g. making sure the amounts and loans and deposits evolve consistently with each other.

The correspondence between credits and debits requires some modelling choices. These are made much more robust and behaviour-independent by the choice of applying the MAP model on an *aggregate* banking sector rather than on individual banks, as many behaviour-driven flows are netted out. For instance, a new loan to customers is automatically matched by a rise in customer deposits, regardless of how the borrowers choose to spend the money, since the exact path of circulation of these deposits amongst banks is irrelevant at the aggregate level.

As the model seeks to derive interest flows from balance sheet information, it can only project the net interest income *excluding derivatives*. Indeed, derivatives do not pay interests in proportion of their carrying value on the balance sheet, which corresponds to their market value — usually orders of magnitude smaller than their notional amount. The net interests paid or received by banks through derivatives are therefore difficult to model; in practice, for eurozone banks, they tend to represent only a small proportion of the NII.

Second, the model is highly **scenario-driven**: it intentionally provides for a large freedom of choice in the design of the input scenario. For instance, no *a priori* relation is imposed between interest rates and volumes, so that any combination of the evolution of rates and activity may be tested.

One natural use of the model is to plug it on the output of a macro-economical model generating a consistent set of scenario variables (e.g. a joint set of rate/volume evolutions) from behavioural assumptions: the MAP then provides the translation of this scenario in terms of banks' balance sheet structure and profitability. However, the large scenario freedom makes it possible to explore the consequences on bank NII of a much wider range of potential circumstances, including low-probability combinations, such as a rate cut accompanied by a rise in deposit costs, or even unprecedented events, e.g. a rapid shrinkage of central bank security holdings.

2.3 Sketch of model computations

Starting from the last known aggregate balance sheet, the model iterates quarter by quarter, generating for each quarter an income statement as well as the end-of-quarter state of the balance sheet, which is then used as the starting point of the next quarter, as illustrated in figure 2.

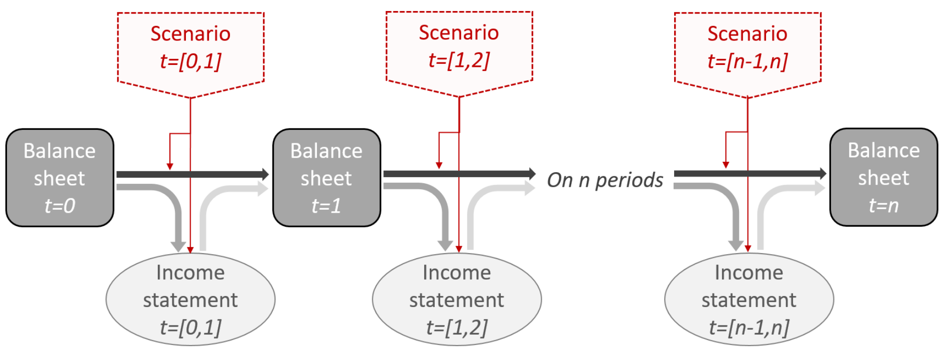


Figure 2: Simplified scheme of the modelling of the aggregate balance sheet and interest income and expense

Per double-entry book-keeping principles, each credit[4] is matched by a corresponding debit. Specifically, each evolution of the financial statement belongs to one of three types:

- Moves between two different balance sheet items (assets, liabilities or equity): e.g. when bank extends a loan to a customer, the balance sheet registers a debit on the asset side (higher outstanding amount of loans) and a credit on the liability side (higher deposit base, assuming the loan is paid into a deposit account within the same banking sector). All banks' business operations are of this type and modify the balance sheet only, as represented by the black arrows in figure 2.
- Generation of income or expense: e.g. when a customer pays interests on a loan, the income statement registers a credit (interest income for the bank) and the balance sheet

[4]The word *credit* in the context of banking can have two unrelated meanings: a business meaning, i.e. the activity of providing funds with an agreement of delayed repayment (hence a bank is also referred to as a "credit institution"), and a technical meaning, i.e. the accounting entry corresponding to the source of a value transfer in double-entry bookkeeping — always matched by a *debit*, which is the entry corresponding to the destination of the same value transfer. In this document, the word *credit* is used in the technical meaning, except in phrases such as "credit institution" or "credit loss", where it has the business meaning.

registers a debit (lower deposit base, likewise assuming the interest is paid from a deposit account within the same banking sector). Such moves are represented by the dark gray arrows in figure 2.

- Allocation of the income statement into the balance sheet: at the beginning of each simulation period, the income statement is reset to zero, which represents an accounting debit if the net income is positive (resp. a credit if it is negative). The balance sheet then registers a corresponding credit (resp. debit), allocated in part to Equity (retained earnings or losses) and in part to the asset or liability items assumed to be used for dividend payment (e.g. customer deposits, assuming bank shareholders to hold deposits within the same banking sector). Such moves are represented by the light gray arrows in figure 2.

2.4 Parameters, scenario variables and state variables

For the purposes of the present exposition, it is helpful to classify the quantities involved in the model into three categories:

- **Model parameters** are quantities that are held constant throughout the simulation and determine the way the financial statements behave. They are estimated *a priori* from historical observations and fully belong to the model design. Among key model parameters are average loan maturities and proportions of variable-rate loans. Model parameters are represented by simple notations such as τ or α.
- **Scenario variables** are time series of financially meaningful quantities over the simulation period that are provided to the model as input. They are chosen at will by the model user and represent a possible future financial environment under which the banking sector is assumed to evolve. Examples include future market interest rates, volumes of customer loans, monetary policy decisions affecting central banks' balance sheets. Scenario variables are written in bold and red colour throughout the exposition, and bear a time index (e.g. $\boldsymbol{r_t}$).
- **State variables** are time series of numbers that encode the structure of the aggregate banking sector at each simulation step. They are computed by the model in a step-by-step approach, using parameters and scenario variables as input. State variables include for instance the size and interest rate structure of each balance sheet item, the interests received and paid by banks at each simulation period, etc. The notations for state variables bear a time index without any particular highlight (e.g. N_t).

2.5 Decomposition of the balance sheet

Figure 3 presents a view of the eurozone banking sector's aggregate balance sheet. The horizontal bars are proportional to the amounts observed at the end of 2022, which are discussed in more details in section 6 and appendix B.1. The balance sheet is dominated by customer loans and deposits, with securities playing a secondary role, and monetary items having a non-negligible weight due to non-standard monetary policy.

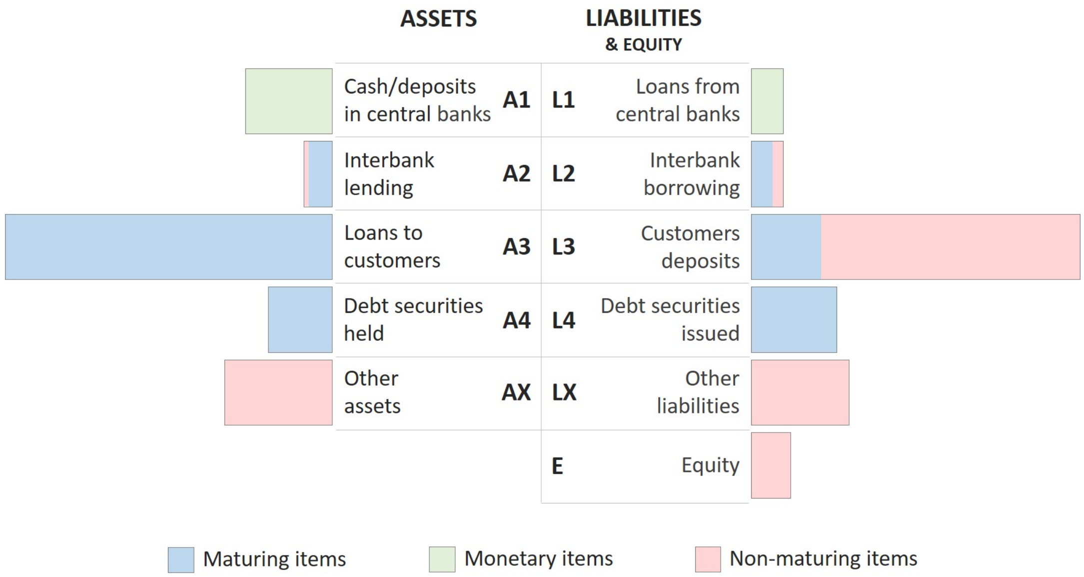


Figure 3: Aggregate balance sheet of the eurozone banking sector, Dec. 2022.

This presentation uses a simplified classification of assets (letter **A**) and liabilities (letter **L**) into five main types:

- Positions with central banks, i.e. banknotes and reserves (**A1**) and refinancing (**L1**);
- Positions with other banks **A2** and **L2**, including interbank financing from the money market, but also currency deposit accounts from *nostro*/*vostro* relationships;
- Positions with customers, i.e. loans to (**A3**) and deposits from (**L3**) non-bank corporations (financial and non-financial), household and public administrations;
- Debt securities held (**A4**) and issued (**L4**) by the banking sector;
- Other assets (**AX**) and liabilities (**LX**) group all items that do not contribute to the net interest income, including shares, non-financial assets and liabilities, derivatives,[5] as well as the interest-bearing products whose interests are registered outside NII.[6]

Each of the items **A1–4** and **L1–4** generates an interest income or expense. The MAP models them separately, which makes it possible to track the net interests between the banking sector and each type of counterparty.

For modelling purposes, it is necessary to further subdivide this aggregate balance sheet into a number of homogeneous items, each of which falls into one of the following categories,

[5] Some derivatives, such as interest rate swaps, generate flows that may technically be considered as interest payments, some of which are registered in the net interest income in the context of hedge accounting. However, as explained in section 2.2, such derivative interests are excluded from the perimeter of the modelled NII; their net contribution is small for the euro area banks, cf. figure 14.

[6] Under the European supervisory reporting rules, banks may merge the interest payments from products registered in fair value into the changes in fair value ("dirty price approach"), in which case they do not contribute to the net interest income as displayed in their income statement. The corresponding portfolios must accordingly be moved to **AX** and **LX** in order to align the modelled NII on the reported NII.

represented by three different colours in figure 3:

- **Maturing items** form the largest part of the banks' balance sheet. These include customers' loans and debt securities on assets side, and debt instruments and term deposits on liabilities side. The model takes into account the maturity term structure of the initial stock, and makes assumptions on the structure of new business lines. The modelling of the maturing items is detailed in section 3.
- **Monetary items** represent commitments between credit institutions and central banks (refinancing operations, bank reserves and banknotes). On aggregate, they are mainly driven by central bank actions such as asset purchase programmes. The model takes this relation into account. Section 4 describes the modelling of the monetary items as a response to central banks' actions.
- **Non-maturing items** cover the rest of the balance sheet, including items which may bear interest, such as sight deposits and liquid savings accounts, but also non-interest-bearing items (non-financial items, equity and derivatives — whose interests are excluded from the model). Their moves are typically accounting counterparts of moves on the first two balance sheet item types. The modelling of these items, including their interactions with the income statement, is described in section 5.

3 Maturing assets and liabilities

Maturing items are interest-paying financial products which remain on the banks' balance sheets for a foreseeable amount of time. They typically form the largest part of the banks' assets, being mainly loans extended to customers (households, companies or Governments) and debt security holdings (items **A3** and **A4** of figure 3); they are also present on the liability side of the balance sheet, under the form of term deposits and issued debt securities (blue part of item **L3** and item **L4** of figure 3). At the aggregate level and taking into account intra-group consolidation, the maturing part of interbank items (**A2**, **L2**) represent a much smaller volume. Maturing assets and liabilities contracted by banks vis-à-vis central banks are treated separately in the monetary section 4; derivatives are excluded and technically treated as non-maturing items, since their balance sheet amount does not represent an interest rate exposure, cf. section 2.2.

The MAP models maturing assets and liabilities under the assumption that their balance sheet value is identical both to the principal amount on which interests are computed and to the amount repaid at maturity.[7] They are represented by a small number of elementary building blocks, referred to hereafter as "exponential accounts", each of which is characterised by maturities following an exponential distribution and by average interest rates exponentially interpolated between short and long maturities.

The model may include an arbitrary number of such exponential accounts on the asset and on the liability side of the aggregate balance sheet, designed to cover together the whole of the

[7]Equivalently, maturing items are assumed to be entirely recorded at amortised cost, without any initial premium or discount. Deviations from this "par-amortised-cost" assumption are discussed in appendix D.

blue part of figure 3. In practice, the aggregate maturity profiles of each of the main categories of maturing assets or liabilities displayed in figure 3 can be approximated to a good degree of accuracy by a combination of only two such exponential accounts, schematically representing long- and short-term products, cf. section B.2.[8] This decomposition allows us to use a single modelling framework while taking into account the diversity of positions.

For each exponential account, the scenario in input of the model specifies the evolution of the amounts on the banks' balance sheet and of the relevant interest rates; in output, the MAP computes the interests, taking into account the progressive replacement of maturing product by newly issued ones.

3.1 Exponential accounts: life cycle

The maturities of the millions of items composing banks' balance sheets can differ widely, ranging from very short-term products due to be repaid almost immediately (up to overnight), to long-term products that will stick in the inventory for ten years or more. In aggregate, only the distribution of these maturities matter; with a large enough number of products issued throughout the year, this distribution becomes nearly continuous, allowing it to be modelled parametrically.

An exponential account's maturity schedule is given by an exponential distribution, characterised by one model parameter, its average maturity τ, and one state variable, the total principal N_t in the exponential account at time t.[9] The exponential model assumes that the amount due to be repaid after time $u > t$ is given by:

$$N_t(u) = N_t e^{-(u-t)\frac{\theta}{\tau}},$$

where θ represents the length of the time step, i.e. one quarter. In other terms, a constant proportion of the outstanding amount is due to be repaid at each time step, cf. figure 4.

In addition to the payment schedule prescribed by the exponential model, the model allows for two other sources of depletion of the existing stock, determined by scenario variables: for each period $t-1 \to t$, a proportion $\boldsymbol{\rho_t}$ of the amounts is considered to be repaid in anticipation, and (for assets only) a proportion $\boldsymbol{\lambda_t}$ is considered to be written off because of default events.[10]

Hence, out of the amount N_{t-1} outstanding at the beginning of the period, the remaining amount at the end of the period after repayment and default is $N_{t-1}e^{-\theta/\tau}\,(1-\boldsymbol{\rho_t})\,(1-\boldsymbol{\lambda_t})$.

On top of that residual amount, the model adds a volume K_t of products issued during the period. This amount is inferred from the variation of outstanding volume $\boldsymbol{\Delta N_t} \equiv N_t - N_{t-1}$,

[8] This observation on very diversified portfolios does not hold for situations where large amounts are concentrated on a small number of maturity dates. This is the case in particular for financing operations initiated by central banks, such as ECB's TLTROs: accordingly, these are modelled separately as part of monetary items, cf. section 4.

[9] In practice, as several exponential accounts coexist in the balance sheet, each account i has its own principal $N_{i,t}$ and its own maturity parameter τ_i. As each account is modelled independently, the indices is are left out of notations for simplicity.

[10] The model assumes that the proportion of defaulting and prepaid amounts is the same across all contractual maturities.

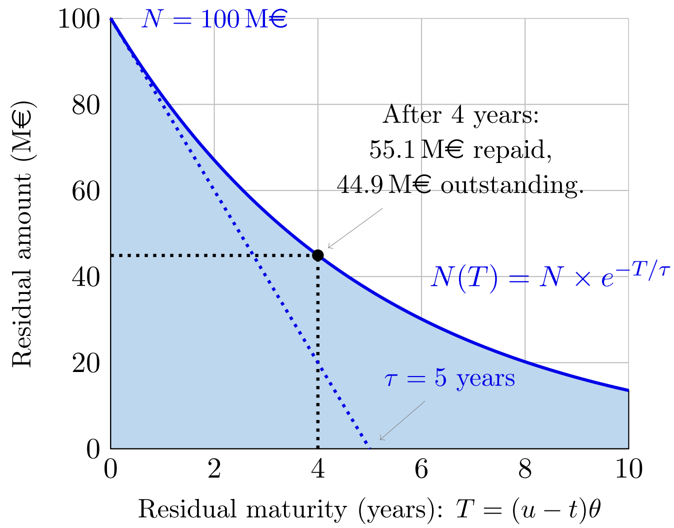


Figure 4: Repayment schedule of the outstanding amounts an exponential account, with $N = 100\,\text{M€}$ and $\tau = 5$ years.

which is a scenario variable, as:

$$K_t = N_{t-1}\left[1 - e^{-\theta/\tau}(1-\boldsymbol{\rho_t})(1-\boldsymbol{\lambda_t})\right] + \boldsymbol{\Delta N_t}.$$

This allows us to decompose the total amount on the balance sheet at the end of the period as the sum of a preexisting amount and a new amount:

$$N_t = \underbrace{N_{t-1}e^{-\theta/\tau}(1-\boldsymbol{\rho_t})(1-\boldsymbol{\lambda_t})}_{\substack{\text{Preexisting amount: residual of } N_{t-1} \\ \text{after repayments and defaults}}} + \underbrace{K_t}_{\substack{\text{New amount: issued} \\ \text{during the period}}}. \tag{1}$$

The model then uses this decomposition to determine the new interest rate structure.

3.2 Exponential accounts: interest rates

The interests received or paid by the banks depend on the individual interest rates of each product contributing to their balance sheets. The MAP relies on an aggregate representation of these interest rates as a volume-weighted average. However, instead of using a single average interest rate for each exponential account, it introduces a term structure. The average interest rate is made to depend on residual maturity, interpolating between a "short" rate, applicable to products that are about to expire, and a "long" rate for products that are due to remain on the balance sheet for a long time. Though not as refined as a granular view taking into account each individual interest rate, this long/short approach allows us to capture some structure at a low computational price.

Specifically, the amount-weighted average as of time t of the interest rates of items maturing at time u inside each exponential account is represented as an interpolation between two state variables, the long-term interest rate R_t^L and the short-term interest rate R_t^S, whose shape is determined by a shape parameter ξ:[11]

$$R_t(u) = R_t^S e^{-(u-t)\frac{\theta}{\xi}} + R_t^L \left(1 - e^{-(u-t)\frac{\theta}{\xi}}\right).$$

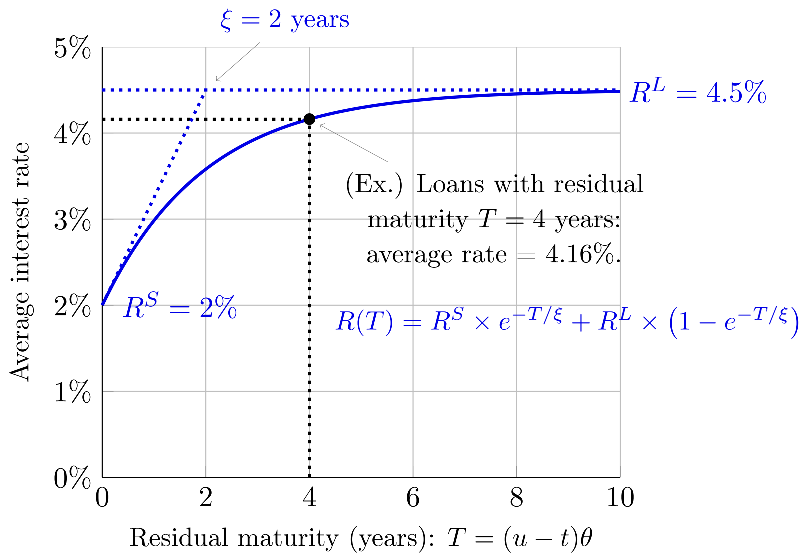


Figure 5: Interest rate structure of an exponential account (portfolio of loans) with $R^S = 2\%$, $R^L = 4.5\%$ and $\xi = 2$ years.

The shape parameter ξ can be thought of as the maturity around which the average interest rates shifts from R^S to R^L. The average interest rate converges to R_t^L for products with long maturities $u \to \infty$, i.e. when $(u-t)\theta \gg \xi$, and to R_t^S for products with short maturities $u \to t$, i.e. when $(u-t)\theta \ll \xi$, cf. figure 5.

In spite of the formal similarity with the exponential distribution of maturities described in section 3.1, ξ bears no relationship with the average maturity τ of the exponential account. However, the comparison between both maturities is meaningful: if the average maturity is much smaller than the shape parameter ($\tau \gg \xi$), then most of the products are short-term and only the short interest rate R_t^S is relevant. In the other extreme ($\tau \ll \xi$), most of the products bear the long-term rate R_t^L and the short-term rate R_t^S only has a very small impact. More generally, the average interest rate of the whole exponential account, all maturities included, is simply:

$$R_t = \frac{\tau R_t^L + \xi R_t^S}{\tau + \xi}, \tag{2}$$

which tends to R_t^L when $\tau \to \infty$ and to R_t^S when $\tau \to 0$. This is demonstrated in appendix C.1.

During a simulation time step $t-1 \to t$, this interest rate structure evolves according to three phenomena:

[11] Likewise, each exponential account i has its own rate variables $R_{i,t}^L, R_{i,t}^S$ and shape parameter ξ_i, even though in practice the ξ_is are chosen equal in the practical application presented in section 6.

Natural ageing Whereas the long-term rate R^L_{t-1}, which applies asymptotically to products with maturity dates $u \to \infty$, remains relevant at time t, the short-term rate R^S_{t-1} applied only to the products maturing at time $t-1$, which have expired. The new value of the short-term rate is the interest rate applicable to products expiring at maturity t, i.e. $R^S_{t-1}e^{-\theta/\xi} + R^L_{t-1}\left(1 - e^{-\theta/\xi}\right)$. The properties of the exponential interpolation ensure that the interest rates applicable to all intermediate maturities are consistent, as demonstrated in appendix C.1.

Effect of variable rates The model assumes that, for all maturities, a constant proportion α of the outstanding amount consists of variable-rate products indexed on a reference rate $\hat{\boldsymbol{r}}_{\boldsymbol{t}}$, typically a short-term benchmark rate such as LIBOR or €STR. Changes in that rate, which is a scenario variable are reflected both on the short- and the long-term interest rates of the products; hence R^S and R^L are shifted by the same amount $\alpha\boldsymbol{\Delta}\hat{\boldsymbol{r}}_{\boldsymbol{t}}$, proportional to the difference $\boldsymbol{\Delta}\hat{\boldsymbol{r}}_{\boldsymbol{t}} \equiv \hat{\boldsymbol{r}}_{\boldsymbol{t}} - \hat{\boldsymbol{r}}_{\boldsymbol{t-1}}$.

Incorporation of new amounts Whereas the above interest rates apply to the preexisting amount, the newly issued amount is assumed to have its own interest rate structure, following the same exponential form as the stock, with long- and short-term interest rates $\boldsymbol{r^L_t}$ and $\boldsymbol{r^S_t}$ prescribed by the scenario[12]. Financially, the interpolated interest rate of new products by maturity, i.e.:

$$r_t(u) = \boldsymbol{r^S_t} e^{-(u-t)\frac{\theta}{\xi}} + \boldsymbol{r^L_t}\left(1 - e^{-(u-t)\frac{\theta}{\xi}}\right),$$

can be interpreted as the prevailing interest rate curve for the exponential account in question.

Knowing the value of the preexisting and new amounts from equation (1), the model can then compute the interest rate structure of the stock at the end of the period by weighted average between the respective long- and short-term interest rates, since the interest rate structure is linear in the parameters (R^L, R^S); hence the following equations:

$$R^L_t = \underbrace{\left(R^L_{t-1} + \alpha\boldsymbol{\Delta}\hat{\boldsymbol{r}}_{\boldsymbol{t}}\right)}_{\substack{\text{Long-term rate of the}\\ \text{preexisting amount}}} \times \underbrace{\left(1 - \frac{K_t}{N_t}\right)}_{\text{Weight}} + \underbrace{\boldsymbol{r^L_t}}_{\substack{\text{Long-term rate of}\\ \text{the new amount}}} \times \underbrace{\frac{K_t}{N_t}}_{\text{Weight}},$$

$$R^S_t = \underbrace{\left[R^S_{t-1}e^{-\theta/\xi} + R^L_{t-1}\left(1 - e^{-\theta/\xi}\right) + \alpha\boldsymbol{\Delta}\hat{\boldsymbol{r}}_{\boldsymbol{t}}\right]}_{\substack{\text{Short-term rate of the}\\ \text{preexisting amount}}} \times \underbrace{\left(1 - \frac{K_t}{N_t}\right)}_{\text{Weight}} + \underbrace{\boldsymbol{r^S_t}}_{\substack{\text{Short-term rate of}\\ \text{the new amount}}} \times \underbrace{\frac{K_t}{N_t}}_{\text{Weight}}.$$

The overall average rate follows from equation (2).

3.3 Model equations for maturing items

To sum up, for each exponential account i contributing to the modelling of maturing assets and liabilities, the model features the following quantities (i index implied everywhere):

[12] In practice, the scenarios are built by deriving the interest rates of new products from benchmark interest rates using a process described in detail in appendix A, which involves in particular an imperfect pass-through from market rates to bank loan and deposit rates.

