Effectiveness of History-Dependent Monetary Policy*

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Summary

In this paper, we evaluate the effectiveness of history-dependent monetary policy, focusing on the design of optimal delegation (targeting regime) and simple policy rules. Our quantitative analysis is based on a small estimated forward-looking model of the Japanese economy with a hybrid Phillips curve in which a fraction of firms use a backward-looking rule to set prices. Using this model, we calculate social welfare losses under the alternative policy regimes, and then compare their performance with that under the optimal commitment policy. Our main findings are summarized as follows:

(1) An important characteristic of history-dependent policy is trend-reverting behavior of the price level. Such price dynamics have an immediate effect of stabilizing inflation against price shocks through the forward-looking expectations of private agents. However, when a fraction of firms use a backward-looking rule to set prices, it is not optimal for the central bank to achieve *full* trend-reversion of the price level. Instead, *partial* trend-reversion of the price level with some degree of drift is desirable under the optimal commitment policy.

(2) Under history-dependent targeting regimes, such as price level targeting and nominal income growth targeting, a discretionary central bank brings the price level back to its initial trend path completely in the face of price shocks. Although such full trend-reversion of the price level is not socially optimal, these targeting regimes achieve lower welfare losses and perform much better than inflation targeting in which history dependence is not introduced. This is because trend-reversion of the price level tends to move equilibrium toward what would be achieved under the optimal commitment policy.

(3) Committing to a simple history-dependent policy rule also achieves lower welfare losses and results in nearly the same performance as the optimal delegation of price level targeting and income growth targeting. Among simple policy rules, the most efficient is the first difference hybrid policy rule in which the change in interest rate responds to current inflation and both current and past output gaps.

(4) By committing to the first difference hybrid policy rule, the central bank can achieve almost the same performance as the optimal commitment policy. This is because the partial trend-reverting behavior of the price level under the optimal policy can be replicated by the hybrid rule.

Keywords : Monetary Policy, History Dependence, Delegation, Policy Rules, Commitment, Discretion, Inflation Targeting, Price Level Targeting, Income Growth Targeting

JEL Classification : E52, E58

1. Introduction

Recent monetary policy studies have emphasized the importance of history dependence in the conduct of monetary policy. Giannoni (2000) and Woodford (2000) point out that when private agents are forward-looking, it is optimal for the central bank not only to respond to current shocks and the current state of the economy, but that it is also desirable to respond to lagged variables. Conducting monetary policy of this kind allows the central bank to appropriately affect private sector expectations. This, in turn, improves the performance of monetary policy, because the evolution of the central bank's target variables depends not only on its current actions, but also on how the private sector foresees future monetary policy.

Optimal commitment policy is history dependent and most efficient. However, it is generally time-inconsistent and therefore not particularly realistic. A lot of previous literature investigated other ways in which the monetary policy decision-making process might incorporate the sort of history dependence required for the optimal commitment policy. The design of optimal delegation is one way. On the assumption that the central bank will pursue its goal in a discretionary fashion, rather than committing itself to an optimal plan, the optimal goal with which to charge the central bank need not correspond to the true social welfare function. It is desirable that the loss function assigned to the central bank depend on lagged as well as current values of the target variables. In this case, discretionary policy is history dependent because the bank's loss function is history dependent even though the true social loss function is not.

Vestin (2000) shows that, within a purely forward-looking Phillips curve, discretionary optimization results in the same equilibrium as the optimal commitment policy, if the central bank is charged with stabilization of the price level rather than the inflation rate. Price level targeting is a history-dependent policy, in the sense that the central bank's loss function depends on the cumulative sum of inflation rates over past periods. Jensen (1999) and Walsh (2001) show that discretionary income growth targeting is also desirable, because the central bank's loss function depends on lagged output gap as well as current values of target variables.

However, there are several problems with the practical implementation of optimal delegation. Optimal delegation regimes require full information with respect to demand and price shocks for implementation by the central bank. When the central bank cannot accurately observe these shocks, which seems to be a realistic assumption, optimal delegation is not so effective as previous literature suggests. Even if the central bank can completely identify these shocks, the policy reaction function implied by optimal delegation is very complicated and not transparent to the public.

Instead of implementing optimal delegation, adopting a simple policy rule is another way to incorporate history dependence into monetary policy. Woodford (1999,2000) and Giannoni (2000) suggest that central banks can introduce desirable history dependence into monetary policy by adopting a policy rule in which the interest rate reacts to its own lagged value, price level, or income growth (i.e. change in output gap). Giannoni (2000) then shows that such a simple history-dependent policy rule performs better than the Taylor rule and results in lower welfare loss. Because simple policy rules involve no explicit dependence on demand and price shocks, and so do not even require that the central bank know these shocks, conducting monetary policy with emphasis on simple rules seems to be a practical and transparent scheme.

Still, there remain several problems with respect to history-dependent policy, which previous literature did not solve.

First, the effectiveness of history-dependent policy critically depends on the forward-looking behavior of private agents. Giannoni (2000) and Vestin (2000) investigate history-dependent policy with a purely forward-looking New Keynesian Phillips curve. However, the New Keynesian Phillips curve has been criticized for failing to match the short-run dynamics exhibited by inflation (see, for example, Mankiw (2000)). Specifically, inflation seems to respond sluggishly and display significant persistence in the face of price shocks, while the New Keynesian Phillips curve allows current inflation to be a jump variable that can respond immediately to any disturbance. Therefore, in order to obtain the practical implication of history-dependent policy, we must use a model with empirically plausible amounts of forward-looking behavior and inflation persistence.

Second, previous literature indicated that history-dependent targeting regimes, such as price level targeting and income growth targeting, outperform inflation targeting, but it is not clear which history-dependent targeting regime performs better and to what degree the performance of each targeting regime differs quantitatively. This is because the competing regimes of delegation have been examined separately over different models.

Third, previous literature does not clarify which history-dependent policy regime performs better, delegation (targeting regime) or a simple policy rule. Although a policy regime based on a simple rule is transparent to the public, if it performs worse than optimal delegation, then the policy rule regime is not desirable. On the other hand, if we can show that simple policy rules which involve no explicit dependence on demand and price shocks are as effective as optimal delegation, then it is more desirable than optimal delegation, based on the realistic assumption that the central bank cannot observe these shocks accurately.

Fourth, Giannoni (2000) derives a policy rule that implements the optimal commitment policy in the purely forward-looking New Keynesian Phillips curve. However, this leaves the problem of whether there is a simple policy rule which achieves the same performance as the optimal commitment policy when endogenous persistence is incorporated in inflation dynamics.

The purpose of this paper is to investigate history-dependent policy and attempt to solve the above unresolved problems in a unified framework. Our quantitative analysis is based on a small estimated forward-looking model of the Japanese economy with a hybrid Phillips curve. This Phillips curve is a modified inflation adjustment equation which incorporates endogenous persistence by including the lagged inflation rate in the New Keynesian Phillips curve. That is, it nests the purely forward-looking Phillips curve as a particular case, and allows for a fraction of firms that use a backward-looking rule to set prices. Using this model, we calculate social welfare losses under the alternative policy regimes, and then compare their performance with that under the optimal commitment policy. Although we do not consider the optimal commitment policy particularly realistic, it will serve as a useful benchmark against which policy under the alternative regimes can be evaluated.

Our main findings are summarized as follows.

(1) According to the estimation results of the hybrid Phillips curve based on Japan's data, about 60% of firms exhibit forward-looking price setting behavior (i.e. the remaining 40% of firms exhibit backward-looking behavior). Although the purely forward-looking Phillips curve is rejected by the data, such a degree of forward-looking behavior seems to be sufficient to make history-dependent policy effective.

(2) Indeed, history-dependent targeting regimes, such as income growth targeting and price level targeting, outperform inflation targeting and result in lower social welfare losses. In addition, the optimal commitment policy can be roughly replicated by delegating income growth targeting to a conservative central bank, which values inflation stability more than society does. The optimal commitment policy can also be roughly replicated by

delegating price level targeting to a liberal central bank, which values output stability more than society does. Price level targeting and income growth targeting perform at almost the same level.

(3) Simple history-dependent policy rules, such as a price level targeting rule and income growth targeting rule, perform much better than the Taylor rule in which history dependence is not introduced. Moreover, committing to such a history-dependent rule results in nearly the same social welfare as the optimal delegation of price level targeting and income growth targeting.

(4) Among simple history-dependent rules, the most efficient is the first difference hybrid rule in which the change in interest rate responds to current inflation and both current and past output gaps. By adopting this rule, the central bank can achieve almost the same performance as the optimal commitment policy. Besides, this hybrid rule may be effective even if the central bank is faced with the zero interest rate bound.

(5) The good performance of the first difference hybrid rule results from partial trend-reverting behavior of the price level. Through the forward-looking expectations of private agents, the trend-reversion of the price level has the immediate effect of stabilizing inflation against price shocks. However, in the hybrid Phillips curve, it is not socially optimal for the central bank to completely bring the price level back to its initial trend path. This is because the central bank must pay the high cost of greater output gap volatility to achieve *full* trend-reversion of the price level when a fraction of firms use a backward-looking rule to set prices. Therefore, *partial* trend-reversion of the price level with some degree of drift is desirable in the hybrid Phillips curve. Such partial trend-reverting behavior of the price level can be replicated by the hybrid rule, but not by other history-dependent rules.

The outline of the remainder of the paper is as follows. Section 2 presents the model of the economy and poses the problem of optimal commitment and discretion policy. Section 3 presents the estimation results of the model. Then, using the estimated model, we show how endogenous variables respond to shocks and calculate welfare losses under optimal policy. Sections 4 and 5 present quantitative analysis of optimal delegation and simple policy rules respectively. Finally, Section 6 provides concluding remarks.

2. Framework for Optimal Monetary Policy

This section first sets out a dynamic general equilibrium model, and then describes the formal monetary policy design problem.

2.1. Model

The model economy is a particularly simple version of a dynamic general equilibrium model that has been extensively applied in literature discussing monetary policy evaluation.[1] The model consists of two relationships: 1) an aggregate demand condition that links output and the real interest rate, and 2) an inflation adjustment equation that links inflation and output.

Aggregate demand is determined by the following intertemporal IS curve:

$$y_t - y_t^* = f(y_{t-1} - y_{t-1}^*) + (1-f)\{E_t[y_{t+1} - y_{t+1}^*] - s\,(i_t - E_t[\Delta p_{t+1}] - ri_t^*)\} + e_t \quad , \qquad (1)$$

$$ri_t^* = s^{-1}E_t[\Delta y_{t+1}^*] + d,\quad 0 \le f < 1,\quad 0 < s,\quad 0 < d,$$

where y_t and y_t^* are real output and potential real output, respectively, both in logs. y_t^* is the level of output that would arise if wages and prices were perfectly flexible, and the difference between y_t and y_t^*, $y_t - y_t^*$, is an output gap. i_t is the short-term nominal interest rate assumed to be the monetary policy instrument. p_t is the log price level, and its first difference Δp_t is the inflation rate. E_t is the expectations operator conditional on all information up to period *t*. e_t is a demand shock representing an autonomous variation in spending such as government spending. In the special case where $f = 0$, equation (1) approximates the path of demand arising from the Euler equation characterizing optimal consumption choices by the representative household. Parameter s can then be interpreted as the rate of intertemporal substitution determining to what extent the real interest rate, $i_t - E_t[\Delta p_{t+1}]$, affects expected spending growth. ri_t^* is the equilibrium real interest rate, which is the sum of the expected potential growth rate $E_t[\Delta y_{t+1}^*]$ (times the inverse of s) and time preference rate d. For mainly empirical reasons, the IS curve specification allows for endogenous persistence in demand when $0 < f < 1$. By appealing to some form of adjustment costs, it may be feasible to explicitly motivate the appearance of lagged output within the IS curve.

The inflation adjustment equation is modeled by the following Phillips curve:

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[1] See, for example, Clarida, Galí and Gertler (1999), Jensen (1999), Walsh (2001), Giannoni (2000), Rudebusch (2000), Smets (2000), Vestin (2000), and Woodford (1999,2000,2001).

$$\Delta p_t = q(\Delta p_{t-1}) + (1-q) b E_t[\Delta p_{t+1}] + l(y_t - y_t^*) + m_t, \quad 0 \le q < 1, \quad l > 0. \qquad (2)$$

In the special case where $q = 0$, the structure of equation (2) resembles what Roberts (1995) has labeled the "New Keynesian Phillips curve", which can be derived from a variety of supply-side models. For example, it can be described as the staggered price determination that emerges from a monopolistically competitive firm's optimal behavior in Calvo's (1983) model. Roughly speaking, firms set nominal prices based on the expectations of future marginal costs. The output gap $y_t - y_t^*$ captures movements in marginal costs associated with variation in excess demand. The shock m_t, which we refer to as price shock, captures anything else that might affect expected marginal costs. This could be caused, for example, by movements in nominal wages that push real wages away from their equilibrium values due to frictions in the wage contracting process.[2] The presence of this price shock m_t is of all importance for the ensuing analyses because it induces the trade-off in monetary policymaking that matters in the design of optimal policy.

