(Un)Conventional Monetary and Fiscal Policy *

Jing Cynthia Wu — *Notre Dame and NBER*

Yinxi Xie — *Bank of Canada*

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Abstract

We build a tractable New Keynesian model to jointly study four types of monetary and fiscal policy. We find quantitative easing (QE) and tax-financed fiscal transfers or government spending have the same effects on the aggregate economy. Compared with these three policies, conventional monetary policy is more inflationary. QE and transfers have redistribution consequences, whereas others do not. Ricardian equivalence breaks: tax-financed fiscal policy is more stimulative than debt-financed policy. Finally, we study optimal policy coordination and find that adjusting two types of policy instruments can stabilize three targets simultaneously: inflation, the aggregate output gap, and cross-sectional consumption dispersion.

Keywords: monetary policy, quantitative easing, fiscal policy, tax finance, debt finance

JEL Codes: E52, E62, E63

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1 Introduction

In response to the Great Recession and COVID-19 pandemic, the Federal Reserve (Fed) has conducted massive quantitative easing (QE) by expanding its balance sheet from $900 billion in August 2008 to $9 trillion by early 2022. Meanwhile, the Treasury has implemented a sequence of fiscal stimuli, including $800 billion of the Paycheck Protection Program and $800 billion of the Economic Impact Payments. How do we compare all the emergency monetary and fiscal policy? Moreover, in light of the recent surge in inflation, which jumped from close to zero in May 2020 to about 9% in June 2022, the Fed has raised the policy rate aggressively in 2022.[1] Around the same time, many state governments have provided another round of fiscal stimulus to help households navigate the increased cost of living. Understanding how an expansionary fiscal policy could potentially interact with monetary tightening is critical.

We propose a tractable New Keynesian model that features four policy instruments (the policy rate, QE, lump-sum fiscal transfers, and government expenditures) to contribute to this discussion. Our model can be reduced to an IS equation, a Phillips curve, and four policy rules. We compare individual policies and have four findings. First, when fiscal policy is fully financed via lump-sum taxes, fiscal transfers and government spending have the same aggregate implications as QE, because the three policy tools enter both the demand side (IS curve) and the supply side (Phillips curve) in the same fashion.

Second, to provide the same amount of stimulus, conventional monetary policy is more inflationary than QE or tax-financed fiscal policy. That is because the policy rate only enters the IS curve, whereas QE and tax-financed fiscal policy enter both the IS and Phillips curve. This result speaks directly to the 2021-2022 inflation surge caused by pandemic-induced global supply chain disruption and the Russian invasion of Ukraine. In 2022, the Fed raised its policy rate sharply without unwinding its balance sheet, while 17 states sent out inflation-

[1] We calculate inflation using the percentage change from a year ago of "Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (CPIAUCSL)" from the Federal Reserve Economic Data.

relief checks to residents to alleviate the higher cost of living. Our model implies the mix of tightening conventional monetary policy and stimulative fiscal policy can potentially lower inflation without putting additional pressure on economic activities.

Third, although the four types of policy can provide similar stimulus to the economy, they have different redistribution consequences. QE and tax-financed lump-sum transfers make a transfer from the unconstrained household to the constrained household, whereas government expenditures and the policy rate do not have a similar redistribution channel.

The fourth result is that the Ricardian equivalence breaks. Our model implies fiscal policy is more stimulative when it is taxed financed than when it is debt financed. In particular, when they are fully debt financed, fiscal transfers or government expenditures become neutral, because the expansionary effect of the fiscal policy cancels out the contractionary effect of issuing debt.

Our model is consistent with three empirical observations in the literature. First, empirical work has focused mostly on the central bank's balance sheet and used it to argue for an expansionary effect of QE since the Great Recession. On the other hand, existing theory suggests the total outstanding debt held by the public should be used to capture the overall government balance sheet policy and the empirical literature's focus on the Fed's balance sheet is one-sided, especially monetary operations are much smaller than fiscal actions in practice. However, our model validates the practice in the empirical literature: the contractionary effect of the fiscal authority's debt issuance is wiped out by the expansionary effect the fiscal policy provides, and thus, only the central bank's balance sheet is relevant to summarize the joint effort made by the monetary and fiscal authorities.

Second, empirical studies present a wide range of estimates for the fiscal multiplier, and show it is lower when it is financed by debt. Consistent with the empirical finding, our model produces a state-dependent fiscal multiplier, which is a function of how government policy is financed. Third, empirical papers show stimulus packages during different economic downturns increase households' consumption, especially for the financially constrained ones. Our model allows such a channel: fiscal transfers stimulate aggregate demand by redistributing wealth from the unconstrained to the constrained household.

In the background, we have a New Keynesian model with two types of households: the unconstrained household behaves similarly to a standard model and saves via one-period deposits, and the constrained household issues long-term bonds to finance its consumption. The market is segmented. The financial intermediary performs maturity transformation by taking in deposits and holding long-term bonds. It faces a leverage constraint, which limits the amount the unconstrained household can borrow. The firms feature the standard price rigidity. The central bank and the government implement monetary and fiscal policy, respectively.

The full model allows us to inspect underlying transmission mechanisms for these different policy instruments. First, although QE, tax-financed transfers, and government expenditures enter the IS and Phillips curves similarly, they work through different channels. QE relaxes the financial intermediary's leverage constraint, which allows the constrained household to borrow and hence consume more. Similarly, fiscal transfers also increase the constrained household's consumption by directly handing over more resources. Different from both of them, government spending does not alter the constrained household's behavior. Second, the breakdown of the Ricardian equivalence results from the fact that the leverage constraint of the financial intermediary limits bond demand. When the government issues debt to finance its policy, it crowds out private bonds issued by the constrained household, which reduces its consumption and hence the aggregate demand. By contrast, financing the government policy by levying lump-sum taxes on the unconstrained household does not have equilibrium consequences.

Finally, we study optimal policy coordination. We start by focusing on dual stability in the aggregate inflation and (welfare-relevant) output gap. The Divine Coincidence holds for the demand shock; that is, the policy rate alone can stabilize both objects. By contrast, it does not hold for the supply shock, which requires two types of policy instruments to stabilize aggregate fluctuations. That is because the flexible-price equilibrium is not efficient due to the financial friction, and the productivity shock acts as a cost-push shock. Moreover, QE or fiscal transfers can fully stabilize the credit shock, a shock to the leverage ratio of the financial intermediary, but not conventional monetary policy.[2]

Next, we derive the micro-founded quadratic welfare loss function, and show it depends not only on fluctuations in the aggregate inflation and output gap, but also the cross-sectional consumption dispersion between the two types of households. Nevertheless, the optimal stabilization policy that uses two types of policy instruments: conventional monetary policy and QE or fiscal transfers, to stabilize aggregate fluctuations against three types of shocks automatically eliminate cross-sectional dispersion of consumption.

Literature Our paper contributes to the recent development in jointly studying monetary and fiscal policy. The literature has mainly focused on conventional monetary policy, forward guidance, fiscal transfers, and government expenditures, and it typically analyzes one or two policies at a time. In contrast, we also model QE and study four types of monetary and fiscal policy in a unified tractable small-scale model, and emphasize the consequence of financing approach for fiscal policy. In the literature, to achieve non-neutrality of fiscal transfers, some papers assume bounded rationality (e.g., Woodford and Xie, 2019, 2022; Gabaix, 2020; Bianchi-Vimercati, Eichenbaum and Guerreiro, 2021), whereas others consider heterogeneous agents with borrowing constraints (e.g., Kocherlakota, 2021). Our paper is closer to the latter. In particular, our model has some similar flavor to Debortoli and Galí (2017) in the sense that the model has two types of households.

We are related to the literature that models QE with financial frictions, for example, Gertler and Karadi (2011, 2013), Carlstrom, Fuerst and Paustian (2017), Sims and Wu (2021, 2022), Sims, Wu and Zhang (2022). Most of the papers in the literature work with medium-scale DSGE models, whereas we focus on a tractable small-scale model. In that sense, our paper is related to Cúrdia and Woodford (2011), Sims and Wu (2020), and Sims,

[2]The planner's problem indicates that zero government spending is optimal.

Wu and Zhang (2021). However, we depart from the sole focus on QE of this literature and incorporate fiscal policy.

Our paper is also related to Angeletos, Collard and Dellas (2021) and Calvo and Velasco (2022) by distinguishing debt-financed fiscal policy from tax-financed policy. The key difference is the transmission mechanism: their models work through a liquidity channel, whereas our model works through both financial frictions and heterogeneous households.

The rest of the paper proceeds as follows. Section 2 discusses three empirical findings in the literature. Section 3 lays out the small-scale model that is consistent with these findings, and discusses its properties. Section 4 presents the full model and discusses the key transmission mechanisms. Section 5 analyzes optimal policy coordination, and Section 6 concludes. Derivations are in Appendices.

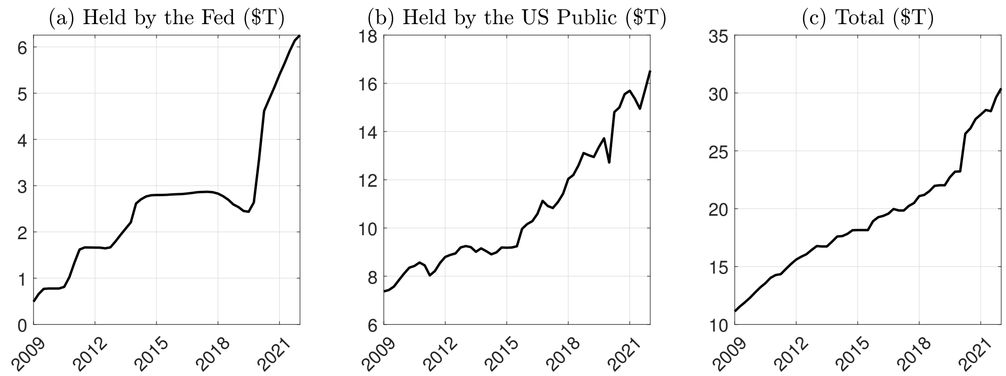
2 Empirical Motivations

In this section, we summarize some empirical observations from the literature regarding the government balance sheet policy and various fiscal policies. In Section 3, we propose a tractable small-scale linear model that is consistent with all these results.

Balance sheet policy. The empirical studies often cite the sharp increase of the central bank's balance sheet between 2009 and 2022 to argue for an expansionary effect of QE (see panel (a) of Figure 1); for example, see Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011), Hamilton and Wu (2012), Greenwood and Vayanos (2014), Wu and Xia (2016), and Wu and Zhang (2019).[3] However, in a theoretical model that studies QE, the relevant summary statistic for government balance sheet policy is the total outstanding debt that is available to the public rather than the central bank's balance sheet; for example, see Gertler and Karadi (2011), Carlstrom, Fuerst and Paustian (2017), and Sims and Wu (2021).

[3] To make the panels comparable, we plot the Fed's holding of public debt instead of its total assets. Although the latter is used more often to capture QE, the two series display similar dynamics.

Figure 1: Federal Debt Breakdown



Notes: Left panel: the federal debt held by the Federal Reserve Banks [FDHBFRBN]; middle: the total federal debt (right panel) less the amount held by the Fed (left panel) and by foreign and international investors [FDHBRIN]; right: the total federal debt of the U.S. [GFDEBTN]. All the numbers are reported in trillions of dollars, and the data are in quarterly frequency. Source: Federal Reserve Economic Data.

Figure 1 shows that between 2009 and 2022, when the Fed has expanded its balance sheet aggressively (see panel (a)), the federal debt that is held by the public in the U.S. has grown significantly as well (see panel (b)). The reason is that the Treasury has issued debt faster than the Fed's purchases (see the comparison between panels (a) and (c)). According to the existing theory, the joint balance sheet policy between the Fed and the Treasury during this period would have been contractionary, despite the sharp growth of the central bank's balance sheet. Does the empirical literature miss the dominant piece of government balance sheet policy by solely focusing on the Fed's balance sheet and completely ignoring the fiscal side of actions? Or is there a theoretical rationale or model that validates the practice of the empirical literature?

Fiscal multiplier. The empirical estimates of the fiscal multiplier display a wide range and mostly vary from 0.3 to 0.8 (see Table 1 in Ramey's (2019) survey article). One could potentially attribute some of the variation to different identification assumptions and estimation strategies. However, even using the same estimation method or within the same study, the estimated fiscal multiplier still varies significantly. For summaries of empirical studies, see Ramey (2019), Chodorow-Reich (2019), and Barnichon, Debortoli and Matthes (2022).

Moreover, the literature shows the fiscal multiplier is potentially state-dependent. In particular, we focus on its dependence on the financing method.[4] For example, Pinardon-Touati (2021) uses micro data and finds when the government finances its expenditures by debt, it crowds out private loans, and consequently reduces the output multiplier. Relatedly, Ilzetzki, Mendoza and Végh (2013) find the multiplier is smaller, even negative, in countries with higher debt-to-GDP ratio.

Fiscal transfers. Studies show the US government's stimulus packages that issue paychecks directly to households during recessions (e.g., tax rebates in 2001, the Economic Stimulus Payments of 2008, and the Economic Impact Payments of 2020) increase households' spending significantly; see, for example, Parker et al. (2013) and Parker et al. (2022). Moreover, some research suggests responses are larger for poorer households with lower liquid wealth or income; see, for example, Broda and Parker (2014).

3 A Tractable Model for Monetary and Fiscal Policy

In this section, we propose a tractable small-scale New Keynesian model that is consistent with the empirical observations discussed in Section 2, highlight its key properties, and relate it to the post-COVID inflation surge. Our model has the scope for both monetary policy (conventional and QE) and fiscal policy (government expenditures and lump-sum transfers).

The full model that gives rise to the tractable small-scale linear model features the following ingredients. A standard unconstrained household saves via one-period deposits and a constrained household finances its consumption by issuing long-term private bonds.[5] A financial intermediary performs maturity transformation and faces a leverage constraint. The central bank implements QE, modeled as the real market value of its long-term bond portfo-

[4]Some other studies show the fiscal multiplier might also depend on downward nominal wage rigidity or incomplete markets, e.g., Shen and Yang (2018), Barnichon, Debortoli and Matthes (2022), and Jo and Zubairy (2022).

[5]We label the second household constrained because the amount it can borrow is limited by the leverage constraint of the financial intermediary. Although our constrained household is not hand-to-mouth as in the two-agent New Keynesian (TANK) literature (e.g., Debortoli and Galí, 2017), it behaves similarly to a hand-to-mouth household.

lio, to relax this constraint. The government can purchase final goods or make a lump-sum transfer to the constrained household. Fiscal policy is financed by either collecting lump-sum taxes from the unconstrained household or issuing long-term government debt. See the details of the full model in Section 4 and derivations in Appendix A.

3.1 The Linear Model

Our small-scale New Keynesian model features an IS curve and a Phillips curve:

$$
\begin{aligned}
\hat{y}_t &= \mathbb{E}_t\,\hat{y}_{t+1} - \frac{\vartheta}{\sigma}(\hat{i}_t - \mathbb{E}_t\,\hat{\pi}_{t+1}) \\
&\quad + \left[\widehat{qe}_t + \eta(\hat{\tau}_t^C + \hat{g}_t)\right] - \mathbb{E}_t\left[\widehat{qe}_{t+1} + \eta(\hat{\tau}_{t+1}^C + \hat{g}_{t+1})\right], && (3.1)\\
\hat{\pi}_t &= \beta\,\mathbb{E}_t\,\hat{\pi}_{t+1} + \gamma\zeta\hat{y}_t - \frac{\gamma\sigma}{\vartheta}\left[\widehat{qe}_t + \eta(\hat{\tau}_t^C + \hat{g}_t)\right], && (3.2)
\end{aligned}
$$

where the lowercase with a hat and t subscript denotes variables in the log deviation from the non-stochastic steady state unless otherwise noted. Variables $\hat{y}_t$ and $\hat{\pi}_t$ represent output and inflation, respectively.