- The state variables N_t (outstanding amount), R_t^L (average long-term interest rate) and R_t^S (average short-term interest rate);
- The constant model parameters τ (average maturity), ξ (characteristic time of the interest rate structure) and α (proportion of variable-rate products);
- The scenario parameters imposed by the user for each time step $t-1 \to t$: $\boldsymbol{\Delta N_t}$ the variation of outstanding amount, $\boldsymbol{r_t^L}$ the average long-term interest rate of the new production during the period, $\boldsymbol{r_t^S}$ the average short-term interest rate of the new production during the period, and $\boldsymbol{\Delta \hat{r}_t}$ the average change of reference rate underlying variable-rate products.[13]

The evolution of the state variables from modelling step $t-1$ to the next step t is as follows:

$$N_t = N_{t-1} + \boldsymbol{\Delta N_t} \tag{3}$$

$$R_t^L = \left(R_{t-1}^L + \alpha \boldsymbol{\Delta \hat{r}_t}\right)\left(1 - \frac{K_t}{N_t}\right) + \boldsymbol{r_t^L}\frac{K_t}{N_t}, \tag{4}$$

$$R_t^S = \left[R_{t-1}^S e^{-\theta/\xi} + R_{t-1}^L\left(1 - e^{-\theta/\xi}\right) + \alpha \boldsymbol{\Delta \hat{r}_t}\right]\left(1 - \frac{K_t}{N_t}\right) + \boldsymbol{r_t^S}\frac{K_t}{N_t}, \tag{5}$$

where K_t, the volume of new products created during the period, is computed as:

$$K_t \equiv N_t - N_{t-1}e^{-\theta/\tau}(1 - \boldsymbol{\lambda_t})(1 - \boldsymbol{\rho_t}). \tag{6}$$

In accounting terms, the variations of N modify an account on the balance sheet, so that the model must attribute a counterpart to that modification to respect double-entry accounting principles. This is split in two parts:

- the defaulted amounts on assets, which represent losses for the banks, are debited on the income statement in a dedicated account C for credit losses:

$$C_t = N_{t-1}e^{-\theta/\tau}\boldsymbol{\lambda_t}; \tag{7}$$

- the net effect of repaid and newly issued amounts ($\boldsymbol{\Delta N_t} - C_t$) has no effect on the income statement; its counterpart is reflected on selected non-maturing accounts of the balance sheet representing the accounts used for payment, typically customer deposits. For instance, a net increase of customer loans on the asset side mechanically results in a creation of customer deposits on the liability side. This is discussed in detail in section 5.

In addition, the model generates accounting entries linked to interest payments. The interests paid or received by banks on each exponential account during the period are computed using a first-order approximation:

$$I_t = N_{t-1}\frac{\tau R_t^L + \xi R_t^S}{\tau + \xi}\theta, \tag{8}$$

[13] Though in theory the triplet of interest rates variables could be chosen independently for each exponential account, in practice they are derived from a small number of benchmark rates reflecting the prevailing interest rate environment for each period, which ensures consistency within each scenario. For the eurozone banking sector model, this derivation reflects both the observed proportion of EUR and USD holdings and the imperfect transmission of market rates changes to bank customers. Details are provided in appendix A.

where $\frac{\tau R_t^L + \xi R_t^S}{\tau + \xi}$ is the average interest rate of the whole exponential account across all maturities. The interests are credited (for assets) or debited (for liabilities) on the income statement, and the corresponding debit (resp. credit) is reflected on non-maturing accounts in the same way as amount variations, as the model assumes immediate settlement of interests. For instance, interests received by banks on customer loans are debited from customer deposits. This is again discussed in section 5.

3.4 Summary of modelling steps

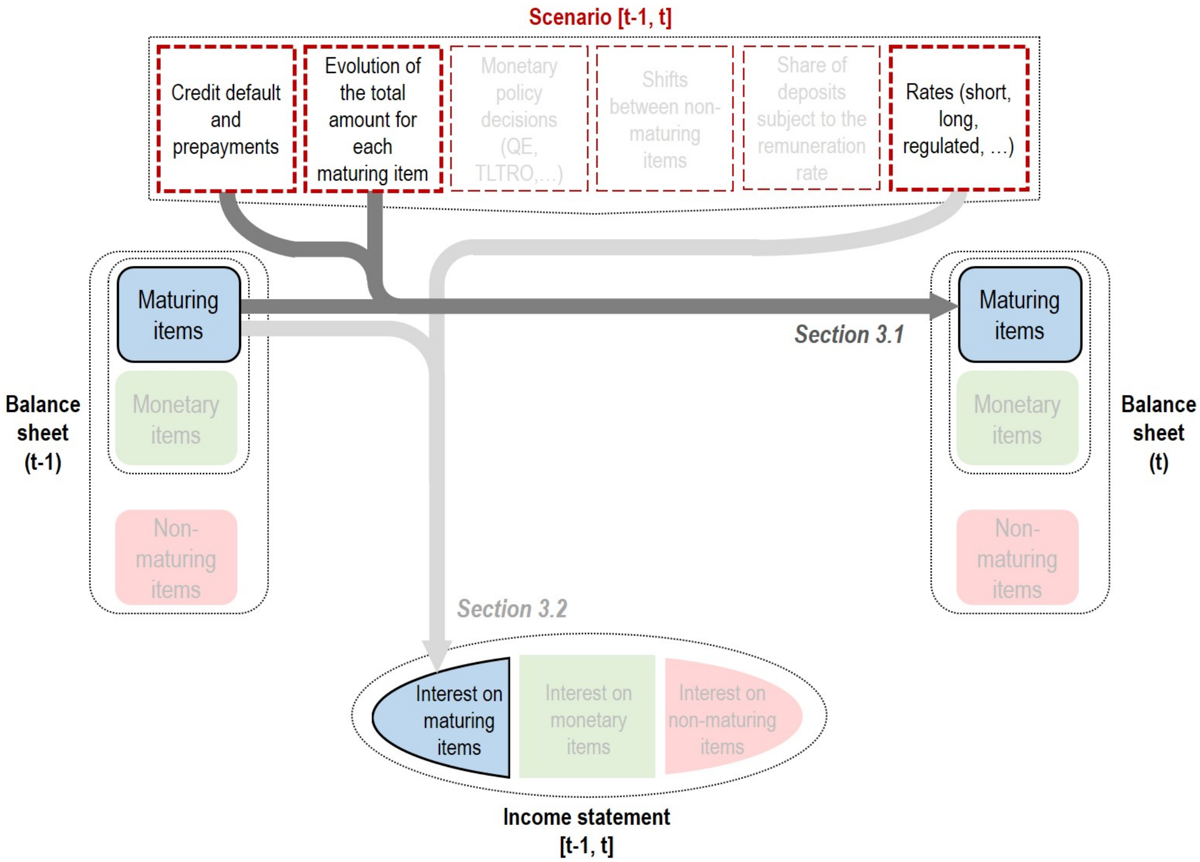


Figure 6: Maturing items: schematic representation of modelling steps

4 Monetary assets and liabilities

The "monetary" assets and liabilities refer in this paper to the mutual financial commitments between credit institutions and central banks, denoted **A1** and **L1** respectively (see figure 3). These amounts have, by definition, a counterpart on central bank balance sheets, **A1** corresponding to central bank liabilities and **L1** to central bank assets. At the aggregate level, their evolution mostly results from central bank decisions, such as asset purchase programmes and refinancing operations. These central bank quantities are financially intuitive variables, for which scenarios can be designed, e.g. leveraging forward guidance or other monetary policy statements.

Monetary items have long played a rather anecdotal role in the banks' balance sheets: before 2008, central banks positions were limited to a few percentage points of the total balance sheet size. The Global Financial Crisis marked the start of a new regime, as central banks stepped in with a succession of "unconventional measures": offering bank funding on favourable terms, and buying large amounts of financial assets ("quantitative easing"), the latter form of action reaching unprecedented volumes during the CoViD-19 recession.

Such unconventional measures have mechanical effects on the banks' aggregate balance sheets. Indeed, when banks resort to central bank funding, they increase their central bank liabilities **L1** and receive funds under the form of central bank assets, or "reserves", in **A1**. Likewise, when the central bank purchases assets, the payment process for these assets results in an increase of reserves in **A1**. This is not only true when the asset seller is a bank, paid in reserves in exchange for an inventory decrease: it also holds in the general case, as the central bank can only pay a non-bank seller by depositing the proceeds with the seller's bank, under the form of reserves in **A1**, upon which the bank transmits the payment to the seller by increasing its deposit account in **L3**.

These increases in **A1** are "sticky": even though an individual bank can seek to get rid of its excess reserves, it can only do so by transferring them to other banks, leaving the total amount unchanged, cf. McLeay et al. (2014), Rule (2015). Within the current operational framework of euro area monetary policy, aggregate excess liquidity resorption can only happen when banks repay their central bank loans, or when the central bank sell its assets or has them repaid at maturity. Hence, as a result of the unconventional monetary measures initiated by the ECB, the central bank assets of the eurozone banking system reached 15% of their consolidated balance sheet in 2021, cf. figure 13. With such sizeable and volatile amounts, and given their interest rate is directly set by central bank decisions, the monetary items have become a major driver of banks' net interest income in the recent context of higher interest rates.

Given this importance, the MAP embeds a refined modelling scheme to deduce **A1** and **L1** from a central bank balance sheet model, leveraging accounting correspondences corrected by some coefficients to account for observed third-party behaviour and perimeter mismatches. This allows the model to project the effect on aggregate banks' balance sheet and NII of monetary policy measures initiated by central banks.

This model of the relation between banks and central banks was specifically designed for the euro area, and therefore reflects the Eurosystem's monetary framework; however, it should be possible to transpose it to other contexts, provided the necessary adaptations are made to reflect the various local operational frameworks of monetary policy.

4.1 Projection of monetary items on banks' balance sheet

4.1.1 Scenario variables

The model input for the evolution of **A1** and **L1** amounts is a set of scenario variables linked to monetary policy decisions. They relate to a "reference central bank", which can be either a single central bank or a group of central banks, that is considered most relevant for the banking sector at stake. For instance, the reference central bank for the eurozone banking sector is the Eurosystem, while the reference central bank for the French banking sector is Banque de France.

Specifically, the evolution of banks' aggregate monetary items can be deduced from the user-prescribed evolution of the following amounts on the reference central bank's balance sheet:

- S_t is the volume of securities in the reference central bank's assets, which may vary according to monetary policy decisions such as quantitative easing or tightening;
- L_t^L is the volume of long-term financing granted to banks in the reference central bank's assets — if fixed-term loans are outstanding, their repayment schedule must be taken into account in the scenario design;
- T_t is the net payment position of the reference central bank with respect to the rest of its monetary zone, if any. This item is *a priori* irrelevant in cases such as the eurozone banking sector, where the reference central bank covers a whole monetary system, but may be useful when modelling a sub-sector: e.g. when modelling the French banking sector, T_t represents the Banque de France's net TARGET position with the Eurosystem.

In addition, the model needs a specification of the ratio m_t of reserve requirements imposed by the reference central bank, i.e. the proportion of certain liabilities (mainly short-term or non-term deposits) each bank is required to hold as reserves. This ratio, which has a limited impact on the outcome, is usually constant: the Eurosystem has changed it only once since the creation of the euro in 1999 (from 2% to 1% in 2012).

4.1.2 Projection mechanism

Starting from a monetary policy scenario as defined above, the MAP projects the amounts **A1** and **L1** of monetary items on the banking sector's balance sheet at each simulation step. Indeed, it is possible to estimate from the scenario variables the evolution of the entire reference central bank balance sheet, that include in particular the deposits of credit institutions (central bank liability); from there, accounting relationships are used to compute the amounts on the banking sector's balance sheet. This two-stage deduction is illustrated in figure 7.

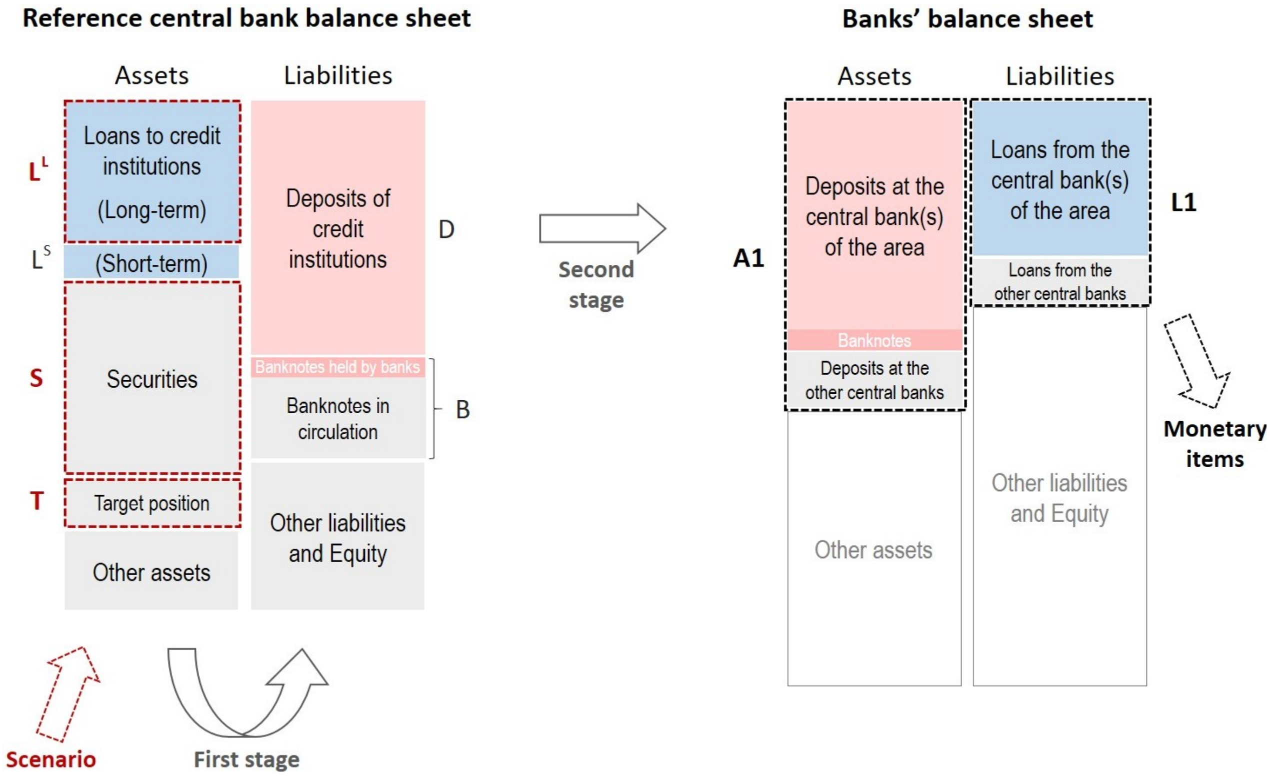


Figure 7: Schematic representation of reference central bank and banks' balance sheets

The **first stage** relies on a set of assumptions on the evolution of the reference central bank's balance sheet to compute, from the scenario variables, both the amount of short-term funding L^S on the asset side, and the amount of deposits D on the liability side. Some assumptions are very simple, but this does not prevent them from correctly reproducing real developments. They are used in particular to take into account the behaviour of third parties (states, bank customers, etc.) and can be summarised as:

- The amount of "other" assets is deemed constant;
- The amount of "other" liabilities, which generally include government deposits, is considered to absorb a constant proportion γ of changes in the central bank assets;
- The volume of banknotes B changes proportionally to changes in the amount of customer deposits **L3** in the banking sector.[14]

Taking this into consideration, the accounting identity between *Assets* and *Liabilities (+Equity)* is as follows:

$$\underbrace{\Delta \boldsymbol{L_t^L} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t} + \Delta L_t^S}_{\Delta Assets_t} = \underbrace{\Delta D_t + \Delta B_t + \gamma{\cdot}\Delta Assets_t}_{\Delta Liabilities_t},$$

[14]This assumption, which is currently well verified for euro, avoids adding banknotes to the list of scenario risk factors. However, if behavioural changes or monetary innovations, such as the issuance of central bank digital currency, were to break the relationship, the model can easily be modified in consequence.

so that the variation in the amount of deposits ΔD_t can be written in terms of the variation in short-term refinancing ΔL_t^S, both remaining to be determined:

$$\underbrace{\Delta D_t}_{\substack{\text{To be}\\\text{determined}}} = (1-\gamma)\cdot\left(\Delta \boldsymbol{L_t^L} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t} + \underbrace{\Delta L_t^S}_{\substack{\text{To be}\\\text{determined}}}\right) - \Delta B_t.$$

To fully solve for both ΔL_t^S and ΔD_t, an assumption on the behaviour of banks is necessary: they are assumed to resort to the lowest amount of short-term bank funding compatible with their reserve requirements, supplemented by a constant minimum amount $\underline{L}^S$. In particular, in times of excess liquidity provided by e.g. long-term funding and/or central bank security purchases, short-term funding will be limited to $\underline{L}^S$. In other words, L_t^S is the minimal positive amount allowing D_t to remain above required reserves (depending on $\boldsymbol{m_t}$ and customer deposits), plus $\underline{L}^S$ (cf. appendix C.2 for explicit formulae).

The **second stage** consists in translating central bank balance sheet items D and L ($\equiv \boldsymbol{L^L} + L^S$) into the monetary items of the banking sector **A1** and **L1**, leveraging the accounting correspondence between them. Indeed, the colour coding in figure 7 illustrates two near-identities:

- In pink, the banks' monetary assets **A1** are *mostly* deposits in the reference central bank, which can be found in its liability item D, as well as banknotes held by banks, which can be found in the reference central bank's liability item B;
- In blue, the banks' monetary liabilities **L1** are *mostly* loans extended by the reference central bank, which can be found in its asset item $L \equiv \boldsymbol{L^L} + L^S$.

However, the match is not perfect: a portion of the banking sector's monetary assets **L1** consists of financing obtained at other central banks than the reference central banks (e.g. contracted by foreign subsidiaries), whereas a portion of reference central bank's loans L is borrowed by credit institutions which do not belong to the modelled aggregated perimeter. The same holds for the correspondence between **A1** and D. This mismatch is illustrated in figure 8. As a consequence, the model introduces a parameter p, close to one, used as a scaling factor to translate changes in reference central bank items into changes in the banking sector's monetary items. For instance, for the aggregate of euro area banks studied in detail in section 6, p is set to 85%, meaning that for 100 € of additional bank deposits in the Eurosystem liabilities, there are 85 € of additional central bank reserves in the assets of the modelled aggregate of banks.

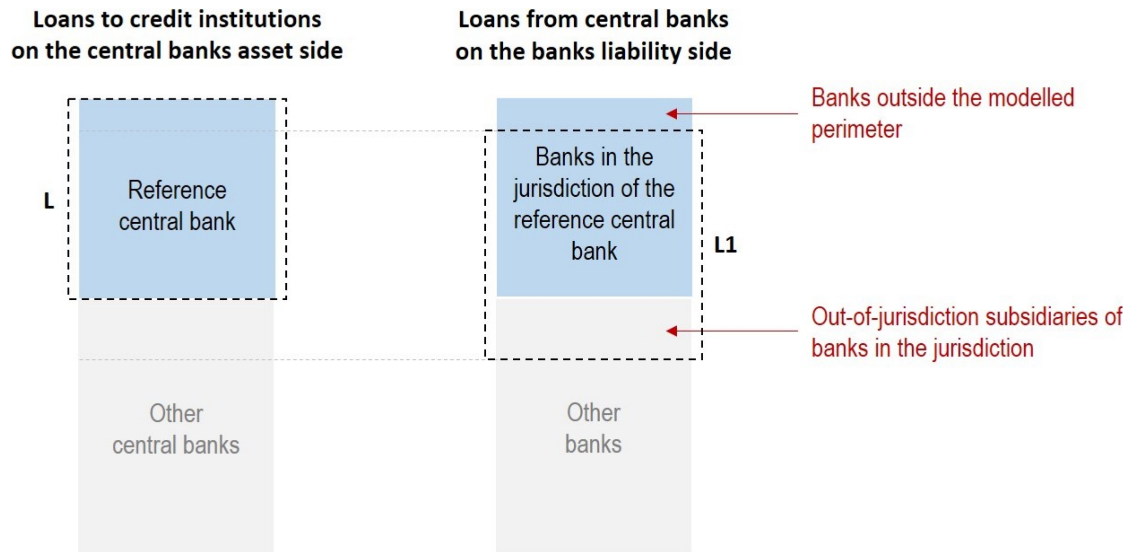


Figure 8: Schematic representation of the different scopes.

4.1.3 Equations

As described above, the projection of banks' monetary items **A1** and **L1** can conceptually be broken down into two successive stages: firstly the projection of the entire balance sheet of the central bank based on a monetary policy scenario, and then the deduction of bank's monetary items from the projected central bank's balance sheet. When putting it into equations, these two successive stages are aggregated, thus linking the following different quantities:

- The state variables composing the monetary assets **A1** of the banking sector: $A1_t^B$ the banknotes held by the banking sector, $A1_t^M$ the amount of minimum reserves and $A1_t^E$ the amount of excess reserves;[15]
- The state variables composing the monetary liabilities **L1** of the banking sector: $L1_t^L$ tracking the amount of long-term financing and $L1_t^S$ the amount of short-term financing;
- A set of constant model parameters:
  - β the scaling factor between changes in banknotes on the reference central bank liabilities and changes in the banking sector's customer deposits ($0 \leq \beta \leq 1$),
  - γ the scaling factor governing changes in the reference central bank's "other" liabilities ($0 \leq \gamma < 1$),
  - δ the fraction of the banking sector's customer deposits subjected to minimum reserve requirements ($0 \leq \delta \leq 1$),

[15] In this document, "excess reserves" means all of the banks' deposits in the central bank excluding required reserves. It does not matter whether these deposits are on the banks' current accounts or on the deposit facility account. Banks are supposed to opt for the account that offers the best remuneration conditions, that is to say the deposit facility account since the key rates are positive and the two-tier system is suspended.

- ϵ the scaling factor between banknotes held by the banking sector and banknotes on the reference central bank liabilities ($0 \leq \epsilon \ll 1$),
- p the scaling factor corresponding to perimeter mismatches,
- $\underline{L}^S$ the minimal short-term refinancing amount on the reference central bank assets,
- $\tilde{L}_0^S = L_0^S - \frac{D_0^E}{1-\gamma}$ the initial theoretical value of short-term refinancing that would result in a zero amount of excess reserve (this value is negative in times of excess liquidity);

- The scenario variables set by the user for each simulated time t: $\boldsymbol{S_t}$ is the volume of central bank security holdings, $\boldsymbol{L_t^L}$ the volume of central bank long-term financing, $\boldsymbol{T_t}$ is the reference central bank's net payment position, and $\boldsymbol{m_t}$ is the reserve requirement ratio.

In addition to the scenario variables, the evolution of the monetary items depends on the amount of customer deposits in the banking sector (balance sheet item **L3** of figure 3), which is used to model both banknote issuance and reserve requirements, with a one-quarter lag, i.e. monetary items at time t depend on customer deposit amounts as at the previous quarter $L3_{t-1}$. As this is directly a banking sector quantity, the scaling coefficient p does not apply on it.