The New Keynesian Phillips curve, where $q = 0$ is assumed in equation (2), has been criticized for failing to match the short-run dynamics exhibited by inflation.[3] Specifically, inflation seems to respond sluggishly and to display significant persistence in the face of price shocks, while the New Keynesian Phillips curve allows current inflation to be a jump variable that can immediately respond to any disturbance. Therefore, we introduce the endogenous inflation persistence in equation (2) by allowing for $0 < q < 1$. The inclusion of the lagged inflation rate could be due to overlapping nominal contracts aimed at securing real wage levels comparable to existing and expected real wages.[4] Alternatively, it could be due to the existence of firms that use a backward-looking rule to set prices.[5] In the latter case, equation (2) can be interpreted as the hybrid curve of the traditional backward-looking Phillips curve and the New Keynesian Phillips curve, where the parameter q depends on the weight of backward-looking firms.

We assume that exogenous disturbances follow AR(1) processes:

$$e_t = y e_{t-1} + \hat{e}_t, \quad -1 < y < 1 \qquad (3)$$

$$m_t = r m_{t-1} + \hat{m}_t, \quad -1 < r < 1 \qquad (4)$$

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[2] See Erceg, Henderson and Levin (2000) for details.

[3] See, for example, Mankiw (2000).

[4] See Fuhrer and Moore (1995).

[5] See Galí and Gertler (1999).

$$\Delta y_t^* = (1-w)t + w\Delta y_{t-1}^* + \hat{x}_t, \quad 0 \le w < 1 \tag{5}$$

t is an average rate of potential growth. All stochastic innovations $\hat{e}_t$, $\hat{m}_t$, and $\hat{x}_t$ are white noises and may be correlated with each other.

Finally, we note that our model may be applied to the open economy. Although our model does not explicitly involve the exchange rate, Clarida, Galí and Gertler (2001) show that the open economy can be simply modeled by equations (1)-(2) with the price of domestic output p_t as well as the terms of trade equation.

2.2. Optimal Monetary Policy

The policy regimes to be considered are evaluated according to a social welfare loss function, which is assumed as follows:

$$E_t\left[\sum_{j=0}^{\infty} b^j\{(1-a)(\Delta p_{t+j})^2 + a(y_{t+j} - y_{t+j}^*)^2\}\right], \quad 0 < b < 1, \quad 0 \le a \le 1, \tag{6}$$

where b is a discount factor. This loss function is the unconditional expectation of the discounted sum of quadratic deviations of output from potential and quadratic deviations of inflation from a target of zero. The weight a reflects society's preference for output stabilization relative to inflation stabilization. As demonstrated by Woodford (2001), such a social loss function can be derived as an approximation from the utility of a representative household in the closed economy, when inflation dynamics can be described as the New Keynesian Phillips curve (i.e. where $q = 0$ in equation (2)). Benigno and Benigno (2001) suggest that where $q = 0$, the social loss function in the open economy can also be approximated as function (6) with the domestic price p_t. Although function (6) may not be an accurate approximation of social welfare in the case of the hybrid Phillips curve ($0 < q < 1$), we simply adopt it because this formulation does not seem out of sync with the way monetary policy operates in practice, at least implicitly. Indeed, function (6) has been applied in virtually all literature on monetary policy issues in the past.[6]

[6] As demonstrated by Steinsson (2000) and Amato and Laubach (2002), in the case of the hybrid Phillips curve, the theoretical loss function includes the variability in the change of inflation as well as inflation rate and output gap. However, the only effect of adding the variability of the change in inflation to the traditional loss function (6) is to increase the weight of inflation stabilization relative to output gap stabilization, since any reduction of inflation variability reduces the variability of the change in inflation as well. We analyze the whole range of society's preference for inflation stabilization by using the traditional loss function (6). Therefore, using the theoretical loss function instead of the traditional one does not substantially influence the results of our analysis.

Given the social welfare loss function, it is possible — and obviously optimal — to completely stabilize the output gap and inflation against demand and productivity shocks. This follows immediately upon inspection of the IS curve (1) and the Phillips curve (2). By adjusting the interest rate, any effect of demand and productivity shocks on the output gap and thus inflation is completely eliminated. That is, these shocks do not constitute a policy trade-off. In contrast, it is impossible to perfectly stabilize the output gap and inflation against price shocks. This is also immediately seen by inspecting the Phillips curve (2). For example, in the case of a positive price shock, inflationary pressures arise in the economy, which can only be dampened through a contractionary policy reducing the output gap. Therefore, the central bank faces a trade-off between stabilization of inflation and stabilization of the output gap. While this mechanism is very straightforward, the optimal way of responding to price shocks differs according to policy regime, as we show below.

2.2.1. Optimal Monetary Policy under Commitment

Consider the case where the central bank has complete credibility and is able to commit. Thus, it can credibly announce any future path for the output gap and thus affect private sector expectations about future inflation. The central bank's problem is to minimize the social loss function (6) subject to the hybrid Phillips curve (2).[7] The first order conditions for this problem are as follows:

$$l(1-a)\Delta p_t = -a\left[(y_t - y_t^*) - qbE_t[y_{t+1} - y_{t+1}^*]\right] \quad , \tag{7a}$$

$$\forall j \geq 1, \quad l(1-a)\Delta p_{t+j} = -a\left[(y_{t+j} - y_{t+j}^*) - (1-q)(y_{t+j-1} - y_{t+j-1}^*) - qbE_{t+j}[y_{t+j+1} - y_{t+j+1}^*]\right] \quad . \tag{7b}$$

It is easy to interpret the economic meaning of these conditions in the case of $q = 0$, that is, when the inflation adjustment equation is described as the New Keynesian Phillips curve. Condition (7a) simply implies that the central bank pursue a 'lean against the wind' policy: Whenever inflation is above the target rate of zero, reduce demand below capacity (by raising the interest rate); and vice versa when below target. How aggressively the central bank should reduce the output gap depends positively on the gain in reduced

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[7] What is critical in this optimization problem is the existence of price shock m_t in equation (2), which induces the trade-off between inflation stabilization and output gap stabilization. Since neither demand shock nor productivity shock induces such a tradeoff, the IS curve (1) is redundant for this problem. After the central bank chooses a path for both the inflation rate and output gap to minimize the social loss function subject to equation (2), it then determines the interest rate implied by the IS equation (1), conditional on the optimal values of inflation rate and output gap. See Clarida, Galí and Gertler (1999) for details.

inflation per unit of output loss, l , and inversely on the relative weight placed on output losses a . Condition (7b) implies, in the case of $q = 0$, that optimal commitment requires adjusting the *change* in the output gap in response to inflation. That is, optimal monetary policy is history dependent in that equation (7b) involves lagged output gap $y_{t+j-1} - y^*_{t+j-1}$. Such history dependence in monetary policy arises as a product of the central bank's ability under commitment to directly manipulate private sector expectations. To see this, keep in mind that the inflation rate depends not only on the current output gap but also on the expected future path of the output gap.[8] Then suppose, for example, that there is a price shock that raises inflation above target at time *t*. Optimal policy continues to reduce output gap $y_{t+j} - y^*_{t+j}$ as long as Δp_{t+j} remains above target. The credible threat to continue to contract the output gap in the future, in turn, has the immediate effect of dampening current inflation, given the dependence of inflation rate on the future output gap.

Note that, in the case of $q = 0$, the first order condition (7b) can be equivalently expressed as

$$\forall j \geq 1, \quad l(1-a)(p_{t+j} - p^*) = -a(y_{t+j} - y^*_{t+j}) \ , \tag{8}$$

where p^* is a constant target price level corresponding to the target rate of inflation, i.e. zero. Thus, for example, the central bank commits to reduce demand when the price exceeds the constant target level; hence, the trend-reverting behavior of the price level. This is an important property of history dependent policy in the case of $q = 0$.

Although it is not easy to interpret the economic meaning of equation (7b) in the case of $q > 0$, optimal monetary policy under commitment is history dependent as long as $1 > q$, because equation (7b) involves lagged output gap $y_{t+j-1} - y^*_{t+j-1}$. This means that the qualitative significance of history dependence in the conduct of monetary policy does exist when expectations play a role (i.e. $1 > q$).

The first order conditions (7) reveal the importance of history dependence in the optimal commitment policy, but this policy is generally not time-consistent in rational expectations models. At time *t*, the central bank sets

$$l(1-a)\Delta p_t = -a\left[(y_t - y^*_t) - qbE_t[y_{t+1} - y^*_{t+1}]\right]$$

and promises to set

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[8] In the case of $q = 0$, it is instructive to iterate equation (2) forward to obtain

$$\Delta p_t = bE_t[\Delta p_{t+1}] + l(y_t - y^*_t) + m_t = E_t \sum_{j=0}^{\infty} b^j [\, l(y_{t+j} - y^*_{t+j}) + m_{t+j}] \ .$$

$$l(1-a)\Delta p_{t+1} = -a\left[(y_{t+1} - y^*_{t+1}) - (1-q)(y_t - y^*_t) - qbE_{t+1}[y_{t+2} - y^*_{t+2}]\right] \;.$$

But, when period t+1 arrives, the central bank that reoptimizes will again obtain

$$l(1-a)\Delta p_{t+1} = -a\left[(y_{t+1} - y^*_{t+1}) - qbE_{t+1}[y_{t+2} - y^*_{t+2}]\right]$$

as its optimal setting for inflation, since the first order condition (7a) updated to t+1 will reappear. Therefore, we do not consider optimal policy under commitment as particularly realistic, but this policy will serve as a useful benchmark against which policy under the ensuing alternative regimes can be evaluated.

2.2.2. Optimal Monetary Policy under Discretion

In contrast to the case of commitment, the central bank that acts in a discretionary manner takes expectations as given. The central bank takes as given the process through which private agents form their expectations, and recognizes that their expectations will depend on the state variables at time t and that these state variables may be affected by policy actions at time t or earlier. In the case of the hybrid Phillips curve, state variables include both lagged inflation Δp_{t-1} and price shock m_t. Expectations of future inflation $E_t[\Delta p_{t+1}]$ will now depend on current inflation Δp_t and expectations of a future price shock, $E_t[m_{t+1}]$. Policy actions that affect Δp_t will also affect $E_t[\Delta p_{t+1}]$, and the central bank will take this dependency into account under optimal discretion. Accordingly, the first order condition of optimal discretion is

$$\forall j \geq 0, \quad l(1-a)\Delta p_{t+j} = -a\left[(1-(1-q)b\,g_p)(y_{t+j} - y^*_{t+j}) - qbE_{t+j}[y_{t+j+1} - y^*_{t+j+1}]\right] \;, \qquad (9)$$

where g_p is a parameter of the equilibrium solution expression $\Delta p_t = g_p \Delta p_{t-1} + g_m m_t$.[9] Although this first order condition is similar to the first period condition (7a) in commitment optimization, it now applies to each period.[10] Such discretionary policy making, i.e. a process that presumes period-by-period reoptimization involving each period's start-up conditions, is time-consistent.

However, note that, unlike the optimal commitment policy, the optimal discretion policy is not history dependent in that condition (9) does not involve lagged output gap $y_{t+j-1} - y^*_{t+j-1}$. This absence of history dependence will result in an inefficiency in the conduct of monetary policy, as Woodford (2000) emphasizes.

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[9] See Appendix for details of derivations.

[10] When the lagged inflation rate does not become an endogenous state variable (i.e. $q = g_p = 0$), condition (9) coincides with condition (7a).

3. Quantitative Significance of History-Dependent Policy

3.1. Estimation Results of the Model

The differences in the first order conditions between optimal commitment and optimal discretion help us identify the *theoretical* significance of history-dependent monetary policy in the forward-looking model. However, to assess the *quantitative* significance of history-dependent policy, we need to calculate welfare losses under commitment and discretion, with the model (1)-(5) using reasonable parameter values and appropriate measures of the uncertainty the central bank faces due to exogenous shocks. The quantitative significance of history dependence critically depends on the value of parameter q in the hybrid Phillips curve (2), and the effectiveness of history-dependent policy increases as q approaches zero, i.e. the proportion of forward-looking firms increases.

To obtain reasonable parameter values for our model, we estimate equations (1)-(2) using GMM methods. We use quarterly historical data for Japan, concentrating on the period from 1975Q1 to 1997Q1.[11] For inflation Δp_t, we use quarterly inflation in the seasonally-adjusted GDP deflator in percent at an annual rate. y_t is seasonally-adjusted real GDP in log (i.e. $y_t = 100 ln(\text{real GDP})$). Potential output y_t^* is estimated by using the Hodrick-Prescott filter on y_t.[12] For the short-term nominal interest rate i_t, we use the call rate in percent at an annual rate.[13] For demand shock e_t and price shock m_t, we use the estimated residuals of equations (1)-(2), and then estimate equations (3)-(5) using least squares.[14]

Estimation results of equations (1)-(5) are shown in Chart 1. Overall, the estimated

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[11] We set the beginning of the sample period at 1975Q1 to avoid the high inflation period of the first-round increases in oil prices, since the expectation formation process of the private sector may have changed after transition to a moderate or low inflation period. We set the end of the sample period at 1997Q1 to avoid a possible structural break which might be caused by the impact of the financial system crisis (credit crunch) that occurred in the autumn of 1997.

[12] As argued in Galí and Gertler (1999), the use of detrended GDP as a proxy for the output gap may not have a theoretical justification. However, we use it because detrended GDP does not seem out of sync with the business cycle in practice, at least implicitly. Indeed, detrended GDP has been applied in many studies on monetary policy issues. See, for example, Gertler (1999), Rudebusch (2000), and Smets (2000).