The system has four policy variables: the short-term nominal interest rate $\hat{i}_t$, QE $\widehat{qe}_t$, government spending $\hat{g}_t$, and a lump-sum transfer to the constrained household $\hat{\tau}_t^C$. Note we define $\widehat{qe}_t$, $\hat{g}_t$, and $\hat{\tau}_t^C$ relative to the steady-state output.[6]

A new policy parameter in our model, $0 \leq \eta \leq 1$, measures the share of fiscal policy (the government's expenditures and transfers) that is financed by collecting lump-sum taxes from the unconstrained household. We focus on two alternative fiscal financing rules: $\eta = 1$ corresponds to the case in which fiscal policy is fully financed by taxes, and $\eta = 0$ corresponds to the case of fully debt-financed fiscal policy.

All the parameters are positive. σ, β, and γ are standard in the New Keynesian literature: σ measures the inverse of elasticity of intertemporal substitution, $\beta < 1$ is a subjective discount factor, and γ is the elasticity of inflation with respect to the real marginal cost.

[6] $\widehat{qe}_t \equiv (QE_t - \bar{QE})/\bar{Y}$, $\hat{g}_t \equiv (G_t - \bar{G})/\bar{Y}$, and $\hat{\tau}_t^C \equiv (T_t^C - \bar{T}^C)/\bar{Y}$, where $\bar{Y}, \bar{QE}, \bar{G}, \bar{T}^C$ denote the steady state output, QE, government spending, and transfers, respectively.

The parameter ϑ measures the share of the unconstrained household's consumption in total output at the steady state. ζ represents the elasticity of real marginal cost with respect to output, which is a function of ϑ. Detailed definition of these parameters can be found in Appendix A.2.

The equilibrium is fully characterized by (3.1) - (3.2), together with four policy rules. For the exposition of this section, we assume the short-term nominal interest rate follows a standard Taylor (1993) rule, that is,

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \hat{\delta}_{i,t}, \tag{3.3}$$

where $\phi_\pi > 1, \phi_y > 0$, and $\hat{\delta}_{i,t}$ represents exogenous monetary disturbances, and the other three policy instruments follow exogenous processes. We will relax some of the assumptions in Section 5, where we study endogenous policy responses to more shocks.

3.2 Monetary Policy vs. Fiscal Policy

In this section, we compare the four policy instruments and discuss their similarities and differences. For a detailed discussion on transmission mechanisms, see Section 4.6. We begin by showing their similarity:

Proposition 1 *The effects of QE, government expenditures, and lump-sum fiscal transfers on output and inflation are the same when fiscal policy is fully tax financed.*

Proof: see Appendix B.1.

Proposition 1 bridges the gap in the literature that typically studies QE and fiscal policy separately, and facilitates a comparison between them. It argues QE has the same effects on aggregate variables as tax-financed fiscal policy. We can see this property using the equilibrium conditions, in particular, (3.1) - (3.2). When fiscal policy is fully financed via lump-sum taxes, we have $\eta = 1$. In this case, $\widehat{qe}_t$, $\hat{\tau}_t^C$, and $\hat{g}_t$ enter the system in the same fashion. Furthermore, (3.1) - (3.2) imply these policy tools affect the economy through both the supply and demand channels.

Next, we compare conventional monetary policy with the three policy tools we discussed in Proposition 1 and focus on their implications on inflation:

Proposition 2 *To provide the same amount of stimulus, conventional monetary policy is more inflationary than QE and tax-financed fiscal policy.*

Proof: see Appendix B.1.

Proposition 2 makes a comparison among the four policy instruments in terms of their implications on inflation. The difference comes from the fact that the policy rate only enters the IS curve, whereas other policy enters both the IS and Phillips curves. For an expansionary policy in QE, transfers, or government expenditures, the positive policy shock puts downward pressure on inflation through the Phillips curve. We discuss the practical relevance of Proposition 2 for the 2021-2022 inflation surge in Section 3.4.

A similar comparison between conventional monetary policy and QE is made in Sims, Wu and Zhang (2021). Moreover, the empirical literature suggests that government spending and fiscal transfers are not that inflationary; for example, see Nakamura and Steinsson (2014), Pennings (2021), Jørgensen and Ravn (2022), and Liu and Xie (2022).

After comparing the aggregate implications of various monetary and fiscal policy, we next move to their redistribution consequences:

Proposition 3 *QE and tax-financed transfers redistribute wealth from the unconstrained household to the constrained household, whereas the policy rate and tax-financed government spending do not have a redistribution effect.*

Proof: see Appendix B.2.

Although all policy tools under discussion could potentially stimulate the aggregate economy, they work through different channels. Both QE and fiscal transfers have a redistribution effect by affecting both types of households. QE allows the constrained household, whose consumption ties up to their borrowing limit, to borrow more, whereas transfers hand resources to it directly. By contrast, the policy rate and government spending stimulate the aggregate demand without affecting the constrained household.

Finally, Propositions 1 and 3 imply that tax-financed lump-sum transfers stimulate the aggregate economy by redistributing wealth from the *unconstrained* household to the *constrained* household. This result speaks directly to the empirical results on fiscal transfers discussed in Section 2. Unlike in a standard textbook model (e.g., Galí 2015, chap. 3), tax-financed lump-sum transfers are not neutral in our model. The reason is that the government transfers are made to the constrained household, which has limited access to financial markets, whereas the lump-sum taxes are imposed on the unconstrained household.[7] Thus, the transfer policy stimulates the economy by effectively redistributing wealth between the two types of agents. This transmission mechanism is similar to the one in the TANK model of Debortoli and Galí (2017) although the constrained household is set up differently.

3.3 Tax Finance vs. Debt Finance

In a standard business cycle model, Ricardian equivalence holds, and whether a fiscal stimulus is (lump-sum) tax-financed or debt-financed is irrelevant. However, this is not true in our model. Whereas in Section 3.2, we discussed that fiscal policy is stimulative when it is financed by taxes, the stimulative effects disappear when it is financed by debt.

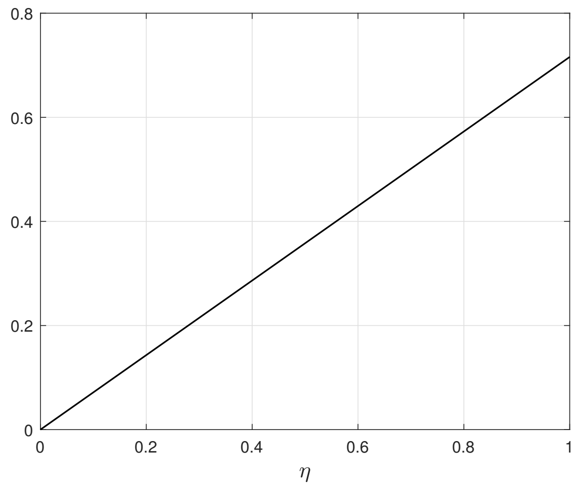
Lemma 1 *The effects of government expenditures and lump-sum fiscal transfers on aggregate output and inflation are neutral when they are fully debt financed.*

Proof: see Appendix B.1.

When the fiscal policy is fully debt financed, that is, when $\eta = 0$, government expenditures $\hat{g}_t$ and lump-sum transfers $\hat{\tau}_t^C$ drop out of the equilibrium conditions; see (3.1) - (3.2). What makes debt-financed fiscal policy have no impact on the aggregate economy? Whereas

[7] We can also assume the government levies lump-sum taxes on both types of households, but as long as the net government transfer to the constrained household is positive, the same result holds.

Figure 2: Cumulative Fiscal Multiplier as a Function of η



Notes: $\eta \in [0, 1]$ is the fraction of government policy that is tax financed. The multiplier is defined as the cumulative response in output relative to the cumulative change in government spending. See calibration in Appendix C.

government spending and transfers are stimulative on aggregate demand, issuing long-term bonds is contractionary, which acts like QE in reverse (or QT). On net, the two effects cancel out.

Lemma 1 rationalizes the empirical literature's focus on the central bank's balance sheet discussed in Section 2. The contractionary effects of government issuing debt is canceled out by the expansionary effects of government expenditures and transfers. Hence, what remains of government balance sheet policy is the expansionary effects of QE, which is summarized by the central bank's balance sheet.

Contrasting the results in Proposition 1 and Lemma 1, we find that how a fiscal stimulus is financed has consequences:

Proposition 4 *Ricardian equivalence breaks: when a larger fraction of fiscal policy is tax financed, government expenditures or transfers are more stimulative.*

Proof: see Appendix B.1.

Proposition 4 makes a contrast to the standard Ricardian equivalence result in the literature and is consistent with the empirical findings on the fiscal multiplier discussed in Section 2. Figure 2 plots the cumulative fiscal multiplier, which measures the cumulative response in output relative to the total change in government spending, as a function of η, which captures the fraction of government spending that is tax financed. Our model can generate a wide range for the fiscal multiplier, which depends on how government policy is financed: when it is fully financed by debt, that is, when $\eta = 0$, the multiplier is 0, and the number goes to 0.72 when it is tax financed, or $\eta = 1$. This range is consistent with the empirical estimates, which vary from 0.3 to 0.8. Moreover, Figure 2 shows the size of the multiplier is increasing in η, which is also consistent with empirical findings.

3.4 Discussion on 2021–2022 Inflation Surge

Proposition 2 is especially relevant for the inflation surge in 2021 and 2022. With persistently high inflation taking center stage, the Fed has raised its policy rate aggressively. From March to November, the target range increased from [0, 0.25] to [3.75, 4] percent. For the same period, it barely winded down its balance sheet. This action is consistent with our model's prediction: tightening the policy rate is more effective at combating inflation.

Meanwhile, to help households alleviate increased cost of living, the fiscal authority provided another round of stimulus to the economy. For example, in late 2022, 17 states sent out inflation-relief checks. Proposition 2 implies the combination of expansionary fiscal policy and tightening conventional monetary policy can potentially lower inflation without contracting the economy.

4 Full Model

In this section, we present the full non-linear model. The economy consists of the following agents: unconstrained household, constrained household, financial intermediary, firms, the central bank, and the government. The unconstrained household and firms are standard as in a textbook New Keynesian model (e.g., Galí 2015, chap. 3). The unconstrained household consumes consumption composites, saves via one-period deposits, supplies labor for production, and owns the firms and financial intermediary. The production side has two layers: a continuum of intermediate good producers hire labor from the unconstrained household, produce differentiated goods, and face price rigidity as in Calvo (1983). A representative final good firm aggregates intermediate goods to produce consumption composites.

The setup of the constrained household and the financial intermediary are similar to Sims, Wu and Zhang (2021). The constrained household is less patient than the unconstrained household and finances its consumption by issuing long-term bonds and receiving transfers from the unconstrained household and the government. The constraint household is structured differently from the hand-to-mouth household in the TANK model of Debortoli and Galí (2017) although they behave similarly. Its borrowing is limited due to the leverage constraint of the financial intermediary, who performs maturity transformation by taking in deposits from the unconstrained household and holding long-term bonds issued by the constrained household and the government.

What is new in our paper is the joint modeling of four types of monetary and fiscal policy. In particular, the government makes a transfer to the constrained household. QE is modeled as the real value of the central bank's bond holdings. Government expenditures and conventional monetary policy are fairly standard. The government can finance its fiscal policy by taxing the unconstrained household or issuing long-term debt.

4.1 Households

4.1.1 Unconstrained Household

The representative unconstrained household is similar to the one in a standard New Keynesian model. It maximizes its life-time utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} \right], \tag{4.1}$$

where C_t is the value of consumption composite and L_t is the labor supply. The parameter χ is the inverse of Frisch elasticity of labor supply, and ψ is a scaling constant.

The budget constraint is

$$P_t C_t + D_t = W_t L_t + I_{t-1} D_{t-1} + P_t T_t^U, \tag{4.2}$$

where P_t is the price of consumption composite and W_t is the nominal wage of labor income. They hold one-period riskless deposits, D_{t-1}, from period $t-1$ to t, which pays a nominal interest rate I_{t-1}. The variable $T_t^U \equiv \Phi_t + \Phi_t^{FI} - T_t^G + T_t^{CB} - X_t^C - X_t^{FI}$ is the total amount of transfers that the unconstrained household receives, which consists of dividends from firms Φ_t and the financial intermediary Φ_t^{FI}, a lump-sum tax paid to the government T_t^G, a lump-sum transfer from the central bank's balance-sheet surplus T_t^{CB},[8] and transfers to the constrained household X_t^C and the financial intermediary X_t^{FI}. None of these transfers are choice variables of the unconstrained household.

The first-order conditions of the household' optimization problem are

$$\psi L_t^{\chi} = C_t^{-\sigma} w_t, \tag{4.3}$$

$$C_t^{-\sigma} = \beta I_t \, \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \right], \tag{4.4}$$

where $w_t \equiv W_t / P_t$ denotes the real wage and $\Pi_t \equiv P_t / P_{t-1}$ is gross inflation. They are the labor supply condition and Euler equation, where the latter is used to price the interest paid on deposits.

4.1.2 Perpetual Bonds

We model long-term bonds as perpetuities with decaying coupon payments, following Woodford (2001) and Carlstrom, Fuerst and Paustian (2017). One unit of perpetuity issued in

[8] We model the central bank's balance-sheet surplus as paid to the unconstrained household via a government transfer. This assumption is purely for convenience and is inconsequential for model implications.

period t pays 1 dollar in period $t+1$, κ dollars in period $t+2$, κ^2 dollars in period $t+3$, and so on. Perpetuities have some convenient features.

B_{t-1} denotes the total nominal liability on past issues due at t, which can be written in terms of past new issues N_{t-j}:

$$B_{t-1} = \sum_{j=1}^{\infty} \kappa^{j-1} N_{t-j}.$$

Consequently, we have

$$N_t = B_t - \kappa B_{t-1}.$$

Let Q_t denote the price of a new issue, and then $\kappa^j Q_t$ is the time-t price of the perpetuity that is issued in period $t-j$. The total value of all outstanding bonds is $Q_t B_t$. Therefore, we can use Q_t, B_t, B_{t-1} to summarize the entire state related to the long-term bonds. Define R_t as the one-period holding-period return of the long-term bonds:[9]

$$R_t = \frac{1+\kappa Q_t}{Q_{t-1}}. \tag{4.5}$$

4.1.3 Constrained Household

The representative constrained household only consumes and does not supply labor. This assumption is only made for convenience so that the system can be reduced to the IS and Phillips curves described in Section 3.1. Our results in Section 3 are robust to the alternative assumption in which the constrained household supplies labor in a similar fashion to the unconstrained household; for details, see Appendix E.

The constrained household maximizes the objective

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^C\right)^t \left[\frac{\left(C_t^C\right)^{1-\sigma} - 1}{1-\sigma}\right], \tag{4.6}$$

[9]The economy has two types of long-term bonds: one issued by the government and one issued by the constrained household. For simplicity, we assume that the private bonds have no excess premium over public bonds and their initial prices are the same. As a result, the two types of bonds have the same price Q_t and return R_t.

where C_t^C is its consumption. Parameter $\beta^C < \beta$ is its subjective discount factor, which implies the constrained household is less patient than the unconstrained household, and makes the constrained household the borrower and the unconstrained household the saver in the equilibrium.