The evolution of the state variables from modelling step $t-1$ to the next step t is then modelled by the following equations (justified in detail in appendix C.2):

$$\begin{aligned}
A1_t^B &= A1_{t-1}^B + \epsilon{\cdot}\beta{\cdot}\Delta L3_{t-1} && (9)\\
A1_t^M &= \delta{\cdot}\boldsymbol{m_t}{\cdot}L3_{t-1} && (10)\\
A1_t^E &= A1_{t-1}^E + p{\cdot}(1-\gamma){\cdot}(\Delta L_t^S + \Delta\boldsymbol{L_t^L} + \Delta\boldsymbol{S_t} + \Delta\boldsymbol{T_t}) - \beta{\cdot}\Delta L3_{t-1} - \Delta A1_t^M && (11)\\
L1_t^L &= L1_{t-1}^L + p{\cdot}\Delta\boldsymbol{L_t^L} && (12)\\
L1_t^S &= L1_{t-1}^S + p{\cdot}\Delta L_t^S && (13)
\end{aligned}$$

where for any quantity X the notation ΔX_t represents the variation $X_t - X_{t-1}$, and where L^S evolves as:

$$L_t^S = \max(\underline{L}^S; \underline{L}^S + \tilde{L}_t^S) \qquad (14)$$

with $\underline{L}^S$ the minimum value of short-term refinancing that banks are supposed to resort at all times and $\tilde{L}_t^S$ the boundary value of short-term refinancing giving a zero amount of excess reserves on the reference central bank liabilities at time t. As elaborated in appendix C.2, $\tilde{L}_t^S$ is computed as:

$$\tilde{L}_t^S = \tilde{L}_{t-1}^S + \Big(\underbrace{\beta{\cdot}\Delta L3_{t-1}}_{\substack{\text{Change in}\\ \text{banknote}\\ \text{amount}}} + \underbrace{\Delta A1_t^M}_{\substack{\text{Change in}\\ \text{required}\\ \text{reserves}}}\Big) \times \underbrace{\frac{1}{p{\cdot}(1-\gamma)}}_{\substack{\text{Transm. coeff.}\\ \text{betw. banks and}\\ \text{central bank}}} - \underbrace{(\Delta\boldsymbol{L_t^L} + \Delta\boldsymbol{S_t} + \Delta\boldsymbol{T_t})}_{\substack{\text{Change in central bank}\\ \text{assets (from scenario)}}}\,. \qquad (15)$$

Box 1: Evolution of bank reserves in a simple theoretical case

By considering the simple case in which $p = 1$, $\beta = 0$ (no banknotes), and $\gamma = 0$ (the central bank has no liabilities other than bank deposits), equation (11) becomes:

$$\underbrace{\Delta A1_t^E + \Delta A1_t^M}_{\Delta A1_t} = \underbrace{\Delta L_t^S + \Delta \boldsymbol{L_t^L}}_{\Delta L_t} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t}$$

This corresponds to the well-known process of creating and destroying bank reserves: one euro of reserve is created/destroyed ($\equiv \Delta A1_t$) when one euro is borrowed/repaid by banks ($\equiv \Delta L_t$), or when one euro of securities is bought/sold/repaid by the central bank ($\equiv \Delta \boldsymbol{S_t}$), or when one euro enters/exits the zone ($\equiv \Delta \boldsymbol{T_t}$).

In the general case, this process is taken into account in the same way. But the amount of reserves created/destroyed at each step is —for example— reduced by the amount used by banks (i) to obtain banknotes and (ii) for payments involving the State having its account at the central bank.

4.2 Interests generated by monetary items

The interests generated by monetary items are computed separately for each state variable composing them, i.e. for required reserves ($A1_t^M$), excess reserves ($A1_t^E$), banknotes ($A1_t^B$), short-term refinancing operations ($L1_t^S$), and long term refinancing operations ($L1_t^L$). Each of these state variables is associated with a different rate r_t, which is either one of the monetary policy rates or a composite rate constructed from these monetary rates. Thus, for one of these given state variables denoted N_t, the interests I_t paid or received during the period $t-1 \to t$ are computed as below:

$$I_t = N_{t-1} \times r_t \times \theta,$$

with θ the length of the time step, i.e. one quarter.

The monetary policy rates used – as they are indicated in the scenario – to construct each rate r_t are the following (cf. appendix A):

- $\boldsymbol{r_t^F}$ the deposit facility rate,
- $\boldsymbol{r_t^M}$ the main refinancing operations rate,
- $\boldsymbol{r_t^T}$ the average rate of long-term refinancing operations.

4.3 Summary of modelling steps

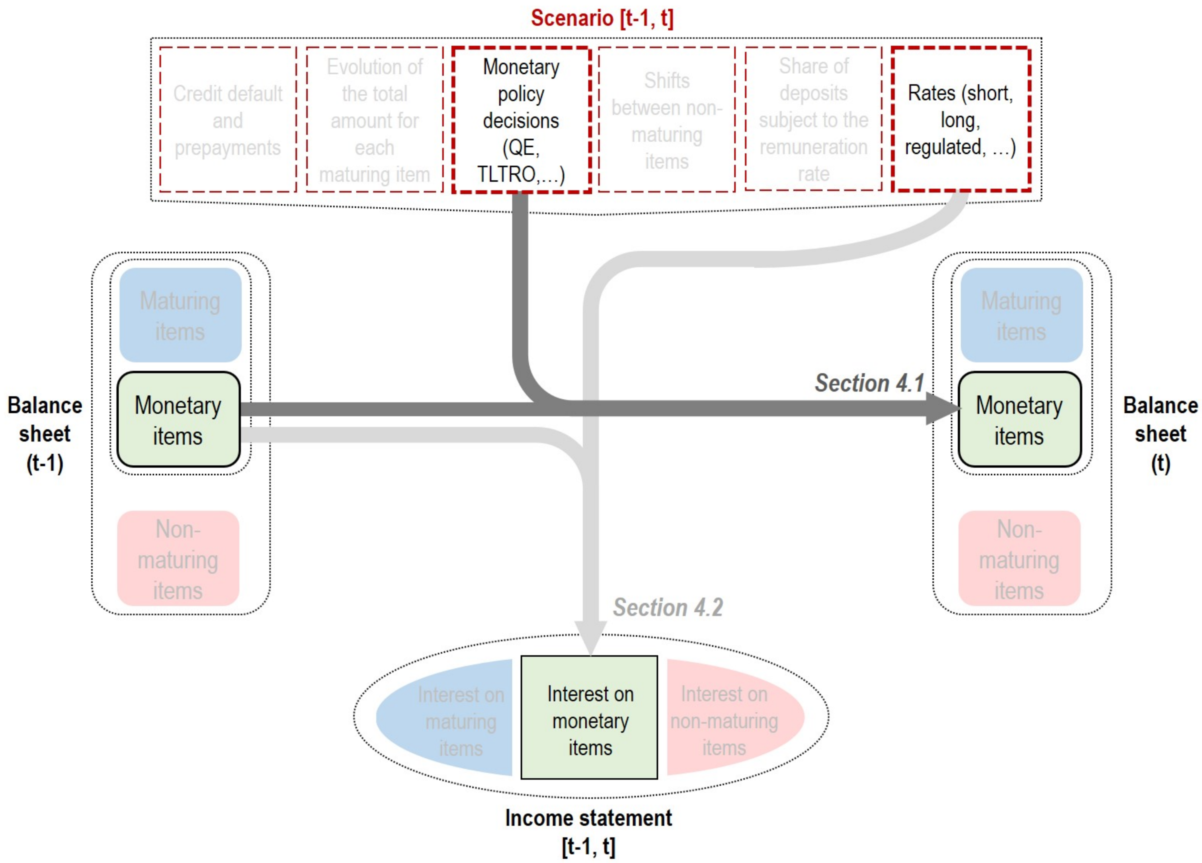


Figure 9: Monetary items: schematic representation of modelling steps

5 Non-maturing assets and liabilities

This section describes the modelling of the balance sheet items representing financial commitments that may evolve freely from one period to another. Since these instruments do not have a contractual maturity, the main challenge is to define hypotheses to properly project their evolution. The MAP leverages double-entry bookkeeping principles and the semi-closed-system property of the aggregate banking sector for that purpose.

The balance sheet items without a defined maturity, outside monetary items modelled in section 4, are mainly customer deposits on the liability side (sight deposits and deposits available at notice, within item **L3** in figure 3) and interbank non-term deposits, which are found both in assets (**A2**) and in liabilities (**L2**). Both of these categories are liable to generate interest. Other balance sheet items are treated as non-maturing items with zero interest rate: this includes equity (item **E**), but also shares, derivatives and non-financial items (items **AX** and **LX**).

The evolution of non-maturing balance sheet items, and in particular customer deposits, is shaped by their use to settle transactions, so that it responds to moves in other parts of the balance sheet. There are five types of operations affecting non-maturing items: settlement of transactions on maturing and monetary items; payment of interests on deposits; settlement of banks' revenues and charges other than interest and credit losses; allocation of banks' net income; and possible additional transfers between non-maturing items. All these are simulated in a way that ensures fully balanced accounting entries.

5.1 Flows induced by maturing and monetary items

Unlike maturing items, the evolution of non-maturing amounts on the balance sheet is not prescribed by the scenario. Instead, it is induced by the evolution of the maturing and monetary items computed by the model as described in the previous sections. In order to respect the accounting consistency, all the changes in outstanding amounts that do not stem from a revenue or expense must be balanced in a non-maturing balance sheet item.

Taking as example an exponential account representing a category of customer loans (i.e. bank maturing assets **A3**), the application of the model for maturing assets (cf. sections 3.3) generates at each time step three accounting entries that must be balanced out:

- The outstanding amount on the balance sheet changes by ΔN_t, which (if positive) represents an accounting debit to customer loans **A3**;
- The loss C_t due to customer defaults represents an accounting debit to the income statement item;
- The interest income I_t represents an accounting credit to the income statement.

The net effect $\Delta N_t + C_t - I_t$ of these three entries is (if positive) an overall debit that must be matched by a credit. In the case of customer loans, the model assigns this credit to non-maturing customer deposits (in **L3**), since payments of loan principal and interests are all settled in customer deposit accounts. Cf. box 2 for a worked-out example.

More generally, for each maturing/monetary items i, an amount $\Delta N_{i,t} + C_{i,t} - I_{i,t}$ must be settled in order to account for the full effects of transactions (note that C_i is nil outside maturing assets). The attribution of this amount is obtained by applying a transition matrix determining how the flows from each maturing and monetary item are channelled to one or several non-maturing items. This transition matrix is a parameter of the model: for each maturing/monetary item represented by an index i, the share of flows attributed to the non-maturing item represented by an index a is given by the matrix element $M_{a,i}$.

Taking into account the flows from all maturing/monetary items i, the induced change in non-maturing item a, denoted $\Delta N^C_{a,t}$, is computed as:

$$\Delta N^C_{a,t} = \sum_i M_{a,i} \cdot (\Delta N_{i,t} + C_{i,t} - I_{i,t})\,. \tag{16}$$

Box 2: Settlement of customer loan flows

An example may be useful to visualise the effect of customer loan events on deposits. Suppose that, during a quarter:

- 90 M€ of existing loans are repaid, i.e. debited from customer deposits;
- 100 M€ of new loans are extended, i.e. credited to customer deposits;
- 2 M€ of loans are written off by banks due to defaults — no impact on deposits;
- 5 M€ of loan interests are paid by borrowers, i.e. debited from their deposits.

The net variation of outstanding loan amounts **A3** on the banks' balance sheet is $\Delta N_t = 100 - 90 - 2 = 8$ M€, being increased by new loans and decreased by both repayments and defaults; meanwhile the net amount credited to customer deposits **L3** is $100 - 90 - 5 = 5$ M€, unaffected by defaults but decreased by interest payments. This net credit is therefore equal to $\Delta N_t + C_t - I_t$.

Thus, this transmission of flows into non-maturing balance sheet items is not an artificial add-on to achieve accounting consistency: it reflects the way banks actually settle operations. The credit to the the customer's deposit account when a bank extends a new loan cannot be separated from the debit on the customer loan asset account, as both moves represent the same operation. The same holds for all operations on banks' balance sheets, cf. appendix E for a detailed discussion.

Accounting consistency (debit=credit) imposes the following condition on the matrix:

$$\forall i, \sum_{a \in \text{Liabilities}} M_{ai} - \sum_{a \in \text{Assets}} M_{ai} = \begin{cases} 1 & \text{if } i \text{ is an asset,} \\ -1 & \text{if } i \text{ is a liability.} \end{cases}$$

The composition of the transition matrix M, as used to model the eurozone banking sector, is represented in figure 10.

M =

		maturing & monetary assets				maturing & monetary liabilities			
		A1	A2	A3	A4	L1	L2	L3	L4
non-maturing assets	A2	0	-1	0	0	0	1	0	0
	AX	0	0	0	0	0	0	0	0
non-maturing liabilities	L2	0	0	0	0	0	0	0	0
	L3	1	0	1	1	-1	0	-1	-1
	LX	0	0	0	0	0	0	0	0
	E	0	0	0	0	0	0	0	0

Figure 10: Flows into non-maturing items from other accounts.

The notations A1, A2, …, LX, E, as well as the colour coding, are those of figure 3. Interpretation: "−1" in column L4 line L3 means that each increase of maturing item L4 (issued debt securities) implies a decrease of the non-maturing part of L3 (customer deposits). Likewise, "1" in column L2 line A2 means that each increase of the maturing part of item L2 (loans from banks) implies an increase on the non-maturing part of A2 (deposits from banks).

In practice, as shown in figure 10, nearly all moves of maturing and monetary items are assumed to be collected by the non-maturing customer deposits **L3**, the only exception being the variations of interbank loans.[16] This modelling choice relies on behavioural assumptions and historical observations, as detailed in appendix E.

5.2 Interests on deposits

Whereas some non-maturing items bear no interest (e.g. shares, real estate, equity...), the deposits generally do. In contrast with maturing items where interest rates are fixed for a period of time, the interests on non-term deposits,[17] i.e. overnight and notice accounts, can adapt immediately to changing circumstances. In practice, interest rates on deposits are very heterogeneous, ranging from zero or below-market rates on some sight deposits to market interest rates on the deposits of sophisticated customers, depending on the competitive structure of banks and depositors.[18] Some regulated types of deposits even bear interest at a government-prescribed rate, which may exceed market interest rates.[19]

The MAP does not seek to account for each of these cases separately, if only because of the difficulty to determine their respective amounts and interest rates. Instead, it models each non-maturing item, and in particular customer deposits, as a bulk, each having a single overall interest rate that seeks to model the weighted average of all the various interest rates composing it, including zeroes, market rates and regulated rates.

This is achieved by applying to each non-maturing item a compound interest rate (computed as a weighted average of benchmark rates read from the scenario), weighted by a remuneration coefficient $\boldsymbol{\eta_t}$, also a scenario variable. In equations, the interests $I_{a,t}$ on non-maturing item a during the period $t-1 \to t$ is computed as:

$$I_{a,t} = \underbrace{N_{a,t-1}}_{\text{Amount}} \times \underbrace{\boldsymbol{\eta_{a,t}}}_{\text{Coeff.}} \times \underbrace{\left(\alpha_a^M \boldsymbol{r_t^M} + \alpha_a^C \boldsymbol{r_t^C} + \alpha_a^R \boldsymbol{r_t^R} + s_a\right)}_{\text{Composite rate}} \times \underbrace{\theta}_{\text{Period}}, \tag{17}$$

where the state variable N_a is the balance sheet amount corresponding to item a, the scenario variables $\boldsymbol{r_t^M}$, $\boldsymbol{r_t^C}$ and $\boldsymbol{r_t^R}$ are respectively the domestic market short-term interest rate, the foreign-currency short-term interest rate, and the regulated interest rate[20], the model parameters $(\alpha_a^M, \alpha_a^C, \alpha_a^R)$ are the respective weights attributed to each type of rates (with $\alpha_a^M + \alpha_a^C + \alpha_a^R = 1$), s_a is a constant spread over the average of these rates, and the scenario

[16] After consolidation of intra-group exposures, interbank items are typically quite small, hence their modelling has little impact on the projected NII.

[17] The remuneration of term deposits follows the general method for maturing items (cf. section 3).

[18] Drechsler et al. (2021), Gambacorta (2008), Berger and Hannan (1991), Gropp et al. (2007) provide empirical evidence on the importance of structural factors in the transmission of money market rates to bank deposit rates and the implicit asymmetric pass-through.

[19] A typical case is France's *Livret A*, a passbook distributed by French banks to households, whose interest rate is determined by the government using a computation formula combining inflation and market rates, with a floor. The interests on *Livret A* represent a sizeable cost for French banks, so that their specific dynamics cannot be ignored in a model of the French banking system, and are still visible at the scale of the eurozone banking system.

[20] Cf. appendix A for the precise construction of these three values.

variable $\eta_{a,t} \in [0,1]$ reflects how much, on average, this compound rate is applied to deposits. θ is the period length (1/4 for quarterly steps).

In extreme cases, $\eta_a = 0$ means that no interest is applied on item a, whereas $\eta_a = 1$ means that the compound interest rate is fully applicable. In practice, $\eta_a = 1$ is used for interbank non-term deposits, which are typically remunerated at full market rate. For non-maturing items a other than deposits (shares, non-financial items...), η_a is set to zero. Customer non-term deposits are modelled with a variable intermediate coefficient $0 < \eta_{a,t} < 1$ reflecting the coexistence of various interest rate regimes, including low- or zero-interest ones: its evolution in the scenario simulates a transformation in the structure of deposits, cf. box 3.

In accounting terms, the interest cash flow $I_{a,t}$ of each non-maturity deposit a is paid into the associated balance sheet item N_a and, as accounting counterpart, credited or debited to the income statement.

5.3 Computation and allocation of the income statement

Since the model intends to produce a coherent simulation of the financial statements, each component of the net income is explicitly accounted for, aggregated and affected to the balance sheet for the next period.

Components of net interest income The net interest income NII is computed at each step of the modelling process by summing interest incomes (II) on assets and subtracting interest expenses (IE) on liabilities:

$$II_t = \sum_{\mu \in \text{Assets}} I_{\mu,t}, \qquad IE_t = \sum_{\mu \in \text{Liabilities}} I_{\mu,t}, \tag{18}$$

$$NII_t = II_t - IE_t, \tag{19}$$

where the index μ stands for both monetary and maturing items i, and non-maturing items a.

Net income The net income NI is computed by subtracting from NII the credit losses C, i.e. impairments on maturing financial assets, as computed from summing all contributions from equation (7), and all other net income items summed into a net cost Q:[21]

$$NI_t = NII_t - C_t - Q_t. \tag{20}$$

Q, which includes very business-dependent items such as commissions, trading revenues or salaries, can be quite variable, but its modelling would exceed the framework of the MAP model, which focuses on the interest rate risk. Hence this amount, which is generally a net

[21] Operating income and expense other than interest bearing revenues such as fees and commissions, gains and losses on trading activities, tax expense and other operating income and expense. As interests from derivatives are excluded from the modelled NII, their effect on the income statement is also included in this category.

charge, is projected proportionally to the evolution of the total balance sheet size B_t with a coefficient q that is a fixed model parameter:

$$Q_t = q \cdot B_t. \tag{21}$$

This net cost, which represents a debit on the income statement, must create a corresponding credit on the balance sheet.[22] Like interests, bank costs are credited to non-maturing items using a vector of weights w_a^Q, which are model parameters, yielding a move on these accounts:

$$\Delta N_{a,t}^Q = w_a^Q \cdot Q_t. \tag{22}$$

Accounting equilibrium imposes $\sum_{a\in\text{Liabilities}} w_a^Q - \sum_{a\in\text{Assets}} w_a^Q = 1$. In practice, the credit is entirely allocated to customer deposits, since it is considered that banks' net expense is paid into customer deposit accounts (cf. figure 11). Appendix E provides the rationale for this choice, which has a very small impact on NII projection.

Distribution of profits Net income being a flow computed on a given period, it must be reintegrated into the balance sheet after each simulation period to preserve the accounting identity. In general, a part of the gains is paid to shareholders, while the remainder is incorporated into Equity as retained earnings. Though the impact of this distribution on projected NII is fairly small, it is necessary to model it in order to keep the accounting picture consistent. The model introduces a distinction between profits and losses, and generates a credit or debit on non-maturing items as follows:

$$\Delta N_{a,t}^D = NI_t \times \begin{cases} w_a^P & \text{if} \quad NI_t \geq 0 \quad \text{(net profits)}, \\ w_a^L & \text{if} \quad NI_t \leq 0 \quad \text{(net losses)}, \end{cases} \tag{23}$$

w_a^P, w_a^L being constant model parameters, each summing at 1 (providing asset items are negated). The equity components of w^P and w^L correspond to the respective proportions of net profits and losses that are capitalised as retained earnings, the rest being distributed. In practice (cf. figure 11 for the eurozone banking sector):

- *net profits* are partly distributed to shareholders, and therefore credited to customer deposits,[23] and partly credited to equity as retained earnings;
- *net losses* are entirely debited from equity.

[22] The other net income components, linked to interests and credit losses, are already balanced out: the former by the attribution to non-maturing items modelling interest payments, the latter by direct credit from the corresponding assets.

[23] The attribution to customer deposits, which is intuitively correct if bank shareholders are all customers of the local banking sector, is less obvious in more general cases, but actually still adequate when induced flows are taken into account, cf. appendix E.

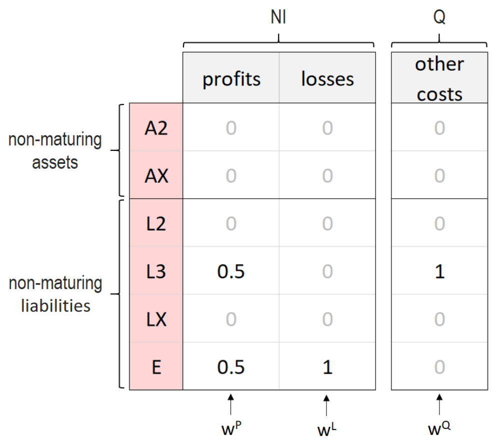


Figure 11: Flows into non-maturing items from net income and from other net costs allocation.

Notes: The notations A2, AX, L2, L3, LX, E, are those of figure 3.

5.4 Flows between non-maturing items

On top of the moves listed above, the model also allows for *ad hoc* shifts between different types of non-maturing items. Examples may include moves between interbank and customer deposits, increases or decreases of derivative values or non-financial assets and liabilities, but also *ad hoc* equity moves such as capital increases or extraordinary distribution of profits.

These flows are exogenous to the model and are a user-defined scenario variable: the change in non-maturing item a induced by such shifts, denoted $\boldsymbol{\Delta N^F_{a,t}}$, is directly part of the scenario. The user must ensure that the shifts on all non-maturing items balances out from an accounting perspective, i.e. $\sum_{a \in \text{Assets}} \boldsymbol{\Delta N^F_{a,t}} = \sum_{a \in \text{Liabilities}} \boldsymbol{\Delta N^F_{a,t}}$ at all time steps t.

In practice, absent any particular reason to believe that such shifts should happen on a large scale, the default value for these transfers is zero. In that case, the non-interest bearing items of the balance sheet (figure 3) are inert components, contributing neither to the net interest income nor to the evolution of the rest of the balance sheet.

Overall, the total evolution of balance sheet amounts of the non-maturing items result from five difference sources:

$$N_{a,t} = N_{a,t-1} + \underbrace{\Delta N^C_{a,t}}_{\substack{\text{Settlt. of} \\ \text{other items} \\ (16)}} + \underbrace{I_{a,t}}_{\substack{\text{Interests on} \\ \text{deposits} \\ (17)}} + \underbrace{\Delta N^Q_{a,t}}_{\substack{\text{Payment of} \\ \text{net bank costs} \\ (22)}} + \underbrace{\Delta N^D_{a,t}}_{\substack{\text{Allocation of} \\ \text{net income} \\ (23)}} + \underbrace{\boldsymbol{\Delta N^F_{a,t}}}_{\substack{\text{Additnl.} \\ \text{moves} \\ \text{(scenario)}}} . \tag{24}$$

5.5 Summary of modelling steps

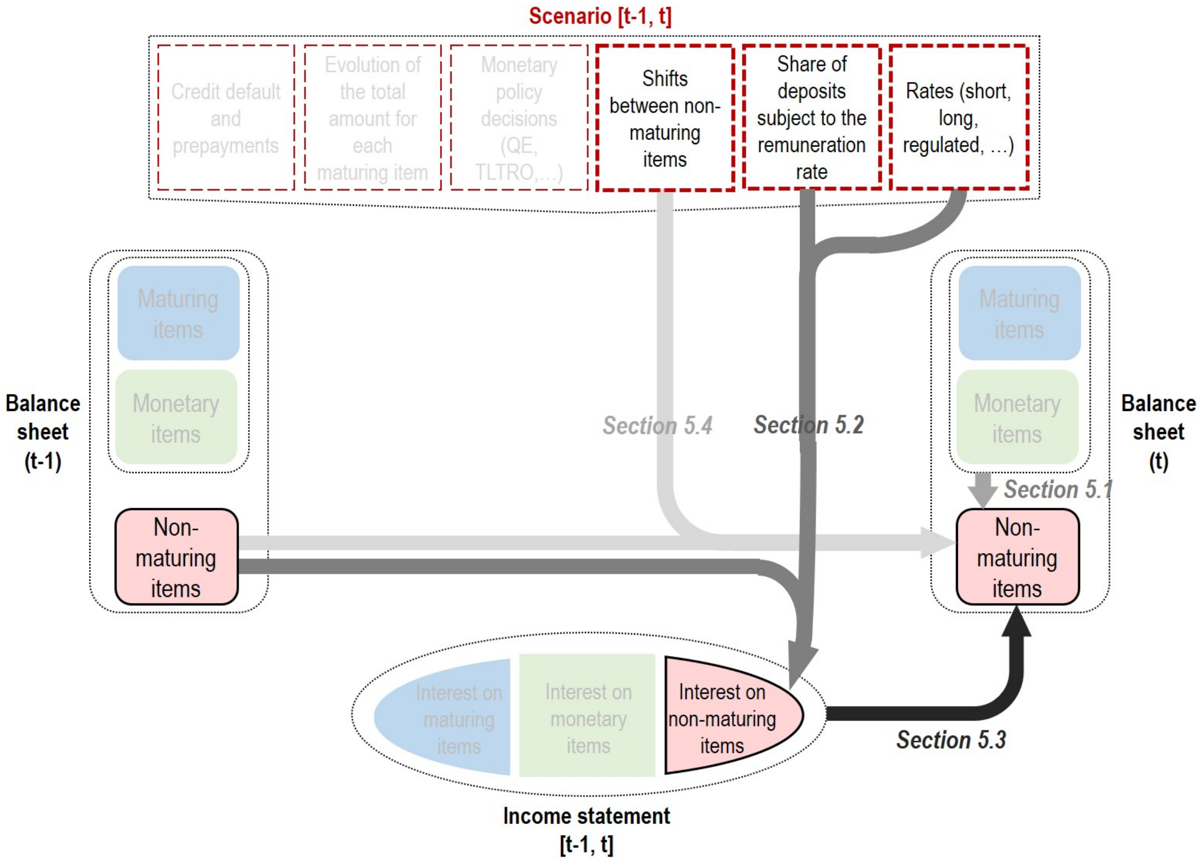


Figure 12: Non-maturing items: schematic representation of modelling steps

6 Application to the euro area banking sector

In this section, we present an application of the MAP to the aggregated euro area banking sector. The scope includes all large European banks directly supervised by the European Central Bank (ECB) via the Single Supervisory Mechanism (SSM) and designated as Significant institutions (SI)[24] which together account for approximately 85% of the euro area's total banking assets.[25] Changes in the list of SI may occur over time mainly due to mergers and acquisitions or reclassifications; however, this remains marginal and the sample over the period 2015–23 is quite stable.