[13] In our analysis, inflation rate, interest rate (including equilibrium real rate), and income growth rate (including potential growth rate) are all measured in percent, at an annual rate.

[14] Note, however, that the estimated residuals of a forward-looking model probably overstate the degree of uncertainty faced by the central bank, since they include forecast errors as well as structural innovations.

parameters have the expected sign and are significant.

With regard to the IS curve, the parameter of the lagged output gap (f) is 0.91, capturing a considerable degree of persistence in output. The sensitivity of the output gap to the real interest rate ($-s$) is significantly negative.

With regard to the Phillips curve, the estimated value of l, the sensitivity of inflation to the output gap, is 0.20. Its estimated standard error suggests a 90% confidence interval for l is between 0.15 and 0.25.[15] This interval is similar to the results of Rudebusch (2000), who estimates for l of between 0.11 and 0.21 using US data. The coefficient of particular interest is q, which measures the degree of persistence in price setting (and thus, by implication, the importance of forward-looking behavior). The estimate of $q = 0.35$ is statistically different from zero, and indicates that about 60% of firms exhibit forward-looking price-setting behavior.[16] We must note, however, that in previous studies the empirical estimates of q have been subject to controversy. Fuhrer (1997) statistically rejects the importance of forward-looking behavior (suggesting $q = 1$), although he acknowledges that inflation dynamics without forward-looking behavior are implausible. Rudebusch (2000) estimates q to equal 0.71, and Roberts (2001) suggests a range for q of between 0.5 and 0.7. Although Rudebusch and Roberts statistically reject the purely backward-looking Phillips curve, their estimates suggest the weight of backward-looking behavior is larger than that of forward-looking behavior. In contrast, the estimates of Galí, Gertler and López-Salido (2001) for q are between 0.03 and 0.27 with Euro area data, and

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[15] Our estimation is based on GMM, therefore, we cannot apply the standard t-test to the estimated parameters. Here, however, we simply show the confidence interval as an approximation.

[16] When we regard Phillips curve (2) as a domestic price adjustment equation in the open economy, the coefficients can be interpreted as follows (see Galí and Gertler (1999), Clarida, Galí and Gertler (2001) for details):

$$\Delta p_t = q(\Delta p_{t-1}) + (1-q)bE_t[\Delta p_{t+1}] + l(y_t - y_t^*) + m_t$$

$$q = \frac{b}{a + b[1 - a(1-b)]}, \quad (1-q)b = \frac{ab}{a + b[1 - a(1-b)]}, \quad l = \frac{(1-b)(1-a)(1-ba)}{a + b[1 - a(1-b)]}\left[d + \frac{s}{1 + g(sh - 1)(2 - g)}\right],$$

where $a =$ probability of price non-adjustment ($0 < a < 1$), $b =$ weight of backward-looking firms ($0 \le b \le 1$), $d =$ elasticity of labor supply, $s =$ intertemporal consumption elasticity, $g =$ share of domestic consumption allocated to imported goods, and $h =$ elasticity of substitution between domestic and foreign goods.

Assuming $d = 1$, $h = 1.5$ (like Galí and Monacelli (2000)), $g = 0.1$ (actual value of SNA statistics) and $b = 1$, then substituting estimated values $s = 1.5$, $q = 0.35$, and $l = 0.2$ into the above coefficient relations, we obtain $a = 0.78$ and $b = 0.42$. $a = 0.78$ implies that the average frequency of price adjustment is 4.5 quarters, i.e. about one year, which seems to be fairly reasonable. $b = 0.42$ implies that about 60% of firms exhibit forward-looking price setting behavior.

about 0.35 with US data, suggesting that the weight of forward-looking behavior is greater than that of backward-looking behavior. At the extreme, Galí and Gertler (1999) conclude that inflation persistence is rather unimportant and that the purely forward-looking Phillips curve is a reasonable first approximation to data (suggesting $q = 0$). Our estimate of $q = 0.35$ using Japan's data is similar to that of Galí, Gertler and López-Salido (2001) using US data.

The standard deviation of demand shocks is 60 basis points, whereas that of price shocks is 98 basis points. There is evidence of a small negative correlation between these two shocks of minus 20%. The AR parameter of the price shock process is slightly negative ($r = -0.26$), which probably indicates that the price shock includes not only structural innovations but also errors in the measurement of the GDP deflator. In contrast, the productivity shock is very persistent, as its AR parameter is very high ($w = 0.95$).

3.2. Optimal Responses to a Price Shock and Comparison of Welfare Losses

Using the above estimated model, we show the impulse response of inflation and output gap to a price shock, to get a firmer grip on the difference between optimal commitment and optimal discretion. Chart 2 shows the simulation results of a negative price shock in period zero where society's preference $a = 0.5$. Under the optimal commitment policy the output gap rises less in period zero but more persistently than under the optimal discretion policy. By persistently keeping the output gap positive, the central bank can raise expectations of future inflation, which partially offsets negative price shocks in the Phillips curve. As a result, under optimal commitment, the central bank achieves inflation stability at the cost of greater output gap volatility. Since negative price shocks are followed by periods of inflation under the optimal commitment policy, such inflation dynamics generates trend-revering behavior of the price level. This is the key characteristic of history-dependent policy, which cannot be seen at all under the discretionary policy. Note, however, that it is not desirable for the central bank to completely bring the price level back to its initial trend path when a fraction of firms use a backward-looking rule to set prices (i.e. $q > 0$). Because the central bank must pay the cost of much greater output gap volatility to achieve *full* trend-reversion of the price level, *partial* trend-reversion with some degree of price drift is desirable in the hybrid Phillips curve.

Now, we numerically calculate welfare losses (6) under both optimal commitment and discretion policies, based on the estimated model (1)-(5). Note that in the limit when the discount factor approaches unity, $b \to 1$, the welfare loss function (6) is proportional to

$$L_t = (1-a)V[\Delta p_t] + aV[y_t - y_t^*] \quad . \tag{10}$$

In what follows, we report this weighted unconditional variance. See Appendix for details of the calculation method of (10).

Chart 3 compares social welfare loss (10) under optimal commitment with that under optimal discretion for a range of society's preferences a. This shows that commitment policy produces smaller losses than discretion policy for all examined values of a. Social loss under discretion is about 15-20% larger than that under commitment. Because the central bank faces a less advantageous inflation-output gap trade-off under discretion, there is an inefficiency relative to commitment. This inefficiency, however, does not result from the traditional inflation bias that was the focus of Barro-Gordon literature. Instead, it results from the absence of history dependence under discretion policy.

4. Optimal Delegation

We now turn to the question of how history dependence in the optimal commitment policy is to be brought about in practice. Perhaps the most straightforward approach is the design of a policy rule to which the central bank may commit itself; this approach is treated in the next section. Another approach is to conceive the choice of monetary policy as a delegation problem. That is, the government delegates monetary policy conduct to an independent central bank which is required to minimize an assigned loss function. Optimal delegation is of practical relevance if we assume that the central bank will pursue its goal in a discretionary fashion, rather than committing itself to an optimal plan, so that the outcome for the economy will be a time-consistent plan associated with that goal. In this case, the optimal goal with which to charge the central bank need not correspond to the true social welfare function; inefficient pursuit of a distorted objective may produce a better outcome, from the standpoint of the true social objective, than inefficient pursuit of the true objective itself. Assigning the central bank the loss function, which depends on lagged as well as current values of target variables, introduces history dependence to discretionary policy, because the bank's loss function is history dependent (even though the true social loss function is not).

4.1. Definition of Targeting Regimes

A targeting regime is understood to be an institutional set-up where the government delegates monetary policy to an independent central bank which is required to minimize an assigned loss function. Moreover, the parameters of the loss function are chosen so as to minimize society's loss function. We focus on the case where the central bank is assigned a loss function depending on inflation, price level, output gap, real income growth, and nominal income growth. More generally, the loss function takes the form

$$E_t\left[\sum_{j=0}^{\infty} b^j\{a_p(\Delta p_{t+j})^2 + a_p(p_{t+j}-p^*_{t+j})^2 + a_y(y_{t+j}-y^*_{t+j})^2 + a_\Delta(\Delta y_{t+j}-\Delta y^*_{t+j})^2 + a_n(\Delta p_{t+j}+\Delta y_{t+j}-\Delta y^*_{t+j})^2\}\right], \quad (11)$$

where Δy_t and $\Delta p_t + \Delta y_t$ are real income growth and nominal income growth, respectively. The target rate of real income growth is the potential growth rate Δy_t^*, and so is that of nominal income growth because the target rate of inflation is zero. The parameters $a_p, a_p, a_y, a_\Delta, a_n$ are chosen at the stage of institutional design.

We focus on the following four alternative targeting regimes, imposing some restrictions on the coefficient of the loss function (11).

Inflation targeting : $a_p = 1 - a^c,\ \ a_p = 0,\ \ 0 \le a_y = a^c \le 1,\ \ a_\Delta = 0,\ \ a_n = 0$ (12)

$$E_t\left[\sum_{j=0}^{\infty} b^j\{(1-a^c)(\Delta p_{t+j})^2 + a^c(y_{t+j}-y^*_{t+j})^2\}\right]$$

Price level targeting : $a_p = 0,\ \ a_p = a^c,\ \ 0 \le a_y = a^c \le 1,\ \ a_\Delta = 0,\ \ a_n = 0$ (13)

$$E_t\left[\sum_{j=0}^{\infty} b^j\{(1-a^c)(p_{t+j}-p^*_{t+j})^2 + a^c(y_{t+j}-y^*_{t+j})^2\}\right]$$

Income growth targeting : $a_p = 1 - a^c,\ \ a_p = 0,\ \ a_y = 0,\ \ 0 \le a_\Delta = a^c \le 1,\ \ a_n = 0$ (14)

$$E_t\left[\sum_{j=0}^{\infty} b^j\{(1-a^c)(\Delta p_{t+j})^2 + a^c(\Delta y_{t+j}-\Delta y^*_{t+j})^2\}\right]$$

Nominal income growth targeting : $a_p = 0,\ \ a_p = 0,\ \ a_y = 0,\ \ a_\Delta = 0,\ \ a_n = 1$ (15)

$$E_t\left[\sum_{j=0}^{\infty} b^j\{(\Delta p_{t+j}+\Delta y_{t+j}-\Delta y^*_{t+j})^2\}\right],$$

a^c is the central bank's preference for output stabilization.

Svensson (1999) defines inflation targeting in terms of the central bank's preference a^c, i.e. he defines the case where $a^c = 0$ as corresponding to strict inflation targeting and $a^c > 0$ as corresponding to flexible inflation targeting. This latter flexibletargeting seems to be in accordance with the understanding of real world practitioners, who generally do not ignore the real side of the economy. Note that the form of the loss function (12) under inflation targeting is the same as the social loss function (6). However, there is no reason why the central bank's preference a^c must equal society's true preference a. Rogoff (1985) suggests that assigning a lower a than society's true value (that is, a more conservative central banker $a^c < a$) reduces inflation bias. Here, there is no inflation bias, as the output gap target is assumed to be consistent with the natural rate of unemployment. However, as will be clear from the results below, different values of a^c will affect the trade-off between inflation and output gap volatility (whereas Rogoff's result is in terms of the level of inflation).

Among the above four targeting regimes, all are history dependent excluding inflation targeting. Price level targeting is a history-dependent regime, in that its loss function effectively depends on the cumulative sum of inflation rates over all past periods, rather than only on the current period's inflation rate. As Vestin (2000) shows, price level targeting has desirable characteristics, because it implies that a positive price shock should cause anticipation of lower inflation in subsequent periods, to an extent that the price level is expected to eventually return to its original level. Both income growth targeting and nominal income growth targeting are also history dependent, in that their loss functions depend on a lagged output gap as well as the current value of target variables. In these regimes, low output gap in the previous period leads the central bank to choose a low output gap and/or deflation in the current period. As a result, a positive price shock results in a more persistent contraction of the output gap, lowering overall inflation and output gap volatility, as Walsh (2001) shows.

As for nominal income growth targeting, we assume the central bank is only required to stabilize nominal income growth, although Jensen (1999) assumes that the bank is required to stabilize both nominal income growth and the output gap. Note that even with the loss function (15), the central bank is concerned about both price stability and output stability, since nominal income growth is the sum of inflation and real income growth. Note also that nominal income growth targeting is not a special case of income growth targeting, as their loss functions do not coincide with each other even in the case of $1-a^c = a^c = 0.5$.[17]

[17] In the case of $a^c = 0.5$, the loss function under income growth targeting is

$$0.5(\Delta p_{t+j})^2 + 0.5(\Delta y_{t+j} - \Delta y^*_{t+j})^2 = 0.5(\Delta p_{t+j} + \Delta y_{t+j} - \Delta y^*_{t+j})^2 - (\Delta p_{t+j})(\Delta y_{t+j} - \Delta y^*_{t+j}) ,$$

where the first term on the right-hand side is the loss function under nominal income growth targeting.

4.2. Quantitative Analysis of Optimal Targeting Regimes

The question of which targeting regime is most desirable is ultimately empirical. To date, the question remains open, because competing regimes have been examined separately with different models in previous literature. We attempt to resolve this question by using a common model framework in examining competing regimes.