The constrained household finances its consumption via issuing long-term private bonds or through transfers it receives from the unconstrained household and the government. Its budget constraint is

$$P_t C_t^C + B_{t-1}^C = Q_t \left(B_t^C - \kappa B_{t-1}^C \right) + P_t X_t^C + P_t T_t^C, \tag{4.7}$$

where B_{t-1}^C is the total coupon liability the constrained household faces, $Q_t \left(B_t^C - \kappa B_{t-1}^C \right)$ is the value of newly issued private bonds, X_t^C is the transfer from the unconstrained household, and T_t^C is the lump-sum transfer from the government.

The first-order condition for the constrained household is

$$\left(C_t^C \right)^{-\sigma} \quad = \quad \beta^C \, \mathbb{E}_t \left[\frac{\left(C_{t+1}^C \right)^{-\sigma} R_{t+1}}{\Pi_{t+1}} \right], \tag{4.8}$$

which is the Euler equation that prices the long-term bonds.

4.2 Financial Intermediary

For tractability, we follow Sims, Wu and Zhang (2021) and assume the financial intermediary (FI) only lives for one period. The FI receives an exogenous transfer from the unconstrained household $P_t X_t^{FI}$, which includes two components: (i) a fixed amount of new equity $P_t \bar{X}^{FI}$ and (ii) the outstanding long-term bonds held by the previous intermediary. Therefore,

$$P_t X_t^{FI} = P_t \bar{X}^{FI} + \kappa Q_t B_{t-1}^{FI}. \tag{4.9}$$

The intermediary also receives deposits D_t^{FI} from the unconstrained household. It uses funds from deposits and transfers to purchase long-term bonds and hold reserves RE_t^{FI} on account with the central bank. Its balance sheet condition is

$$Q_t B_t^{FI} + RE_t^{FI} = D_t^{FI} + P_t X_t^{FI}. \tag{4.10}$$

In period $t+1$, the FI returns lump-sum dividends Φ_{t+1}^{FI} to the unconstrained household:

$$P_{t+1}\Phi_{t+1}^{FI} = \left(R_{t+1} - I_t\right) Q_t B_t^{FI} + \left(I_t^{RE} - I_t\right) RE_t^{FI} + I_t P_t X_t^{FI}, \tag{4.11}$$

where I_t^{RE} is the interest rate on reserves. The FI chooses their asset holdings to maximize the expected one-period-ahead dividends payment in (4.11) subject to the following leverage constraint:

$$Q_t B_t^{FI} \leq \Theta P_t \bar{X}^{FI}, \tag{4.12}$$

where Θ is the leverage ratio. The FI maximizes the dividends on behalf of the unconstrained household, and thus discounts the objective by its stochastic discount factor:

$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+1}}. \tag{4.13}$$

The first-order conditions with respect to assets B_t^{FI} and RE_t^{FI} are

$$\mathbb{E}_t \, \Lambda_{t,t+1} \left(R_{t+1} - I_t\right) = \Omega_t, \tag{4.14}$$

$$\mathbb{E}_t \, \Lambda_{t,t+1} \left(I_t^{RE} - I_t\right) = 0, \tag{4.15}$$

where Ω_t is the Lagrange multiplier on the leverage constraint. When the leverage constraint is not binding ($\Omega_t = 0$), (4.14) implies the FI will purchase private or public long-term bonds up to the point where the expected return on long bonds equals the cost of funds. When the leverage constraint is binding, that is, when $\Omega_t > 0$, it generates excess returns on long-term bonds. The condition (4.15) says the returns on short-term assets are the same.

4.3 Firms

The supply side of the economy is the same as in a standard New Keynesian model. We consider two stages of production: a continuum of intermediate firms indexed by $j \in [0, 1]$ that produce differentiated goods and sell them to the representative final good firm, which aggregates the intermediate goods into the final consumption composites.

The final good firm produces consumption composites Y_t using intermediate outputs $Y_t(j)$ with a CES aggregator:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ is the elasticity of substitution across differentiated intermediate goods. The static cost-minimization problem of the final good firm yields a demand function for each variety of intermediate output

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t, \tag{4.16}$$

and the aggregate price index satisfies

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}. \tag{4.17}$$

Firms that produce intermediate goods face monopolistic competition with the demand function given by (4.16) and are subject to price rigidity per Calvo (1983). In any given period, with a probability $1-\phi$, a firm is able to re-adjust its price to the desired level, where $0 < \phi < 1$. In that case, it sets the optimal real price p_t^* to maximize the present discounted value of expected profits, discounting using the unconstrained household's stochastic discount factor (4.13). The optimal reset price is common across all the firms due to symmetry:

$$p_t^* = \frac{\epsilon}{\epsilon - 1} \frac{x_{1,t}}{x_{2,t}}, \tag{4.18}$$

$$x_{1,t} = mc_t Y_t + \phi \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon+1} x_{1,t+1}, \tag{4.19}$$

$$x_{2,t} = Y_t + \phi \, \mathbb{E}_t \, \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon} x_{2,t+1}, \tag{4.20}$$

where $x_{1,t}$ and $x_{2,t}$ are two auxiliary variables, and $mc_t \equiv MC_t/P_t$ denote the real marginal cost. The optimal price-setting behavior also implies the law of motions for inflation Π_t and the price dispersion $v_t \equiv \int_0^1 (P_t(j)/P_t)^{-\epsilon} dj$:

$$1 = (1 - \phi)(p_t^*)^{1-\epsilon} + \phi \Pi_t^{\epsilon-1}, \tag{4.21}$$

$$v_t = (1 - \phi)(p_t^*)^{-\epsilon} + \phi \Pi_t^{\epsilon} v_{t-1}. \tag{4.22}$$

Firm j produces differentiated intermediate goods $Y_t(j)$ using labor as inputs with a constant-returns-to-scale technology of production:

$$Y_t(j) = A_t L_t(j), \tag{4.23}$$

where A_t is an exogenous productivity disturbance, and the real marginal cost of production is

$$mc_t = \frac{w_t}{A_t}. \tag{4.24}$$

4.4 Central Bank and Fiscal Authority

The central bank sets the policy rate following a standard Taylor rule, that is,

$$\ln I_t - \ln \bar{I} = \phi_\pi (\ln \Pi_t - \ln \bar{\Pi}) + \phi_y (\ln Y_t - \ln \bar{Y}) + \hat{\delta}_{i,t}, \tag{4.25}$$

where $\bar{I}$, $\bar{\Pi}$, and $\bar{Y}$ denote the steady-state values of the nominal interest rate, inflation rate, and output, respectively. $\hat{\delta}_{i,t}$ represents exogenous monetary policy shocks.

The central bank can hold a portfolio of long-term bonds by issuing reserves RE_t and has the following balance sheet condition:

$$Q_t B_t^{CB} = RE_t. \tag{4.26}$$

It acquires bonds via QE, which we refer to as the real value of its balance sheet:

$$QE_t = Q_t b_t^{CB}, \tag{4.27}$$

where $b_t^{CB} \equiv B_t^{CB}/P_t$. The central bank can make a profit from its balance sheet policy, and it returns the surplus to the unconstrained household by a lump-sum transfer via the fiscal authority:

$$P_t T_t^{CB} = R_t Q_{t-1} B_{t-1}^{CB} - I_{t-1}^{RE} RE_{t-1}.$$

The assumption that the unconstrained household receives this surplus is purely for convenience.

The fiscal authority chooses its spending G_t and the lump-sum transfer T_t^C it makes to the constrained household. Its budget constraint is given by

$$P_t T_t^C + P_t G_t + B_{t-1}^G = Q_t B_t^G - \kappa Q_t B_{t-1}^G + P_t T_t^G, \tag{4.28}$$

where $T_t^G \equiv T_t + \varrho Q_{t-1} b_{t-1}^G$ summarizes the lump-sum tax collected from unconstrained household. It has two components: T_t is the portion of the taxes used to finance fiscal stimulus, and it satisfies

$$T_t \equiv \eta(T_t^C + G_t).$$

The second term, $\varrho Q_{t-1} b_{t-1}^G$, represents fiscal responsibility in the sense that the government is mandated to collect additional taxes that amounts to a fraction ϱ of outstanding debt as of $t-1$. A similar term appears in Woodford (2003, chap. 4.4). The following condition guarantees the full model to have a determinate equilibrium:

$$\frac{1}{\beta^C} - 1 < \varrho < \frac{1}{\beta^C} + 1; \tag{4.29}$$

for derivation, see Appendix D. Note this condition does not affect the reduced linear model in Section 3.

4.5 Equilibrium

The market-clearing condition on final goods is

$$Y_t = C_t + C_t^C + G_t. \tag{4.30}$$

The labor market clearing condition is

$$L_t = \int_0^1 L_t(j)dj,$$

and the aggregate production function on final goods is

$$Y_t v_t = A_t L_t. \tag{4.31}$$

The asset market also clears. The FI holds all the reserves and deposits: $RE_t = RE_t^{FI}$ and $D_t = D_t^{FI}$. The market clearing condition for long-term bonds is[10]

$$B_t^G + B_t^C = B_t^{FI} + B_t^{CB}. \tag{4.32}$$

[10] Since the public and private bonds are equivalent, only one market-clearing condition exists for long-term bonds, and the portfolio compositions of the central bank and the FI are indetermined and irrelevant.

To close the model, we need to make an assumption on the transfer from the unconstrained household to the constrained household. Our assumption is similar to the "full bailout" assumption in Sims, Wu and Zhang (2021). With this assumption, the constrained household's budget constraint (4.7) becomes

$$C_t^C = \Theta \bar{X}^{FI} + QE_t + T_t^C - (1-\eta)\left[T_t^C + G_t\right], \tag{4.33}$$

where $\Theta \bar{X}^{FI}$ comes from the leverage constrained in (4.12), and in the equilibrium, it always binds. The consumption of the constrained household is determined by several policies: QE, transfers, and government spending. Note, this assumption is purely for the purpose of tractability, and is inconsequential for our results; see Appendix F for a robustness check.

In summary, given the exogenous processes of $\{A_t, QE_t, T_t^C, G_t, \hat{\delta}_{i,t}\}$, the equilibrium is fully characterized by the following system of equations:

1. unconstrained household: (4.3) - (4.4);
2. constrained household: (4.5) and (4.8);
3. financial intermediary: (4.9) - (4.10) and (4.12) - (4.15);
4. firms: (4.18) - (4.22) and (4.24);
5. government: (4.25) - (4.28);
6. market-clearing conditions: (4.30) - (4.33).

Denote the real value of reserves and deposits as $re_t \equiv RE_t/P_t$ and $d_t \equiv D_t/P_t$, respectively. The equilibrium system includes 24 equations with 24 endogenous variables $\{L_t, C_t, w_t, \Lambda_{t-1,t}, I_t, \Pi_t, C_t^C, R_t, Q_t, b_t^C, X_t^{FI}, b_t^{FI}, re_t, d_t, I_t^{RE}, \Omega_t, p_t^*, x_{1,t}, x_{2,t}, mc_t, Y_t, v_t, b_t^{CB}, b_t^G\}$. Under zero steady-state inflation, this system can be reduced to the IS curve (3.1) and Phillips curve (3.2); see Appendix A for derivations.

4.6 Discussion of Transmission Mechanisms

After setting up the full model, this section discusses the transmission mechanisms of the results we highlighted in Section 3.

4.6.1 Propositions 1 and 3: Similarities and Redistribution

With tax-financed fiscal policy, $\eta = 1$, (4.33) becomes

$$C_t^C = \Theta \bar{X}^{FI} + QE_t + T_t^C. \tag{4.34}$$

The constrained household's consumption is financed by issuing long-term bonds (held by the financial intermediary and by the central bank through QE), and receiving fiscal transfers. The leverage constraint limits bond demand, which in turn restricts how much the constrained household can borrow to consume. When the central bank implements QE, it allows the household to increase its consumption by issuing more private bonds.

Similarly, tax-financed fiscal transfers also relax the constrained household budget constraint in (4.34). This result is different from a standard model, in which transfer policy has no effect on the aggregate economy. However, Section 2 shows that transfer policy is non-neutral in the data. What makes our model feature a non-neutrality result for fiscal transfer? In our model, the government makes a transfer to the *constrained* household, which allows it to increase consumption one for one and hence stimulates the aggregate demand. Meanwhile, the lump-sum tax is imposed on the *unconstrained* (standard) household, which does not alter the equilibrium conditions.

On the other hand, government expenditures do not alter the constrained household's behavior. Instead, it enters the aggregate resource constraint directly. If we plug (4.34) into (4.30),

$$Y_t = C_t + \Theta \bar{X}^{FI} + QE_t + T_t^C + G_t. \tag{4.35}$$

Equation (4.35) implies tax-financed government expenditures affect the aggregate demand similarly to QE or lump-sum transfers, although they work through different channels.

Similar to government spending, conventional monetary policy does not have such a redistribution effect. The reason is that the conventional monetary policy affects only the unconstrained household via its Euler equation (4.4) but not the constrained household; see (4.34). In principle, changing the policy rate redistributes wealth between the unconstrained household and the financial intermediary. But since the household owns the intermediary, this redistribution is inconsequential.

4.6.2 Proposition 2: Implication on Inflation

The reason the policy instruments other than conventional monetary policy—QE, tax-financed fiscal transfers, and government spending—put downward pressure on inflation through the Phillips curve is that stimulative policy crowds out the consumption of the unconstrained household; see (4.35). The income effect makes the household supply more labor, which drives down the wage and thus the real marginal cost of production.

4.6.3 Proposition 4: The Breakdown of the Ricardian Equivalence

When the fiscal policy is fully debt financed, $\eta = 0$. Impose this condition on the consumption of the constrained household in equation (4.33), and the aggregate resource constraint (4.30) becomes

$$Y_t = C_t + \Theta \bar{X}^{FI} + QE_t. \tag{4.36}$$

The terms for government lump-sum transfers and expenditures drop out, and the equilibrium output only depends on the consumption of unconstrained household and QE policy. Therefore, debt-financed fiscal policy, whether it's the transfer policy or government spending, has no impact on the economy. This result provides an explanation for Lemma 1. By contrast, we have argued tax-financed fiscal policy stimulates the aggregate demand in a similar manner to QE in Section 4.6.1.

The debt-financed fiscal policy is less effective because the FI is subject to a leverage constraint. This assumption, together with exogenous QE, makes the total demand of long-term bonds exogenous. Whereas fiscal policy is stimulative on aggregate demand, issuing long-term bonds to finance fiscal stimulus crowds out private bonds issued by the constrained household one for one, which reduces its consumption and hence is contractionary. The debt issuance of the government acts like QE in reverse (or quantitative tightening).

5 Optimal Policy Coordination

Whereas we compared individual ad hoc policies in Section 3, in this section, we study the optimal policy coordination. First, we characterize the first-best efficient allocation for a social planner's problem. Then, we use policy instruments to correct the steady-state distortion from the financial friction, which results in an efficient steady state. Next, we derive a quadratic welfare loss function. Finally, we characterize the optimal stabilization policy that minimizes the welfare loss.