At the end of Q4–2022 the ECB sample of SI counts 111 entities from 18 euro area countries, including banking groups headquartered in the euro area and subsidiaries of foreign groups (i.e. controlled by either an EU or a non-EU parent) with a total assets of 25,848 billion

[24] All banks deemed systemic at the euro area level, with over 30 billion euros in total assets or 20% of national GDP. The respect of this criteria is checked annually and can give rise to very marginal changes.

[25] According to **supervisory data** published by the ECB as of December 2022.

euros. In this paper, we use a slightly different sample since we exclude 12 among the smallest entities for data quality reasons. The total assets of the sample used by the MAP is of 25,358 billion euros (i.e. less than 2% lower than the whole sample of SI of the euro area).

The aggregation is obtained by summing the balance sheets of banking groups at the highest level of consolidation.[26] Therefore, the scope reflects the activity of these institutions in the euro area and abroad via subsidiaries and branches.[27]

6.1 Aggregate structure of the banking sector

6.1.1 Perimeter and aggregate balance sheet

The application of the model in its current form requires exhaustive data to cover all balance sheet items. The MAP needs to run on a consistent aggregate balance sheet of the banking sector, for which the financial characteristics in terms of maturities and interest rates can be determined: this requires a data source with a good level of granularity and with a high consistency from bank to bank and from date to date.

The main data set corresponds to the banks' financial reports provided to supervisors (FINREP for eurozone banks). It covers balance-sheet exposures and profit and losses accounts for all SI in the euro area at a quarterly frequency. This data provides a detailed breakdown by product, counterparty (central banks, governments, credit institutions, other financial corporations, non-financial corporations and households) and accounting category according to IFRS standards. At this stage of the model, the balance sheet items are grouped into five major categories on the asset side (numbered **A1** to **A4** plus **AX**) and six on the liability side (numbered **L1** to **L4** plus **LX** and equity), as defined in figure 3.

In reality, the model uses a more detailed decomposition for each item of the balance sheet by maturity profiles or type of product (for example, **L3** is split into overnight, notice accounts and term deposits) which is not illustrated here for the sake of readability. It is worth pointing out that degree of granularity for both balance sheet and income statement items to be modelled is not fixed: with appropriate parameter choices, the MAP could separately project the interest incomes and expenses for different types of customers (households, non-financial corporations, financial corporations and governments).[28]

Figure 13 illustrates the evolution of the aggregate balance sheet of euro area banking sector between 2015 and 2022 (end of year values). The size of the aggregate balance sheet has increased of 24% since 2015 to reach 25,358 billion euros at end 2022 while the number of institutions in the modelled perimeter has increased of only 9% (from 93 to 99).

[26] On the basis of the IFRS scope of consolidation.

[27] This aggregation method does not consolidate away the operations between euro area banking groups, which therefore remain included in the interbank items **A2** and **L2**. This is does not affect the outcome since the corresponding interests net out when computing the aggregate NII.

[28] The main difficulty in such a split is that a model of deposit circulation is needed to determine the transition matrix of equation (16): when non-financial corporations take out new loans, what proportion of the resulting deposits ends up in household deposits? in financial corporation deposits?... The aggregate view into a single "customer" category makes it possible to dispense with such subjective choices.

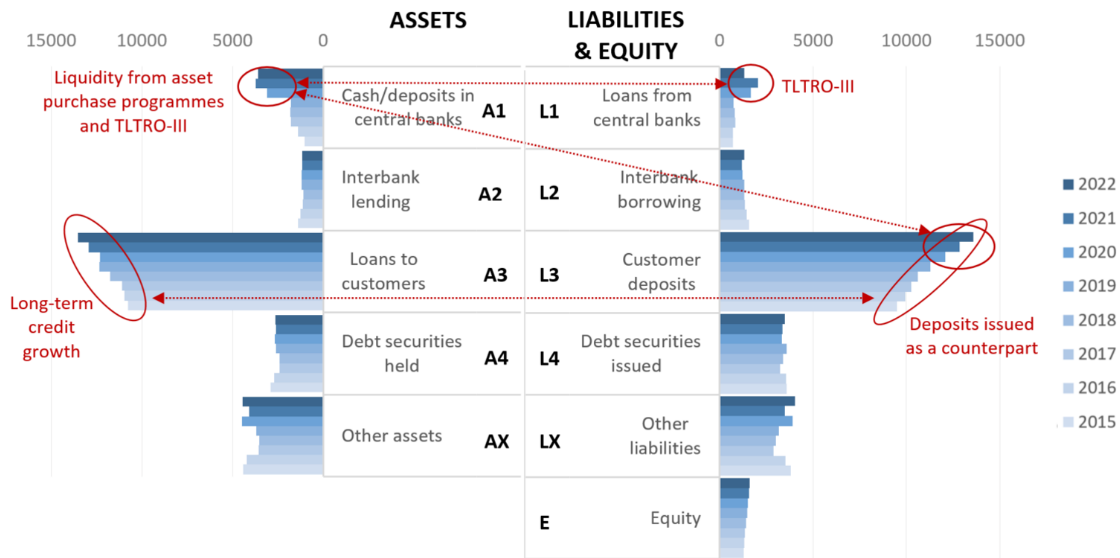


Figure 13: Evolution of the euro area aggregate banking sector balance sheet

Notes: End of year data in billion euros. Positions with central banks include banknotes and reserves in A1 and while refinancing in L1. A2 and L2 include interbank financing from the money market and respectively currency deposit accounts. Loans (A3) and deposits (L3) from customers (i.e. non-bank corporations, both financial and non-financial, household and governments). Debt securities held and issued by the banking sector in A4 and respectively L4. Other assets AX and liabilities LX include derivatives, real estate, tax assets and liabilities etc.

Historically, customer loans and customer deposits dominate the balance sheet on the asset side and respectively on the liabilities side. They experienced a similar evolution between 2015 and 2019 but starting with 2020 one can notice a particular development that is mainly driven by the evolution in monetary policy items **A1** and **L1**. In particular, the strong liquidity flows from central banks (**A1**) driven by the launch of asset purchase programs set up by the Eurosystem generated a sharper increase in deposits accounts on the liability side (**L3**) compared to the progression of loans on the asset side (**A3**), cf. Adam et al. (2023). Interbank and securities items evolved in a more stable way while equity increased progressively in order to comply with regulatory requirements.

6.1.2 Recent income structure

The aggregate income statement of banks is also built on the basis of supervisory financial reporting data. The reporting provides, for both interest income and expense, the same breakdown by type of product and counterparty as for the balance sheet and allows for a detailed decomposition of the NII — our main interest variable. There are two main differences between the "official" NII as published by the ECB[29] for the sample of significant institutions in the euro area and the modelled NII used in our model. First, the global aggregate NII for the euro area banking system is slightly lower than the one published by the ECB since we

[29]Data for the aggregate balance sheet and income statement of euro area SI is published on the ECB Supervisory banking statistics page.

exclude 12 small banks from the sample for data quality reasons. For example, as of end-2022, the total NII for the whole sample of euro area SI is 298 billion euros while the total NII for our sample is of 294 billion euros (i.e. 1.3% lower). Second, the modelled NII excludes the net interests paid or received by banks through derivatives. The net interests from derivatives represent only a very small proportion of the NII (2% of the total NII in average over 2015–2022, see figure 14) and is relatively stable over the period. For the purposes of the MAP, the derivative interests are included in the net cost Q (see figure 1).

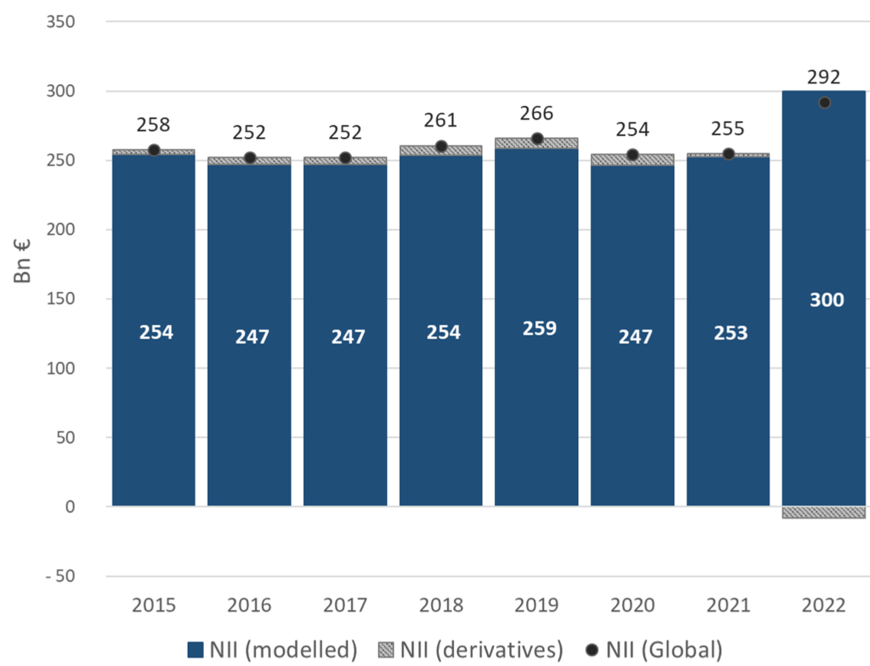


Figure 14: Evolution of the net interest income of the euro area aggregate banking sector

Notes: Annual values in billion euros. The net interest income (NII) is the difference between interest income (II) and expense (IE). The net income is the sum of the NII, credit losses C and all other net income items summed into a net cost Q.

The NII is the main component of banks' net income and is relatively stable over the time horizon covered by our study. However, the rising interest rates during 2022 have been favourable for the NII and led to increasing the aggregate net income of euro area banking sector.

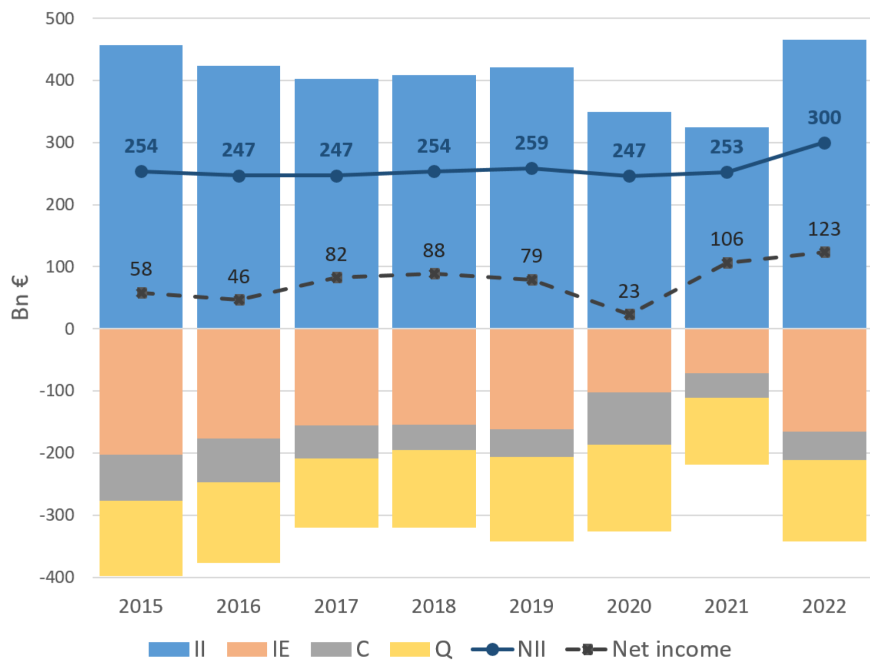


Figure 15: Evolution of the main components of the net income of the euro area aggregate banking sector

Notes: Annual values in billion euros. The net interest income (NII) is the difference between interest income (II) and expense (IE). The net income is the sum of the NII, credit losses C and all other net income items summed into a net cost Q.

6.1.3 ALM structure of the balance sheet

The input data set is enriched by two additional data sources. Firstly, we use statistics publicly available *via* the Statistical Data Warehouse (SDW) platform on i) the ECB's balance sheet holdings, used to model the monetary assets and liabilities (**A1** and **L1**) in banks' balance sheet, and ii) BSI/MIR data on balance sheet and respectively interest rates for monetary financial institutions in the euro area in order to get complementary data on structural characteristics of banks' balance sheet and interest rates level. Secondly, we use granular data collected by the Eurosystem (such as AnaCredit for loans, CSDB for securities etc.) to get detailed information allowing us to perform an analytical decomposition of the balance sheet and to calibrate the model parameters. The complete list of these parameters and the estimation methods are described in appendix B.[30]

It is useful to summarise the outcome of this analysis by grouping together instruments with similar characteristics on both sides of the balance sheet: figure 16 shows this allocation with

[30]The data issued from granular sources generally have a different coverage scope, so that extrapolation techniques are needed to revert to the modelled perimeter. For example, AnaCredit covers loans to firms, including small and medium-sized enterprises (SMEs), in the euro area countries, larger than 25,000 euros. In turn, households loans are excluded and so are loans granted by subsidiaries and branches located outside the euro area.

three different colours.[31] The interests generated by the instruments with a variable rate or a short-maturity fixed rate (dark blue) are very reactive to changes in interest rates, while those generated by instruments with long fixed rates (light blue) have a greater inertia. The non-interest bearing instruments (white) are inherently insensitive to interest rate fluctuations. Customer loans, securities holdings or issued as well as interbank items are distributed between fixed and variable rate, while monetary assets and liabilities are primarily remunerated at a variable rate. Non-remunerated items concern mainly customer deposits, equity, derivatives as well as non-financial assets and liabilities.

At the aggregate level, the structural breakdown indicates a positive net holding of interest-paying products: the surplus of long fixed-rate assets (22%) and the surplus of short and variable rate assets (12%) are both effectively funded by non-interest-bearing liabilities. This allocation is expected to be favourable to euro area banks in case of raising interest rates since almost a half of liabilities are insensitive to interest rate fluctuations.

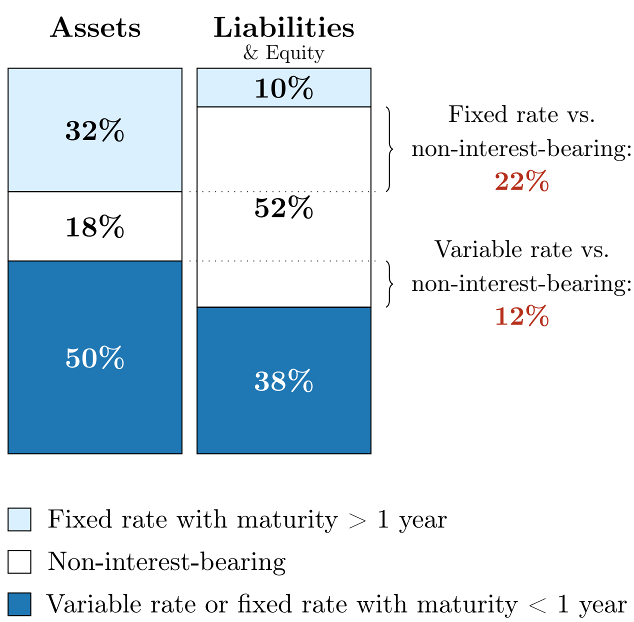


Figure 16: Estimated balance sheet structure of the euro area aggregate banking sector, breakdown by interest rate and maturity as of Dec. 2022

The starting point for the MAP is given by the aggregate balance sheet and income statement at the end of 2022. In order to project the balance sheet and the interest income and expense over the next quarter and up to a 5-year horizon, a scenario must be defined: the next subsection explains the construction of the scenario for the eurozone case study.

[31]The allocation of each maturing item of the balance sheet by type of interest rate takes into account both the variable-rate proportion α and the maturity schedule determined by τ. Monetary items are by nature allocated in whole to variable rates. Non-maturing items are split between the variable-rate and the non-interest-bearing part according to the remuneration coefficient η.

6.2 Defining a scenario

The scenario is an exogenous entry of the MAP. A complete scenario must include the quarterly evolution over five years, not only of interest rates, but also of the volumes of maturing items (loans, securities and term deposits), central bank security holdings and refinancing operations, as well as a number of more minor parameters. In order to build consistent scenarios, the various rates used by the model are automatically derived from only four benchmark interest rates, as detailed in appendix A.

For illustrative purposes, we present in this paper a scenario for the aggregated euro area banking sector, spanning the five years 2023–2027. Table 1 provides an overview of the scenario variables and of the method chosen to build this scenario.

Type of variable	Scenario variables	Value
Interest rates	e_t – €STR	Risk-neutral expectation derived from interest rate curves.
	y_t – 10-year AAA yield	
	f_t – SOFR	
	i_t – French inflation	Banque de France projections.
Central bank volumes	S_t – Eurosystem security holdings	ECB indications.
	L_t^L – TLTRO-III outstanding amounts	Rapid repayment in 2023.
	T_t – net TARGET payment position	Zero (near-closed system).
Bank volumes	ΔN_t^{A3} – loan volumes	Moderate decline, then stagnation.
	ΔN_t^{L3} – customer term deposit volumes	Rapid rise (higher attractiveness).
	$\Delta N_t^{\cdot\cdot}$ – other volumes (securities, interbank)	Zero (no change).
	ΔN_a^F – transfers between non-maturing items	Zero (no net transfer).
Technical variables	η_t – deposit remuneration coefficient	Rapid rise (higher attractiveness).
	ρ_t^{A3} – customer loan prepayment rate	1% per quarter.
	$\rho_t^{A4,L4}$ – security prepayment rate	Share of amounts held for trading.
	$\rho_t^{\cdot\cdot}$ – other prepayment rates	Zero.
	λ_t^{A3} – customer loan default rate	0.1% per quarter.
	$\lambda_t^{\cdot\cdot}$ – other defaults (securities, interbank)	Negligible.
	m_t – reserve requirement ratio	Constant (1%).

Table 1: Scenario variables

Interest rate scenario The future evolution of short- and long-term euro interest rates e_t and y_t is derived from interest rate futures market prices and the AAA-rated euro yield curve published by the ECB, with a cut-off date of 1 September 2023: standard curve construction techniques (cf. e.g. Svensson (1994)) are applied to build risk-neutral expectations of future interest rates. The SOFR rate f_t for US dollars is derived from the same approach using SOFR swap rate market data. For the French inflation rate i_t, which is only used to project the future evolution of the regulated *Livret A* interest rate r_t^R, the scenario follows the medium-term macroeconomic projection provided by Banque de France.[32] The evolution of these rates

[32]That formula is overridden till the first quarter of 2025 to account for the French Government's July 2023 decision to freeze the *Livret A* interest rate on a one-off basis at 3% for 18 months. After that date, the rate is determined using the regulatory formula based on inflation and €STR.

in the scenario is illustrated in figure 17.

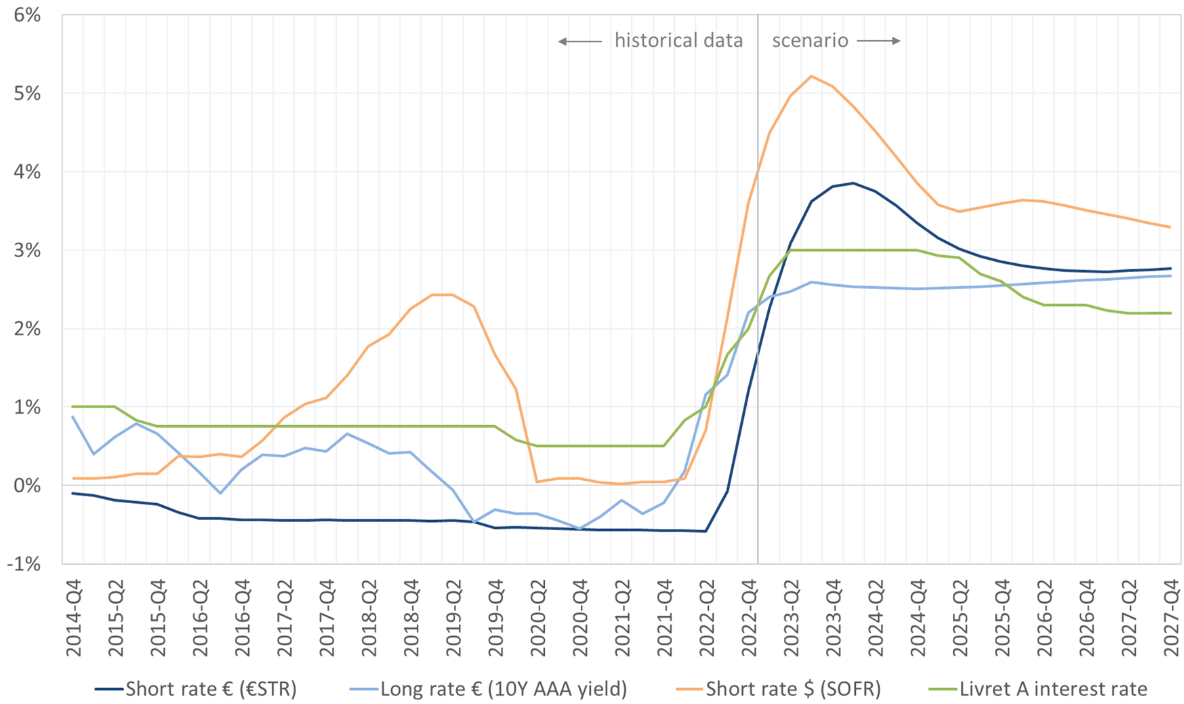


Figure 17: Benchmark interest rates in the scenario.

Notes: Quarterly average values. As per the French Government's July 2023 decision, the French Livret A interest rate is maintained at 3% until January 2025. The formula described in appendix A.4 is applied afterwards.

Central bank scenario The main scenario input for monetary items is the volume of central bank asset portfolios $\boldsymbol{S_t}$, for which the ECB's monetary policy strategy as of 15 June 2023 can be leveraged: *"The Governing Council will discontinue the reinvestments under the APP as of July 2023. As concerns the PEPP, the Governing Council intends to reinvest the principal payments from maturing securities purchased under the programme until at least the end of 2024."* As the ECB also communicates on the expected monthly redemption amounts for the APP over a rolling 12-month horizon, the scenario is easily determined until mid-2024: on average, $\boldsymbol{S_t}$ drops by 80 billion euros per quarter. For the rest of the scenario, this trend is extended until the end of 2024, and slightly accentuated after.

Long-term refinancing volumes $\boldsymbol{L_t^L}$ are projected under the assumption that banks repay the remaining loans taken during the pandemic (TLTRO-III) during the third quarter of 2023. Indeed, taking into account the new remuneration rules announced by the ECB in October 2022,[33] banks no longer have a financial incentive to keep excess reserves in the deposit facility instead of repaying the TLTRO-III loans. Even though some volume may actually remain outstanding until final maturity (December 2024), the identical interest rate applicable to

[33]Since 23 November 2022, the effective interest rate paid by banks on their TLTRO-III loans has been set to the deposit facility rate (or more if the optimal conditions are not met). Cf. ECB/2022/37: **Decision (EU) 2022/2128 of the European Central Bank** *of 27 October 2022 amending Decision (EU) 2019/1311 on a third series of targeted longer-term refinancing operations (ECB/2019/21).*

TLTRO-III and deposits means that the impact on the NII would be limited, within the model, to the second-order effects of the absorption by non-bank deposits described in section 4.1.2.

$\boldsymbol{T_t}$, which represents the net payment position of the sector vis-à-vis the rest of the monetary zone, is irrelevant for the euro area as a whole and set to zero.