To compare the performance of competing targeting regimes, we define the optimal targeting regime as the case where weight a^c in the associated loss function assigned to the central bank is chosen optimally to minimize the social loss function (6).[18] The solution of optimal targeting regimes is derived under the assumption that the central bank cannot commit to any policy path beforehand, but rather reoptimizes each and every period to minimize the assigned loss function. Appendix outlines the numerical solution.

4.2.1. Comparison of Social Welfare Losses

In Chart 4, we show the associated losses of society and optimal weight a^c under each targeting regime, comparing their losses with those under the optimal commitment and discretion policies. The following three results are of particular interest.

First, there is a gain from moving from the optimal discretion policy to delegating inflation targeting to a conservative central bank ($a^c < a$).[19] By placing greater weight on stabilizing inflation, such a central bank achieves more stable inflation at the cost of greater output gap volatility. This tends to move equilibrium toward what would be achieved under the optimal commitment policy. However, the gain from delegating inflation targeting to a conservative central bank is rather small, and the ratio of social losses under inflation targeting to those under optimal commitment is still high (around 1.15-1.20). This is because assigning the same form of social loss function to the central bank even with a different weight does not change the fundamental characteristic of inflation targeting, i.e. it is not a history-dependent regime.

Second, both price level targeting and income growth targeting drastically reduce social losses. It can be seen that these targeting regimes produce much smaller losses than inflation targeting for all examined values of a. In addition, the ratio of social losses under these targeting regimes to those under the optimal commitment policy is below 1.03. In

[18] Note that under nominal income growth targeting the central bank is assigned to put equal weight on each component of nominal income growth.

[19] Walsh (2001) shows the same result as we do.

other words, a discretionary central bank, whose objective is price level stabilization or income growth stabilization, can implement a monetary policy that roughly replicates the optimal commitment policy. Vestin (2000) shows that price level targeting results in the same equilibrium as the optimal commitment policy, using the purely forward-looking New Keynesian Phillips curve ($q = 0$). Walsh (2001) also shows that, in the case of $q = 0$, income growth targeting results in the same equilibrium as the optimal commitment policy, when implemented by a discretionary and myopic central bank which is only concerned with minimizing its current period loss function. However, our quantitative analysis indicates that even in the case of the hybrid Phillips curve ($q > 0$), a discretionary and non-myopic central bank can roughly achieve the welfare-optimal allocation by conducting either price level targeting or income growth targeting.

Third, nominal income growth targeting also produces smaller losses than inflation targeting for a wide range of society's preferences ($0.2 < a < 0.8$). In addition, when society is equally concerned about price stability and output stability (i.e. $a = 0.5$), nominal income growth targeting roughly achieves similar performance to the optimal commitment policy. The ratio of social losses under nominal income growth targeting to those under optimal commitment is 1.025, i.e. slightly above unity.

4.2.2. Conservative Central Bank and Liberal Central Bank

As shown in Chart 4, optimal weight a^c under both price level targeting and income growth targeting is less than society's preference a.[20] However, unlike inflation targeting, this result does not necessarily mean that appointing a conservative central bank is desirable, because a^c and a have different meanings as seen from comparing the social loss function (6) and the central bank's loss functions (13)(14). In the social loss function, a can be interpreted as the relative weight placed on the volatility of the *output gap*, compared to the volatility of *inflation*. In the case of price level targeting, a^c measures the relative weight placed on the volatility of the *output gap* compared to the volatility of the *price level*. In the case of income growth targeting, a^c measures the relative weight placed on the volatility of

[20] Contrary to our result, Walsh (2001) suggests that the optimal weight a^c under income growth targeting is larger than society's preference a. This is probably because the parameter l (slope of the Phillips curve) in Walsh's model is much smaller than our estimated value. When l is small, the central bank must pay considerable cost of output gap volatility to stabilize inflation against price shocks, which results in large social losses. Therefore, in such a case, the optimal weight a^c would exceed society's preference a.

real income growth rate compared to the volatility of *inflation*. In this sense, these weights have different benchmarks, and comparing a^c and a does not provide any meaningful information.[21]

To obtain the implication from optimal weight a^c under price level targeting and income growth targeting, we need to transform the central bank's loss functions (13)(14) to the following weighted unconditional variances.

Price level targeting :

$$(1-a^c)V[p_t - p^*] + a^c V[y_t - y_t^*] = (1-a^c)\frac{V[p_t - p^*]}{V[\Delta p_t]}V[\Delta p_t] + a^c V[y_t - y_t^*] \tag{13a}$$

Income growth targeting :

$$(1-a^c)V[\Delta p_t] + a^c V[\Delta y_t - \Delta y_t^*] = (1-a^c)V[\Delta p_t] + a^c \frac{V[\Delta y_t - \Delta y_t^*]}{V[y_t - y_t^*]}V[y_t - y_t^*] \tag{14a}$$

This modified loss function (13a) implies that the relative weight placed on the volatility of the *output gap*, compared to the volatility of *inflation*, can be regarded as

$$a' = \frac{a^c}{(1-a^c)\frac{V[p_t - p^*]}{V[\Delta p_t]} + a^c}$$

under price level targeting. Similarly, the modified loss function (14a) implies that the relative weight placed on the volatility of the *output gap*, compared to the volatility of *inflation*, can be regarded as

$$a' = \frac{a^c \frac{V[\Delta y_t - \Delta y_t^*]}{V[y_t - y_t^*]}}{(1-a^c) + a^c \frac{V[\Delta y_t - \Delta y_t^*]}{V[y_t - y_t^*]}}$$

under income growth targeting.

Now it is meaningful to compare the central bank's implied preference a' with society's preference a, as they have the same benchmarks. If $a' < a$, then appointing a *conservative* central bank is desirable. While if $a' > a$, then appointing a *liberal* central bank, which places greater weight on stabilizing the output gap than society, is desirable. Chart 5 indicates that delegating income growth targeting to a conservative central bank is desirable. On the other hand, delegating price level targeting to a liberal central bank is

[21] Walsh (2001) compares the central bank's preference a^c with society's preference a, and obtains the result of $a^c > a$ under income growth targeting. Then, he concludes that it is desirable for a government to delegate income growth targeting to a liberal central bank. However, the above reason shows the fallacy of such an interpretation.

desirable.[22]

4.2.3. Trend-Reverting Behavior of the Price Level

Chart 6 shows the impulse response of the economy to a negative price shock in period zero where society's preference $a = 0.5$. Under inflation targeting, there is no trend-reversion of the price level. In contrast, under the three history-dependent targeting regimes, trend-reverting behavior of the price level is observed, although the degree of reversion differs according to the regime. While both price level targeting and nominal income growth targeting require the central bank to completely bring the price level back to its initial trend path, it is not socially optimal for the bank to do so in the hybrid Phillips curve ($q > 0$), as noted earlier.[23] Nevertheless, these targeting regimes perform much better than inflation targeting. This is because trend-reversion of the price level induced by history dependence tends to move equilibrium toward what would be achieved under the optimal commitment policy.

The above analysis allows us to conclude that history-dependent targeting regimes are very effective in an economy with a hybrid Phillips curve. A discretionary and conservative central bank, whose objective is the stabilization of income growth, can implement monetary policy that roughly replicates the optimal commitment policy. A discretionary and liberal central bank whose objective is the stabilization of the price level can do the same. A central bank whose objective is the stabilization of nominal income growth also performs well, when society is equally concerned about price stability and output stability. These history-dependent targeting regimes outperform inflation targeting.

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[22] Our result opposes that of Vestin (2000), who suggests that delegating price level targeting to a conservative central banker is desirable. Why the results differ is not clear, but may be because our analysis is based on the hybrid Phillips curve while Vestin's is based on the purely forward-looking Phillips curve.

[23] Under nominal income growth targeting, condition $\Delta p_t + \Delta y_t - \Delta y_t^* = 0$ always holds. This condition can be equivalently expressed as $(p_t - p^*) + (y_t - y_t^*) = 0$, when we assume an initial state is at equilibrium, i.e. $p_{-1} - p^* = y_{-1} - y_{-1}^* = 0$. This means that the price level necessarily reverts to its initial level.

5. Optimal Simple Rules

History-dependent targeting regimes are very effective, but there are several problems in their practical implementation. Under a targeting regime, the central bank chooses a path for the price level and output gap to minimize the loss function subject to both the Phillips curve (2) and price shock process (4). Then it determines the interest rate implied by the IS curve (1) and demand shock process (3), conditional on the optimal values of the price level and output gap (see Appendix for details). Therefore, targeting regimes require full information with respect to demand and price shocks for implementation by the central bank. When the central bank cannot observe these shocks accurately, which is a realistic assumption, optimal delegation is not so effective as our simulation indicates. Even if the central bank can identify these shocks completely, the policy reaction function implied by the IS curve is very complicated and not transparent to the public.

In this section, we investigate another way in which the monetary policy decision process might incorporate the sort of history dependence required for the optimal commitment policy. The previous section assumes that the central bank cannot credibly commit and that monetary policy is conducted under full discretion. However, does this assumption always hold in the real world? Whether such an assumption holds or not depends on the degree of monetary policy's transparency. In other words, sufficient transparency can make the central bank's commitment credible. Recently, many central banks have improved transparency through the publication of periodic 'inflation reports' and minutes that detail their forecasts, judgments, and the conclusions drawn from them. Conducting monetary policy with emphasis on a simple policy rule also improves transparency. The transparency of simple policy rules may increase the visibility of discretionary policy actions and thereby reduce their effectiveness, diminishing the central bank's incentive to deviate from the rule.

In this section, we assume that the central bank can credibly commit to a simple policy rule for the entire future. As many previous studies suggest, policy rules allow the central bank to achieve good monetary policy performance by taking advantage not only of the gains from commitment, but also of the effect of a credible commitment on the way the private sector forms expectations of future variables.[24] In addition, when the central bank takes the "timeless perspective" proposed by Woodford (1999), policy rules are time-

[24] See, for example, Taylor ed. (1999) , Levin, Wieland and Williams (1999), and Williams (1999).

consistent. A simple policy rule is a feedback rule expressing the short-term nominal interest rate (the central bank’s instrument) as a function of current and lagged values of a few variables that can be observed by the central bank, and lagged values of the interest rate itself. Although a targeting regime (optimal delegation) requires full information for implementation by the central bank, a simple policy rule does not. It is possible for the central bank to adopt a simple rule that involves no explicit dependence on demand and price shocks, and so does not even require the bank to know these shocks. In adopting such a simple rule, the central bank relies on private sector awareness of the current underlying shocks to bring about the desired response of endogenous variables to these shocks.

Here, we investigate the effectiveness of simple history-dependent policy rules. The following three questions are of particular interest:

- How can the central bank introduce history dependence into simple policy rules?
- By adopting a simple history-dependent policy rule, can the central bank implement a monetary policy that replicates the optimal commitment policy?
- Which history-dependent policy regime performs better, simple policy rule or optimal delegation?

Giannoni (2000) examined the first two questions with a purely forward-looking Phillips curve. Here, we instead examine these questions with a hybrid Phillips curve, which describes the short-run dynamics of inflation in the real world better than a purely forward-looking Phillips curve. The third question is a very important issue which previous literature has not addressed. If our simulation indicates that a simple policy rule which involves no explicit dependence on demand and price shocks is as effective as optimal delegation, we can conclude that the simple policy rule is more desirable, based on the realistic assumption that the central bank cannot observe shocks accurately.

5.1. Definition of Optimal Simple Rules

Our analysis incorporates a wide variety of policy rules, in which the interest rate may depend on its own lagged values as well as current and lagged values of the price level and output gap. More generally, a policy rule takes the form

$$i_t = g i_{t-1} + (1-g) ri_t^* + g_p (p_t - p^*) + g_p (p_{t-1} - p^*) + g_y (y_t - y_t^*) + g_y (y_{t-1} - y_{t-1}^*) \; . \qquad (16)$$

This general form of policy rule nests three kinds of rules according to the lagged interest rate coefficient: level rules ($g = 0$), partial adjustment rules ($0 < g < 1$), and first difference rules ($g = 1$). In the first difference rule, the change in interest rate responds to the current and past state of the economy. We shall use the term 'interest rate smoothing' to refer to both partial adjustment and first difference rules. Since we assume the policy rule is followed exactly, interest rate smoothing is identical to specifying the policy rule in terms of the level of interest rate responding to a weighted sum of the current and past price level and output gap. As a result, policy rules with interest rate smoothing introduce history dependence to monetary policy.

We focus on the following five simple rules, imposing some restrictions on the coefficient of the general form (16).