We consider three types of exogenous shocks. Besides the productivity shock $\hat{a}_t \equiv \ln A_t - \ln \bar{A}$, we introduce a shock to the leverage ratio in (4.12) and label it credit shock $\hat{\theta}_t \equiv \ln \Theta_t - \ln \bar{\Theta}$. Finally, we add an aggregate demand shock $\hat{\xi}_t$ to the IS curve (3.1). We assume they all follow AR(1) processes.

5.1 The First-Best Efficient Allocation

A social planner maximizes a weighted sum of the expected utilities of both types of households with the timeless perspective following Woodford (2003):

$$W = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} \right] + \delta \frac{(C_t^C)^{1-\sigma} - 1}{1-\sigma} \right\}, \tag{5.1}$$

subject to the resource constraint

$$C_t + C_t^C + G_t = A_t L_t, \tag{5.2}$$

where δ captures the weight the social planner puts on the constrained household relative to the unconstrained household.

The first-order conditions yield

$$C_t^{-\sigma} = \delta(C_t^C)^{-\sigma}, \tag{5.3}$$

$$\frac{\psi L_t^{\chi}}{C_t^{-\sigma}} = A_t, \tag{5.4}$$

$$G_t = 0. \tag{5.5}$$

Equation (5.3) describes the efficient consumption allocation between the two households, and (5.4) characterizes the optimal level of production. Equation (5.5) requires zero government spending because it crowds out private consumption, increases labor supply, while does not provide any additional utility to either household.[11]

Equations (5.2) - (5.5), together with $Y_t = A_t L_t$, imply the following efficient level of output after log-linearization:

$$\hat{y}_t^e \equiv \frac{1+\chi}{\sigma+\chi}\hat{a}_t. \tag{5.6}$$

For derivation, see Appendix G.1.

5.2 Steady State and Flexible-Price Equilibrium

For the steady state to be efficient efficient, we need to correct for two types of distortion: one from monopolistic competition and the other from financial friction. Without price rigidity (that is, $\phi = 0$), the firm's problem in (4.18) - (4.20) and (4.24) implies the following steady-state relationship:

$$\bar{P} = \frac{\epsilon}{\epsilon - 1}(1-\tau_p)\frac{\bar{W}}{\bar{A}}. \tag{5.7}$$

[11] Alternatively, one could introduce government spending into the utility function; for instance, see Woodford (2011). In this case, optimal government spending would be non-zero. Nevertheless, all our main conclusions regarding optimal stabilization policy are robust in this alternative scenario.

Note, we follow the literature and add the new term $1 - \tau_p$, which captures the government subsidy paid to firms to correct the distortion of markup from monopolistic competition. The government finances this subsidy through contemporaneous lump-sum taxes on the unconstrained household. Common in the literature, we assume $\tau_p = 1/\epsilon$, which implies $\bar{A} = \frac{\bar{W}}{\bar{P}}$. Consequently, the labor supply at the steady state implied by (4.3) now achieves its efficient level characterized by (5.4). We also impose $\bar{G} = 0$ per the efficient allocation in (5.5). To satisfy the efficient allocation described by (5.3), we further impose a condition on the steady-state QE or fiscal transfer, which determines the constrained household's consumption per (4.33) and hence its marginal utility. This condition corrects for the financial market distortion at the steady state.

Although the steady state is efficient with the aforementioned conditions, the output in the flexible-price equilibrium (under only productivity shocks)

$$\hat{y}_t^f \equiv \frac{(1+\chi)(1-z)}{(1-z)\chi + \sigma} \hat{a}_t \tag{5.8}$$

differs from its efficient level in (5.6). $z \equiv \bar{C}^C/(\bar{C} + \bar{C}^C)$ represents the share of the constrained household's consumption in total private expenditure at the steady state. The flexible-price output becomes efficient only when the share of constrained household approaches zero, $z = 0$. The flexible-price equilibrium is not efficient in our model because the constrained household can only borrow up to a limit. Specifically, under flexible prices, the constrained household's consumption does not achieve its efficient level characterized by (5.3). For detailed derivations, see Appendix G.1.

5.3 Utility-Based Quadratic Welfare Loss Function

Following Woodford (2003), maximizing the second-order approximation to the social planner's objective (5.1) is equivalent to minimizing the following quadratic welfare loss function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \lambda_{agg} (\hat{y}_t - \hat{y}_t^e)^2 + \lambda_{disp} var_i(\hat{c}_t^i) \right], \tag{5.9}$$

where $\lambda_{agg} \equiv \gamma(\chi + \sigma)/\epsilon$, $\lambda_{disp} \equiv \sigma\gamma/\epsilon$, and $var_i(\hat{c}_t^i)$ measures the cross-sectional dispersion of consumption between two types of households, which is defined as[12]

$$var_i(\hat{c}_t^i) \equiv (1 - z)(\hat{c}_t - \hat{y}_t)^2 + z(\hat{c}_t^C - \hat{y}_t)^2. \tag{5.10}$$

The welfare loss function (5.9) includes three terms: aggregate inflation, aggregate (welfare-relevant) output gap, and cross-sectional consumption dispersion. As a result of household heterogeneity, the optimal coordinated policy needs to stabilize not only aggregate fluctuations but also cross-sectional dispersion. Note the weight δ in the social planner's objective (5.1) does not affect the welfare loss function and hence the optimal stabilization policy, though it does affect the steady-state efficient allocation. Details of deriving (5.9) can be found in Appendix G.2.

5.4 Dual Stability

Let us first consider the scenario in which the policymakers only care about stabilizing aggregate fluctuations, which are captured by the first two variances in (5.9), as in a standard representative-agent New Keynesian model.

Rewrite the IS curve (3.1) and the Phillips curve (3.2) by incorporating the three types

[12]Note $G_t = 0$, as a necessary condition of optimal stabilization policy, implies $C_t + C_t^C = Y_t$, which allows us to use $\hat{y}_t$ to capture the average consumption. Specifically, we have $\hat{y}_t = (1 - z)\hat{c}_t + z\hat{c}_t^C$.

of exogenous shocks $\{\hat{a}_t, \hat{\xi}_t, \hat{\theta}_t\}$ and imposing $\hat{g}_t = 0$:

$$\begin{aligned}
\hat{y}_t &= \mathbb{E}_t\, \hat{y}_{t+1} - \frac{\vartheta}{\sigma}(\hat{i}_t - \mathbb{E}_t\, \hat{\pi}_{t+1}) \\
&\quad + \left[\mathcal{Q}\hat{\theta}_t + \widehat{qe}_t + \eta\hat{\tau}_t^C\right] - \mathbb{E}_t\left[\mathcal{Q}\hat{\theta}_{t+1} + \widehat{qe}_{t+1} + \eta\hat{\tau}_{t+1}^C\right] + \hat{\xi}_t, \qquad (5.11) \\
\hat{\pi}_t &= \beta\, \mathbb{E}_t\, \hat{\pi}_{t+1} + \gamma\zeta\hat{y}_t - \frac{\gamma\sigma}{\vartheta}\left[\mathcal{Q}\hat{\theta}_t + \widehat{qe}_t + \eta\hat{\tau}_t^C\right] - \gamma(1+\chi)\hat{a}_t, \qquad (5.12)
\end{aligned}$$

where $\mathcal{Q}$ represents the ratio of the value of bonds held by the financial intermediary to output at the steady state. Details of deriving (5.11) - (5.12) can be found in Appendix G.3.

By (5.11) - (5.12) and the definition of parameter ζ, dual stability $\hat{\pi}_t = 0$ and $\hat{y}_t = \hat{y}_t^e$ requires

$$\widehat{qe}_t + \eta\hat{\tau}_t^C = \frac{\vartheta}{\sigma}\left[\zeta\hat{y}_t^e - (1+\chi)\hat{a}_t\right] - \mathcal{Q}\hat{\theta}_t \qquad (5.13)$$

and

$$\hat{i}_t = \frac{\sigma}{\vartheta}\hat{\xi}_t - \sigma\left[\hat{y}_t^e - \mathbb{E}_t\, \hat{y}_{t+1}^e\right] = \frac{\sigma}{\vartheta}\hat{\xi}_t - \sigma(1-\rho_a)\hat{y}_t^e, \qquad (5.14)$$

where ρ_a is the persistence of productivity shock. The optimal policies characterized by (5.13) and (5.14) fully stabilize aggregate fluctuations, and (5.13) further suggests QE and fiscal transfers are isomorphic. These policy prescriptions have the following implications for the Divine Coincidence:

Lemma 2

1. *The Divine Coincidence holds for the demand shock, and the policy rate can fully stabilize aggregate fluctuations.*

2. *The Divine Coincidence breaks for the supply shock, and it requires at least two types of policy instruments to stabilize aggregate fluctuations: an interest rate policy together with QE or fiscal transfers.*

The first part of Lemma 2 is a standard result in the literature. By contrast, the second part deviates from the literature because the flexible-price equilibrium is not efficient and the gap between y_t^e and y_t^f acts as a cost-push shock. As a result, the Divine Coincidence does not hold, which works similarly to a cost-push shock in a standard New Keynesian model.

Next, we turn to the credit shock, which does not exist in a standard New Keynesian model:

Lemma 3 *Whereas the conventional interest rate policy cannot fully stabilize the aggregate fluctuations caused by the credit shock, QE and fiscal transfers can.*

5.5 Triune Stability

The micro-founded welfare loss function (5.9) argues policy-makers should not only stabilize aggregate fluctuations but also cross-sectional dispersion. To achieve all three objects, we have the following result:

Proposition 5 *Two types of policy instruments, that is, the policy rate together with QE or fiscal transfers, can stabilize three types of shocks and achieve three targets: zero inflation, output gap, and cross-sectional consumption dispersion.*

Proof: see Appendix G.4.

The optimal coordinated policies to achieve dual stability (5.13) and (5.14), together with the constrained household's consumption (4.33) and the resource constraint (5.2), directly lead to zero consumption dispersion $var_i(\hat{c}_t^i) = 0$. In particular, QE and fiscal transfers gauge the redistribution of consumption between the two types of households, as emphasized in Proposition 3, and the condition in (5.13) ensures zero consumption dispersion. Therefore, the policy mix that achieves dual stability leads automatically to triune stability.

6 Conclusion

We propose a tractable New Keynesian model that features monetary policy (the policy rate and QE) and fiscal policy (lump-sum transfers and government spending). In our model, we have two types of households: the unconstrained household saves via one-period deposits, and the constrained household borrows by issuing long-term bonds. The financial market is segmented. A financial intermediary performs maturity transformation and faces a leverage constraint. The government makes transfers to the constrained households or purchases final goods. To finance its policy, the government either collects taxes from the unconstrained household or issues long-term bonds. The central bank implements monetary policy by varying the policy rate and purchasing assets.

Comparing individual policies yields four findings, which are consistent with the empirical literature. First, when the fiscal policy is financed by lump-sum taxes, transfers and government spending have the same effects on aggregate variables as QE. The non-neutrality result of fiscal transfers is consistent with the empirical findings. Second, conventional monetary policy is more inflationary than other policy tools. The policy responses of the 2021-2022 inflation surge, a tightening conventional monetary policy and a stimulative fiscal policy, could lower inflation without interfering much with real activity. Third, QE and tax-financed transfer policy have direct redistribution effects, whereas government spending and conventional monetary policy do not. Fourth, Ricardian equivalence breaks, and tax-financed fiscal policy is more expansionary than debt-financed policy. This result is consistent with the empirical finding of a state-dependent fiscal multiplier. It also reconciles with the QE literature that the central bank's balance sheet is the sufficient statistic for the joint balance sheet policy between the monetary and fiscal authorities.

Finally, we discuss optimal stabilization policy coordination. We begin with dual stabilization on inflation and the welfare-relevant output gap, and we find the following: the Divine Coincidence holds for the demand shock but not for the productivity shock; in response to the credit shock, the government ought to respond with QE or fiscal transfers instead of conventional monetary policy. Further, achieving dual stability directly leads to triune stability in our model, that is, two types of policy instruments, the policy rate together with QE or transfers, can stabilize three targets in the micro-founded welfare loss function:

variances of inflation, the output gap, and cross-sectional consumption dispersion.

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A Derivation of the Small-Scale Linear Model

In this appendix, we log-linearize the full nonlinear model in Section 4 and reduce it to the small-scale linear model in Section 3.

A.1 Log-Linearization

We first linearize the full model around the steady state with zero trend inflation (i.e., $\bar{\Pi} = 1$). A variable with an overline but without a t subscript stands for its non-stochastic steady state. We use lowercase with a hat for the log deviation from the steady state with a few exceptions: we define policy instruments and asset positions relative to the steady-state level of output, that is,

$$\hat{\tau}_t^C \equiv \frac{T_t^C - \bar{T}^C}{\bar{Y}}, \qquad \hat{g}_t \equiv \frac{G_t - \bar{G}}{\bar{Y}}, \qquad \widehat{qe}_t \equiv \frac{QE_t - \bar{QE}}{\bar{Y}},$$

$$\hat{b}_t^C \equiv \frac{b_t^C - \bar{b}^C}{\bar{Y}}, \qquad \hat{d}_t \equiv \frac{d_t - \bar{d}}{\bar{Y}}, \qquad \widehat{re}_t \equiv \frac{re_t - \bar{re}}{\bar{Y}},$$

and similarly for $\hat{b}_t^G$, $\hat{b}_t^{CB}$, and $\hat{b}_t^{FI}$. Further, define the following parameters:

$$\varphi \equiv \frac{\bar{G}}{\bar{Y}}, \qquad z \equiv \frac{\bar{C}^C}{\bar{C} + \bar{C}^C}.$$