Bank volume scenario Following the recent trend of lower loan creation volumes, it is assumed that the bank loan supply, after a slight initial contraction (–0.7% until 2024), will switch to a stagnation regime with a minor increase of +0.2% per year in 2027.

The scenario also anticipates that the higher interest rate environment may trigger a progressive increase in the share of term deposits (maturing part of **L3**), rising from 21% of all customer deposits in 2022 to 36% in 2027, symmetrically with the past decline of this proportion (from 39% in 2014 to a minimum of 18% at the end of 2021).

The volumes of securities, interbank positions and non-financial assets and liabilities is projected to be constant.

Technical scenario variables For the same reason as the term deposit volume increase, the scenario features a progressive rise in the deposit remuneration coefficient $\boldsymbol{\eta_{L3,t}}$ of customer deposits, from 29% to over 40% over the five-year period, representing a higher appetite of depositors for interest-paying forms of non-term deposits, including regulated deposits.

Prepayment $\boldsymbol{\rho_t}$ is assumed to remain at a relatively low value (1% per quarter) for customer loans, as a high-rate environment does not encourage early repayment. Prepayment is set to zero for other products, except for debt securities, where it is used to model the sale of securities before maturity and their replacement by newly bought securities. Indeed, both events have the same effect on interest income and expense, i.e. resetting the interest rate to the prevailing market rate.[34] The scenario arbitrarily assumes that securities within banks' trading portfolios rotate on average every quarter, so that the proportion of trading portfolio holdings in debt securities **A4** and **L4** is used as constant quarterly prepayment rate.

6.3 Projection results

From this scenario, the MAP projects the aggregate interest income and expense of the eurozone banking sector, as displayed in figure 18. Both quantities undergo a large upward shift in the year 2023, consistently with the interest rate surge in the scenario.

[34] The effects of a security sale differ from those of an early payment in that a sale is generally not performed at par value; however, this difference relates to realised capital gains or losses, which for the sake of this model are gathered into the "other net costs" item Q; the effects on interests are identical.

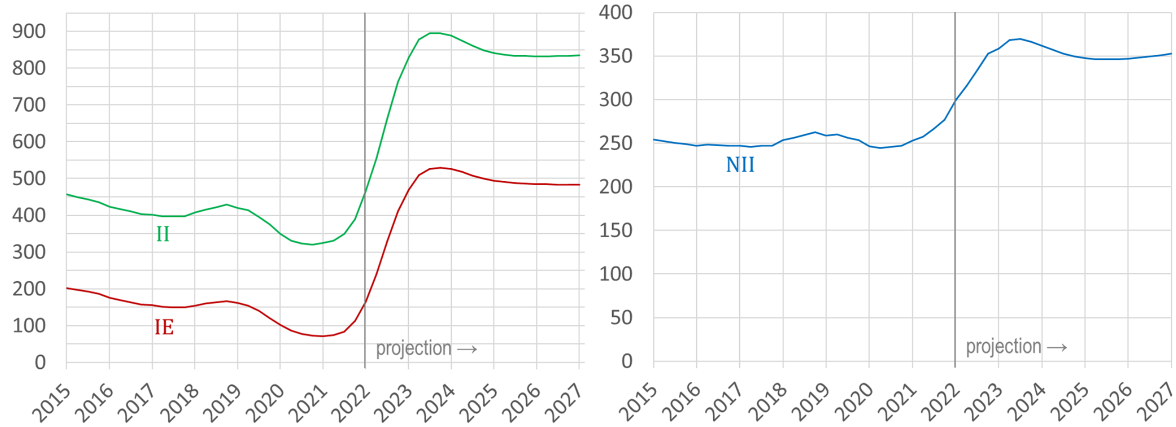


Figure 18: Historical and projected interest income (green curve), interest expense (red curve) and net interest income (blue curve) of the aggregated euro area banking sector (billion euros).
Notes: Both graphs represent annual revenues and costs over 4 rolling quarters. E.g. the June 2024 point represents the sum of the last two 2023 quarters and the first two 2024 quarters. This convention provides a smooth evolution from one full-year quantity to the next.

Interestingly, in spite of the structural difference between assets and liabilities highlighted in figure 16, the shapes of the respective evolutions of interest income and expense look qualitatively similar: both of them undergo a slower increase in 2024, followed by a slight decline in 2025. This common feature is the reflection of the evolution of the €STR short rate in the scenario, as it directly affects the variable-rate and short-maturity parts of the balance sheet (dark blue areas in figure 16). In parallel, the long-maturity parts of the balance sheet (light blue areas), predominantly on the asset side, progressively incorporate the 2022 shock on long-term interest rates, but the longer time scale (5–7 years) makes this process less spectacular.

The net effect on the evolution of NII is consistent with the structural analysis of figure 16: after the dust settles, the long-term projected effect of the 2022–23 interest rate shock is a frank NII increase, reflecting the overall excess of interest-bearing assets of the banking sector effectively backed by non-interest-bearing liabilities. The 2023–24 rate overshoot in the scenario translates into a temporarily higher NII during that period, as expected given the higher amount of variable-rate and short-term assets compared to liabilities.

The MAP also makes it possible to analyse this projected evolution in terms of bank counterparties. Figure 19 shows how the projected NII can be decomposed into four "partial NIIs"[35] facing the four types of counterparties used in this version of the model.

[35] These "partial NIIs" are not strictly speaking interest margins, since they do not correspond to self-financed parts of the balance sheet. For instance, the positive net interests facing central banks reflect the fact that banks are, as a whole, net creditors to central banks: they do *not*, in themselves, mean that banks earn margins off their central bank positions, since the liabilities that fund this imbalance have a cost.

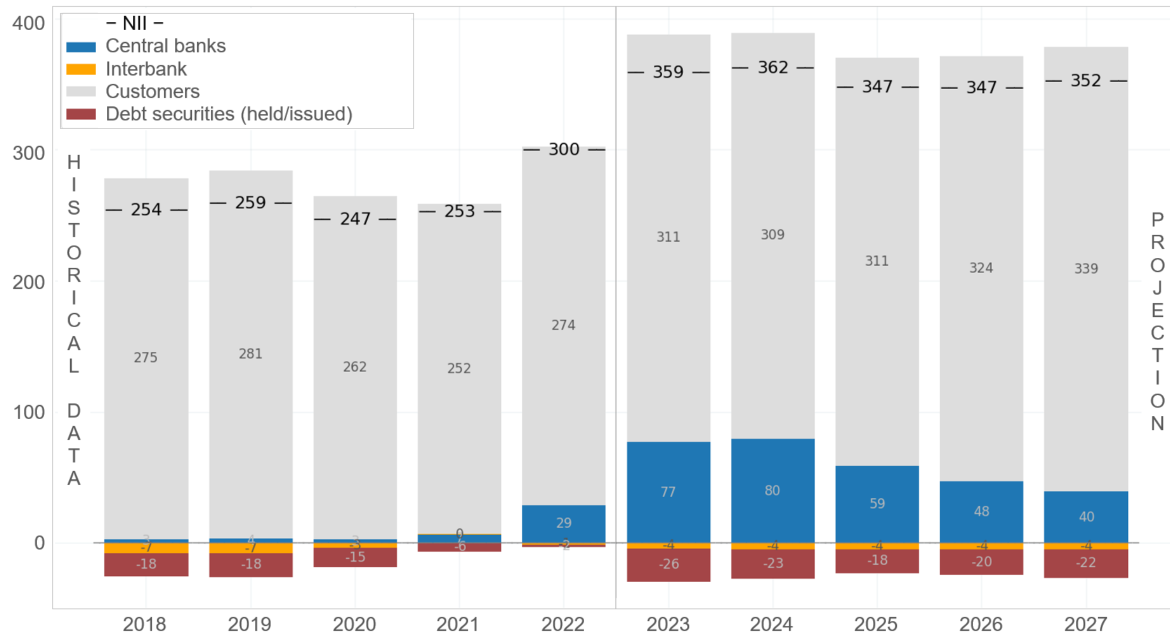


Figure 19: Breakdown of the aggregate NII of the euro area banking sector, by counterparty (annual values, billion euros).

On the long term, the increase in NII is largely borne by customers, who pay a higher interest rate on loans while collecting only part of the rate hike on deposits. Debt securities contribute negatively, reflecting the larger volumes of issued securities in banks' liabilities compared to securities held on the asset side. As stated before, the net contribution of the interbank part of the aggregate balance sheet is negligible.

The remarkable component in this analysis is the piece of NII attributable to central banks: historically negligible, it is projected to surge in 2023 with the unprecedented combination of large excess assets produced by the ECB's security purchase programmes, with relatively high interest rates, hence becoming for the first time a major component of banks' NII. In this example scenario, the progressive extinction of the central bank asset portfolio absorbs bank reserves, leading to a gradual decrease of the net interests paid by the Eurosystem to the banking sector as soon as interest rates stop rising.

6.4 Sensitivity analysis

The large flexibility of the model in terms of scenario design makes it possible to measure the individual influence of various elements on the projected NII by computing sensitivities. To illustrate this use case, the model is run on three alternative scenarios, designed to single out the respective effects of the level of interest rates, of the pace of the Eurosystem's Quantitative Tightening, and of the speed and magnitude of the rise in customer deposit costs.

More precisely, the alternative scenarios are derived by altering the central scenario (denoted S0) in three different directions:

- Higher interest rates (S1): starting at the beginning of 2024, interest rates (€STR, 10Y

AAA yield, SOFR and inflation) jump 100 basis points higher than in scenario S0 (the *Livret A* rate remains at its frozen 3% level until the end of 2024).

- Fast-paced quantitative tightening (S2): starting at the beginning of 2024, the reduction in the stock of securities held by the Eurosystem is greatly accelerated: the stock decreases by around 560 billion euros per quarter and thus disappears completely at the end of 2025.
- Accelerated rise in customer deposit costs (S3): starting at the beginning of 2024, the pace of migration towards customer term deposits is doubled compared to scenario S0, as well as the pace of change in remuneration coefficient $\eta_{L3,t}$. As a result, term deposits end up representing 41% of all customer deposits in 2027, and the final remuneration coefficient of non-term deposits is 51% (instead of 36% and 42% respectively in scenario S0).

Figure 20 compares the outcome of the scenarios in terms of interest income, expense and margin, to the equivalent quantities for the central scenario S0. The difference between each scenario and the central scenario materialises the sensitivity of the banks' interest income to the individual factor at play.

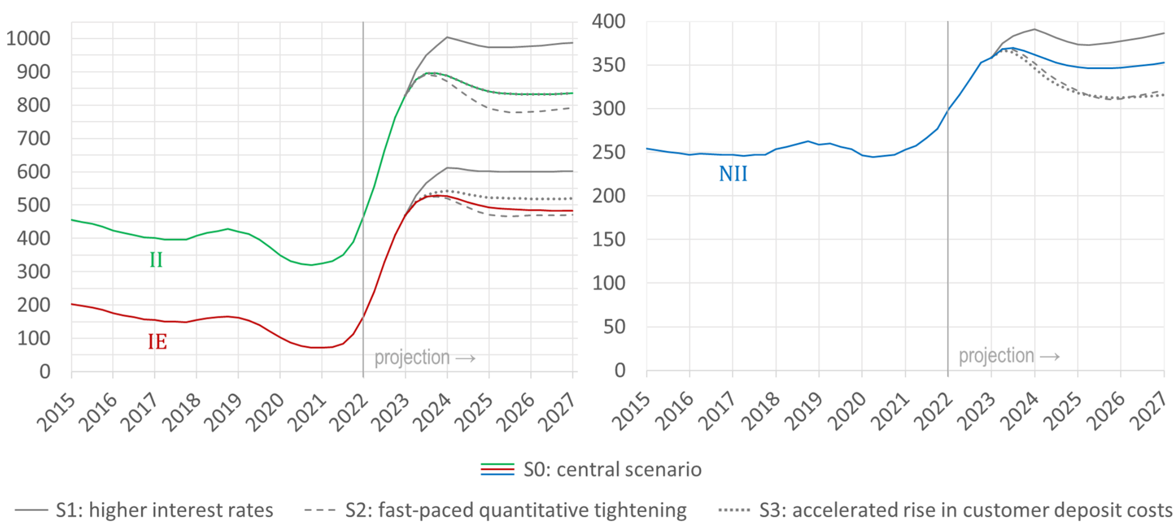


Figure 20: Historical and projected interest income (green curve), interest expense (red curve) and net interest income (blue curve) of the aggregated euro area banking sector (billion euros).
Notes: Both graphs illustrate annual values over 4 rolling quarters. Grey lines correspond to alternative scenarios.

Effect of an interest rate shock. The NII with higher rates (scenario S1) is larger than that of scenario S0. The effect of the shock is visible in both interest income and expense, but is consistently higher on income, in line with the structure of the banks' balance sheet (see figure 16). One can distinguish an immediate windfall in 2024, linked to the rise in short-term rates, and temporarily amplified by the fixation of *Livret A* rates, and a longer-term effect kicking off in 2025–27 due to the cumulative benefit of higher long-term rates. The overall benefit for banks' NII of a parallel upward shift in rates, all else being held equal, shows in particular that this income does not solely rely on maturity transformation.

Effect of an accelerated Quantitative Tightening. Scenario S2, with its fast unwind of central bank asset portfolios, leads to a lower NII than scenario S0. This negative sensitivity of banks' NII to quantitative tightening is however rather small, as the fairly extreme scenario S2 produces bearish effect on the NII of a magnitude comparable to the bullish effect obtained with an increase in rates of 100 basis points. This relatively low sensitivity is explained by a negative impact on expense, partially offsetting the negative impact on income: indeed, the drop in central bank reserves **A1** induced by Quantitative Tightening results in an equivalent drop in customer deposits **L3**. The overall effect remains negative since central bank reserves have a higher interest rates than customer deposits.

Effect of higher customer deposit costs. The NII in case of higher customer deposit costs (scenario S3) is notably lower than that of the scenario S0, comparable to the effect of a –100 basis point shift on interest rates. The impact is, as expected, concentrated on interest expense. In spite of the material shift in deposit structure envisioned in scenario S3, the relative loss to banks remains lower than the NII benefit of the 2022–23 interest rate rise. Though less impactful than large changes in interest rate levels, fragilities in the deposit cost structure are a significant risk factor in the evolution of the banks' NII, all the more so that their prediction raises specific challenges, cf. box 3.

Box 3: Forecasting deposit costs

Potential changes in the deposit cost structure, represented in the MAP by the remuneration coefficient $\boldsymbol{\eta_t}$, are difficult to forecast given their behavioural nature. The design of realistic stressed scenarios for this risk factor makes for a research topic in its own.

Micro-founded methods could be built to estimate how much deposits costs adjust to shifts within the banking sector, by leveraging the expected dependence of deposit spreads on interest rate levels, but also on the market power of banks (Drechsler et al., 2017). For instance, a scenario in which depositors entrust large amounts of money to a few wholesale agents such as funds, who then demand an interest mark-up, may erode banks' market power and result in higher values of $\boldsymbol{\eta_t}$.

More refined methods might account for the role of uninsured deposits as a factor of vulnerability for banks' deposit franchise, especially in times of rising interest rates (Drechsler et al., 2023). Such studies, and more generally research in consistent scenario-generation methods, are left for future works.

These examples demonstrate the interest of the scenario-driven nature of the MAP, in its capacity to provide separate information on the various factors affecting bank profitability, without being tied by interdependence assumptions.

6.5 Back-testing the model

The credibility of the MAP projections can be assessed by a back-testing method. As the model is, ultimately, a relation between input (initial balance sheet data and scenario) and output (aggregate bank balance sheet and interests), the point of back-testing is to assess the robustness of that relation by retroactively running the model on the past aggregate balance sheet structure of the euro area banks, using realised interest rates and volumes as scenario, and comparing the model output with actual data.

We use the financial and economic conditions actually observed over the period 2016–23 to build, for each end-of-quarter date in this period, an initial balance sheet for the aggregated euro area banks, and a so-called "realised" 5-year scenario starting from the said date — for recent start dates, the scenario is cut at the last available observation date, i.e. June 2023.

Thus, for each start date, the model computes the quarterly projections for the amounts of each balance sheet item and each income statement account over the next five years. These projections can then be compared to the corresponding actual values published by banks over the period 2016–23. As the model parameters used by the MAP have been estimated using data running till the end of 2022, the results of this comparison on 2016–22 must be considered as "in-sample" back-testing, whereas the two first quarters of 2023 are "out-of-sample".[36]

The back-testing results for the NII are illustrated in figure 21, with the decompositions between income and expense displayed in figures 22 and 23 respectively. In each figure, the orange curves represent the different projections made by the MAP from the successive end-of-quarter dates using the realised scenario, and the black curve represents the actual data published by the banks. In a perfect model, the orange curves should match the black curve.

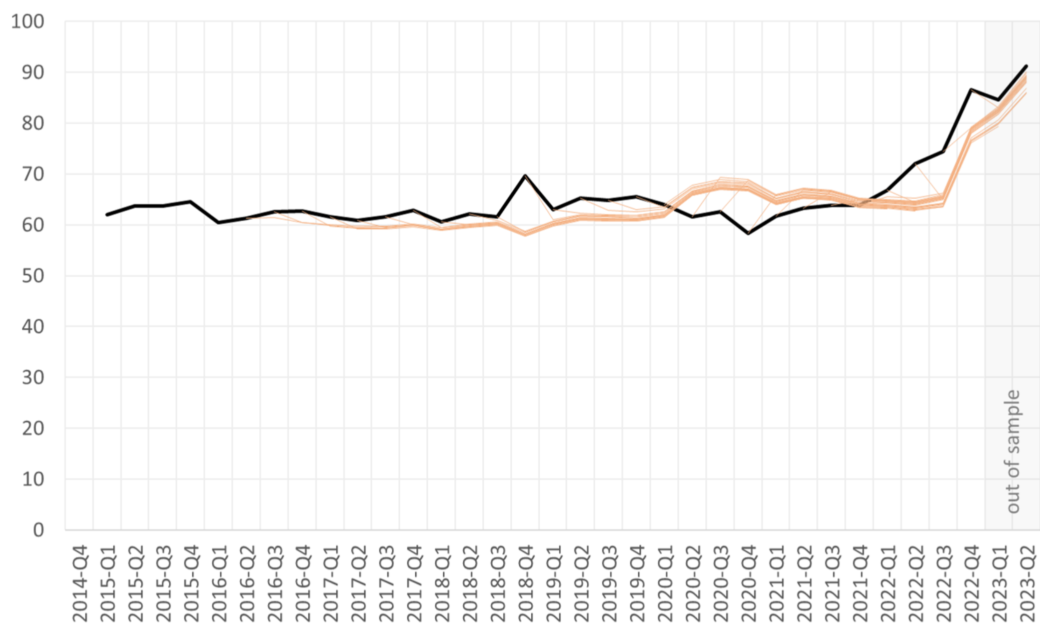


Figure 21: Back-testing of the aggregate NII of the euro area banking sector (quarterly values, billion euros).

[36] A fully "out-of-sample" back-testing would necessitate a re-estimation of all model parameters at each past date using only data available at that date, which would avoid the classical risk of parameter over-fitting, but would require a considerably more complex process. As things stand, this mostly "in-sample" back-testing already provides useful information on the reliability of model assumptions over a 7-year period.

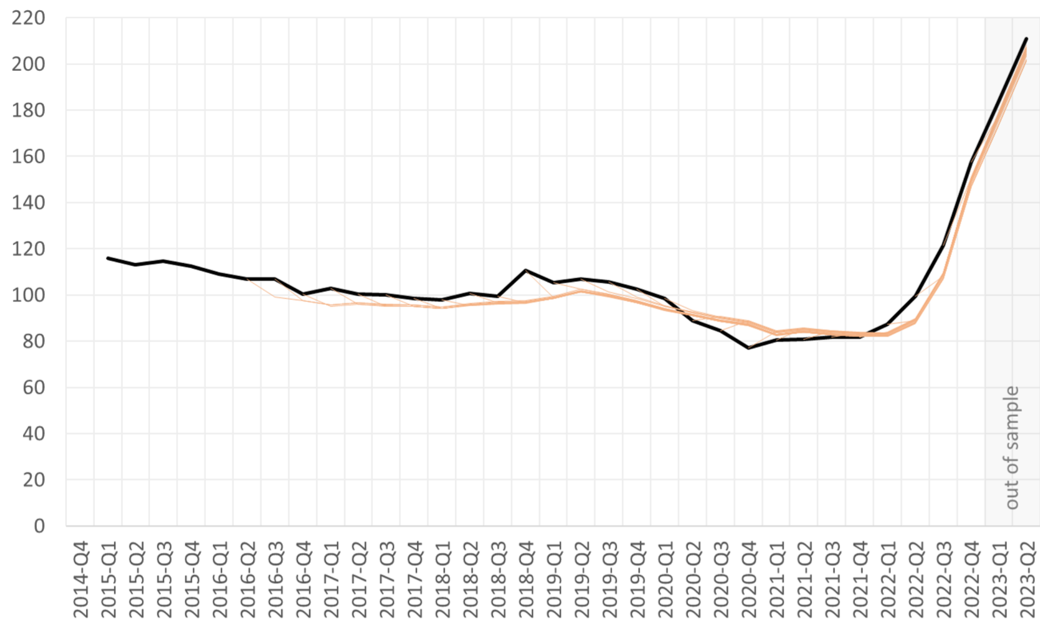


Figure 22: Aggregate interest income (II) of the euro area banking sector: projected vs. realised (quarterly values, billion euros).

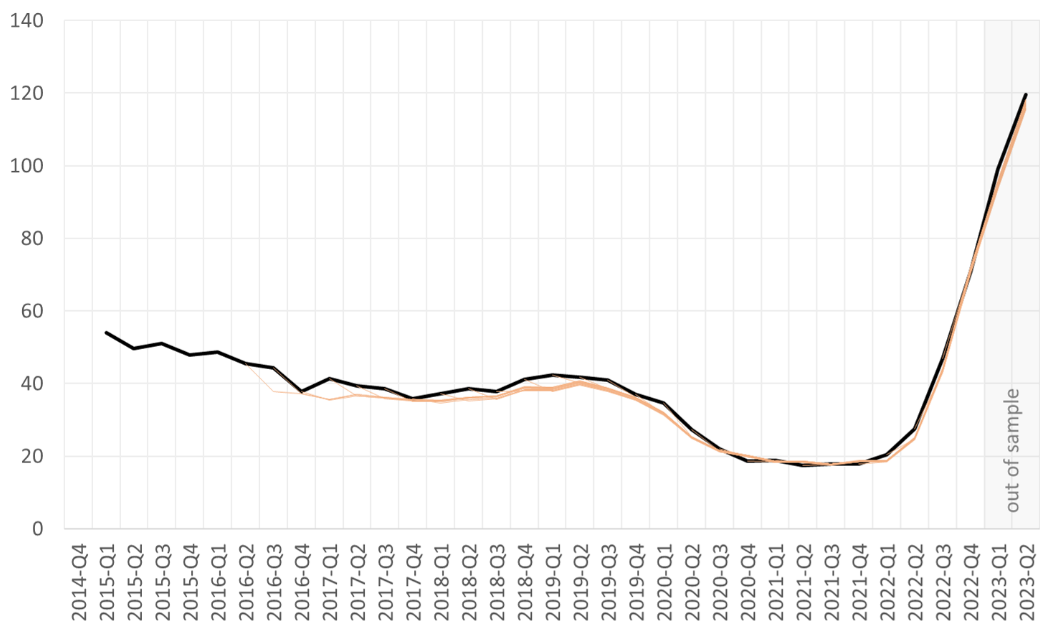


Figure 23: Aggregate interest expense (IE) of the euro area banking sector: projected vs. realised (quarterly values, billion euros).

The results are overall satisfactory, replicating the pattern of stability observed until 2021, and the significant upward move in 2022–23. It can be observed that the actual rise in NII in 2022–23 starts earlier than in the MAP projection, but this deviation remains small, especially when considering the separate back-testing results for the interest income from assets (figure 22) and the interest expense from liabilities (figure 23), which show a good fit of the large upward moves driven by recent rate hikes.[37]

Given the reliance of the exponential model for maturing items on a "par-amortised-cost" assumption, whereas a significant proportion of the debt securities held in **A4** are registered at fair value (cf. appendix D), it is interesting to focus on the back-testing results for the part of the interest income generated by these securities, displayed in figure 24.