Taylor-type rule (TT rule) : $\gamma_p = -\gamma_{lp} > 0, \quad \gamma_y > 0, \quad \gamma_{ly} = 0$ (17)

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i) ri_t^* + \gamma_p \Delta p_t + \gamma_y (y_t - y_t^*)$$

Price level targeting rule (PL rule) : $\gamma_p > 0, \quad \gamma_{lp} = 0, \quad \gamma_y > 0, \quad \gamma_{ly} = 0$ (18)

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i) ri_t^* + \gamma_p (p_t - p^*) + \gamma_y (y_t - y_t^*)$$

Income growth targeting rule (IG rule) : $\gamma_p = -\gamma_{lp} > 0, \quad \gamma_y = -\gamma_{ly} > 0$ (19)

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i) ri_t^* + \gamma_p \Delta p_t + \gamma_y (\Delta y_t - \Delta y_t^*)$$

Nominal income growth targeting rule (NI rule) : $\gamma_p = -\gamma_{lp} = \gamma_y = -\gamma_{ly} > 0$ (20)

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i) ri_t^* + \gamma_p [\Delta p_t + \Delta y_t - \Delta y_t^*]$$

Hybrid rule : $\gamma_p = -\gamma_{lp} > 0, \quad \gamma_y > -\gamma_{ly} > 0$ (21)

$$\begin{aligned} i_t &= \gamma_i i_{t-1} + (1-\gamma_i) ri_t^* + \gamma_p \Delta p_t + \gamma_y (y_t - y_t^*) + \gamma_{ly} (y_{t-1} - y_{t-1}^*) \\ &= \gamma_i i_{t-1} + (1-\gamma_i) ri_t^* + \gamma_p \Delta p_t + (\gamma_y + \gamma_{ly})(y_t - y_t^*) - \gamma_{ly} (\Delta y_t - \Delta y_t^*) \end{aligned}$$

In the Taylor-type rule (TT rule), the interest rate responds to inflation and the output gap. In the price level targeting rule (PL rule), the interest rate responds to price level and output gap. In the income growth targeting rule (IG rule), the interest rate responds to a weighted sum of inflation and real income growth rate. In the nominal income growth targeting rule (NI rule), the interest rate responds literally to the nominal income growth rate. The NI rule is a special case of the IG rule in which the parameter restriction $\gamma_p = \gamma_y$ is imposed. The hybrid rule, in which the interest rate responds to inflation, output gap, and real income growth rate, can be interpreted as a linear combination of the TT rule and IG rule.[25] Targeting either price level or real income growth (i.e. changes in the output gap) introduces history dependence into policy rules. In other words, PL, IG, NI, and hybrid rules even

[25] FED economists often use this hybrid rule to estimate the policy reaction function of the FED. See, for example, Orphanides and Wieland (1998), Levin, Wieland and Williams (1999), and Williams (1999).

without interest rate smoothing are all history-dependent. Note that the TT rule with interest rate smoothing is also history dependent, but the TT rule without interest rate smoothing is not.

For a given functional form of the simple policy rule, we assume that the optimal policy rule is chosen to solve the following optimization problem:

$$\min_{g_i, g_p, g_{lp}, g_y, g_{ly}} (1-a)Var[\Delta p_t] + aVar[y_t - y_t^*] \tag{22}$$

subject to IS curve (1), hybrid Phillips curve (2), and shock processes (3)(4)(5)
simple policy rule (17),(18),(19),(20), or (21)
$\sqrt{Var[i_t]} \leq k$

This means that the central bank sets the coefficients of the simple policy rule to minimize the social loss subject to several constraints. The first set of constraints is the law of motion of the economy modeled by equations (1)-(5). The second constraint is that the interest rate always be set according to the simple policy rule. The third constraint is for the standard deviation of the interest rate not to exceed the specified value of k. Although the volatility of interest rates has no direct effect on the economy in the model, the constraint on interest rate volatility is important in practice. For example, due to the non-negativity constraint on nominal interest rates, policy rules with considerable interest rate volatility cannot be implemented by the central bank. In addition, as Williams (1999) suggests, the hypothesized invariance of the model parameters to changes in policy rules is likely to be stretched to the breaking point under policies that differ so dramatically in terms of interest rate volatility from those seen historically.

Here, we set the upper bound of interest rate volatility (k) to 3%, the actual standard deviation of the call rate over the sample period 1975Q1-1999Q4. Note, however, that k= 3% is much lower than the standard deviation of the interest rate under both optimal commitment and discretion policies (see Chart 7). To fairly compare the performance of simple policy rules with that of benchmark policy, k should be set close to the standard deviation of the interest rate under the optimal commitment and discretion policies. Therefore, we also set k=4%.

5.2. Quantitative Analysis of Optimal Simple Rules

We now present empirical analysis of optimal simple rules. Throughout our analysis, we only consider simple policy rules that generate a unique stationary rational expectations solution. To compute social losses under each policy rule, we determine policy rule parameters which minimize the social losses for each value of a over the range zero to unity. In all cases this is performed numerically by grid search where parameters g, g_p, g_p, g_y, g_y are varied within the relevant ranges with a grid of 0.1 (see Appendix for the numerical solution method).

5.2.1. Interest Rate Smoothing and History Dependence

In Chart 8, we show the lagged interest rate coefficient of optimal rules, and in almost all cases it exceeds zero. This means that either the partial adjustment rule ($0 < g < 1$) or the first difference rule ($g = 1$) outperforms the level rule ($g = 0$). The result that optimal rules generally smooth the interest rate reaction to a change in economic conditions stems from several factors: constraining interest rate volatility ($(Var[i_t])^{0.5} \leq k$) favors interest rate smoothing; the lagged interest rate provides a measure of the existing state of the economy in models with output and inflation persistence.[26] However, these reasons are insufficient to explain a very high degree of smoothing, such as that associated with first difference IG, NI, and hybrid rules. In addition to the above reasons, interest rate smoothing plays a very important role in introducing history dependence to monetary policy, as Woodford (1999) and Giannoni (2000) suggest. We can see the relationship between history dependence and interest rate smoothing by writing the first difference rule equivalently as one in which the level of the interest rate reacts to the sum of current and past values of inflation and the output gap.[27]

First difference IG rule

$$i_t = i_{t-1} + g_p \Delta p_t + g_y (\Delta y_t - \Delta y_t^*) \qquad (23)$$
$$= ri_{-1}^* + g_p (p_t - p^*) + g_y (y_t - y_t^*)$$

First difference NI rule

$$i_t = i_{t-1} + g_p [\Delta p_t + \Delta y_t - \Delta y_t^*] \qquad (24)$$
$$= ri_{-1}^* + g_p [(p_t - p^*) + (y_t - y_t^*)]$$

First difference hybrid rule

$$i_t = i_{t-1} + g_p \Delta p_t + (g_y + g_y)(y_t - y_t^*) - g_y (\Delta y_t - \Delta y_t^*) \qquad (25)$$
$$= ri_{-1}^* + g_p (p_t - p^*) + g_y (y_t - y_t^*) + (g_y + g_y) \sum_{j=1}^{t} (y_{t-j} - y_{t-j}^*)$$

This shows that the first difference IG and NI rules are equivalent to the PL rule without interest rate smoothing. The first difference hybrid rule also involves price level targeting.

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[26] See Williams (1999) and Levin, Wieland and Williams (1999) for these explanations.

[27] Deriving equations (23)-(25), we assume that an initial state is at equilibrium, i.e. $i_{-1} = ri_{-1}^*$ and $p_{-1} - p^* = y_{-1} - y_{-1}^* = 0$.

Therefore, first difference IG, NI, and hybrid rules can be interpreted as ones in which the price level is indirectly targeted. Such price level targeting, by introducing desirable history dependence to monetary policy, results in lower volatility of inflation and lower welfare loss, as noted earlier.

5.2.2. Comparison of Social Welfare Losses

In Charts 9 and 10, we show associated social losses under optimal simple rules, and compare them with those under the optimal commitment and discretion policies. When comparing the performance of simple policy rules with that of optimal commitment or discretion policy, we restrict our attention to the case where $0.2 \le a \le 0.9$. As shown in Chart 7, the standard deviations of the interest rate under both optimal commitment and discretion policies are extremely high in the case of $a < 0.2$. Therefore, in this case, it is not fair to compare such policies with simple policy rules where the constraint of interest rate volatility (k=3% or 4%) is imposed. In the case of $a = 1$, i.e. when society is exclusively concerned about stabilization of the output gap, the central bank which can observe demand shocks stabilizes the output gap completely. Therefore, social losses under both optimal commitment and discretion policies is zero in this case. However, with simple policy rules which involve no explicit dependence on demand shocks, the central bank cannot completely eliminate the effects of these shocks. This results in finite social losses under simple rules, and then the ratio of losses under simple rules to those under optimal policies becomes infinite in the case of $a = 1$. Such a ratio does not provide any meaningful information. This is the reason why we restrict our attention to the case where $0.2 \le a \le 0.9$.

The following four results obtained from Charts 9 and 10 are of particular interest.

First, PL, IG, and hybrid rules outperform the TT rule (see Chart 9). Social losses under the former rules are much smaller than those under the latter rule for all examined values of society's preference a. The NI rule also outperforms the TT rule in the case of $0.3 \le a \le 0.7$, i.e. unless society's preference is biased toward either price stability or output stability. These results hold for both $k = 3\%$ and $k = 4\%$. As Chart 8 shows, history dependence is introduced to the TT rule by some degree of interest rate smoothing, but the TT rule is not so efficient as the other four rules in which history dependence is introduced by targeting the price level directly or indirectly.

Second, PL, IG, and hybrid rules outperform the optimal discretion policy in the case of k=4% (see Chart 9). The ratio of social losses under these rules to those under optimal discretion ranges between 0.83 and 0.91. Even in the case of k=3% (i.e. imposing a severe constraint on interest rate volatility), these rules perform better than the optimal discretion policy unless society's preference is biased toward either price stability or output stability.

Third, in the case of $k=4\%$, the hybrid rule outperforms history-dependent targeting regimes for a wide range of society's preferences (see Chart 10). Both IG and PL rules also achieve roughly the same performance as those targeting regimes. In the case of $k=3\%$, these rules perform appreciably worse than history-dependent targeting regimes, but this is because the latter policy regimes are allowed to generate larger interest rate volatility than the former rules (see Chart 7).

Fourth, by committing to the hybrid rule, the central bank can nearly achieve the same performance as the optimal commitment policy in the case of $k=4\%$ (see Chart 9). The ratio of social losses under the hybrid rule to those under the optimal commitment policy is very close to unity. The hybrid rule with interest rate smoothing is the most efficient among simple policy rules.

The above analysis allows us to conclude that the performance of simple history-dependent rules is very high.

5.2.3. Partial Trend-Reverting Behavior of Price Level

In Chart 11, we show the impulse response of the economy to a negative price shock under simple policy rules. From this simulation result, we can see why the first difference hybrid rule performs best among simple rules and nearly achieves the same performance as the optimal commitment policy. As noted earlier, the trend-reverting behavior of the price level is the key characteristic of history-dependent policy. In the purely forward-looking Phillips curve ($q = 0$), it is optimal for the central bank to completely bring the price level back to its initial trend path. In the hybrid Phillips curve ($q > 0$), however, it is not desirable to do so. This is because the central bank must pay the high cost of greater output gap volatility to achieve full trend-reversion of the price level, when a fraction of firms exhibit backward-looking price-setting behavior. Therefore, as Chart 2 shows, partial trend-reversion of the price level with some degree of drift is socially optimal in the hybrid Phillips curve ($q > 0$). Chart 11 also shows similar behavior of the price level under the hybrid rule. In other words, the price level eventually ends up below its initial level under this rule. The mechanism of the partial trend-reversion of the price level stems from the structure of the first difference hybrid rule itself. Equation (25) indicates that the following relation holds in the equilibrium ($i_t = ri^*_{-1}, y_t = y^*_t$).

$$p_t = p^* - \frac{g_y + g_y}{g_p} \sum_{j=1}^{t} (y_{t-j} - y_{t-j}^*) \quad (26)$$

Because the central bank keeps the output gap positive persistently to the negative price shock, the cumulative value of past output gaps becomes positive. This, in turn, leads to $p_t < p^*$ in the equilibrium, i.e., the partial trend-reverting behavior of the price level. Neither the PL rule nor the first difference IG rule can replicate such partial reverting behavior. This is why the first difference hybrid rule performs best among simple rules.

Although the effectiveness of history-dependent policy may vary according to the degree of forward-looking behavior (parameter q), we expect robust results where the hybrid rule almost replicates the optimal commitment policy as long as the above mechanism works. Just to make it sure, we conduct a simple sensitivity analysis by choosing alternative values for several of the coefficients in the hybrid Phillips curve. In one case, we assume that the weight of forward-looking firms is 30% higher than the benchmark case. In this case, we set the coefficients in the Phillips curve as $q = 0.11$ and $l = 0.44$ according to the structural equation.[28] In another case, we assume that the weight of forward-looking firms is 30% lower than the benchmark case. In this case, we set the coefficients in the Phillips curve as $q = 0.47$ and $l = 0.09$.[29] We do not have enough space to explain the details of the results, therefore we omit them.[30] However, the results suggest that our main conclusion does not change. As the weight of forward-looking firms decreases, the lagged interest rate coefficient of hybrid rule falls slightly, but does remain high. In other words, history dependence introduced by interest rate smoothing is important in both cases. The most important result is that the ratio of social losses under the hybrid rule to those under the optimal commitment policy is very close to unity under both cases. Therefore, the optimal commitment policy can be replicated by the hybrid rule in both cases, and we can conclude that our main conclusion of "hybrid rule for hybrid Phillips curve" is robust.

5.2.4. History-Dependent Policy Rules under the Zero Interest Rate Bound

In our analysis, taking the zero interest rate bound into account, we impose some

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[28] Instead of $b = 0.42$, substituting $b = 0.1$ into the structural equation in footnote 16 leads to $q = 0.11$ and $l = 0.44$.

[29] Instead of $b = 0.42$, substituting $b = 0.7$ into the structural equation in footnote 16 leads to $q = 0.47$ and $l = 0.09$.