Log-linearize the equilibrium conditions in Section 4.5, and we get the following:

$$\chi \hat{l}_t = -\sigma \hat{c}_t + \widehat{w}_t \tag{A.1}$$

$$-\sigma \hat{c}_t = -\sigma E_t(\hat{c}_{t+1}) + \hat{i}_t - E_t \hat{\pi}_{t+1} \tag{A.2}$$

$$\hat{r}_t = \frac{\kappa}{\bar{R}} \hat{q}_t - \hat{q}_{t-1} \tag{A.3}$$

$$-\sigma \hat{c}_t^C = -\sigma E_t(\hat{c}_{t+1}^C) + E_t \hat{r}_{t+1} - E_t \hat{\pi}_{t+1} \tag{A.4}$$

$$\hat{d}_t = \frac{(1-\kappa)\bar{Q}\bar{b}^{FI}}{\bar{Y}} \hat{q}_t + \bar{Q}\hat{b}_t^{FI} - \kappa \bar{Q}\hat{b}_{t-1}^{FI} + \frac{\kappa \bar{Q}\bar{b}^{FI}}{\bar{Y}} \hat{\pi}_t + \widehat{re}_t \tag{A.5}$$

$$0 = \frac{\bar{b}^{FI}}{\bar{Y}}\hat{q}_t + \widehat{b}_t^{FI} \tag{A.6}$$

$$\hat{\lambda}_{t,t+1} = -\sigma(\hat{c}_{t+1} - \hat{c}_t) - \hat{\pi}_{t+1} \tag{A.7}$$

$$\hat{\omega}_t = E_t\hat{\lambda}_{t,t+1} + \frac{\bar{R}}{\bar{R} - \bar{I}}E_t\hat{r}_{t+1} - \frac{\bar{I}}{\bar{R} - \bar{I}}\hat{i}_t \tag{A.8}$$

$$\hat{i}_t^{RE} = \hat{i}_t \tag{A.9}$$

$$\widehat{p}_t^* = \widehat{x}_{1,t} - \widehat{x}_{2,t} \tag{A.10}$$

$$\widehat{x}_{1,t} = (1 - \phi\beta)\widehat{mc}_t + (1 - \phi\beta)\hat{y}_t + \phi\beta E_t\hat{\lambda}_{t,t+1} + (1 + \epsilon)\phi\beta E_t\hat{\pi}_{t+1} + \phi\beta E_t\widehat{x}_{1,t+1} \tag{A.11}$$

$$\widehat{x}_{2,t} = (1 - \phi\beta)\hat{y}_t + \phi\beta E_t\hat{\lambda}_{t,t+1} + \epsilon\phi\beta E_t\hat{\pi}_{t+1} + \phi\beta E_t\widehat{x}_{2,t+1} \tag{A.12}$$

$$\hat{\pi}_t = \frac{1 - \phi}{\phi}\widehat{p}_t^* \tag{A.13}$$

$$\hat{v}_t = 0 \tag{A.14}$$

$$\hat{w}_t = \widehat{mc}_t + \hat{a}_t \tag{A.15}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t + \hat{\delta}_{i,t} \tag{A.16}$$

$$\frac{\bar{Q}\bar{b}^{CB}}{\bar{Y}}\hat{q}_t + \bar{Q}\hat{b}_t^{CB} = \widehat{re}_t \tag{A.17}$$

$$\widehat{qe}_t = \widehat{re}_t \tag{A.18}$$

$$(1 - \eta)[\hat{\tau}_t^C + \hat{g}_t] = \bar{Q}\hat{b}_t^G + (\varrho - \frac{1}{\beta^C})\bar{Q}\hat{b}_{t-1}^G + \frac{\bar{Q}\bar{b}^G}{\bar{Y}}\hat{q}_t + \frac{\bar{Q}\bar{b}^G}{\bar{Y}}(\varrho - \frac{1}{\beta^C})\hat{q}_{t-1} - \frac{\bar{Q}\bar{b}^G}{\beta^C\bar{Y}}(\hat{r}_t - \hat{\pi}_t) \tag{A.19}$$

$$(1 - \varphi)[(1 - z)\hat{c}_t + z\hat{c}_t^C] + \hat{g}_t = \hat{y}_t \tag{A.20}$$

$$\hat{v}_t + \hat{y}_t = \hat{a}_t + \hat{l}_t \tag{A.21}$$

$$\hat{b}_t^G + \hat{b}_t^C = \hat{b}_t^{FI} + \hat{b}_t^{CB} \tag{A.22}$$

$$(1 - \varphi)z\hat{c}_t^C = \widehat{qe}_t + \hat{\tau}_t^C - (1 - \eta)[\hat{\tau}_t^C + \hat{g}_t]. \tag{A.23}$$

Given the exogenous processes of $\{\hat{a}_t, \widehat{qe}_t, \hat{\tau}_t^C, \hat{g}_t, \hat{\delta}_{i,t}\}$, (A.1) - (A.23) is a system of 23 equations with 23 variables $\{\hat{l}_t, \hat{c}_t, \hat{w}_t, \hat{\lambda}_{t-1,t}, \hat{i}_t, \hat{\pi}_t, \hat{c}_t^C, \hat{r}_t, \hat{q}_t, \hat{b}_t^C, \hat{b}_t^{FI}, \widehat{re}_t, \hat{d}_t, \hat{i}_t^{RE}, \hat{\omega}_t, \hat{p}_t^*, \hat{x}_{1,t}, \hat{x}_{2,t}, \widehat{mc}_t,$ $\hat{y}_t, \hat{v}_t, \hat{b}_t^{CB}, \hat{b}_t^G\}$. Note that for simplicity, we have eliminated the variable X_t^{FI} using (4.9).

To complete the model, we assume the exogenous variables follow an AR(1) process:

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + s_a \varepsilon_{a,t} \tag{A.24}$$

$$\widehat{qe}_t = \rho_q \widehat{qe}_{t-1} + s_q \varepsilon_{q,t} \tag{A.25}$$

$$\hat{\tau}_t^C = \rho_\tau \hat{\tau}_{t-1}^C + s_\tau \varepsilon_{\tau,t} \tag{A.26}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + s_g \varepsilon_{g,t} \tag{A.27}$$

$$\hat{\delta}_{i,t} = \rho_i \hat{\delta}_{i,t-1} + s_i \varepsilon_{i,t}, \tag{A.28}$$

where $\varepsilon_{a,t}$, $\varepsilon_{q,t}$, $\varepsilon_{\tau,t}$, $\varepsilon_{g,t}$, and $\varepsilon_{i,t}$ are productivity, QE, fiscal transfer, fiscal spending, and monetary policy shocks, respectively. Parameters s_a, s_q, s_τ, s_g, and s_i capture the associated standard deviations.

A.2 System Reduction

Next, we reduce the system and derive the IS curve (3.1) and the New Keynesian Phillips curve (3.2).

Combining (A.10) - (A.13) yields the standard New Keynesian Phillips curve of the form

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \gamma \widehat{mc}_t, \tag{A.29}$$

where $\gamma \equiv (1-\phi)(1-\beta\phi)/\phi$. Further combining (A.1), (A.14), (A.15), and (A.21), we have

$$\widehat{mc}_t = \chi \hat{y}_t + \sigma \hat{c}_t - (1+\chi)\hat{a}_t. \tag{A.30}$$

Note that (A.20) and (A.23) imply the consumption of the constrained household is given by

$$\hat{c}_t^C = \frac{1}{(1-\varphi)z}\{\widehat{qe}_t + \hat{\tau}_t^C - (1-\eta)\left[\hat{\tau}_t^C + \hat{g}_t\right]\}, \tag{A.31}$$

and the consumption of the unconstrained household is given by

$$\hat{c}_t = \frac{1}{(1-\varphi)(1-z)}\{\hat{y}_t - \eta\left[\hat{\tau}_t^C + \hat{g}_t\right] - \widehat{qe}_t\}. \tag{A.32}$$

Substitute $\hat{c}_t$ into (A.30), and we get

$$\widehat{mc}_t = \zeta\hat{y}_t - \frac{\sigma}{(1-z)(1-\varphi)}\left[\widehat{qe}_t + \eta(\hat{\tau}_t^C + \hat{g}_t)\right] - (1+\chi)\hat{a}_t, \tag{A.33}$$

where $\zeta \equiv \chi + \frac{\sigma}{(1-\varphi)(1-z)}$. Then, substituting $\widehat{mc}_t$ into (A.29) yields

$$\hat{\pi}_t = \beta\,\mathbb{E}_t\,\hat{\pi}_{t+1} + \gamma\zeta\hat{y}_t - \frac{\gamma\sigma}{(1-z)(1-\varphi)}\left[\widehat{qe}_t + \eta(\hat{\tau}_t^C + \hat{g}_t)\right] - \gamma(1+\chi)\hat{a}_t. \tag{A.34}$$

Abstracting from the productivity shock, we get the New Keynesian Phillips curve in (3.2), where $\vartheta \equiv (1-z)(1-\varphi)$.

Combining (A.2), (A.20), and (A.31) yields

$$\begin{aligned}\hat{y}_t - E_t\hat{y}_{t+1} &= \hat{g}_t - E_t\hat{g}_{t+1} + (1-\varphi)(1-z)\left[\hat{c}_t - E_t\hat{c}_{t+1}\right] + (1-\varphi)z\left[\hat{c}_t^C - E_t\hat{c}_{t+1}^C\right] \\ &= \hat{g}_t - E_t\hat{g}_{t+1} - \frac{(1-z)(1-\varphi)}{\sigma}\left[\hat{i}_t - \mathbb{E}_t\,\hat{\pi}_{t+1}\right] + \{\widehat{qe}_t + \hat{\tau}_t^C - (1-\eta)\left[\hat{\tau}_t^C + \hat{g}_t\right]\} \\ &\quad - E_t\{\widehat{qe}_{t+1} + \hat{\tau}_{t+1}^C - (1-\eta)\left[\hat{\tau}_{t+1}^C + \hat{g}_{t+1}\right]\}.\end{aligned}$$

Rearranging gives the New Keynesian IS curve in (3.1).

B Proofs

B.1 Tractable Linear Model

We first solve the tractable linear model characterized by (3.1) - (3.3) and (A.25) - (A.28). To facilitate a comparison across policy instruments, we further assume the autoregressive coefficients of all exogenous processes are the same: $\rho \equiv \rho_i = \rho_q = \rho_\tau = \rho_g \in (0,1)$.

Conjecture a solution of the following form:

$$\hat{y}_t = \mu_i \hat{\delta}_{i,t} + \mu_q \widehat{qe}_t + \mu_\tau \hat{\tau}_t^C + \mu_g \hat{g}_t,$$

$$\hat{\pi}_t = \nu_i \hat{\delta}_{i,t} + \nu_q \widehat{qe}_t + \nu_\tau \hat{\tau}_t^C + \nu_g \hat{g}_t.$$

Substitute these expressions into (3.1) - (3.3) and solve for the coefficients as follows:

$$\mu_q = \frac{\gamma\sigma(\phi_\pi - \rho) + \sigma(1-\rho)(1-\beta\rho)}{(1-\beta\rho)\left[\sigma(1-\rho) + \vartheta\phi_y\right] + \gamma\vartheta\zeta(\phi_\pi - \rho)} \tag{B.1}$$

$$\mu_\tau = \mu_g = \eta\mu_q \tag{B.2}$$

$$\mu_i = -\frac{\vartheta(1-\beta\rho)}{(1-\beta\rho)\left[\sigma(1-\rho) + \vartheta\phi_y\right] + \gamma\vartheta\zeta(\phi_\pi - \rho)} \tag{B.3}$$

$$\nu_q = \frac{\gamma\sigma\left[(1-\rho)(\vartheta\zeta - \sigma) - \vartheta\phi_y\right]}{\vartheta(1-\beta\rho)\left[\sigma(1-\rho) + \vartheta\phi_y\right] + \gamma\vartheta^2\zeta(\phi_\pi - \rho)} \tag{B.4}$$

$$\nu_\tau = \nu_g = \eta\nu_q \tag{B.5}$$

$$\nu_i = \frac{\gamma\zeta}{1-\beta\rho}\mu_i. \tag{B.6}$$

B.1.1 Proof of Proposition 1

When fiscal policy is fully tax financed, that is, $\eta = 1$, we have $\mu_q = \mu_\tau = \mu_g$ and $\nu_q = \nu_\tau = \nu_g$. Therefore, the effects of QE, government expenditures, and lump-sum fiscal transfers on output and inflation are the same. Moreover, given all the parameters are positive, and $\phi_\pi > 1, 0 < \rho, \beta, z < 1, 0 \leq \varphi < 1, \phi_y > 0, \sigma > 0$, we obtain $0 < \vartheta < 1$ and $\mu_q > 0$.■

B.1.2 Proof of Proposition 2

For the nominal interest rate, (B.3) and (B.6) yield

$$\frac{d\hat{\pi}_t/d\hat{\hat{\delta}}_{i,t}}{d\hat{y}_t/d\hat{\hat{\delta}}_{i,t}} = \frac{\nu_i}{\mu_i} = \frac{\gamma\zeta}{1-\beta\rho}.$$

For QE and tax-financed fiscal policy ($\eta = 1$), (B.1), (B.2), (B.4), and (B.5) yield

$$\frac{d\hat{\pi}_t/d(\widehat{qe}_t)}{d\hat{y}_t/d(\widehat{qe}_t)} = \frac{d\hat{\pi}_t/d\hat{\tau}_t^C}{d\hat{y}_t/d\hat{\tau}_t^C} = \frac{d\hat{\pi}_t/d\hat{g}_t}{d\hat{y}_t/d\hat{g}_t} = \frac{\nu_q}{\mu_q} = \frac{\gamma\zeta}{1-\beta\rho} - \frac{\gamma\sigma}{\vartheta(1-\beta\rho)\mu_q}. \tag{B.7}$$

We have shown $\vartheta > 0$ and $\mu_q > 0$ in Appendix B.1.1. Together with positive parameters and $0 < \rho, \beta < 1$, we have

$$\frac{\gamma\sigma}{\vartheta(1-\beta\rho)\mu_q} > 0.$$

Therefore,

$$\frac{d\hat{\pi}_t/d\hat{\hat{\delta}}_{i,t}}{d\hat{y}_t/d\hat{\hat{\delta}}_{i,t}} > \frac{d\hat{\pi}_t/d(\widehat{qe}_t)}{d\hat{y}_t/d(\widehat{qe}_t)} = \frac{d\hat{\pi}_t/d\hat{\tau}_t^C}{d\hat{y}_t/d\hat{\tau}_t^C} = \frac{d\hat{\pi}_t/d\hat{g}_t}{d\hat{y}_t/d\hat{g}_t}.$$

That is, to provide the same amount of stimulus on output, conventional monetary policy is more inflationary than QE and tax-financed fiscal policy.■

B.1.3 Proof of Lemma 1

When fiscal policy is fully debt financed, that is, $\eta = 0$, we have $\mu_\tau = \mu_g = 0$ and $\nu_\tau = \nu_g = 0$. Thus, debt-financed government expenditures and lump-sum transfers have no impact on aggregate output and inflation.■

B.1.4 Proof of Proposition 4

The effect of government expenditures on output is

$$\frac{d\hat{y}_t}{d\hat{g}_t} = \mu_g = \eta\mu_q. \tag{B.8}$$

This effect is an increasing function of η because $\mu_q > 0$, which we proved in Appendix B.1.1. One can also prove for transfers via the same logic.■

B.2 Proof of Proposition 3

For tax-financed fiscal policy, $\eta = 1$, the consumption of constrained household in (A.31) becomes

$$\hat{c}_t^C = \frac{1}{(1-\varphi)z}\left[\widehat{qe}_t + \hat{\tau}_t^C\right].$$

Therefore, only QE and transfers affect the constrained household's consumption.

Moreover, the consumption of the unconstrained household (A.32) becomes

$$\hat{c}_t = \frac{1}{(1-\varphi)(1-z)}\left[\hat{y}_t - \hat{\tau}_t^C - \hat{g}_t - \widehat{qe}_t\right].$$

Thus, conditional on aggregate output $\hat{y}_t$, QE and tax-financed fiscal transfers reduce the consumption of the unconstrained household by the amount of $\widehat{qe}_t$ and $\hat{\tau}_t^C$, respectively, while they increase the consumption of the constrained household by the same amount.■

C Calibration

We calibrate our model to a quarterly frequency; see Table C.1. We set the steady-state values of government spending and the fiscal lump-sum transfer to be zero, $\bar{T}^C = \bar{G} = 0$, and thus, $\varphi = 0$.