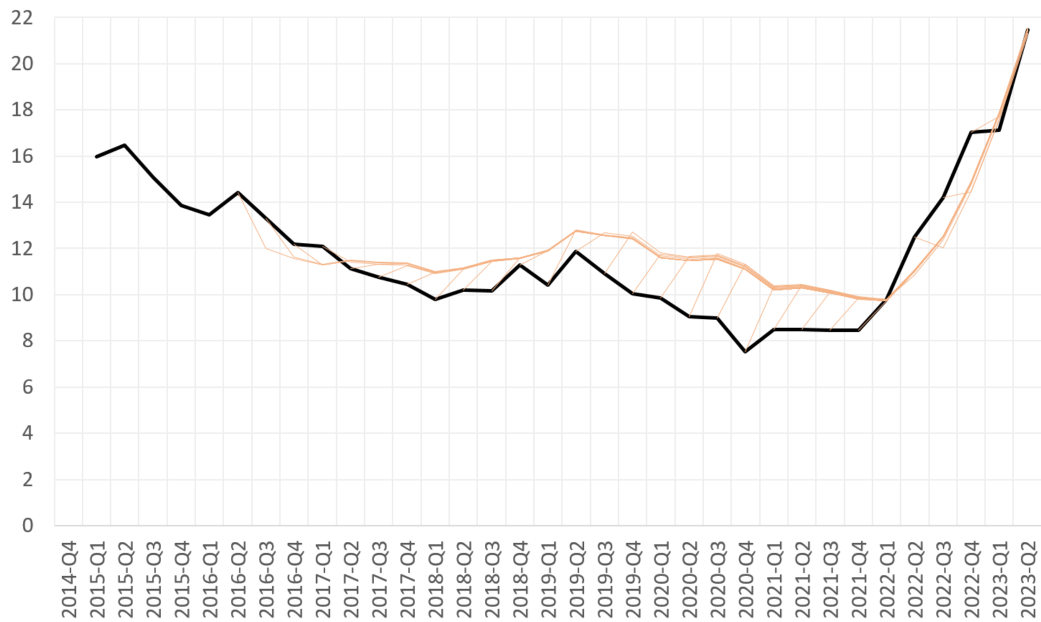


Figure 24: Aggregate interest income from the debt securities held by of the euro area banking sector: projected vs. realised (quarterly values, billion euros).

The performance is noticeably less good than for the rest of overall interest income; in particular, debt securities account for a significant part of the income over-estimation observed in 2020–21. However, the general shape of the evolution of these interests is still correctly accounted for by the model and deviations remain limited.

Overall, the back-testing exercise confirms the relevance of the modelling choices driving interest computations, i.e. the exponential model for maturing items (section 3), and the composite rate method for deposits (equation 17), while pointing to possible improvements discussed in appendix D.

To get a more detailed view of how well the other model components perform, it is useful to consider the back-testing results of the balance sheet amounts. The main aggregates whose evolution is derived from the model are the monetary ones (**A1** and **L1**) and customer deposits (**L3**). As shown in figure 3, the other aggregates are essentially made up of maturing items (see **A2**, **L2**, **A3**, **A4**, and **L4**), whose volumes are directly driven by the scenario, or of non-maturing items that do not generate interest (**AX**, **LX**, and **E**).

[37] The NII is a net difference between two larger and much more dynamic quantities. As a result, any deviation in the projection of either quantity is amplified on a relative basis when applied to the NII: e.g. a 10 bn€ difference on the quarterly interest income represents a 5% relative deviation on that quantity (worth 210 bn€ as of June 2023), but becomes a 11% deviation on the NII (91 bn€).

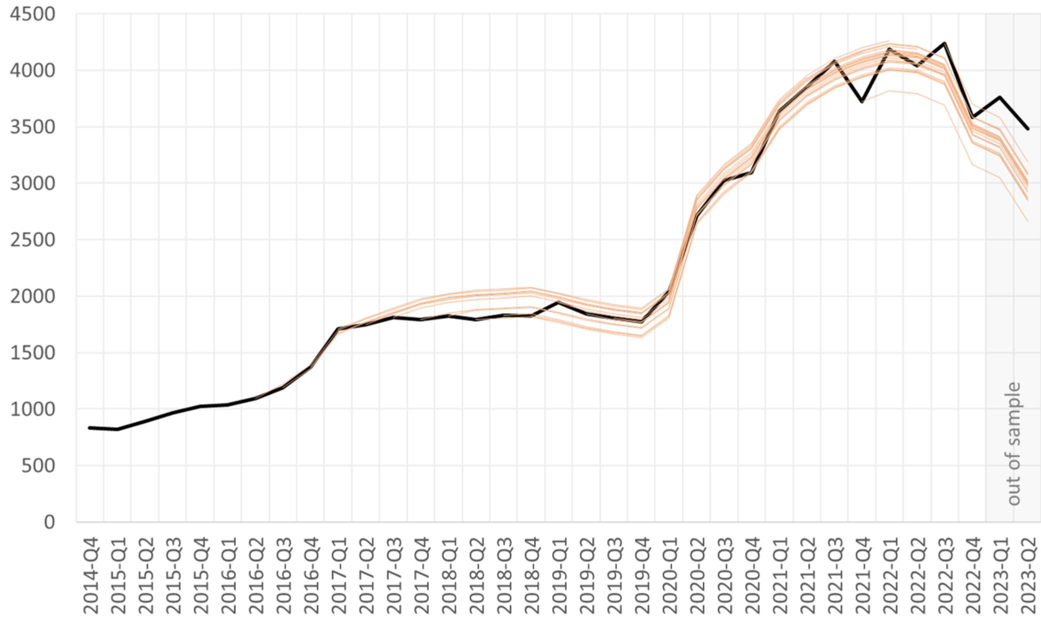


Figure 25: Aggregate central bank asset amounts (item **A1**) of the euro area banking sector: projected vs. realised (quarterly values, billion euros).

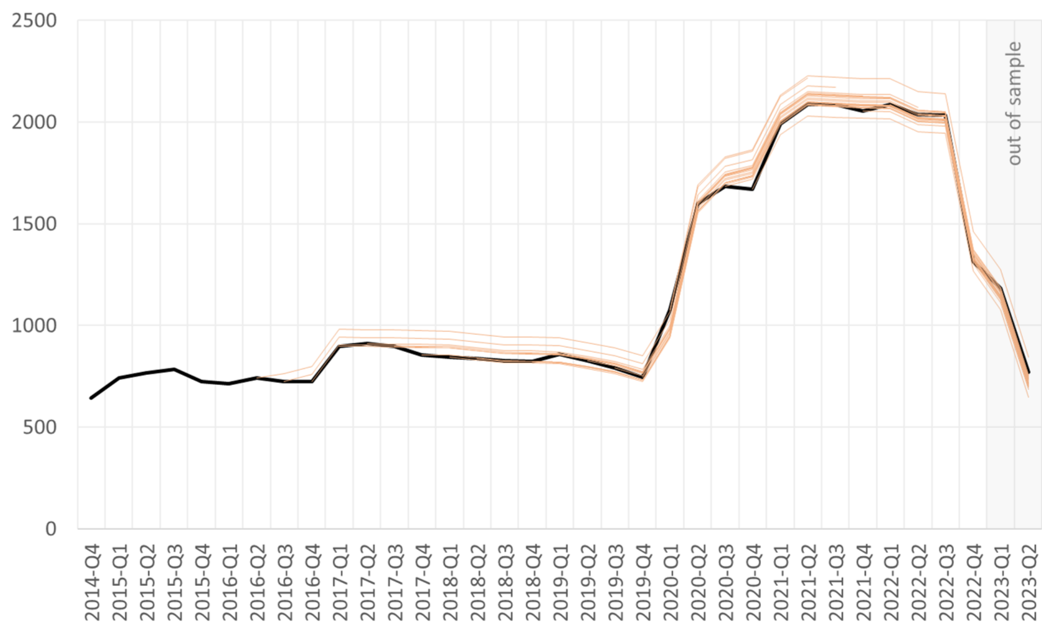


Figure 26: Aggregate central bank liability amounts (item **L1**) of the euro area banking sector: projected vs. realised (quarterly values, billion euros).

The back-testing results of the monetary items **A1** and **L1** (figures 25 and 26) quantify the performance of the specific central-bank-driven model described in section 4: it shows that the large changes in central banks amounts, in particular in the period 2020–21, are reproduced in the right proportions. On the asset side, the average model error is wider at the end of the back-testing period: this may be the result of a one-off event such as the move from negative to positive rates. Indeed, when rates are positive, deposits in central banks from e.g. governements no longer bear interest. Thus, there may be a punctual deviation from the model assumption of stable average behaviour of central bank depositors (other than banks), or even a slightly smoother effect over time due to recent decisions taken by the Eurosystem.[38] If it persists, such a behaviour could be integrated in the model, e.g. by setting non-zero values to the variable $\boldsymbol{T_t}$: originally designed to represent the net payment position of the reference central bank in the case where it does not cover the whole monetary zone, that variable can equivalently represent a deviation of other central bank depositors from their default modelled behaviour.

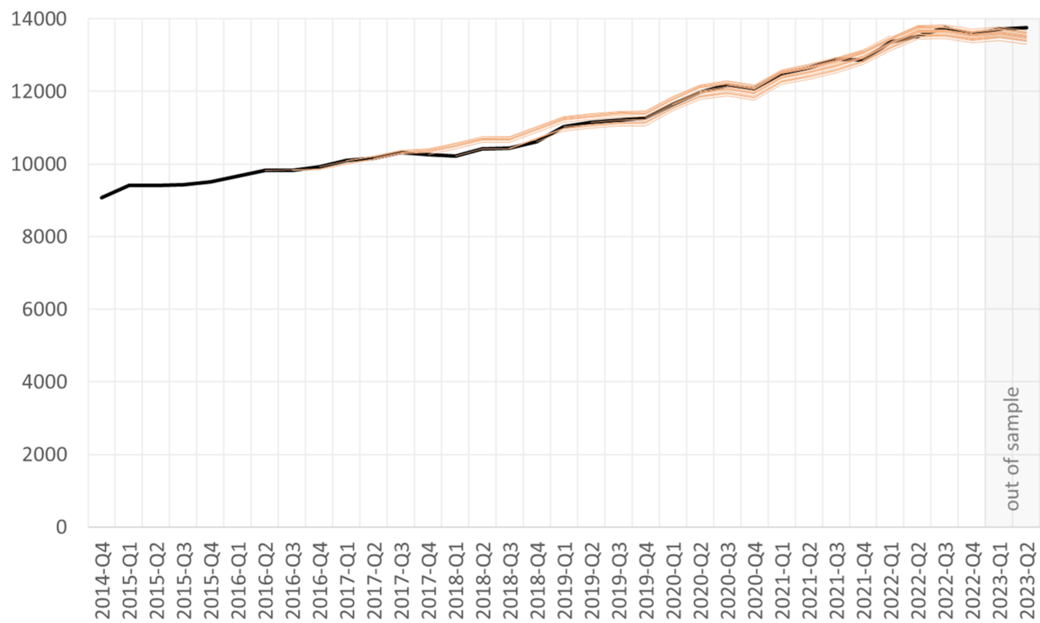


Figure 27: Aggregate customer deposit amounts (item **L3**) of the euro area banking sector: projected vs. realised (quarterly values, billion euros).

The volume of customer deposits **L3** (figure 27) is also projected with a good accuracy, which confirms the robustness of the model driving the evolution of non-maturing items as described in section 5, in particular the choice of accounting counterparties for the moves in maturing/monetary items (matrix M of figure 10) and the allocation of the income statement (see figure 11). Note that in the scenarios used here, ad-hoc flows between non-maturing items (cf. section 5.4) were left at their default zero value.

Overall, the back-testing results indicate a decent performance of the MAP, which is able to account both for the trajectories of banks' interest flows and for the evolution of their balance sheet structure over the past 7 years, including the first two quarters of 2023 which were not used for parameter estimation.

[38] See the decisions of **8 September 2022** and **7 February 2023**.

7 Conclusion

In this paper, we have presented an asset-liability approach for projecting the evolution of the balance sheet and the net interest income of an aggregated banking sector under various financial and macro-economic scenarios. The model, called MAP, is essentially based on financial and accounting principles and is highly scenario-driven: it provides a wide freedom to choose the design of the input scenario illustrating consistent behavioural assumptions on interest rates and lending volumes.

Using detailed financial data for the euro area banking sector in 2022, the model's projections in several scenarios over 2023–2027 evidence in particular a clearly positive sensitivity of aggregate NII to interest rate levels. However, changes in the structure of the balance sheet, such as shifts in deposit costs, may have significant adverse impacts. Such information on the main source of profitability of the banking sector, though abstracted from individual vulnerabilities, is significant for financial stability. As such, the MAP's ability to provide a long-term view in different scenarios should be of interest for policy makers such as central banks, governments and supervisory authorities.

Overall, the back-testing results indicate a robust performance of the model, for both the trajectories of banks' interests and the evolution of their balance sheet structure over the period 2016–23.

Many challenges remain and our future work will aim to refine the definition and the calibration of model parameters. At each quarter, as banks publish their financial data, the back-testing tool allows us to monitor the performance of the model on an ongoing basis by comparing the model output with actual NII numbers, highlighting deviations from model assumptions if any. In terms of extensions, the model can be transposed to other banking sectors than the euro area by adjusting the parameters and adapting the assumptions to respond to the local operational frameworks of monetary policy. An even more challenging future path of development is the extension of the model to other sectors of the financial system, in order to model all bilateral interest flows between banks, central banks, insurance companies, investments firms, governments etc., in order to map the financial interactions between the major groups of stakeholders.

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A Derivation of individual interest rates within a scenario

The model needs a large number of interest rates to be specified at each time step, as each modelled item has, in theory, its own interest rates. In order to build consistent scenarios, all of the scenario rates (r_t^S, r^L, t, $\hat{r}_t$, monetary rates and deposit rates) are derived from only four benchmark rates, or more precisely their respective average values over each time step $t-1 \to t$, namely:

- e_t is the average value of €STR, the short-term euro money market rate published daily by the ECB,
- y_t is the average value of the 10-year par yield of euro AAA-rated bonds published daily by the ECB,
- f_t is the average value of SOFR, the short-term money market rate for US dollars published daily by the Federal Reserve Bank of New York,
- i_t is the average value of the French year-on-year inflation rate published monthly by the national statistic bureau (INSEE).

Then first two interest rates represent the yield curve for euro products, i.e. about 80% of the eurozone banks' aggregate balance sheet. SOFR is introduced to take into account foreign-currency positions, which are predominantly in USD; USD curve slope effects are neglected and a single rate is used for all maturities. The French inflation rate is used only for regulated deposit rates, dominated by France's *Livret A*, whose interest rate is partly indexed on inflation.

From a scenario on the future evolution of these four quantities, the model then infers the evolution of all other interest rates in the following way.

A.1 Euro yield curve

A euro yield curve is first built using the exponential interpolation model, i.e. the yield for maturity T is interpolated between a short-term rate and a long-term rate using:

$$r_t(T) = r_t^S e^{-\frac{T}{\xi}} + r_t^L \left(1 - e^{-\frac{T}{\xi}}\right).$$

The curve shape parameter ξ is determined using recent interest rate curves provided by the ECB — $\xi = 2.4$ years provides a reasonable fit across maturities as at end-2022. Then the values of r_t^S and r_t^L are determined from the values of €STR ($T = 0$) and of the 10-year par yield ($T = 10$ years), i.e.:

$$r_t(0) = e_t, \quad r_t(10 \text{ years}) = y_t,$$

so that:

$$r_t^S = e_t, \quad r_t^L = \frac{y_t - e_t e^{-\frac{10}{\xi}}}{1 - e^{-\frac{10}{\xi}}}. \tag{25}$$

Long and short interest rates of new products Each exponential account i of maturing items (loans, securities, term deposits) has its own value for the interest rates of newly issued products, with a short-term value $\boldsymbol{r^S_{i,t}}$ and a long-term value $\boldsymbol{r^L_{i,t}}$ (cf. section 3.3). These are derived from the euro and USD rates in the following way:

$$\boldsymbol{r^S_{i,t}} = \kappa_i \left[(1-\sigma_i) r^S_t + \sigma_i \boldsymbol{f_t}\right] + s^S_i, \tag{26}$$
$$\boldsymbol{r^L_{i,t}} = \kappa_i \left[(1-\sigma_i)\left((1-\alpha_i) r^L_t + \alpha_i r^S_t\right) + \sigma_i \boldsymbol{f_t}\right] + s^L_i. \tag{27}$$

σ_i is the share of US dollar products in the exponential account, which are driven by SOFR, the complement being driven by euro rates.

κ_i is the transmission coefficient of the exponential account (between 0 and 1). Indeed, it can be observed that banks do not transmit the variations of market interest rates one for one, in particular on customer loans: when lower than one, κ_i represents the fraction of market interest rate variations that is effectively transmitted to new products. Though κ_i applies identically to $\boldsymbol{r^S_t}$ and $\boldsymbol{r^L_t}$ for simplicity, some structure can be introduced by using different values of κ_i for exponential accounts with different average maturities.

s^S_i and s^L_i are spreads applied to short- and long-term rates respectively.

α_i is the proportion of floating-rate products in the exponential account. Equation (27) accounts for the expectation that the initial interest rate on new long-term floating-rate products is driven, not by long-term market rates (which are fixed rates), but by short-term ones, that will then be periodically reset.

All these are constant model parameters determined for each exponential account.

A.2 Reference rates for floating-rate products

Each exponential account i also needs a reference rate $\boldsymbol{\hat{r}_{i,t}}$ determining the evolution of floating rates. This is simply computed as:

$$\boldsymbol{\hat{r}_{i,t}} = (1-\sigma_i)\boldsymbol{e_t} + \sigma_i \boldsymbol{f_t}, \tag{28}$$

i.e. the reference rate for the share σ_i of US-denominated products is SOFR, while €STR is used for the remainder.

A.3 Central bank interest rates

Depending on monetary policy rules, each type of central bank asset or liability (monetary items) may have its own interest rate, cf. section 4.2. The central bank rules on interest rates being public, it is in theory possible to account in detail for each of them, at least for euro holdings. The model builds interest rates for each class of euro monetary item from the scenario value of €STR. All US dollar amounts are considered to yield the a single interest rate, i.e. SOFR.

Specifically, the model computes by default the following central bank interest rates for euro:

- Deposit facility rate (DFR): $r_t^F = e_t + 0.1\%$,
- Main refinancing operation rates (MRO): $r_t^M = \max(0, e_t + 0.6\%)$,
- Long-term refinancing operation rates (TLTRO-III): $r_t^T = r_t^F$,
- Interest rate on minimum reserves: $r_t^I = h_t^F \cdot r_t^F + h_t^M \cdot r_t^M$.

The DFR formula follows from the observation that are derived from the fact that, in the current situation of excess liquidity, the deposit facility rate is the marginal cost of liquidity for banks, and therefore drives the money market rate €STR, which however tends to be slightly inferior as non-bank lenders do not have access to the deposit facility. The MRO formula leverages the observed relationship between DFR and MRO rates since 2019.
Each of these computed rates can be overridden manually, which allows users to test for scenarios where the ECB may alter in the future the relationships between its interest rates.[39]

The coefficients in the r_t^I equation reflect the ECB remuneration policy for minimum reserves: set at $(h^F, h^M) = (1, 0)$ for the first two quarters of 2023, when the DFR remuneration applied, they then shift to $(h^F, h^M) = (0, 0)$ to reflect the decision to apply a 0% interest rate from 20 September 2023 onwards — the third quarter of 2023 gets *pro rata temporis* weights, i.e. $(h^F, h^M) = (0.88, 0)$.

The US dollar amounts, represented by a proportion σ_{A1} (resp. σ_{L1}) for monetary assets (resp. liabilities), are all deemed to pay the SOFR rate f_t, and all other currencies are neglected. Overall, the interest rate applied to the different monetary items within **A1** and **L1** are represented in table 2.

Monetary item		**Interest rate**
$A1^B$	Banknotes	0
$A1^O$	Minimum reserves	$(1 - \sigma_{A1})r_t^I + \sigma_{A1} f_t$
$A1^E$	Excess reserves and deposits	$(1 - \sigma_{A1})r_t^{F\,*} + \sigma_{A1} f_t$
$L1^S$	Short-term financing	$(1 - \sigma_{L1})r_t^M + \sigma_{L1} f_t$
$L1^L$	Long-term financing	$(1 - \sigma_{A1})r_t^T + \sigma_{L1} f_t$

Table 2: Interest rates applied to monetary items.

*Notes : * The model can account for tiering on excess reserve interests, such as was implemented by the ECB during the negative rates period. This feature, inactive since the summer of 2022, is not shown here for simplicity.*

A.4 Deposit interest rates

For each non-monetary item, the scenario should specify a domestic market short-term interest rate r_t^M, a foreign-currency interest rate r_t^C and a regulated short-term interest rate r_t^R, cf. section 5.2. In the application to the eurozone, the following choices were made:

- $r_t^M = e_t$, the €STR rate, driving the interests on unregulated euro deposits;

[39] This has happened for instance in late 2022, when the ECB modified the rules determining the interest rates r_t^T on long-term operations, which now follow the deposit facility rate.

- $r_t^C = f_t$, the SOFR rate, driving the interests on US dollar deposits, other currencies being negligible;
- r_t^R is the *Livret A* interest rate, set every six months by the French government, which is the main interest rates on regulated euro deposits given the weight of French household deposits in the euro area. By default, the model projects this rate by applying the legal formula, which defines it as the mean value between the 6-month average values of €STR and of the French inflation rate, floored at 0.5%,[40] i.e., given the quarterly time step:

$$r_t^R = \max\left[\frac{1}{4}\left(e_{t-2} + e_{t-1} + i_{t-2} + i_{t-1}\right),\ 0.5\%\right].$$

  This formula applies for the second and fourth quarter of each year. As interest rate changes apply on 1 February and 1 August each year, a $(1/3, 2/3)$-weighted average between the old and new values of r_t^R is computed for the first and third quarter.

  As for central bank rates, this computed rate can be overridden manually and replaced by explicit values in some scenarios, to account for possible deviations from the formula.[41]

The weight α^R for the regulated rate is only non-zero for customer deposits **L3**: the interests on interbank deposits **A2** and **L2** are only driven by €STR with a SOFR contribution, whereas all other non-maturing items have zero overall interest (zero weights on all three rates). Cf. table 6 (appendix B.4) for the eurozone values.

[40] **Arrêté du 27 janvier 2021** *relatif aux taux d'intérêt des produits d'épargne réglementée*, art. 1, I. 1°.

[41] *Ibid.*, art. 1, II, 2°. The French Government has used this possibility several times since 2022, notably in July 2023, when the rate has been frozen at 3% till January 2025, in a context of high rates and high inflation, to avoid imposing excessively off-market costs on the French social housing sector funded by *Livret A*, cf. **Arrêté du 28 juillet 2023** *relatif aux taux d'intérêt des produits d'épargne réglementée*.

B Initial state variables and estimation of parameters

B.1 Initial balance sheet

Table 3 illustrates the numbers corresponding to the breakdown by item and by the nature of their components, as illustrated in figure 3 from section 2. This breakdown is computed on the basis on the banks' granular financial reports collected by supervisors.

Assets

	Monetary	Maturing	Non-mat.	Total
A1	3,586			**3,586**
A2		968	200	**1,168**
A3		13,517		**13,517**
A4		2,649		**2,649**
AX			4,438	**4,438**
Total	**3,586**	**17,134**	**4,638**	**25,358**

Liabilities & Equity

	Monetary	Maturing	Non-mat.	Total
L1	1,314			**1,314**
L2		894	416	**1,311**
L3		2,881	10,700	**13,581**
L4		3,508		**3,508**
LX			4,031	**4,031**
E			1,614	**1,614**
Total	**1,314**	**7,283**	**16,762**	**25,358**

Table 3: Aggregate balance sheet of the eurozone banking sector as of Dec. 2022.

Notes: End of year data in billion euros. A1 correspond to banknotes and deposits in central banks, L1 to central bank refinancing. A2 and L2 are money market interbank loans and currency deposit accounts. A3 and L3 are loans and deposits of customers, i.e. public administrations, non-bank corporations (both financial and non-financial) and households. A4 (resp. L4) corresponds to debt securities held (resp. issued) by the banking sector. Other assets AX and liabilities LX include derivatives, non-debt securities, real estate, tax assets and liabilities etc.

On assets side, loans and advances to credit institutions are included in **A2** maturing items while balances receivable on demand with credit institutions are classified in **A2** non-maturing deposits. For liabilities **L2** and **L3**, the deposits with agreed maturity and repurchase agreements are classified as maturing items, while the deposits redeemable at notice, current accounts and overnight deposits are classified as non-maturing.

The total amounts by item **A1**–**AX**, **L1**–**LX** and **E** match the balance sheet presented in figure 13 for the end of 2022.

B.2 Model parameters for maturing items

For the purpose of the MAP, the maturing items in banks' aggregate balance sheet are split into a number of "exponential accounts", cf. section 3. In practice, the aggregate maturity profiles of the main categories of maturing assets or liabilities of the euro area banks, in particular loans and securities, can be approximated to a good degree of accuracy by a combination of two such exponential accounts, one with shorter maturity (< 1 year), the "short exponential account", and the other with longer maturity (> 1 year), the "long exponential account". For instance, the MAP models customer loans (asset **A3**) as the sum of two exponential accounts called "short customer loans" (**A3S**) and "long customer loans" (**A3L**), with two different average values τ_i. Each of these components is modelled separately, and their outputs are summed to provide the model results for customer loans as a whole. As an exception, interbank assets and liabilities (**A2**, **L2**) are modelled each as a single exponential account given their comparatively small sizes.