[30] The results of sensitivity analysis are available on reader's request.

restrictions on interest rate volatility (*k*=3%,4%). However, we do not explicitly incorporate the non-negativity constraint on interest rates into policy rules. To provide more practical implications of policy rules, we must examine the effect of the zero interest rate bound. This is because the central bank may be constrained in its ability to engineer a negative real interest rate to dampen the output loss due the zero bound, if the economy is faced with a recession in an environment of low inflation.[31] Even in such a case, however, committing to a history-dependent policy rule is the key to minimizing welfare loss.

Here, we focus on the effect of imposing the zero bound on outcomes from the first difference hybrid rule (25) computed under the constraint of k =3%. To take account of the possibility that the zero bound binds, the interest rate prescribed by the first difference hybrid rule (25) would have to be set equal to either (27) or (28).

$$i_t = MAX\left[0,\ i_{t-1} + g_p \Delta p_t + (g_y + g_y)(y_t - y_t^*) - g_y(\Delta y_t - \Delta y_t^*)\right] \tag{27}$$

$$i_t = MAX\left[0,\ \ ri_{-1}^* + g_p(p_t - p^*) + g_y(y_t - y_t^*) + (g_y + g_y)\sum_{j=1}^{t}(y_{t-j} - y_{t-j}^*)\right] \tag{28}$$

These two descriptions are identical as long as the zero bound never binds; but when it does, they are not identical because under the first difference specification any past constraint on the interest rate is perpetuated through policy response to the lagged interest rate. Indeed, simulations indicate that specifications (27) and (28) lead to rather different outcomes. In Chart 12, we show the dynamic response of the economy with the zero interest rate bound to a negative demand shock. (We assume the equilibrium real interest rate to be 2%.) Under specification (27), the detrimental effect of the zero bound on inflation volatility is quite substantial. In contrast, under specification (28) the central bank achieves rather small inflation volatility. This is because the central bank implicitly takes into account past constraints on policy by setting the interest rate to react to the price level and the cumulative past output gap. Such a history-dependent policy tends to be easy — relative to the level dictated by the current state of the economy alone — in periods following a contraction. Given the forward-looking agent's behavior, anticipation of this monetary easing would affect current spending, as higher expectations of future inflation would lower real interest rates even if nominal rates cannot be lowered.

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[31] See Reifschneider and Williams (1999) for details of analysis on the zero interest rate bound.

6. Concluding Remarks

This paper has investigated the effectiveness of history-dependent policy in a unified framework based on a hybrid Phillips curve. Overall, our analysis suggests that history dependence deserves serious attention in discussions of practical policy conduct. Our main findings are summarized as follows: (1) History-dependent targeting regimes, such as price level targeting and income growth targeting, outperform inflation targeting and roughly replicate the optimal commitment policy. (2) Committing to a simple history-dependent policy rule results in nearly the same social welfare as the optimal delegation of price level targeting and income growth targeting. (3) Moreover, a central bank can achieve almost the same performance as the optimal commitment policy by adopting the first difference hybrid policy rule.

The result that simple policy rules perform as well as (or even outperform) optimal delegation regimes provides important implications for policy design. On the realistic assumption that the central bank cannot accurately observe demand and price shocks, this result implies that simple policy rules which involve no explicit dependence on these shocks are more desirable than optimal delegation which requires full information concerning shocks.

The result that the first difference hybrid rule nearly achieves the welfare-optimal allocation is also of particular importance. When inflation exhibits endogenous persistence, as suggested by the estimation result of the hybrid Phillips curve, the central bank must pay the high cost of greater output gap volatility to completely stabilize the price level against price shocks. Therefore, partial trend-reverting behavior of the price level is socially optimal. Partial trend-reversion of the price level can be replicated by the hybrid rule, but not by optimal delegation, such as price level targeting and nominal income growth targeting. Accordingly, this result can also be regarded as an endorsement of simple policy rules.

Finally, we would like to remark on future research. Although the partial trend-reversion of the price can be replicated by the hybrid rule, the degree to which the central bank should bring the price level back to its initial trend path depends on the degree of forward-looking behavior in the inflation dynamics. In practice, however, central banks are uncertain about the weight of forward-looking firms. By analyzing various specifications of the hybrid Phillips curve, a central bank may reduce the uncertainty about inflation dynamics, but cannot eliminate it completely. When there is uncertainty about the persistence of inflation, how should a central bank conduct monetary policy? Should it respond more aggressively to shocks than under certainty equivalence, or more cautiously? Should a bank decrease the degree of trend-reversion of the price, or not? Although some studies have analyzed the uncertainty about the persistence of inflation, their analyses are based on the backward-looking Philips curve, but not the hybrid Phillips curve.[32] Analysis of the uncertainty about the hybrid Phillips curve remains to be solved in future.

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[32] See, for example, Söderstrom (2000).

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(Chart 1)

Estimation Results of Model

IS curve Equation (1)		Hybrid Phillips curve Equation (2)		Demand shock Equation (3)	Price shock Equation (4)	Productivity shock Equation (5)
f	s	q	l	y	r	w
0.91 (0.02)	1.53 (0.46)	0.35 (0.04)	0.20 (0.03)	-0.04 (0.10)	-0.26 (0.09)	0.95 (0.002)
$\bar{R}^2 = 0.80,\ SE = 0.60$		$\bar{R}^2 = 0.80, SE = 2.51$				

$e_t = (\hat{e}_t, \hat{m}_t, \hat{x}_t)'$, $E[e_t] = 0$, $V[e_t] = \Sigma = L \cdot L'$

$$L = \begin{pmatrix} 0.60 & 0 & 0 \\ -0.19 & 0.95 & 0 \\ -0.01 & 0.01 & 0.05 \end{pmatrix}$$

Σ is the variance-covariance matrix of e_t.

Notes:

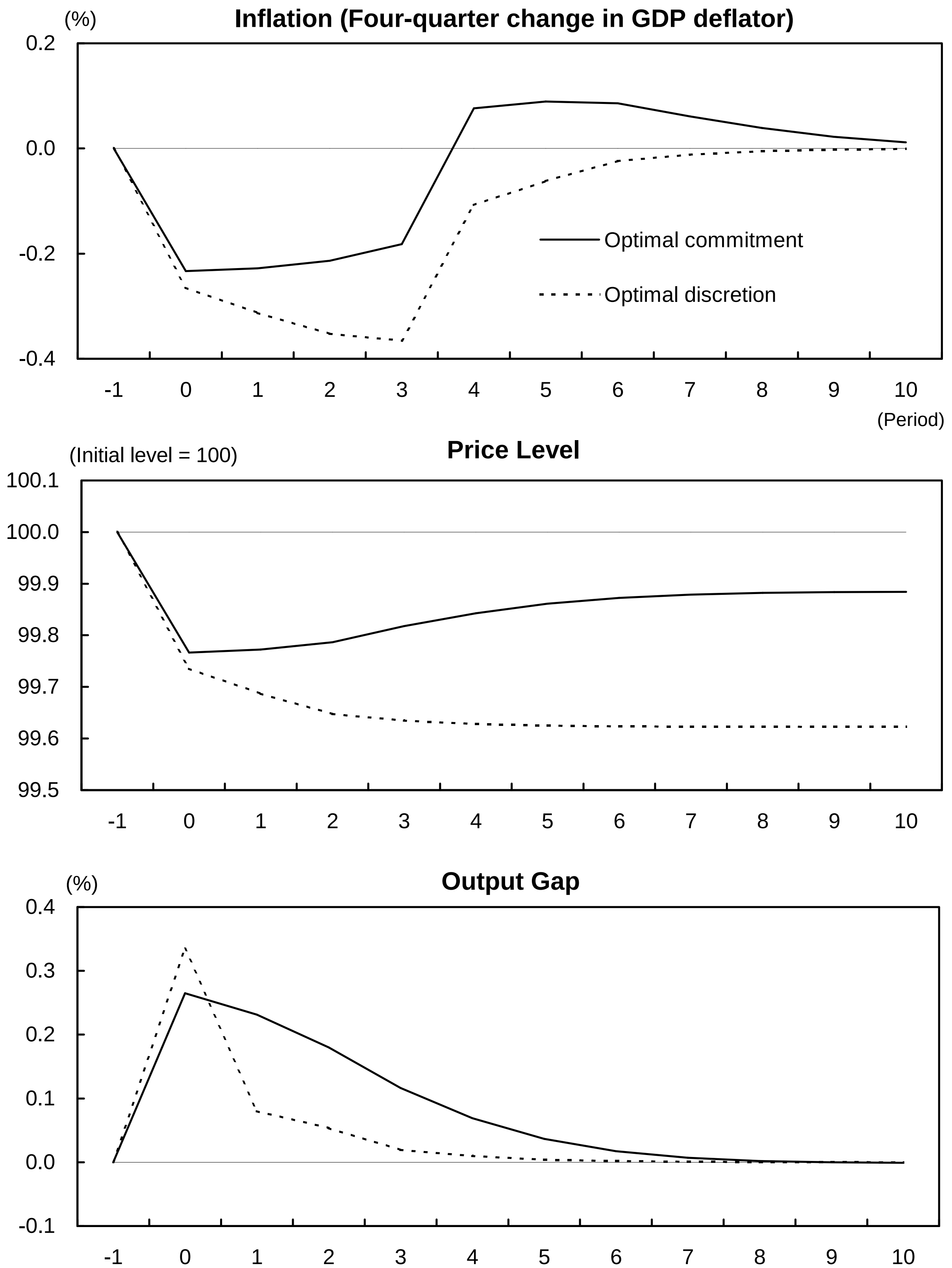
1. Standard errors are in parentheses.
2. We estimate equations (1)-(2) using GMM methods and lagged variables as instruments. Based on Hansen test, we do not reject the overidentifying restrictions (J statistic = 24.57, with associated p-value of 0.94).
3. We estimate equations (1)-(2), including constant terms and imposing $b = 1$. The constant term in equation (1) is statistically significant, but the one in equation (2) is not.
4. Since potential growth rate shifted down gradually in the 1990s, we estimate equation (5) with a smooth underlying trend

$$t_t = c_1 - \frac{c_2}{1 + \exp(-c_3(t - T))}, \quad 0 < c_1, c_2, c_3 .$$

For very large c_3, t_t essentially collapses to a step function that equals c_1 when $t < T$ and equals c_1-c_2 when $t > T$. In this case, the estimate of T offers the best estimated location of the structural break. We estimate equations (3)-(5) with the above smooth trend by non-linear least squares. The estimated midpoint of the transition (determined by T) is 1989Q2.

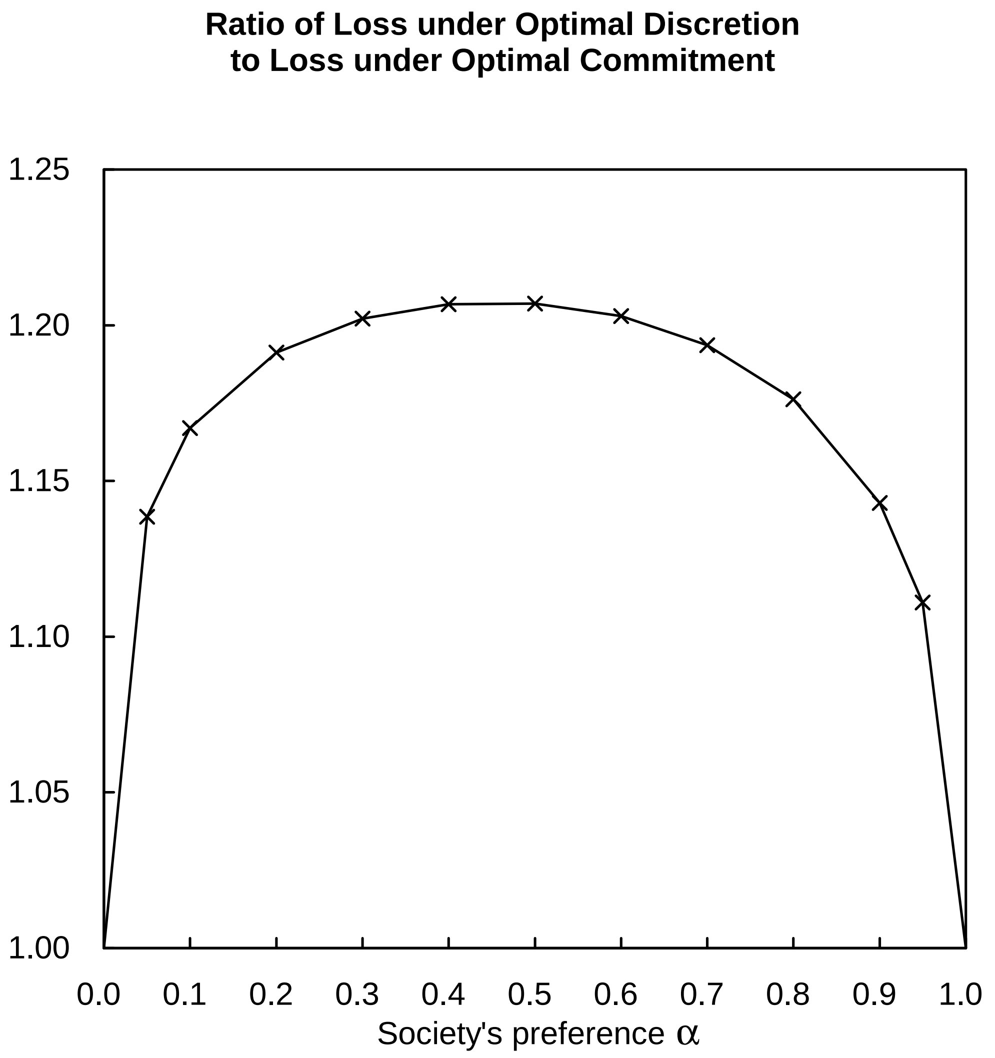
(Chart 2)