The following parameters are standard in the literature. We assume a zero inflation rate at the steady state $\bar{\Pi} = 1$, which implies the steady-state price dispersion $\bar{v}^p = 1$ and the optimal adjustment price $\bar{p}^* = 1$. The elasticity of substitution between goods is $\epsilon = 11$, implying a steady-state price markup of ten percent. The discount factor of the unconstrained household is $\beta = 0.995$, implying the annualized steady-state nominal interest of 2%. We set the relative risk aversion σ and the inverse Frisch elasticity of labor supply χ

Table C.1: Calibrated Parameter Values

Parameter	Value	Description
β	0.995	Discount factor, unconstrained household (HH)
σ	1	Inverse of intertemporal elasticity of substitution
χ	1	Inverse Frisch elasticity of labor supply
ψ	1.36	Labor disutility scaling parameter (target $\bar{L} = 1$)
β^C	0.99	Discount factor, constrained HH (target yearly spread of 200 b.p.)
$\bar{\Pi}$	1	Steady state inflation
ϵ	11	Elasticity of substitution between goods
κ	$1 - 40^{-1}$	Coupon decay (target duration ten years)
ϕ	0.75	Price inertia (Calvo parameter)
$\bar{QE}$	0.1	Steady state central bank bond portfolio
z	0.33	Steady state share of constrained HH's consumption
φ	0	Steady state government spending relative to output
$\bar{T}^C/\bar{Y}$	0	Steady state fiscal transfer relative to output
ϕ_π	1.5	Taylor rule inflation
ϕ_y	0	Taylor rule output
ϱ	0.2	Response of government surplus to debt
ρ_i	0.8	AR monetary policy shock
ρ_a	0.8	AR productivity
ρ_q	0.8	AR QE
ρ_τ	0.8	AR fiscal transfer
ρ_g	0.8	AR government spending
s_i	0.0025	SD monetary policy shock
s_a	0.0125	SD productivity
s_q	0.01	SD QE
s_τ	0.01	SD fiscal transfer
s_g	0.01	SD government spending

both to be a standard value of one. The Calvo parameter of price stickiness $\phi = 0.75$ implies an average duration of four quarters between two consecutive price adjustments.

Similar to Sims, Wu and Zhang (2021), we calibrate $\beta^C = 0.99$ and $\bar{R} = 1.01$, so that the annualized steady-state term spread between long-term bonds and short-term bonds is 200 basis points. We assume a steady-state share of the constrained household's consumption in total consumption $z = 1/3$. We calibrate the labor-disutility scaling parameter $\psi = 1.36$ to normalize steady-state labor to unity. The size of the central bank's asset holdings at the steady state is ten percent of output, that is, $\bar{QE} = 0.1\bar{Y}$. The steady-state risk-weighted leverage ratio is $\Theta = 5$. We calibrate the duration of the long-term bond to be ten years; that is, $\kappa = 1 - 40^{-1}$.

We assume parameters in the standard Taylor rule $\phi_\pi = 1.5$ and $\phi_y = 0$, and the response of government surplus to debt $\varrho = 0.2$ (following Cochrane (2021)). The autoregressive parameters are all set to $\rho_a = \rho_q = \rho_\tau = \rho_g = \rho_i = 0.8$.

D Determinacy of the Full Linear Model

In this section, we discuss the determinacy condition for the full linear model described in Appendix A.1. First, in Appendix D.1, we derive the determinacy condition for a reduced linear system, which consists of (3.1) - (3.3) and the equation for public debt evolution (A.19), for endogenous variables $\{\hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{b}_t^G\}$. Then, in Appendix D.2, we discuss how to pin down the rest of the variables.

D.1 Reduced System

In the steady state, $\bar{T}^C = \bar{G} = 0$, and thus, $\bar{b}^G = 0$ from (4.28). Consequently, $\hat{b}_t^G$ only depends on its lag and exogenous variables in (A.19). Therefore, the reduced linear system can be separated into two independent parts. First, (A.19) is determined if $|1/\beta^C - \varrho| < 1$, which is equivalent to the condition in (4.29). This condition together with (D.4) guarantees the determinancy of the reduced linear system.

Second, we can rewrite the system of (3.1) - (3.3) into

$$\mathbb{E}_t\, \boldsymbol{z}_{t+1} = A\boldsymbol{z}_t + B_t, \tag{D.1}$$

where $\boldsymbol{z}_t = [\hat{y}_t\ \hat{\pi}_t]'$ and B_t includes all exogenous variables. Note we have substituted $\hat{i}_t$ using (3.3), and the matrix A is

$$A = \begin{pmatrix} 1 + \frac{\vartheta\phi_y}{\sigma} + \frac{\vartheta\gamma\zeta}{\sigma\beta} & \frac{\vartheta\phi_\pi}{\sigma} - \frac{\vartheta}{\sigma\beta} \\ -\frac{\gamma\zeta}{\beta} & \beta^{-1} \end{pmatrix}.$$

With two non-predetermined endogenous state variables, the system (D.1) is determinate if and only if both eigenvalues of A are outside the unit cycle. The eigenvalues of A are the two roots of the characteristic equation

$$\mathcal{P}(\lambda) = \lambda^2 + A_1\lambda + A_0 = 0, \tag{D.2}$$

where $A_1 \equiv -\operatorname{tr} A$ and $A_0 \equiv \det A$. Given the signs of parameters, we have $\operatorname{tr} A > 0$ and $\det A > 0$, and thus, as shown in Woodford (2003, app. C), $\mathcal{P}(\lambda)$ has two roots outside unit cycle if and only if

$$\det A > 1, \qquad \det A - \operatorname{tr} A > -1, \qquad \det A + \operatorname{tr} A > -1, \tag{D.3}$$

which yields the condition

$$\gamma\zeta(\phi_\pi - 1) + \phi_y(1 - \beta) > 0. \tag{D.4}$$

This result is standard and is satisfied by the two restrictions we imposed on the model: $\phi_\pi > 1$ and $\phi_y > 0$.

Therefore, the determinacy condition of the reduced linear system (3.1) - (3.3) and (A.19) is (4.29) and (D.4), which is a sufficient condition for determinacy in the full model.

D.2 The Rest

Next, we show how the rest of the system in (A.1) - (A.23) is determined. Given $\{\hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{b}_t^G\}$ are determined, $\hat{c}_t^C$ is uniquely determined by (A.31), $\hat{c}_t$ is uniquely determined by (A.20), $\hat{p}_t^*$ is uniquely determined by (A.13), $\hat{q}_t$ and $\hat{r}_t$ are uniquely determined by (A.3) and (A.4), $\hat{b}_t^{FI}$ is uniquely determined by (A.6), $\hat{\lambda}_{t,t+1}$ is uniquely determined by (A.7), and $\hat{\omega}_t$ is uniquely determined by (A.8). Noting $\widehat{re}_t$ is uniquely determined by (A.18); we then have that $\hat{b}_t^{CB}$ is uniquely determined by (A.17), $\hat{b}_t^C$ is uniquely determined by (A.22), and $\hat{d}_t$ is uniquely determined by (A.5). Furthermore, because we have $\hat{v}_t = 0$ in (A.14), $\hat{l}_t$ is uniquely determined by (A.21), $\hat{w}_t$ is uniquely determined by (A.1), $\widehat{mc}_t$ is uniquely determined by (A.15),

$\hat{x}_{1,t}$ is uniquely determined by (A.11), and $\hat{x}_{2,t}$ is uniquely determined by (A.12).

E Robustness: Constrained Household's Labor Supply

In this section, we consider the alternative scenario in which the constrained household is also able to supply labor. We demonstrate that our main results are robust to this alternative assumption.

Similar to (4.1), we modify the objective function of the constrained household as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^C\right)^t \left[\frac{\left(C_t^C\right)^{1-\sigma} - 1}{1-\sigma} - \psi \frac{\left(L_t^C\right)^{1+\chi}}{1+\chi}\right].$$

The constrained household's first order condition for labor supply satisfies

$$\psi (L_t^C)^{\chi} = (C_t^C)^{-\sigma} w_t,$$

and (4.33) becomes

$$C_t^C = \Theta \bar{X}^{FI} + QE_t + T_t^C - (1-\eta)\left[T_t^C + G_t\right] + w_t L_t^C.$$

The aggregate production function on final goods is

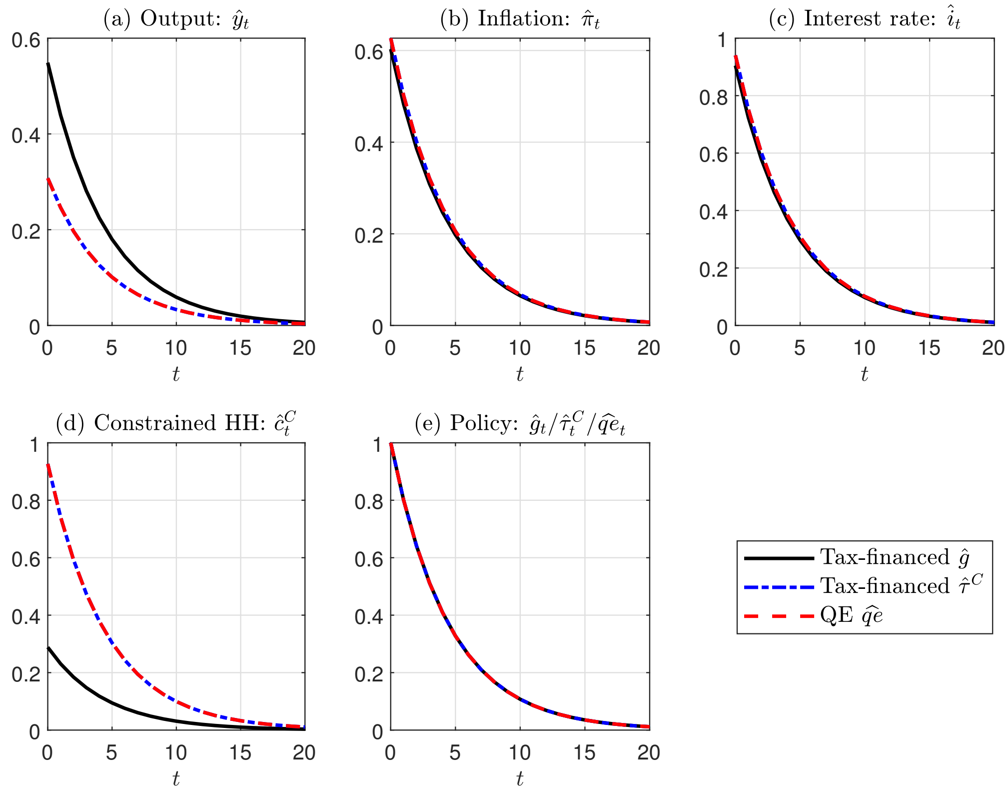
$$Y_t v_t = A_t (L_t + L_t^C).$$

All other equilibrium conditions remain the same as in Section 4.

E.1 QE, Government Spending, and Transfers

Figure E.1 shows the impulse response functions to a one-percentage-point shock to tax-financed government spending $\hat{g}_t$ (solid black lines), fiscal transfers $\hat{\tau}_t^C$ (dash-dotted blue

Figure E.1: IRFs to Tax-Financed Government Spending Shock, Fiscal Transfer Shock, and QE shock

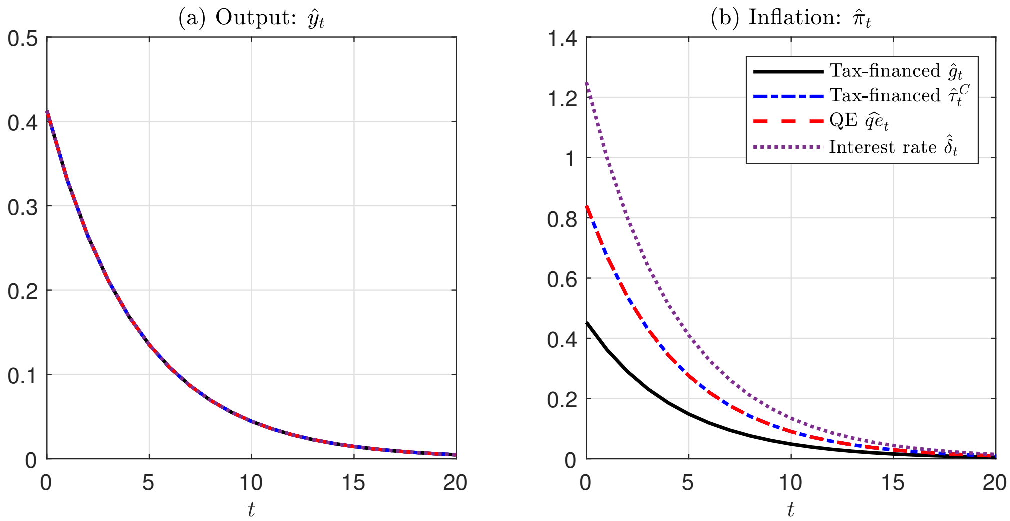


Notes: This figure shows the IRFs to a one-percentage-point shock to fully tax-financed government spending ($\hat{g}_t$), fiscal transfer ($\hat{\tau}_t^C$), and QE ($\widehat{qe}_t$), respectively. Output and inflation are reported in percentage points, while inflation and nominal interest rate are reported in annualized percentage points. The three types of policy instruments share the same AR(1) process with persistence $\rho = 0.8$.

lines), and QE $\widehat{qe}_t$ (dashed red lines).

Panels (a), (b), and (c) suggest that the effects of QE and fiscal transfers are the same on aggregate output, inflation, and the short-term nominal interest rate. Compared with these two, government spending is more stimulative on aggregate output but shares a similar effect on inflation and interest rate. This result is consistent with Proposition 1. On the other hand, one can observe from panel (d) that government spending has a smaller impact on the constrained household's consumption compared with fiscal transfers and QE, which is consistent with Proposition 3.

Figure E.2: Effects on Inflation with Tax-Financed Government Spending Shock/Transfer Shock, QE shock, and Interest Rate Shock



Notes: This figure shows the IRFs to shocks to fully tax-financed government spending ($\hat{g}_t$), fiscal transfer ($\hat{\tau}_t^C$), QE ($\widehat{qe}_t$), and nominal interest rate ($\hat{\delta}_t$), respectively. The size of the shock on nominal interest rate is minus one percentage point, and the sizes of shocks to other policy instruments are adjusted so that the initial responses of aggregate output are the same. Output and inflation are reported in percentage points, while inflation and nominal interest rate are reported in annualized percentage points. The four types of policy instruments share the same AR(1) process with persistence $\rho = 0.8$.

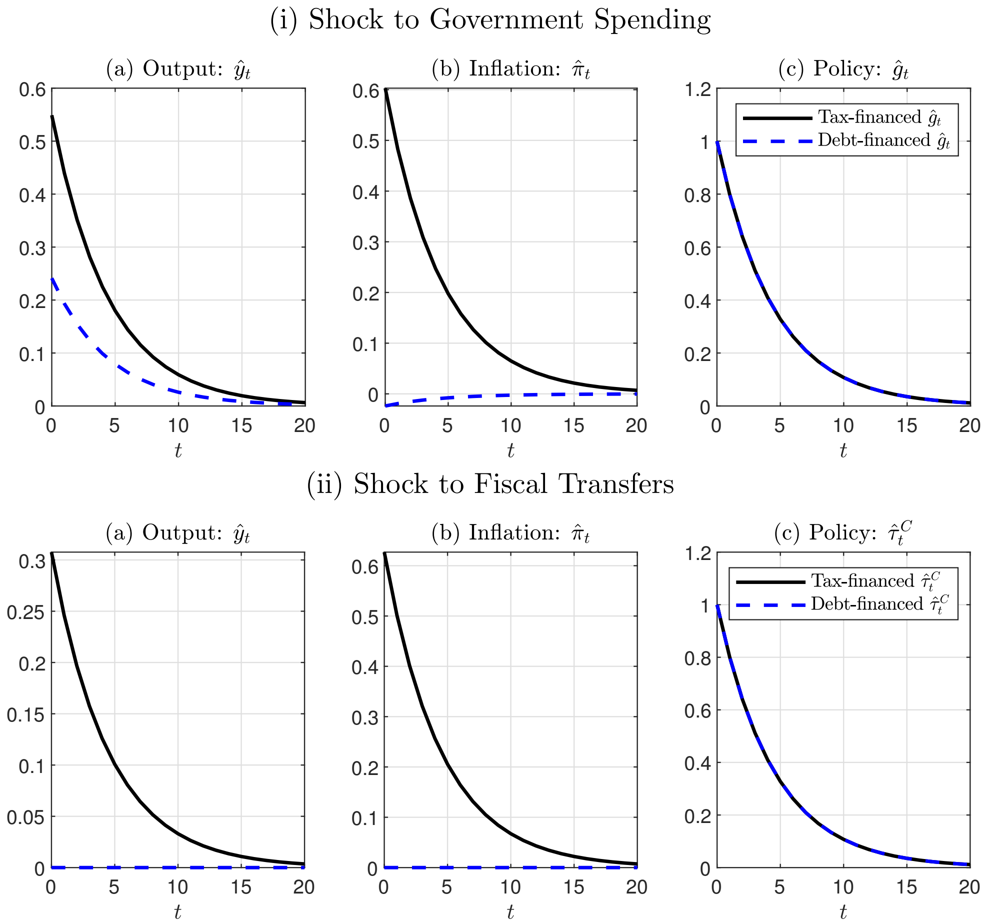
E.2 Implication on Inflation

Figure E.2 compares the effects of four policy shocks on inflation when they provide the same amount of stimulus to the aggregate output. The solid black, dash-dotted blue, dashed red, and dotted purple lines correspond to a shock to government spending, fiscal transfers, QE, and the short-term nominal interest rate, respectively. Figure E.2 suggests that for the same amount of increase in output, conventional monetary policy is more inflationary than QE and tax-financed fiscal policy, which is consistent with Proposition 2.