Figure 28 shows, for the two largest maturing items on the aggregate balance sheet (customer loans **A3** and issued securities **L4**), the comparison between the sum of the two exponential accounts and the observed amortisation profile deduced from aggregate liquidity reports. Table 4 provides the full set of model parameters for each exponential account.

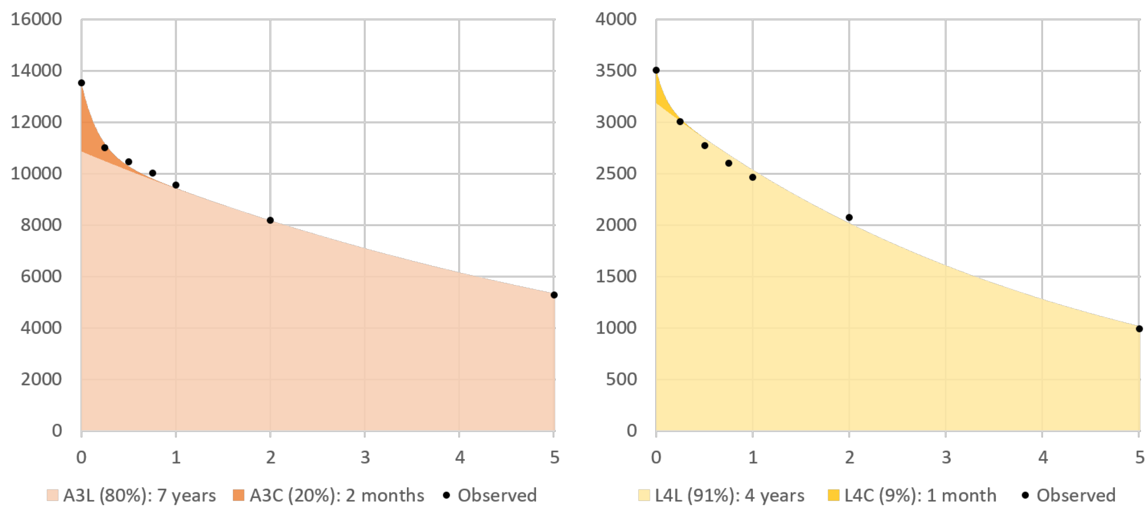

Figure 28: Calibration of maturity profiles by exponential accounts for loans (left) and issued securities (right) as of end-2022.

Notes: (x) Residual maturity in years, (y) Residual outstanding amounts in billion euros. The starting point (0) shows the outstanding amount in each item as of end of Q4-2022; the ending point (5 years) is the part that will still be due at the end of Q4-2027. In both cases, the "long" exponential account provides a good fit for the amortisation profile after one year, while the "short" one captures the special behaviour on the short term.

		A2	**A3S**	**A3L**	**A4S**	**A4L**	**L2**	**L3S**	**L3L**	**L4S**	**L4L**
	Weight (short/long)	100%	19.7%	80.3%	3.1%	96.9%	100%	78.9%	21.1%	9.2%	90.8%
τ	Maturity (years)	0.9	0.2	7.0	0.1	6.0	0.9	0.2	4.2	0.1	4.4
ξ	Shape (years)	2.4									
α	Variable rates	81.6%	36.9%	36.9%	6.2%	6.2%	72.4%	45.3%	45.3%	21.1%	21.1%
σ	USD	11.4%	28.4%	5.0%	13.4%	16.0%	6.9%	17.3%	3.6%	67.5%	16.6%
κ	Transmission coeff.	100%	76.2%	99.4%	100%		100%	87.9%	84.8%	100%	
s^S	Short-term spread	1.0%	2.5%	2.3%	2.0%	2.0%	0.9%	1.0%	0.8%	0.4%	1.0%
s^L	Long-term spread	1.0%	1.5%	2.1%	1.5%	1.5%	0.9%	–1.4%	0.9%	1.3%	1.3%
R_0^S	Short-term rate	2.2%	4.2%	2.9%	3.5%	1.9%	1.9%	2.4%	2.2%	3.2%	1.6%
R_0^L	Long-term rate	2.4%	3.7%	3.0%	3.8%	2.2%	2.1%	0.5%	2.5%	4.3%	2.1%

Table 4: Model parameters for the maturing items of the euro area banks, calibrated as of end-2022.

These parameters are obtained using a wide variety of data and estimation methods. In an nutshell:

- The weights, average maturities (τ) and dollar proportion (σ) of each exponential account are determined by fitting the aggregate maturity schedule by currency reported by eurozone banks in the context of the liquidity risk part of the common supervisory reporting process (COREP), using error minimisation techniques;
- The shape parameter (ξ), which drives the transition between short- and long-term interest rates, is the same for all items; it is chosen so as to best account for the shapes of euro interest rate curves observed in 2022;
- The proportion of variable rates (α) is estimated by crossing financial information from eurozone bank statistics (BSI) and granular data (AnaCredit, CSDB);
- The transmission coefficient (κ) is assumed to be 100% for interbank loans and securities; its value for customer loans and term deposits is inferred from the historical behaviour of the average interest rates of new production available in eurozone bank statistics (MIR);
- The long/short structure of spreads s is derived from financial information (MIR statistics and/or granular data). As the perimeter of these bases usually differs from the modelled perimeter, an overall spread shift is then applied on each item based on the observed historical behaviour of the corresponding aggregate interests in supervisory financial reports. The initial interest rate structure as of end-2022 is then computed by retro-active application of the model.

B.3 Model parameters for monetary items

The parameters used to project the central bank amounts on the aggregate balance sheet of euro area banks, as detailed in section 4, are calibrated to the following values:

p	Perimeter mismatch	85.4%
β	Banknote sensitivity to deposits	11.4%
γ	Proportion of deposits leaking outside banks	18.3%
δ	Deposit scaling factor for reserves requirements	102.1%
ϵ	Proportion of banknotes held by banks	8.0%
$\underline{L}^S$	Min. bank short-term financing	1 bn€

Table 5: Model parameters for the monetary items of the euro area banks, calibrated as of end-2022.

The five coefficients are estimated by historical regression between quarterly central bank balance sheet amounts published by the ECB, and eurozone bank balance sheet amounts from supervisory financial reports on the period 2015–2022. All regressions are of good quality ($R^2 > 98\%$) except for the estimation of ϵ ($R^2 = 48\%$), as banknotes holding by banks is highly seasonal — but the relation remains robust on average. Figure 29 shows the quality of this regression for the coefficient γ, which has the largest impact on projected NII: a constant coefficient correctly accounts for variations in the combined volume of banknotes and bank deposits in central banks.

The technical parameter $\underline{L}^S$, which determines the minimal volume of banks' short-term central bank loans, is chosen arbitrarily small.

In addition, the USD weights in monetary items (cf. table 2) are estimated, using banks' liquidity risk reports, at:

$$\sigma_{A1} = \sigma_{L1} = 10\%.$$

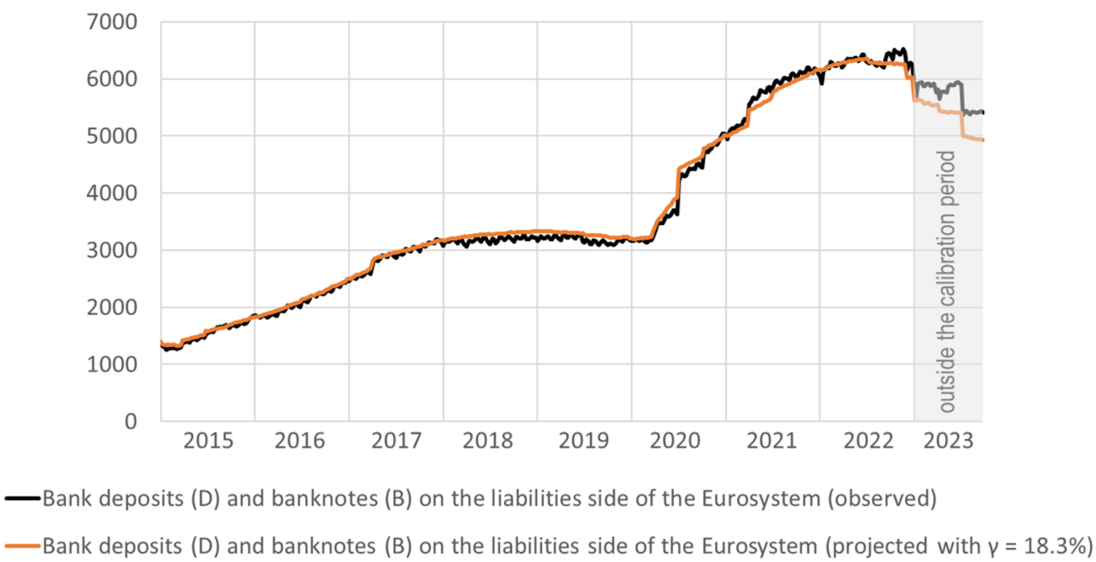

Figure 29: Bank deposits (D) and banknotes (B): observed values and projected values with γ as calibrated at the end of 2022

Notes: weekly data in billion euros. The black curve represents the evolution of the aggregate amount of banknotes (B_t) and bank deposits (D_t) on the liabilities side of the Eurosystem balance sheet. The orange curve represents the same quantity, as projected with $\gamma = 18.3\%$ and using the equation $\Delta D_t + \Delta B_t = (1-\gamma) \cdot (\Delta L_t + \Delta S_t + \Delta T_t)$ with the actual observed values of L_t, S_t, and T_t. A constant γ provides a good fit until the end of 2022; the 2023 deviation is commented in section 6.5 within the back-testing of A1.

B.4 Model parameters for non-maturing items

The parameters used to determine the interest rates of non-maturing deposits (cf. section 5.2) are as follows:

		A2	**L2**	**L3**
α^M	€STR weight	20.8%	35.1%	40.4%
α^C	SOFR weight	79.2%	64.9%	27.6%
α^R	*Livret A* weight	0	0	32.1%
s	Spread	1.0%	0.9%	0.7%
η_0	Remuneration coeff.	100%	100%	28.7%

Table 6: Model parameters for the interests of non-maturing items of the euro area banks, calibrated as of end-2022. $\alpha^M = 1 - \alpha^C - \alpha^R$. The parameters are set to zero for other assets and liabilities (**AX**, **PX**) and for equity (**E**).

The SOFR weight α^C is derived from the currency information available in the banks' liquidity risk reports, in the same way as the κ parameter of maturing items.[42] For non-maturing interbank deposits **A2** and **L2**, the spreads are chosen equal to their maturing counterparts; the remuneration coefficient is assumed to be 100%. For non-maturing customer deposits **L3**, the *Livret A* weight α^R, spread s and initial remuneration coefficient η_0 are estimated from the historical behaviour of the corresponding interests in supervisory financial reports.[43]

Last, the net bank costs outside interests and defaults (cf. section 5.3) are driven by the following coefficient:

q	Other net costs (% of balance sheet size, per quarter)	0.5%

Table 7: Model parameter for the non-interest non-default net costs of the euro area banks, calibrated as of end-2022.

This is estimated from historical values of non-interest non-default net costs as reported in financial reports.

[42]The large weight of USD in interbank deposits reflects the fact that most of the interbank loans in domestic currency are term loans. Interbank sight deposits are mostly used for currency holding purposes, e.g. in the context of correspondent banking.

[43]In spite of its purely national nature, the French *Livret A* interest rate has a noticeable effect on the average cost of eurozone bank deposits. Table 6 implies that the sensitivity of eurozone bank deposit interests to the *Livret A* rate at the end of 2022 was equivalent to $\alpha^R \times \eta_0 \simeq 9.3\%$ of the non-maturing part of **L3**, i.e. around 1,000 billion euros. The actual amounts of regulated deposits then held by French banks were close to 500 billions at year end, but large volumes of non-regulated deposits are heavily influenced by the *Livret A* rate, which effectively acts as an unofficial benchmark for retail saving interest rates in France, hence the higher figure used for modelling purposes.

C Derivation of model equations

C.1 Equations for the exponential model (section 3.3)

As explained in section 3, the maturing balance sheet items in the MAP are modelled as combinations of "exponential accounts", each of which is characterised by:

- an outstanding amount N_t,
- an exponentially decaying maturity structure with average maturity τ (cf. figure 4),
- an exponentially interpolated average interest rate structure with short-term rate R_t^S, long-term rate R_t^L and shape parameter ξ (cf. figure 5).

Such an ideal structure can be thought of as a continuum of products spanning all residual maturities $T > 0$, where the products in each infinitesimal maturity interval $[T, T + \mathrm{d}T]$ are characterised by:

- an infinitesimal outstanding amount $N_t e^{-T/\tau} \frac{\mathrm{d}T}{\tau}$,
- an average interest rate $R_t^S e^{-T/\xi} + R_t^L \left(1 - e^{-T/\xi}\right)$.

Derivation of average interest rates — equation (2)

The above decomposition allows us to compute the average interest rate of the exponential account as a weighted average over all maturities, as follows:

$$
\begin{aligned}
R_t &= \frac{\int_0^\infty N_t e^{-T/\tau} \frac{\mathrm{d}T}{\tau} \times \left[R_t^S e^{-T/\xi} + R_t^L \left(1 - e^{-T/\xi}\right)\right]}{\int_0^\infty N_t e^{-T/\tau} \frac{\mathrm{d}T}{\tau}} \\
&= \left(R_t^S - R_t^L\right) \int_0^\infty \frac{\mathrm{d}T}{\tau} e^{-T/\tau} \times e^{-T/\xi} + R_t^L \int_0^\infty \frac{\mathrm{d}T}{\tau} e^{-T/\tau} \\
&= \left(R_t^S - R_t^L\right) \frac{1}{\tau\left(\frac{1}{\tau} + \frac{1}{\xi}\right)} + R_t^L \\
&= \left(R_t^S - R_t^L\right) \frac{\xi}{\xi + \tau} + R_t^L \\
&= \frac{\xi R_t^S + \tau R_t^L}{\xi + \tau}.
\end{aligned}
$$

This result, stated in equation (2), is used to compute interests with equation (8).

Derivation of the ageing effect — equations (4) and (5)

At the end of a simulation period ($t-1 \to t$) of length θ, the products whose residual maturities at the beginning of the period were shorter than θ have expired, while the other products have their residual maturities shortened by θ. Before taking into account the effects of variable rates and new products, the average interest rate of each surviving product is unchanged. As a result, the financial structure of the aged "exponential account" is, again, a continuum of products spanning all residual maturities $T > 0$, where the products in each infinitesimal maturity interval $[T, T + \mathrm{d}T]$ are those that were in the interval $[T + \theta, T + \theta + \mathrm{d}T]$ at time $t - 1$, i.e.:

- infinitesimal outstanding amount $N_{t-1} e^{-(T+\theta)/\tau} \frac{\mathrm{d}T}{\tau}$,
- average interest rate $R^S_{t-1} e^{-(T+\theta)/\xi} + R^L_{t-1} \left(1 - e^{-(T+\theta)/\xi}\right)$.

Now these two quantities can be rewritten as:

$$N_{t-1} e^{-(T+\theta)/\tau} \frac{\mathrm{d}T}{\tau} = \left[N_{t-1} e^{-\theta/\tau}\right] e^{-T/\tau} \frac{\mathrm{d}T}{\tau},$$

and

$$\begin{aligned} R^S_{t-1} e^{-(T+\theta)/\xi} \quad &+ \quad R^L_{t-1} \left(1 - e^{-(T+\theta)/\xi}\right) \\ = \quad & R^S_{t-1} e^{-\theta/\xi} e^{-T/\xi} + R^L_{t-1} \left(1 - e^{-T/\xi} + e^{-T/\xi} - e^{-(T+\theta)/\xi}\right) \\ = \quad & \left[R^S_{t-1} e^{-\theta/\xi} + R^L_{t-1} \left(1 - e^{-\theta/\xi}\right)\right] e^{-T/\xi} + R^L_{t-1} \left(1 - e^{-T/\xi}\right). \end{aligned}$$

Otherwise said, the aged structure is exactly identical to that of an exponential account with total outstanding amount $N_{t-1} e^{-\theta/\tau}$, short-term interest rate $R^S_{t-1} e^{-\theta/\xi} + R^L_{t-1} \left(1 - e^{-\theta/\xi}\right)$ and long-term interest rate R^L_{t-1}, using unchanged parameters τ and ξ.

As the interest rate structure is left unchanged by the early repayments and defaults, which are assumed to affect all maturities in the same proportion, this justifies the use of the expression R^L_{t-1} on the right-hand side of equation (4) and of the expression $R^S_{t-1} e^{-\theta/\xi} + R^L_{t-1} \left(1 - e^{-\theta/\xi}\right)$ on the right-hand side of equation (5).

C.2 Equations for the monetary items (section 4)

Evolution of banknotes ($A1^B_t$) — equation (9)

On the banks' balance sheet, $A1^B_t$ is the amount of banknotes on the asset side. Under the assumptions set out in section 4, its evolution is proportional to the evolution of the amount of banknotes B_t on the balance sheet of the reference central bank, with the scaling factor p to take into account perimeter mismatches, and the scaling factor ϵ reflecting the fact that only a small part of the banknotes is held by the banks:

$$\Delta A1^B_t = \epsilon \cdot p \cdot \Delta B_t.$$

A further assumption is that B_t changes proportionally to changes in the amount of customer deposits $L3_{t-1}$, with a scaling factor β, combined with the inverse of p to account again for perimeter mismatches:

$$\Delta B_t = \frac{\beta}{p} \cdot \Delta L3_{t-1}.$$

Combining both equations yields the value of $\Delta A1^B_t$ as written in equation (9):

$$\Delta A1^B_t = \epsilon \cdot \beta \cdot \Delta L3_{t-1}.$$

Evolution of excess reserves ($\boldsymbol{A1_t^E}$) — equation (11)

The evolution of excess reserves ($A1_t^E$) can be written as the evolution of the total reserves minus the evolution of required reserves:

$$\Delta A1_t^E = (\Delta A1_t^M + \Delta A1_t^E) - \Delta A1_t^M.$$

Considering in addition that the total reserves on banks' balance sheet evolves like the reserves on the balance sheet of the reference central bank (except that perimeter mismatches must be taken into account with the parameter p), the equation becomes:

$$\Delta A1_t^E = p{\cdot}\Delta D_t - \Delta A1_t^M.$$

Now the expression of ΔD_t is $(1-\gamma){\cdot}(\Delta \boldsymbol{L_t^L} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t} + \Delta L_t^S) - \Delta B_t$ as shown in section 4.1.2, so that:

$$\Delta A1_t^E = p{\cdot}[(1-\gamma){\cdot}(\Delta \boldsymbol{L_t^L} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t} + \Delta L_t^S) - \Delta B_t] - \Delta A1_t^M.$$

Substituting for ΔB_t its expression $\frac{\beta}{p}{\cdot}\Delta L3_{t-1}$ leads to equation (11):

$$\Delta A1_t^E = p{\cdot}(1-\gamma){\cdot}(\Delta \boldsymbol{L_t^L} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t} + \Delta L_t^S) - \beta{\cdot}\Delta L3_{t-1} - \Delta A1_t^M.$$

Evolution of the boundary value of short-term refinancing giving a zero amount of excess reserves ($\boldsymbol{\tilde{L}_t^S}$) — equation (15)

On the balance sheet of the reference central bank, $\tilde{L}_t^S$ is the boundary value of short-term refinancing giving a zero amount of excess reserves, i.e the value of L_t^S such that $D_t^E = 0$.

The asset-liability equilibrium of the reference central bank's balance sheet gives the following equation (see section 4.1.2):

$$(1-\gamma){\cdot}(\Delta \boldsymbol{L_t^L} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t} + \Delta L_t^S) = \Delta D_t^E + \Delta D_t^M + \Delta B_t,$$

which gives the evolution of L_t^S:

$$\Delta L_t^S = \frac{\Delta D_t^E + \Delta D_t^M + \Delta B_t}{1-\gamma} - (\Delta \boldsymbol{L_t^L} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t}).$$

This being valid for all previous time steps $i \in [1;t]$, it can be asserted that:

$$\sum_{i=1}^{t} \Delta L_i^S = \sum_{i=1}^{t} \left[\frac{\Delta D_i^E + \Delta D_i^M + \Delta B_i}{1-\gamma} - (\Delta \boldsymbol{L_i^L} + \Delta \boldsymbol{S_i} + \Delta \boldsymbol{T_i}) \right],$$

which sums to:

$$L_t^S - L_0^S = \frac{(D_t^E - D_0^E) + (D_t^M - D_0^M) + (B_t - B_0)}{1-\gamma} - (\boldsymbol{L_t^L} - L_0^L + \boldsymbol{S_t} - S_0 + \boldsymbol{T_t} - T_0).$$

So the value of L_t^S such that $D_t^E = 0$ is:

$$\tilde{L}_t^S = \left(L_0^S - \frac{D_0^E}{1-\gamma} \right) + \frac{(D_t^M - D_0^M) + (B_t - B_0)}{1-\gamma} - (\boldsymbol{L_t^L} - L_0^L + \boldsymbol{S_t} - S_0 + \boldsymbol{T_t} - T_0). \quad (*)$$

It is useful to compute $\tilde{L}^S_t - \tilde{L}^S_{t-1}$, which yields the simpler expression:

$$\tilde{L}^S_t = \tilde{L}^S_{t-1} + \frac{\Delta D^M_t + \Delta B_t}{1-\gamma} - (\Delta \boldsymbol{L^L_t} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t}). \qquad (**)$$

Finally, the value of $\tilde{L}^S_0$ can be deduced from $(*)$ in the case $t = 0$:

$$\tilde{L}^S_0 = L^S_0 - \frac{D^E_0}{1-\gamma}.$$

This determines the initial boundary value of short-term financing by comparing the actual initial amounts of short-term financing with the initial level of excess reserves.

From $(**)$, the evolution of $\tilde{L}^S_t$ can be deduced by exploiting the links between the balance sheet of the banks and that of the reference central bank ($\Delta D^M_t = \frac{\Delta A1^M_t}{p}$ and $\Delta B_t = \frac{\beta . \Delta L3_{t-1}}{p}$), as established in section 4. This leads to equation (15):

$$\tilde{L}^S_t = \tilde{L}^S_{t-1} + \left(\beta \cdot \Delta L3_{t-1} + \Delta A1^M_t\right) \times \frac{1}{p \cdot (1-\gamma)} - \left(\Delta \boldsymbol{L^L_t} + \Delta \boldsymbol{S_t} + \Delta \boldsymbol{T_t}\right).$$

Thus, the need for short-term financing increases with $L3$ (inasmuch as additional deposits are assumed to result in increase customer demand for banknotes) and with $A1^M_t$ (higher reserve requirements). Both these effects are affected by factors reflecting the perimeter mismatch between the central bank and the banking sector (p) and the partial absorption of any additional funds borrowed by banks into "other" liabilities ($1 - \gamma$).

Conversely, the need for short-term financing decreases on a one-for-one basis with increases in $\boldsymbol{L^L_t}$ (long-term financing), $\boldsymbol{S_t}$ (central bank security purchases) and $\boldsymbol{T_t}$ (inflows from the rest of the monetary zone), as these represent direct payments into the banks' central bank accounts.

D Note on the "par-amortised-cost" assumption

As discussed in section 3, the exponential model for maturing products relies on the assumption that changes in outstanding amounts stem only from repayments, new issuances and defaults. In reality, additional changes in amounts can be triggered by accounting conventions outside portfolio variations, in the following cases:

- **Amortisation of an issuance premium or discount.** When a debt security is purchased at e.g. 90 whereas the principal paid at maturity is 100, the amortised cost accounting method results in a progressive increase of the balance sheet amount from 90 to 100 (accounting debit); the corresponding credit is an interest income.
- **Indexed securities.** Under the amortised cost method, an inflation-linked bond's balance sheet value increases (debit) with the underlying price index, the corresponding credit being again the interest part of the income statement.

- **Fair-value accounting** involves remarking the financial instruments at their market price at each new financial statement, leading to increases or decreases of outstanding amounts when the market price moves up or down. The accounting counterpart of these changes is either the fair value change account of the income statement, or directly equity through other comprehensive income. In particular, the market value of debt securities generally declines when interest rates go up.

In all three situations, although the balance sheet change can be represented by a scenario change in outstanding amounts, the par-amortised-cost assumption of the exponential model leads to mis-classifying the resulting increase (resp. decrease) as a purchase (resp. sale) of new products, yielding an over- (resp. under-)estimation of the proportion of new products, and therefore of the evolution of the average interest rates in equations (4, 5).[44]

In the case of eurozone banks, such cases are likely to be concentrated in debt security assets (**A4**), as banks routinely buy securities on the secundary market at prices which may depart from par, and account for a significant part of them at fair value. The financial reports show that nearly half (49%) of the debt securities owned by euro area banks are registered at fair value as of end-2022, mostly through other comprehensive income. As some governments issue inflation-indexed bonds, it is also likely that a small proportion of **A4** is indexed.