Dynamic Response to Price Shock under Optimal Policy



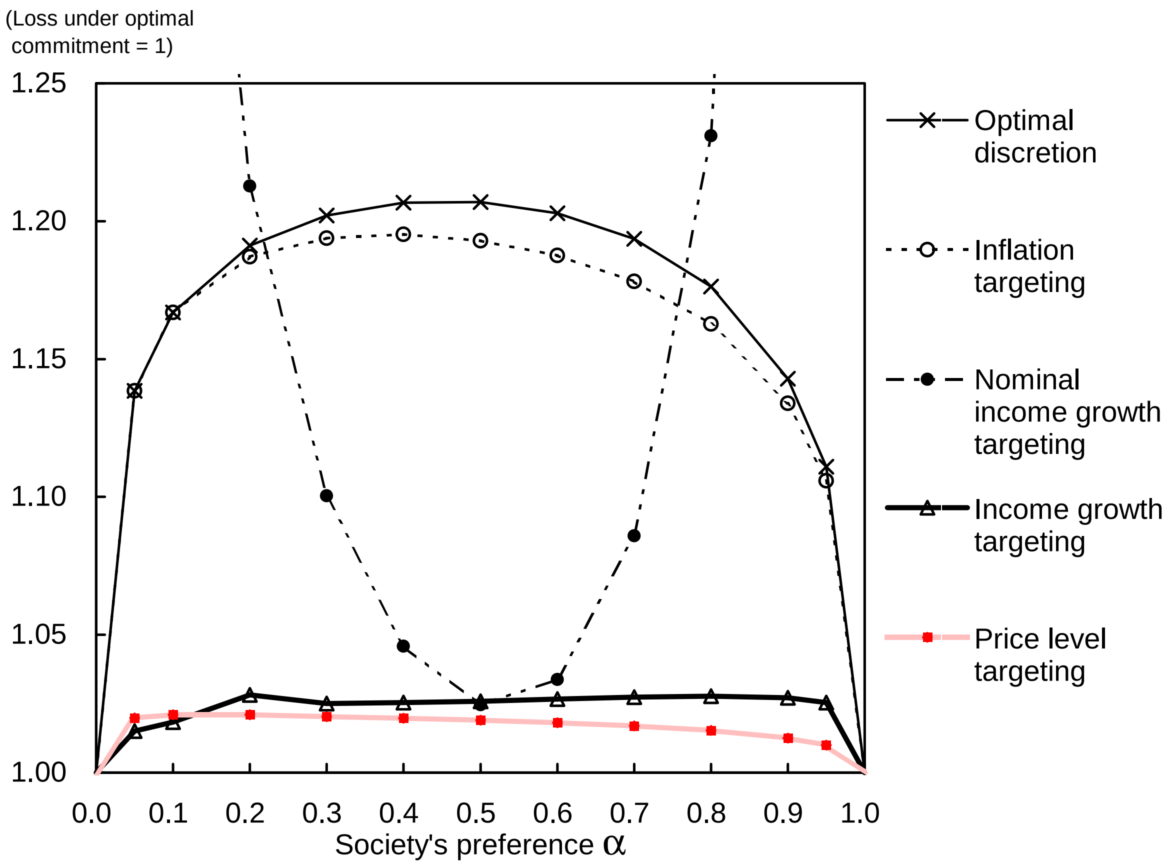
Note: The above figures plot the dynamic response of the endogenous variables following a one standard deviation negative realization of the price shock in period zero. Society's preference α is set to 0.5.

Social Welfare Loss under Optimal Policy

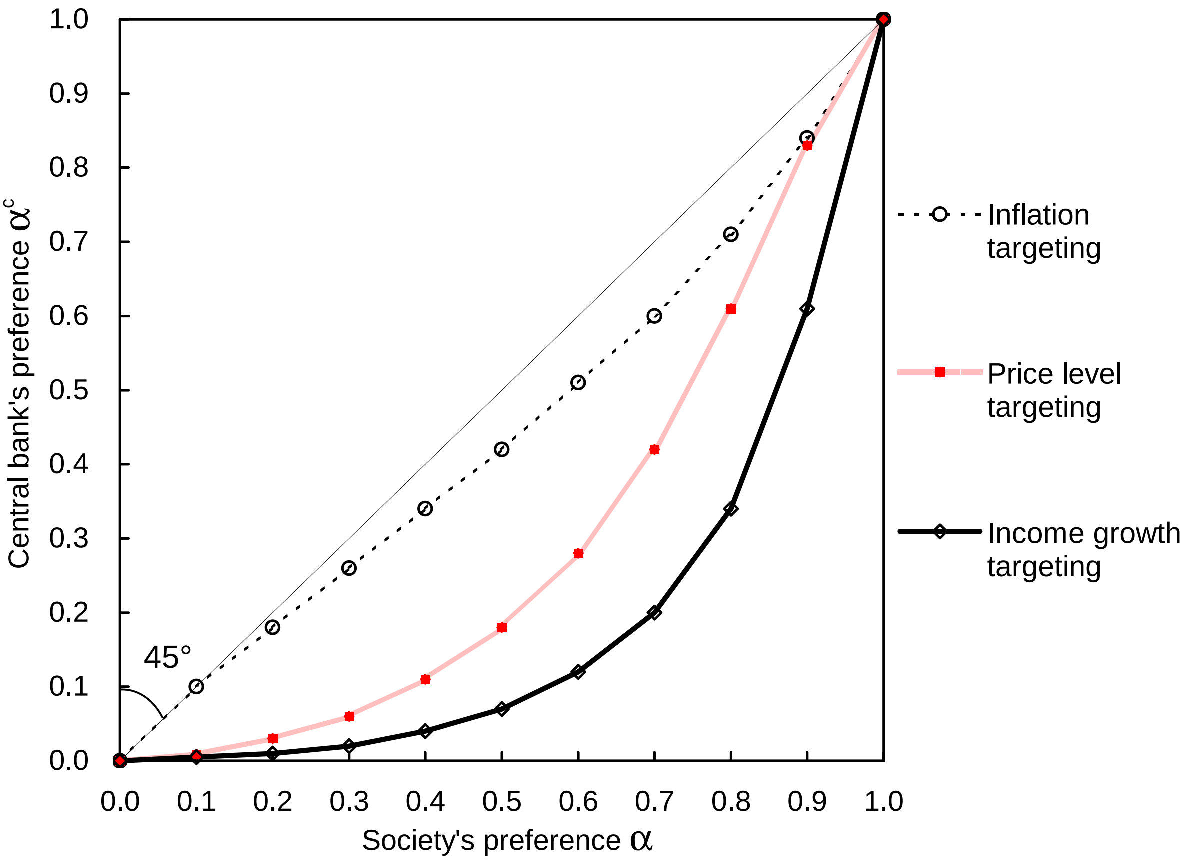


(Chart 4)

Social Welfare Loss Relative to Optimal Commitment

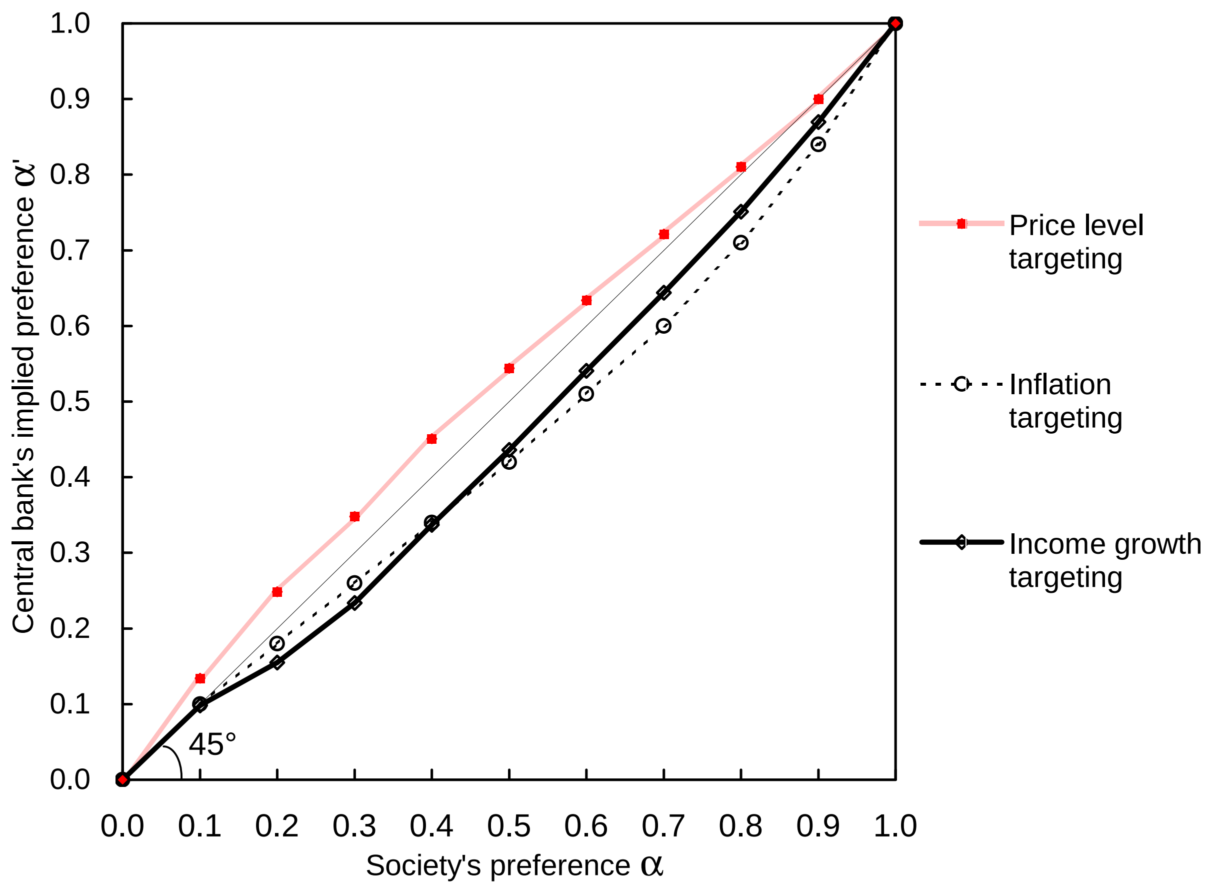


Optimal Weight a^c under Discretion



(Chart 5)

Conservative Central Bank and Liberal Central Bank



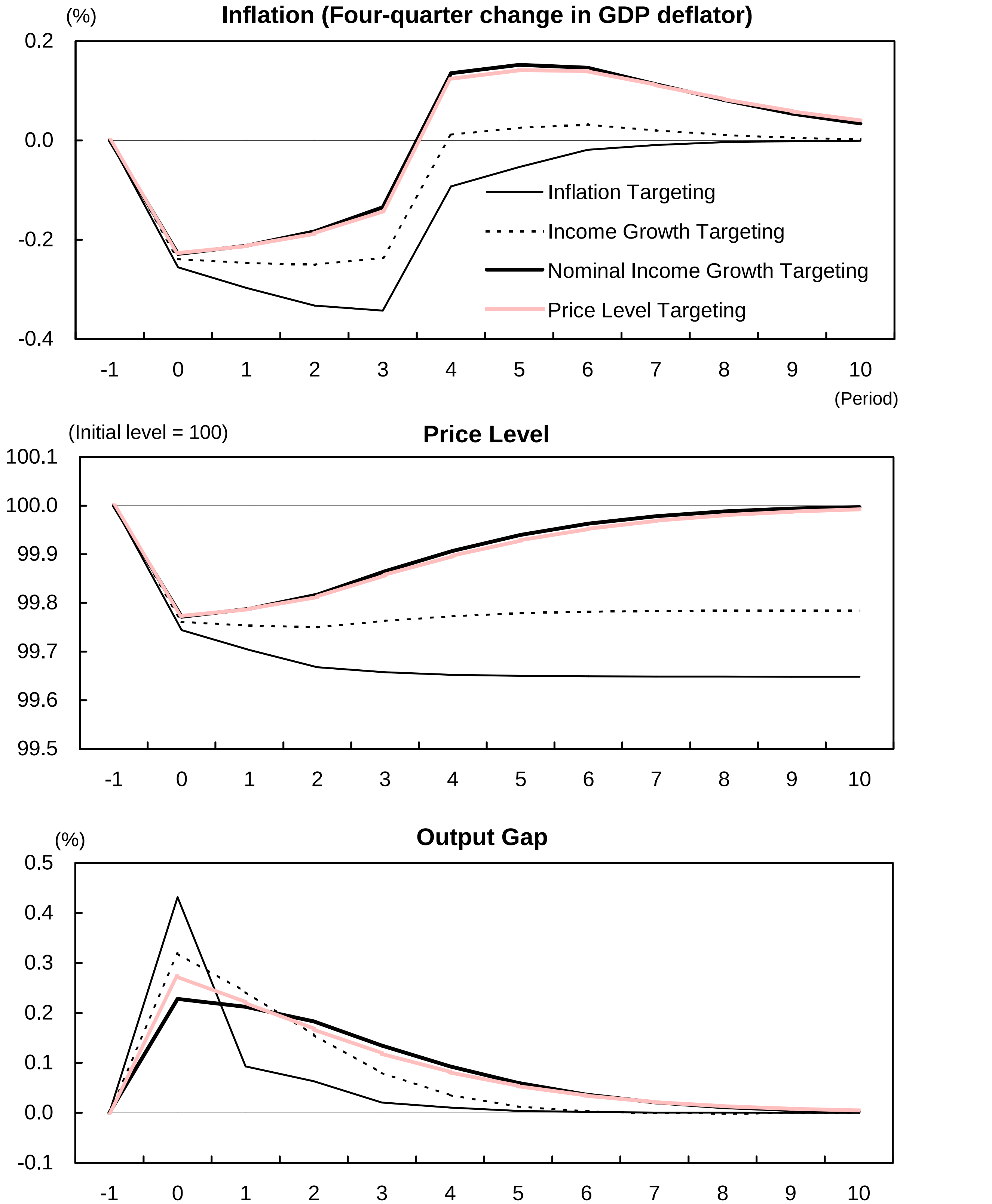
Inflation targeting: $a' = a^c$

Price level targeting:
$$a' = \frac{a^c}{(1-a^c)\dfrac{V[p_t - p^*]}{V[\Delta p_t]} + a^c}$$