E.3 Tax Finance vs. Debt Finance

Figure E.3 compares the effects of fiscal policy under alternative financing approaches. It illustrates the impulse responses to a one-percentage-point shock to government spending $\hat{g}_t$ (top row) and fiscal transfers $\hat{\tau}_t^C$ (bottom row). The black solid lines represent the case of fully tax-financed policy ($\eta = 1$), and the blue dashed lines represent the fully debt-financed

Figure E.3: IRFs to Shocks on Fiscal Policy under Alternative Financing Approaches



Notes: This figure shows the IRFs to a one-percentage-point shock to government spending ($\hat{g}_t$) and fiscal transfer ($\hat{\tau}_t^C$) under tax financing and debt financing, respectively. The first row shows the IRFs to the spending shock, and the second row shows the IRFs to a transfer shock. Output and inflation are reported in percentage points, whereas inflation and the nominal interest rate are reported in annualized percentage points. The shock on fiscal transfer follows an AR(1) process with persistence $\rho = 0.8$.

case ($\eta = 0$). Comparing the black lines with the blue lines, we can see the stimulative effects are larger when fiscal policy is tax financed. This result is consistent with Proposition 4.

F Robustness: Transfer Between Households

In Appendix F.1, we describe the simplifying assumption on the transfer between households, which is used to reduce the system. In Appendix F.2, we demonstrate that our main results are robust to an alternative assumption.

F.1 Simplifying Assumption

The assumption on the transfer between the households to obtain (4.33) is

$$X_t^C = (1+\kappa Q_t)\Pi_t^{-1}(b_{t-1}^C + b_{t-1}^G) - \varrho Q_{t-1} b_{t-1}^G. \tag{F.1}$$

This transfer amounts to the total outstanding debt from the previous period plus all coupon liabilities minus the passive debt repayment by the government. This "debt payoff" condition is similar to the "full bailout" assumption in Sims, Wu and Zhang (2021).

Substituting X_t^C into the budget constraint of the constrained household (4.7) and utilizing the government's budget constraint (4.28) and the market-clearing condition of bond market (4.32), the constrained household's consumption can be rewritten as

$$C_t^C \;=\; Q_t b_t^{FI} + Q_t b_t^{CB} + T_t^C - (1-\eta)\left[T_t^C + G_t\right].$$

This equation, together with the leverage constraint of the FI (4.12) and the definition of QE_t in (4.27), implies (4.33).

F.2 Alternative: Constant Transfer between Households

For robustness checks, we assume a constant transfer $\bar{X}^C$ between the households instead of in the form of equation (F.1).

F.2.1 Equivalence between QE and Tax-Financed Fiscal Transfers

With a constant transfer between households, the constrained household's budget constraint (4.7) becomes

$$C_t^C = Q_t b_t^C - (1+\kappa Q_t)\Pi_t^{-1} b_{t-1}^C + \bar{X}^C + T_t^C.$$

Utilizing the government's budget constraint (4.28) and the bond market clearing condition (4.32), the constrained household's consumption can be rewritten as

$$\begin{aligned} C_t^C \; = \; & \left[Q_t b_t^{FI} - (1+\kappa Q_t)\Pi_t^{-1} b_{t-1}^{FI}\right] + \left[Q_t b_t^{CB} - (1+\kappa Q_t)\Pi_t^{-1} b_{t-1}^{CB}\right] \\ & + \bar{X}^C + T_t^C - (1-\eta)\left[T_t^C + G_t\right] + \varrho Q_t b_t^G. \end{aligned} \tag{F.2}$$

Since government spending G_t and fiscal transfer T_t^C are both flow variables, we redefine QE in terms of a flow:[13]

$$QE_t = Q_t b_t^{CB} - (1+\kappa Q_t)\Pi_t^{-1} b_{t-1}^{CB}. \tag{F.3}$$

With this definition, together with the leverage constraint of the FI in (4.12), we can rewrite equation (F.2) into

$$C_t^C = \left[1 - R_t \Pi_t^{-1}\right] \Theta \bar{X}^{FI} + QE_t + T_t^C - (1-\eta)\left[T_t^C + G_t\right] + \varrho Q_t b_t^G + \bar{X}^C. \tag{F.4}$$

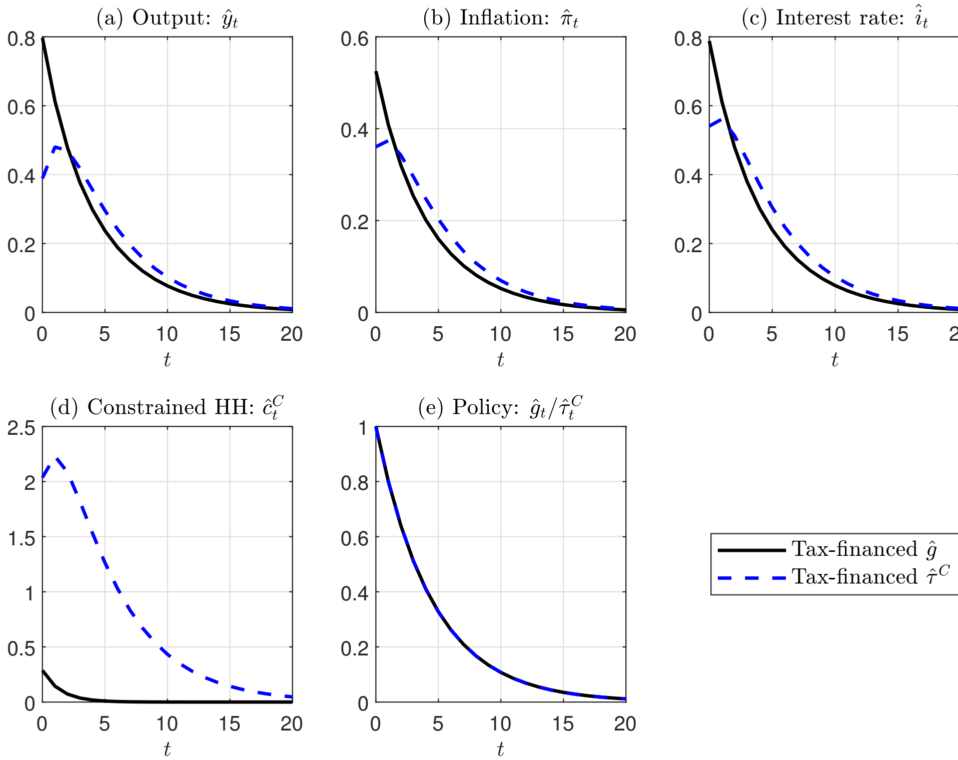
When fiscal transfers are fully tax financed, $\eta = 1$ and $b_t^G = 0$ for any t. Therefore,

$$C_t^C = \left[1 - R_t \Pi_t^{-1}\right] \Theta \bar{X}^{FI} + QE_t + T_t^C + \bar{X}^C. \tag{F.5}$$

Substituting equation (F.5) into (4.8), we can solve the real return of long-term bonds $R_t \Pi_t^{-1}$ as a function of QE and fiscal transfers. Consequently, we can solve the consumption of the constrained households as a function of QE and fiscal transfers. The resulting linear equation is analogous to (A.23), and the derivations in Appendix A.2 follow. Note, QE and transfers enter (F.5) in the same fashion, which proves their equivalence. This result shows that Proposition 1 holds exactly for QE and fiscal transfers.

[13] For the model in Section 4, to reduce the system, we do not distinguish between flows and stocks. Also note, with a flow definition of QE, to ensure determinacy under active monetary policy, we define the QE shock on the stock of reserves instead of flow QE.

Figure F.1: IRFs to Government Spending Shock/Transfer Shock under Tax Financing



Notes: This figure shows the IRFs to a one-percentage-point shock to government spending ($\hat{g}_t$) and fiscal transfer ($\hat{\tau}_t^C$), respectively, under the case of fully tax-financing $\eta = 1$. Output and inflation are reported in percentage points, while inflation and nominal interest rate are reported in annualized percentage points. The two types of policy instruments share the same AR(1) process with persistence $\rho = 0.8$.

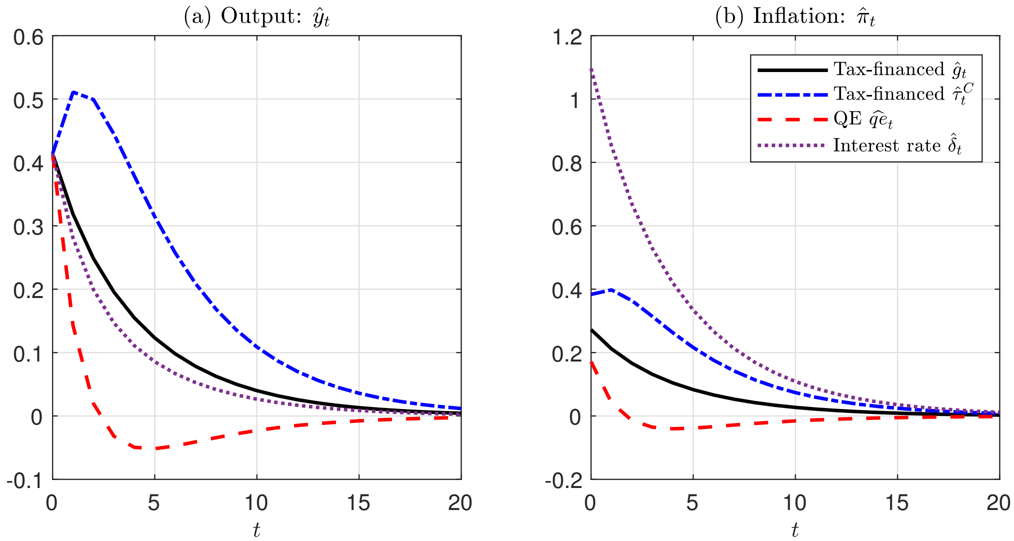
F.2.2 Government Spending vs. Fiscal Transfers

Figure F.1 shows the impulse response functions to a one-percentage-point shock to tax-financed government spending $\hat{g}_t$ (solid black lines) and fiscal transfers $\hat{\tau}_t^C$ (dashed blue lines) when the transfers between the households are constant.[14]

Panels (a), (b), and (c) suggest that the effects of these two fiscal policies are similar on aggregate output, inflation, and the interest rate, especially after the initial period, which shows Proposition 1 holds approximately without the simplifying assumption discussed in Appendix F.1. On the other hand, one can observe that government spending has much less impact on the constrained household's consumption (panel (d)) compared to fiscal transfers.

[14] We assume $\bar{X}^C$ equals the steady-state value of the transfer of the form in equation (F.1).

Figure F.2: Effects on Inflation with Tax-Financed Government Spending Shock/Transfer Shock, QE shock, and Interest Rate Shock



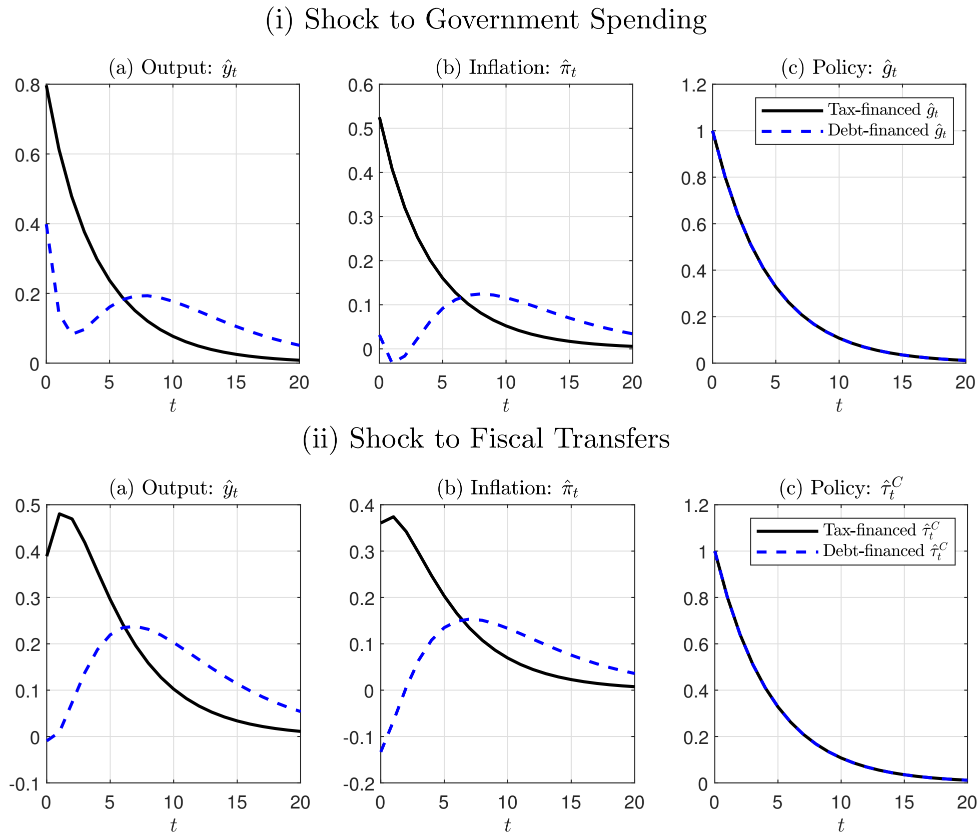
Notes: This figure shows the IRFs to shocks to fully tax-financed government spending ($\hat{g}_t$), fiscal transfer ($\hat{\tau}_t^C$), QE ($\widehat{qe}_t$), and nominal interest rate ($\hat{\delta}_t$), respectively. The size of the shock on nominal interest rate is minus one percentage point, and the sizes of shocks to other policy instruments are adjusted so that the initial responses of aggregate output are the same. Output and inflation are reported in percentage points, while inflation and nominal interest rate are reported in annualized percentage points. The four types of policy instruments share the same AR(1) process with persistence $\rho = 0.8$.