By contrast, the par-amortised-cost assumption is probably much less of an issue for other items: absent a secundary market, the loans, term deposits and issued debt securities are likely to be mostly issued at par, and inflation-indexing is not a widespread practice. The share of fair value accounting is rather low (9% for issued debt securities); for loans and deposits it is concentrated on repurchase agreements, whose short maturities guarantee a limited price volatility.

Overall, the model performance on the interest income from debt securities **A4** is a good indicator of the overall modelling error linked to such accounting effects. The back-testing results shown in figure 24 is indeed overall less convincing than the rest of the interest income, showing that a model refinement on this point would be beneficial. However, the overall effect at balance sheet level appears to be limited. A more refined projection of these accounting effects would require additional data (aggregate premium/discount, structure of fair-value-accounted products, indexing information) and, ideally, a pricer to simulate fair value changes. Such an improvement is left for future developments.

[44] An early redemption, as modelled by the parameter $\boldsymbol{\rho}$, has the same effects than a premium amortisation, so that the discount-premium effects could be modelled by changing the meaning of this parameter; however, such a solution does not work for inflation indexing or fair value change effects, which create different interest rate dynamics.

E Accounting for transaction settlement

The model's respect of double-entry accounting principles requires some assumptions regarding the systematic pairing of debits and credits. Specifically, the model has to account for the way transactions are settled. The central question is therefore: how do banks pay, or get paid, for the various financial events generated by the other modelling assumptions (new loans, interest payments, security purchases, dividend distribution etc.)?

The financial intermediation service provided by the banking sector to the economy makes it possible to use simple assumptions for that purpose, that turn out to be quite robust.

E.1 Preliminary note: payments from and to the banking sector

A payment is a transfer of money. An ordinary economic agent pays by using up some of the money it owns, and transferring it so that it becomes money owned by the payee. The operation can range from a simple physical transfer of paper and coins (cash payments) to a series of accounting entries in the books of various intermediaries (wire payments), but the final outcome is always that the payer has less money while the payee has more.

The picture is a little different for banks, who actually deal with two different kinds of money, referred to below as "central" and "commercial" money:[45]

- **Central money** consists in deposits at the central bank. Also called "reserves" by reference to the minimum reserve requirements set by some central banks, it is an *asset* of the depositor banks, and is, properly speaking, the banks' own "money", the central bank playing the role of the "banks' bank". In the balance sheet classification of figure 3, central money sits in item **A1**.
- **Commercial money** consists in deposits made by customers at the banks themselves. It is simultaneously an asset for depositors (their own "money") and a *liability* for the banks. It belongs to item **L3**, and more precisely to its non-maturing part.

Accordingly, considering at first the simple case of a closed, single-currency economy,[46] and representing the "deposit" relationship as a network, the banking sector is sitting between two separate groups of agents, as in figure 30: in red, the agents who use commercial money, i.e. who accept bank deposits **L3** as payment; in green, agents who do not: the central bank itself, but also the agents, such as public administrations, who bank directly with the central bank. If the perimeter chosen to represent the "banking sector" is not complete, the green group also includes the remaining banks and their customers.[47]

[45] A more extensive discussion of the various types of money and their relation to the banking system can be found in McLeay et al. (2014).

[46] The effects of foreign currencies on the banking sector will be discussed in sections E.3 and E.4.

[47] The customers of the banks that are not included in the perimeter chosen to represent the "banking sector", e.g. of less significant institution in the eurozone, are ordinary households and corporations: they use *commercial* money, not central money. However, as this money is not deposited with the modelled banking sector, it does not belong to **L3**. To pay them, the (incomplete) banking sector must use central money **A1** to pay their banks, who then credit their deposit accounts. Hence they sit on the **A1** side of the network of deposits, as represented in figure 30.

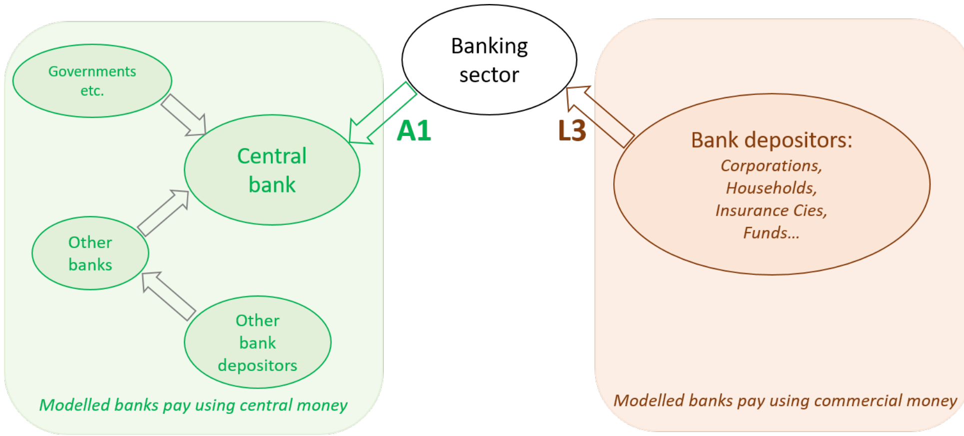

Figure 30: The banking sector in the network of deposits: two types of money.

Notes: the arrows represent deposits. Ordinary agents (right) hold deposits with banks (middle), who hold deposits with the central banks (left). ***L3*** *is money for ordinary agents (commercial money), while* ***A1*** *is the banks' own money (central money). The banking sector is involved in both kinds of money: it uses central money (****A1****) for payments to and from the green group, and commercial money (****L3****) for payments to and from the green group.*

When banks issue a payment to a member of the green group, they must deplete their stock of central money (**A1**), so that the central bank can attribute it to the payee, either directly or through its own bank outside the modelled sector. In this case, banks behave exactly as any agent in the naïve case, using their own money to pay.

However, because of the double-faced nature of commercial money, a payment between the banking sector and the red group, which represents most of the economy, is very different: it is performed by adding an extra amount to the customer deposit account on the liability side of the balance sheet (**L3**). Such a move is sufficient for the recipient to consider itself fully paid, having received commercial money. The asset side of the banking sector balance sheet is not involved; in particular, unlike the intuitive picture, a payment from the banking sector to the general economy does not affect its own money, in the sense of the central money **A1**. This holds at aggregate level: individual transactions do require the payer bank to transfer central money to the bank of the payee, affecting both banks' individual balances, but the aggregate amount is unchanged.

The exact same holds, in reverse, when banks receive a payment from customers: the payment is fully settled by the aggregate bank balance sheet registering a depletion of deposit liabilities **L3**. No inflow of central money **A1** is involved at the aggregate level.

Depending on the destination, the banking sector thus uses two different modes of payment to settle a transaction: either in central money, which involves subtracting from (resp. adding to) **A1** on the asset side, or in commercial money, which involves adding to (resp. subtracting from) **L3** on the liability side.

E.2 Determining the settlement coefficients

When the model generates an event triggering an accounting credit (resp. debit), either on a balance sheet item or on the income statement, a matching debit (resp. credit) is simultaneously generated on non-maturing items of the balance sheet using equations (16), (22) and (23), relying on a settlement matrix $M_{a,i}$ illustrated in figure 10, and on vectors w_a^P, w_a^L, w_a^Q in figure 11.

The natural interpretation of this set of coefficients is that they represent the way the banking sector pays, or is paid, for the various events simulated by the model: inventory changes, interests, other costs etc. Given the above discussion, the matching entries are therefore likely to occur partly in **A1** (payment in central money) and partly in **L3** (payment in commercial money). However, the evolution of **A1** is already completely determined by the model (section 4), which effectively specifies which events generate payments in central money.

Thus, the settlement coefficient $M_{a,i}$, for a maturing or monetary item i and a non-maturing item a, represents the effect on item a of a unit change in item i *all other monetary/maturing items* $j \neq i$ *being held equal*, and outside any net cost or income distribution. Likewise, the coefficient w_a^Q represents the effect on a of a net cost incurred by the bank without any move in monetary/maturing items, etc.

This *ceteris paribus* clause, which is necessary to prevent double counting in equations (16), (22) and (23), is key to the correct determination of the settlement coefficients. In particular, as **A1** is included in the monetary items held constant, the coefficients must be determined by assuming that payments are made entirely in commercial money, hence the predominance of **L3** as counterpart to most moves in the matrix and vectors. As this outcome may look unintuitive in some cases, the following paragraphs elaborate on it on a case by case basis. The special case of interbank positions is discussed in a later section.

E.2.1 Settlement for central bank deposits

The first and most important case to tackle is that of the **A1** aggregate itself, representing central money. The coefficients of the matrix $M_{a,i}$ when i is the monetary asset **A1**, represent the evolution of the accounts a in the case of an increase of **A1** *ceteris paribus*, meaning that banks receive a payment on their central bank accounts that does *not* result from any action of the banks such as borrowing from the central bank (unchanged **L1**), selling or issuing securities (unchanged **A4** and **L4**), or from any income or expense.

Such a situation arises mainly when receive central money in the context of a payment made to their customers. In particular, the monetary part of the model in section 4.1.3 generates **A1** moves when the central bank buys securities to a bank customer ($\Delta \boldsymbol{S}$ in equation (11)), when the banks receive a payment from outside the modelled banking sector ($\Delta \boldsymbol{T}$ in equation (11)) or when customers deposit banknotes (modelled as $-\beta \cdot \Delta L3$ in equation (11)).

In all these cases, the banking sector collects money on behalf of its customers, so that they credit the same amount to the customer current accounts in the non-maturing part of **L3**, allowing the customers to receive the proceeds of sales to the central bank or to out-of-perimeter customers, or of their banknote deposits.

It could be objected some of the moves in **A1** generated by the model correspond central bank security purchases from the banking sector itself. However, such a transaction generates a move in **A4**: to satisfy the *ceteris paribus* condition, it must be coupled with a simultaneous purchase of securities from customers to keep **A4** unchanged: such an overall transaction is equivalent to the case where the central bank buys the securities directly from bank customers, cf. (Adam et al., 2023). If no such equivalent purchase actually happens, then the effects of the resulting change in **A4** must also be accounted for, cf. section E.2.2.

Overall, the model assumes that changes in **A1**, all else being held equal, generate equivalent changes in the non-maturing part of **L3**, hence the +1 number in the **A1** column of figure 10).

E.2.2 Settlement for other maturing and monetary items

Outside the case of interbank items, which will be tackled in section E.3, the model considers that moves in all other maturing and monetary items (**A3**, **A4**, **L1**, **L3**, **L4**) are matched by moves in the non-maturing part of **L3**. The reasoning behind this choice is the same in all five cases: transactions that affect any of these items are settled by the banking sector partly in central money **A1** and partly in commercial money **L3**; as the settlement coefficients must be determined *ceteris paribus*, the move in **A1** must be cancelled out, triggering an equivalent move in **L3** as per section E.2.1; therefore, the settlement matrix column is concentrated at 100% on **L3**.

This "all-deposits" modelling choice is corroborated by the back-testing exercise, showing (cf. figure 27) that it correctly explains the recent evolution of deposit volumes.

Beyond this empirical check, the following worked-out examples are useful to illustrate the consistency of this choice. Not all of them correspond to events individually generated by the model, which is only concerned with aggregate behaviour; however, they illustrate the conceptual robustness of the settlement matrix taken by itself.

- **New bank loans**: the banking sector lends an extra 10 M€ to customers, who use the proceeds to buy goods and services from other bank customers (for 9 M€) and to pay taxes (for 1 M€).

  This increases **A3** (10 M€ debit representing the loan inventory increase), but also decreases **A1** (1 M€ credit representing the payment to the government, in central bank money). Both items having a +1 coefficient on **L3** in the settlement matrix, the net modelled settlement effect of these operations given by equation (16) is a 9 M€ credit on the non-maturing part of **L3**, which correctly represents the 9 M€ earned by the other bank customers.

- **Central bank financing**: the banking sector borrows 10 M€ from the central bank financing facilities.

  This increases both **A1** (1 M€ debit representing the inflow of central money) and **L1**

(1 M€ credit representing the extra debt to the central bank). Such a move is already balanced, and is not expected to have any direct effect on the rest of the balance sheet. Indeed, as the coefficients for **L3** are +1 for **A1** and −1 for **L1**, the modelled settlement effect on non-maturing items is zero and customer deposits **L3** are not affected.

- **Government bond issuance**: the banking sector buys 10 M€ worth of debt securities issued by the Government on the primary market.

  This increases **A4** (10 M€ debit representing the security inventory increase) and decreasing **A1** (10 M€ credit representing the payment to the government, in central bank money). Again, this move is already balanced, and should not affect on the rest of the balance sheet. Indeed, as the coefficients on **L3** are +1 for both **A1** and **A4**, the modelled settlement effect of this move is zero.

- **Central bank security purchase**: the central bank sector buys 10 M€ worth of debt securities, of which 7 M€ are sold by bank customers, and 3 M€ by banks themselves.

  This increases **A1** (10 M€ debit representing the inflow of central money received from the central bank as settlement both for the bank and for its customers) and decreases **A4** (3 M€ credit representing the security inventory decrease). As the coefficients on **L3** are +1 for **A1** and **A4**, the modelled settlement effect of this move is a 7 M€ credit on **L3**, which correctly represents the sale proceeds received by customers in commercial money.

- **Bank bond issuance**: the banks issue 10 M€ worth of debt securities, 9 M€ of which are purchase by bank customers (e.g. funds), and 1 M€ by the central bank.

  This increases both **L4** (10 M€ credit representing the issued debt inventory increase) and **A1** (1 M€ debit representing the inflow of central bank money received as settlement). As the coefficients on **L3** are −1 for **L4** and +1 for **A1**, the modelled settlement effect of this move is a 9 M€ debit on **L3**, which correctly represents the disbursement of commercial money made by the customers to buy their share of bonds.

- **Repayment of maturing term deposits**: Customer term deposits worth 10 M€ mature and are not replaced.

  This decreases the maturing part of **L3** (10 M€ debit representing the term deposit inventory decrease). As the coefficient on **L3** is −1 for **L3**, the modelled settlement effect of this move is a 10 M€ credit on the non-maturing part of **L3**, which adequately reflects the payment made by banks to the customers' deposit accounts in commercial money as their term deposits mature.

E.2.3 Settlement of banks' net expense and profit distribution

A similar rationale backs the choice to settle all of the banks' non-interest non-default net expense Q_t on the non-maturing customer deposits **L3**, hence the single +1 coefficient in equation (22)'s vector w_a^Q as illustrated in the right-hand side of figure 11. To the extent that the sources of income and expense making up Q_t do not correspond to value changes in maturing items, they are mostly services provided by the banking sector (for which customers must pay), or to the banking sector (for which the banking sector must pay).

The net payment can in theory be made partly in central money and partly in commercial money; however, the *ceteris paribus* clause must still hold in the determination of the model coefficients, which must be determined in a scenario where **A1** is held constant. Any payment in central money must therefore be cancelled out by a reverse move in **A1** which, per the settlement matrix for monetary items, is matched by a customer deposit move. Overall, the whole payment is attributed to **L3**.

For exactly the same reason, profit distribution is entirely made through **L3**. The coefficients here, i.e. equation (23)'s vectors w_a^P and w_a^L, displayed in the left-hand side of figure 11, are not +1, since not all profits are distributed: a proportion is kept in equity as retained earnings.

As for balance sheet items, some worked-out examples may be helpful.

- **Tax payment**: the banking sector pays 10 M€ taxes to the government.

  The contribution to net expense Q_t is a 10 M€ debit, and the payment process itself, in central money, depletes **A1** by 10 M€ (credit). The overall modelled effect on non-maturing items is zero, as +10 M€ from equation (22) is balanced out by −10 M€ from equation (16). As expected, customer deposits are not affected. A more intricate scenario where the government immediately spends the tax proceeds leads to unchanged **A1** and to a 10 M€ credit on **L3** — the latter reflecting the payments in commercial money received by customers from the government.

- **Dividend distribution**: the banking sector, having made a 10 M€ profit, keeps half of it as retained earnings and distributes the rest. Its shares are owned by customers (80%) and by the government (20%), who receive a *pro rata* dividend payment.

  The net income NI is therefore 10 M€. The dividend distribution to the government implies that the central money stock **A1** is depleted by 1 M€. The overall modelled effect on non-maturing items is therefore, in addition to the 5 M€ credit to equity, a net 4 M€ credit to **L3**, i.e. 5 M€ from equation (23) minus 1 M€ from equation (16), which is consistent with a 4 M€ dividend payment in commercial money to customer shareholders.

- **Valuation changes**: the banking sector makes a 10 M€ profit due higher market valuations of debt securities recognised at fair value.

  The profit decreases the "net other costs" account Q_t (10 M€ credit), and the change in valuation increases **A4** (10 M€ debit).[48] The modelled effect on **L3** is +10 M€ from equation (22) and −10 M€ from equation (16), netting out to zero: indeed, such a profit does not involve any payment in commercial money.

[48]The model for maturing items can handle such a scenario by manually prescribing a positive evolution of **A4** volumes, though it will not correctly infer the consequences on the interest rate structure of the item, as pointed out in appendix D. The point here is that the immediate consequences of such a scenario in terms of *settlement* are correctly modelled.

E.3 Interbank items

All of the above relies on the simplifying assumption that payments between the banking sector and the outside world (non-customers) are always settled in central money, i.e. central bank deposits **A1**. Though this accounts for the majority of flows, a complete picture of the balance sheet must introduce the secondary role played by interbank deposits (non-maturing parts of **A2** and **L2**) in payments.

Banks are free to open deposit accounts at other banks. Between banks within the same currency area, such accounts can be used as temporary buffers to net out retail cash flows, or as a form of interbank loan, or for various technical purposes, such as tracking cash collateral transfers in the context of derivative trading. Across currency areas, interbank deposits are also used for correspondent banking, where a domestic bank grants access to its currency to a foreign bank through a deposit account on its balance sheet (a "vostro" account of the domestic bank, a "nostro" account of the foreign bank) — such accounts may explain the high proportion of USD in interbank deposits shown in table 6.

Hence, a bank paying another bank may in some cases credit an interbank deposit, which is technically a form of commercial money, instead of a central bank deposit. Such a deposit may be on the asset or liability side of its balance sheet, and may therefore belong to the non-maturing parts of **A2** or **L2**.

To account for such moves, the model assumes that interbank loans (maturing parts of **A2** and **L2**) are settled using the non-maturing part of **A2**, cf. figure 10. The choice of **A2** instead of **L2** is arbitrary; it has limited consequences on the model outcome given the small size of historical variations of interbank items on both sides of the balance sheet, cf. figure 13.

E.4 Some remarks on flows in and out of the banking system

Remarkably, the modelled evolution of deposits does not rely on any behavioural assumptions on the depositors themselves: it is only necessary to use assumptions on behaviour of the banking sector, the central bank, the other central bank depositors, and on the volume of loans. The volume of deposits follows. This property is due to the near-closed nature of the deposits in the banking sector as a whole: depositors may transfer their deposits between each other, but they mostly cannot, on their own, act in a way that lowers the total amount deposited in the banking sector — outside loan repayment (i.e. lower **A3** volumes), the only exceptions are cash withdrawal and sale of locally held foreign currency, both of which are by nature limited in volume.

This conservation law may look puzzling from the point of view of an individual depositor, who has many available options to seemingly transfer value out of the banking sector: buying securities, subscribing to mutual funds, opening an account in a foreign bank, converting local currency into foreign currency, or even digital assets... However, seen from a global perspective, most of these transactions are actually transfers from one depositor to another, that conserve the overall amount deposited in the banking sector.

It may be useful to go through these examples one by one to describe their effects on the aggregate balance sheet of the banking sector, which, for the purposes of the discussion, is assumed to correspond to the eurozone — the word "foreign" refers to non-eurozone entities.

- **Wire transfer of local currency to a foreign account**. A eurozone bank customer may open a euro-denominated account in a foreign bank, and wire funds from the local to the foreign deposit account. The eurozone bank then has to make a euro payment to the foreign bank. It flows to the foreign bank's *nostro* account in euros, which is ultimately an interbank deposit with a eurozone bank — possibly *via* a number of intermediary institutions.

  From the point of view of the aggregate eurozone banking sector, the debit from customer deposits **L3** is thus matched by a credit to interbank deposits **L2**, so that the overall deposit base is left unchanged. The same hold with any transfer of local currency between local and foreign accounts.

- **Security purchase**. What happens in such a transaction depends on the nature of the seller. As long as the selling entity is not a local bank (unchanged security amounts **A4**/**L4**), nor the central bank (unchanged security holdings $\boldsymbol{S_t}$), nor other central bank depositors (whose aggregate behaviour is already explicitly modelled), it can only belong to two groups:

  - another bank customer: the deposit is shifted from one customer account to another;
  - non-customers, e.g. foreign entities without a local bank account: the payment to the seller is then equivalent to the foreign account case above, with a transfer from **L3** to **L2**, and no net deposit outflow.

  The purchase of digital assets or "cryptocurrencies" is exactly similar, since the nature of the asset underlying the transaction has no effect on banks' balance sheets.

- **Mutual fund subscription**. A customer subscribing to a mutual fund is effectively buying shares of that fund, making the transaction equivalent to a security purchase. The use of the proceeds by the fund is generally also a security purchase, or a direct loan to another entity, which, as far as deposit flows are concerned, is identical. However, money market funds may resort to two types of transactions deserving further discussion:

  - The fund may enter a repurchase agreement with a bank: this is still a form of deposit, which, having an agreed maturity, is classified, for the purposes of the model, as a maturing deposit: such a transaction can therefore be described by the evolution of the maturing part of **L3**.
  - The fund may enter a repurchase agreement with the *central bank*. Some central banks (though not the Eurosystem) maintain a repo facility available to some non-bank entities, the most notable case being the Federal Reserve. Deposits are then transferred out of the banking sector (lower **A1** and **L3**). However, the deposit outflow in this case can ultimately be described as driven by a *central bank* action, equivalent to a temporary sale of assets, i.e. lower $\boldsymbol{S_t}$.

- **Foreign currency conversion**. A customer holding euro deposits may seek to convert them into a foreign currency — e.g. US dollars (USD). In practice, what is usually called “conversion” is actually an exchange: the customer has to find a willing USD seller, and the transaction consists in a simultaneous transfer of euros from the buyer to the seller, and of USD from the seller to the buyer. Whereas the euro transfer is exactly similar to the case of a security purchase discussed above, the nature of the USD transfer depends on where the buyer and seller hold USD:

– If both have their USD deposit accounts in foreign banks, then the USD transfer happens entirely out of the local banking sector’s balance sheet.

– If both have their USD deposit accounts in local banks, then the USD transfer is a shift from one customer to another, leaving total deposits unchanged.

– If the buyer has a local USD deposit account and not the seller, then the local bank receives a USD payment from the foreign bank, and credits it to the customer USD account. The incoming payment may be either in the *nostro* account of the eurozone bank in a US bank (interbank deposits **A2**), or, in the case of a banking group including a US subsidiary with an access to the Federal Reserve, directly in that central bank deposit account, which is part of **A1**. Overall, the banking sector’s deposit base *increases*, with a debit on **A1**/**A2** matching the credit on **L3**.

– The obverse holds if the seller’s USD deposit account is local and not the buyer’s.

Thus, the “conversion” of local-currency deposits into locally-held foreign currency increases the overall volume of deposits in the banking sector: the local currency does not leave the aggregate deposit base, while the foreign currency enters it. On the flip side, the banking sector is exposed to aggregate liquidity outflows on account of the foreign-currency deposits it holds, as the depositors may make them exit the aggregate balance sheet by wiring them to foreign accounts, or equivalently selling them for local currency.

Overall, the banking sector’s near-exclusive access to the central bank’s balance sheet ensures that customer actions cannot result in large aggregate outflows of local-currency deposits. An “aggregate bank run” is impossible in practice, as the local currency always end up being deposited in the banking sector, except for small-volume cases such as cash or, potentially, central bank digital currencies. No customer behavioural assumption is therefore needed to model the volumes of local-currency deposits.

As that near-exclusivity does not apply to foreign currencies, nothing prevents large amounts of customers’ foreign-currency deposits from moving in and out of the local banking sector. Such deposits are by nature a minority of the banks’ deposit base: according to the liquidity risk data reported by banks, non-euro currencies represent 18% of the aggregate deposits of eurozone banking groups as of the end of 2022, of which an overwhelming majority is in USD. That proportion has oscillated between 16.5% and 20% in 2018–22. The model’s behavioural assumption that it will remain reasonably constant has limited consequences in terms of NII as long as interest rates do not widely diverge between currencies.