The result is consistent with Proposition 3.

F.2.3 Implication on Inflation

Figure F.2 compares the effects of four policy shocks on inflation when they provide the same amount of stimulus to the aggregate output. Specifically, we normalize the shocks such that the initial responses of output are the same. The solid black, dash-dotted blue, dashed red, and dotted purple lines correspond to a shock to government spending, fiscal transfers, QE, and the short-term nominal interest rate, respectively. Figure F.2 suggests that conventional monetary policy is more inflationary than QE and tax-financed fiscal policy, which is consistent with Proposition 2.

Figure F.3: IRFs to Shocks on Fiscal Policy under Alternative Financing Approaches



Notes: This figure shows the IRFs to a one-percentage-point shock to government spending ($\hat{g}_t$) and fiscal transfer ($\hat{\tau}_t^C$) under tax financing and debt financing, respectively. The first row shows the IRFs to the spending shock, and the second row shows the IRFs to a transfer shock. Output and inflation are reported in percentage points, whereas inflation and the nominal interest rate are reported in annualized percentage points. The shock on fiscal transfer follows an AR(1) process with persistence $\rho = 0.8$.

F.2.4 Tax Finance vs. Debt Finance

Figure F.3 compares the effects of fiscal policy under alternative financing approaches when the transfer between households is a constant. It illustrates the impulse responses to a one-percentage-point shock to government spending $\hat{g}_t$ (top row) and fiscal transfers $\hat{\tau}_t^C$ (bottom row). The black solid lines represent the case of fully tax-financed policy ($\eta = 1$), and the blue dashed lines represent the fully debt-financed case ($\eta = 0$). Comparing the black lines with the blue lines, we can see both the initial responses and stimulative effects are larger when fiscal policy is tax financed. This result is consistent with Proposition 4.

G Optimal Policy

G.1 The Efficient-Level and Flexible-Price Output

We derive the efficient level of output via equations (5.3) and (5.4). By noting $Y_t = A_t L_t$, log-linearizing (5.4) around its associated steady-state values yields

$$-\sigma \hat{c}_t = \chi \hat{y}_t - (1+\chi)\hat{a}_t. \tag{G.1}$$

Replacing C_t using (5.3), we have

$$-\sigma \hat{c}_t^C = \chi \hat{y}_t - (1+\chi)\hat{a}_t. \tag{G.2}$$

Given that $G_t = 0$ at the efficient allocation, the resource constraint (5.2) implies

$$\hat{y}_t = (1-z)\hat{c}_t + z\hat{c}_t^C. \tag{G.3}$$

Plugging (G.1) and (G.2) into (G.3), the efficient level of output is then given by (5.6).

We next derive the output in the flexible-price equilibrium only subject to productivity shocks. Under flexible prices, the log-deviation of the real marginal cost is zero. Thus, by noting $\bar{G} = 0$ and hence $\varphi = 0$, (A.33) implies the flexible-price output is given by (5.8).

G.2 Welfare Loss Function

In this section, we derive the welfare loss function with a second-order approximation to the social planner's objective (5.1) around the efficient steady state, following the approach in Woodford (2003). Denote

$$U_{C,t} = \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \qquad U_{L,t} = \psi \frac{L_t^{1+\chi}}{1+\chi}, \qquad U_{C,t}^C = \frac{(C_t^C)^{1-\sigma} - 1}{1-\sigma},$$

and

$$W_t^P = [U_{C,t} - U_{L,t}] + \delta U_{C,t}^C.$$

Then, the social planner's objective function (5.1) can be rewritten as

$$W = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t^P.$$

Up to a second-order Taylor expansion, we have

$$\begin{aligned} W_t^P - \bar{W}^P &= U_C' \bar{C} \left[\hat{c}_t + \frac{1-\sigma}{2} \hat{c}_t^2 \right] - U_L' \bar{L} \left[\hat{l}_t + \frac{1+\chi}{2} \hat{l}_t^2 \right] + \delta U_C^{C'} \bar{C}^C \left[\hat{c}_t^C + \frac{1-\sigma}{2} (\hat{c}_t^C)^2 \right] \\ &= U_C' \bar{C} \left[\hat{c}_t + \frac{1-\sigma}{2} \hat{c}_t^2 \right] - U_C' \bar{Y} \left[\hat{l}_t + \frac{1+\chi}{2} \hat{l}_t^2 \right] + U_C' \bar{C}^C \left[\hat{c}_t^C + \frac{1-\sigma}{2} (\hat{c}_t^C)^2 \right], \end{aligned}$$

where the second equality has used the steady-state relationship $U_L' \bar{L} = U_C' \bar{Y}$ and $\delta U_C^{C'} = U_C'$. Note $\bar{G} = 0$ at the steady state (i.e., $\varphi = 0$), $\bar{C}/\bar{Y} = 1 - z$, and $\bar{C}^C / \bar{Y} = z$. Then, we have

$$\frac{W_t^P - \bar{W}^P}{U_C' \bar{C}} = \left[\hat{c}_t + \frac{1-\sigma}{2} \hat{c}_t^2 \right] - \frac{1}{1-z} \left[\hat{l}_t + \frac{1+\chi}{2} \hat{l}_t^2 \right] + \frac{z}{1-z} \left[\hat{c}_t^C + \frac{1-\sigma}{2} (\hat{c}_t^C)^2 \right]. \quad \text{(G.4)}$$

We next aim to express the term $\hat{l}_t + \frac{1+\chi}{2} \hat{l}_t^2$ in terms of output and price dispersion. The production function (4.31) yields

$$\hat{a}_t + \hat{l}_t = \hat{y}_t + \hat{\nu}_t,$$

where $\hat{\nu}_t \equiv \log v_t$ measures the price dispersion. As shown in Woodford (2003, chap. 6), the price dispersion $\hat{\nu}_t$ satisfies

$$\hat{\nu}_t = \frac{\epsilon}{2} var_i \{ p_t(i) \}.$$

Thus, we have

$$\hat{l}_t + \frac{1+\chi}{2} \hat{l}_t^2 = \hat{y}_t + \frac{\epsilon}{2} var_i \{ p_t(i) \} + \frac{1+\chi}{2} (\hat{y}_t - \hat{a}_t)^2 + t.i.p., \quad \text{(G.5)}$$

where $t.i.p.$ represents the terms that are independent of policy.

Taking $Y_t = C_t + C_t^C + G_t$ to a second-order approximation and noting $\bar{G} = 0$ yield

$$\hat{y}_t + \frac{1}{2}\hat{y}_t^2 = (1-z)\hat{c}_t + z\hat{c}_t^C + \hat{g}_t + \frac{1}{2}\left[(1-z)\hat{c}_t^2 + z(\hat{c}_t^C)^2 + \hat{g}_t^2\right]. \tag{G.6}$$

Then, by substituting (G.5) into (G.4) and utilizing (G.6), we get

$$\begin{aligned}
\frac{W_t^P - \bar{W}^P}{U_C'\bar{C}} =& [\hat{c}_t + \frac{1-\sigma}{2}\hat{c}_t^2] + \frac{z}{1-z}[\hat{c}_t^C + \frac{1-\sigma}{2}(\hat{c}_t^C)^2] \\
& - \frac{1}{1-z}[\hat{y}_t + \frac{\epsilon}{2}var_i\{p_t(i)\} + \frac{1+\chi}{2}(\hat{y}_t - \hat{a}_t)^2] + t.i.p. \\
=& - \frac{\hat{g}_t}{1-z} + \frac{1}{2(1-z)}\{\hat{y}_t^2 - [(1-z)\hat{c}_t^2 + z(\hat{c}_t^C)^2 + \hat{g}_t^2]\} + \frac{1-\sigma}{2}\hat{c}_t^2 + \frac{(1-\sigma)z}{2(1-z)}(\hat{c}_t^C)^2 \\
& - \frac{\epsilon}{2(1-z)}var_i\{p_t(i)\} - \frac{1+\chi}{2(1-z)}(\hat{y}_t - \hat{a}_t)^2 + t.i.p. \\
=& - \frac{\hat{g}_t}{1-z} - \frac{\hat{g}_t^2}{2(1-z)} + \frac{1}{2(1-z)}\{\hat{y}_t^2 - \sigma[(1-z)\hat{c}_t^2 + z(\hat{c}_t^C)^2]\} \\
& - \frac{\epsilon}{2(1-z)}var_i\{p_t(i)\} - \frac{1+\chi}{2(1-z)}(\hat{y}_t - \hat{a}_t)^2 + t.i.p. \\
\equiv& - \frac{\mathcal{L}_t}{2(1-z)} + t.i.p.
\end{aligned}$$

In the last line, we define $\mathcal{L}_t$ as the period welfare loss. Maximizing the social planner's objective (5.1) is equivalent to minimizing the welfare loss $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t$.

Given the textbook result (see Woodford (2003, chap. 6)) that[15]

$$\sum_{t=0}^{\infty} \beta^t var_i\{p_t(i)\} = \frac{1}{\gamma}\sum_{t=0}^{\infty} \beta^t \hat{\pi}_t^2,$$

the period welfare loss function $\mathcal{L}_t$ can be rewritten as

$$\mathcal{L}_t = 2\hat{g}_t + \hat{g}_t^2 + \sigma\left[(1-z)\hat{c}_t^2 + z(\hat{c}_t^C)^2\right] + (1+\chi)(\hat{y}_t - \hat{a}_t)^2 - \hat{y}_t^2 + \frac{\epsilon}{\gamma}\hat{\pi}_t^2. \tag{G.7}$$

Given that $\bar{G} = 0$ and hence $\hat{g}_t \geq 0$, the terms related to (exogenous) $\hat{g}_t$ in (G.7) are

[15] We have assumed $\bar{\Pi} = 1$ at the steady state, and hence, $\hat{\pi}_t = \pi_t$.

non-negative: $2\hat{g}_t + \hat{g}_t^2 \geq 0$. Thus, an optimal stabilization policy necessarily requires

$$\hat{g}_t = 0. \tag{G.8}$$

Next, with $\hat{g}_t = 0$ and using (G.3) and (5.10), we have

$$\begin{aligned}
\mathcal{L}_t =& \sigma[(1-z)\hat{c}_t^2 + z(\hat{c}_t^C)^2] + (1+\chi)(\hat{y}_t - \hat{a}_t)^2 - \hat{y}_t^2 + \frac{\epsilon}{\gamma}\hat{\pi}_t^2 + t.i.p. \\
=& \sigma[\hat{y}_t^2 + var_i(\hat{c}_t^i)] + (1+\chi)(\hat{y}_t - \hat{a}_t)^2 - \hat{y}_t^2 + \frac{\epsilon}{\gamma}\hat{\pi}_t^2 + t.i.p. \\
=& \frac{\epsilon}{\gamma}\Big\{\hat{\pi}_t^2 + \lambda_{agg}(\hat{y}_t - \hat{y}_t^e)^2 + \lambda_{disp} var_i(\hat{c}_t^i)\Big\} + t.i.p.,
\end{aligned} \tag{G.9}$$

where $\lambda_{agg} = \gamma(\chi + \sigma)/\epsilon$ and $\lambda_{disp} = \sigma\gamma/\epsilon$. Therefore, (G.9) gives the quadratic welfare loss function (5.9).

G.3 The Extended IS Curve and Phillips Curve with Shocks

In this section, we extend the reduced IS curve and Phillips curve in (3.1) and (A.34) by including credit shocks and demand shocks.

For the equilibrium conditions in Appendix A.1, only (A.6) and (A.23) are affected by the credit shock. (A.6) now becomes

$$\frac{\bar{b}^{FI}}{\bar{Y}}\hat{\theta}_t = \frac{\bar{b}^{FI}}{\bar{Y}}\hat{q}_t + \widehat{b}_t^{FI}, \tag{G.10}$$

and (A.23) becomes

$$(1-\varphi)z\hat{c}_t^C = \mathcal{Q}\hat{\theta}_t + \widehat{qe}_t + \hat{\tau}_t^C - (1-\eta)[\hat{\tau}_t^C + \hat{g}_t], \tag{G.11}$$

where $\mathcal{Q} \equiv \bar{Q}\bar{b}^{FI}/\bar{Y}$. By combining (A.20) and (G.11), the unconstrained household's consumption is

$$\hat{c}_t = \frac{1}{(1-\varphi)(1-z)}\{\hat{y}_t - \mathcal{Q}\hat{\theta}_t - \eta\left[\hat{\tau}_t^C + \hat{g}_t\right] - \widehat{qe}_t\}. \tag{G.12}$$

Substituting the above expression of $\hat{c}_t$ into the derivations in Appendix A.2, we have the following New Keynesian Phillips curve:

$$\hat{\pi}_t = \beta\,\mathbb{E}_t\,\hat{\pi}_{t+1} + \gamma\zeta\hat{y}_t - \frac{\gamma\sigma}{\vartheta}\left[\mathcal{Q}\hat{\theta}_t + \widehat{qe}_t + \eta(\hat{\tau}_t^C + \hat{g}_t)\right] - \gamma(1+\chi)\hat{a}_t. \tag{G.13}$$

Adding a demand shock $\hat{\xi}_t$ to the IS curve, it becomes

$$\begin{aligned} \hat{y}_t - \mathbb{E}_t\,\hat{y}_{t+1} \;=\; & -\frac{(1-z)(1-\varphi)}{\sigma}\left[\hat{i}_t - \mathbb{E}_t\,\hat{\pi}_{t+1}\right] \\ & + \left[\mathcal{Q}\hat{\theta}_t + \widehat{qe}_t + \eta\left(\hat{\tau}_t^C + \hat{g}_t\right)\right] \\ & - \mathbb{E}_t\left[\mathcal{Q}\hat{\theta}_{t+1} + \widehat{qe}_{t+1} + \eta\left(\hat{\tau}_{t+1}^C + \hat{g}_{t+1}\right)\right] + \hat{\xi}_t. \end{aligned} \tag{G.14}$$

Further imposing $\hat{g}_t = 0$ for all t in (G.13) - (G.14) gives (5.11) and (5.12).

G.4 Proof of Proposition 5

Here, we show the policy mix that achieves dual stability, characterized by (5.13) and (5.14), also yields no cross-sectional consumption dispersion. Due to $\varphi = 0$, $\zeta = \chi + \sigma/(1-z)$ and $\vartheta = 1 - z$. Then, plugging equation (5.13) into the constrained household's consumption (G.11) and noting $\hat{y}_t^e = \frac{1+\chi}{\chi+\sigma}\hat{a}_t$, we have

$$\begin{aligned} \hat{c}_t^C &= \frac{1}{z}\left[\mathcal{Q}\hat{\theta}_t + \widehat{qe}_t + \hat{\tau}_t^C - (1-\eta)(\hat{\tau}_t^C + \hat{g}_t)\right] \\ &= \frac{\vartheta}{\sigma z}\left[\zeta\hat{y}_t^e - (1+\chi)\hat{a}_t\right] \\ &= \frac{\vartheta(1+\chi)}{\sigma z}\left[(\chi + \frac{\sigma}{1-z})\frac{1}{\chi+\sigma} - 1\right]\hat{a}_t, \end{aligned}$$

which implies $\hat{c}_t^C = \hat{y}_t^e$ by the definition of efficient level of output in (5.6). Together with the resource constraint (G.3), it also implies $\hat{c}_t = \hat{y}_t^e$. Therefore, $var_i(\hat{c}_t^i) = 0$ by its definition in (5.10).