Income growth targeting:
$$a' = \frac{a^c \dfrac{V[\Delta y_t - \Delta y_t^*]}{V[y_t - y_t^*]}}{(1-a^c) + a^c \dfrac{V[\Delta y_t - \Delta y_t^*]}{V[y_t - y_t^*]}}$$

(Chart 6)

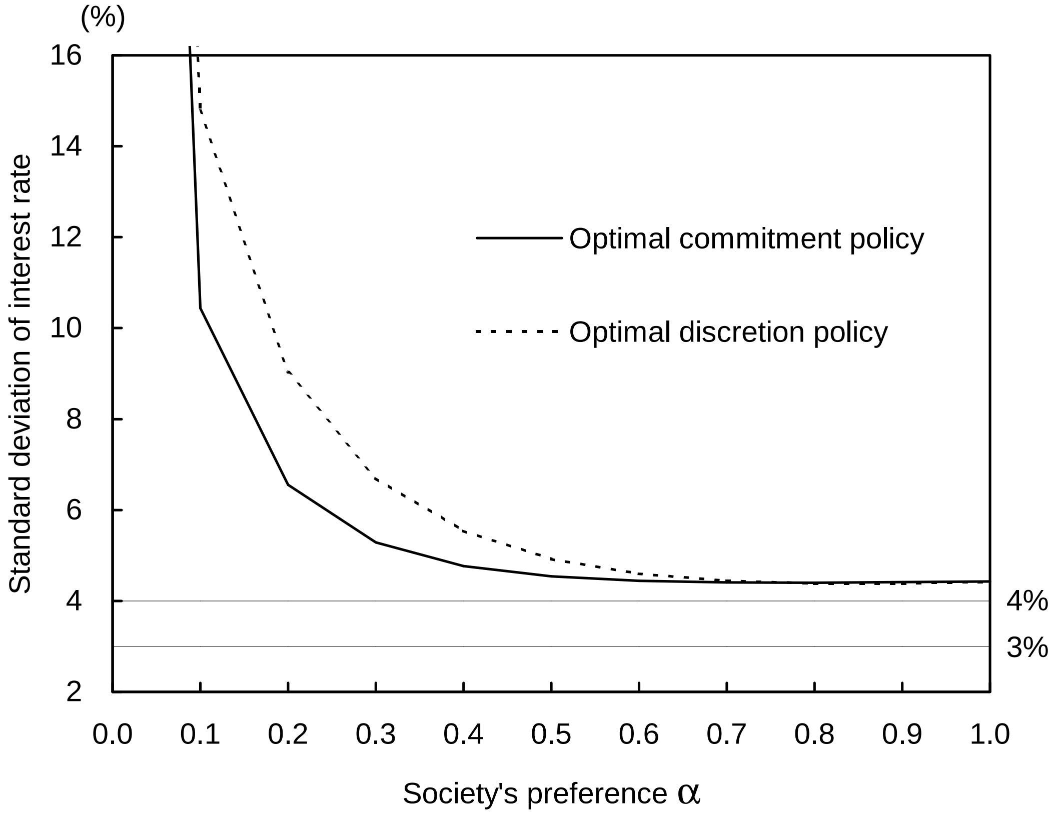
Dynamic Response to Price Shock under Optimal Delegation

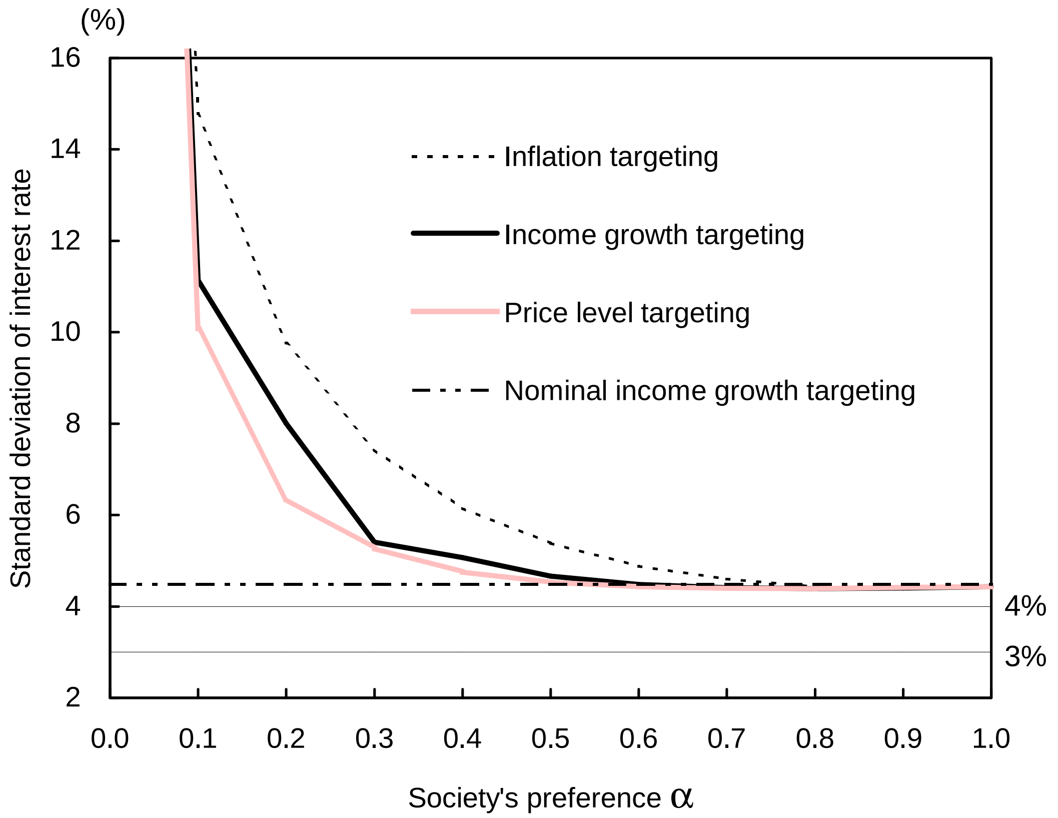


Note: The above figures plot the dynamic response of the endogenous variables following a one standard deviation negative realization of the price shock in period zero. Society's preference α is set to 0.5.

(Chart 7)

Interest Rate Volatility

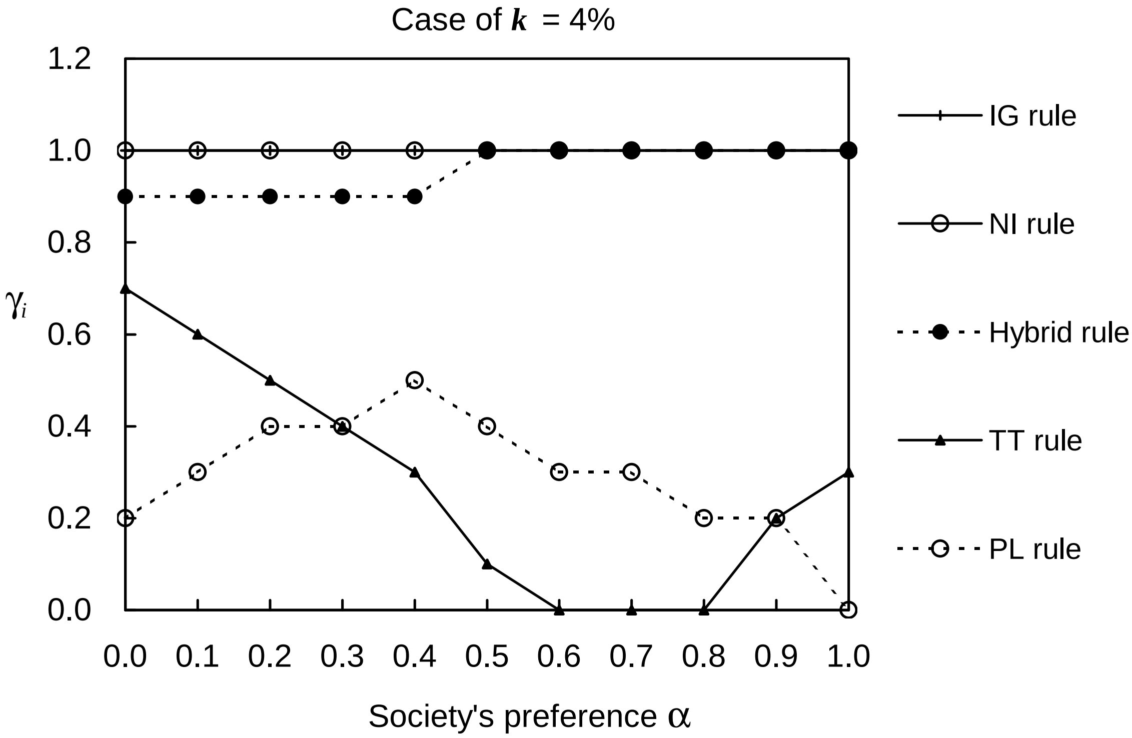


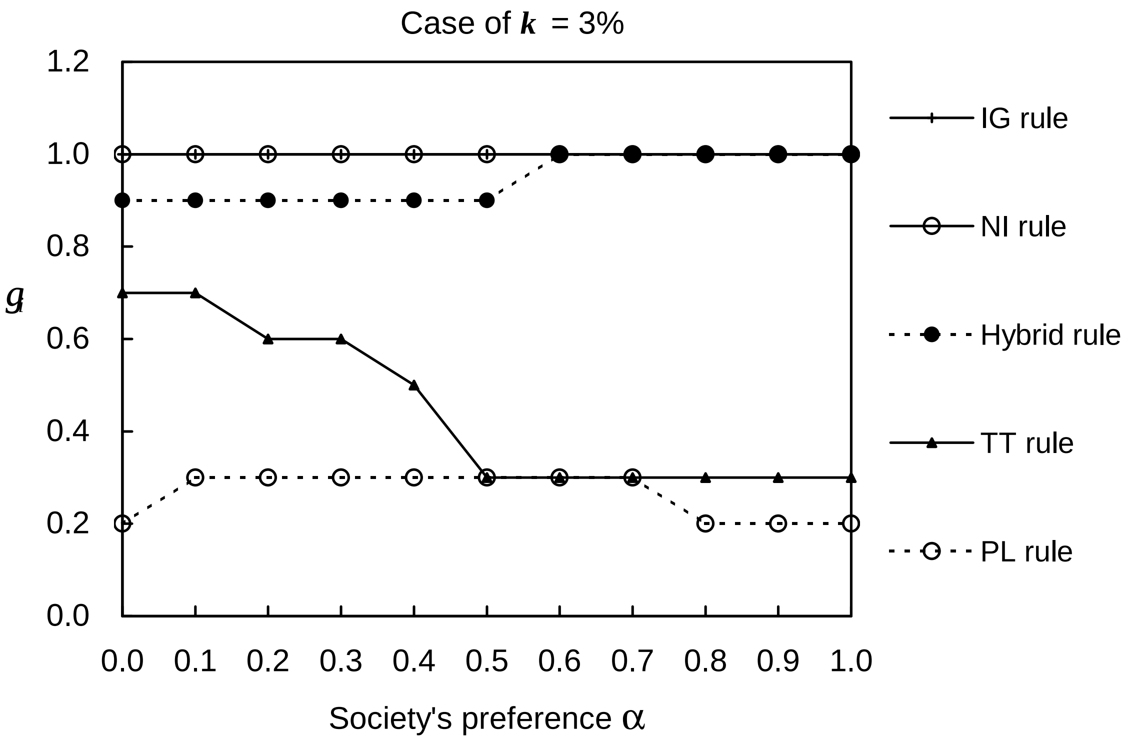


(Chart 8)

Interest Rate Smoothing in Optimal Simple Policy Rules

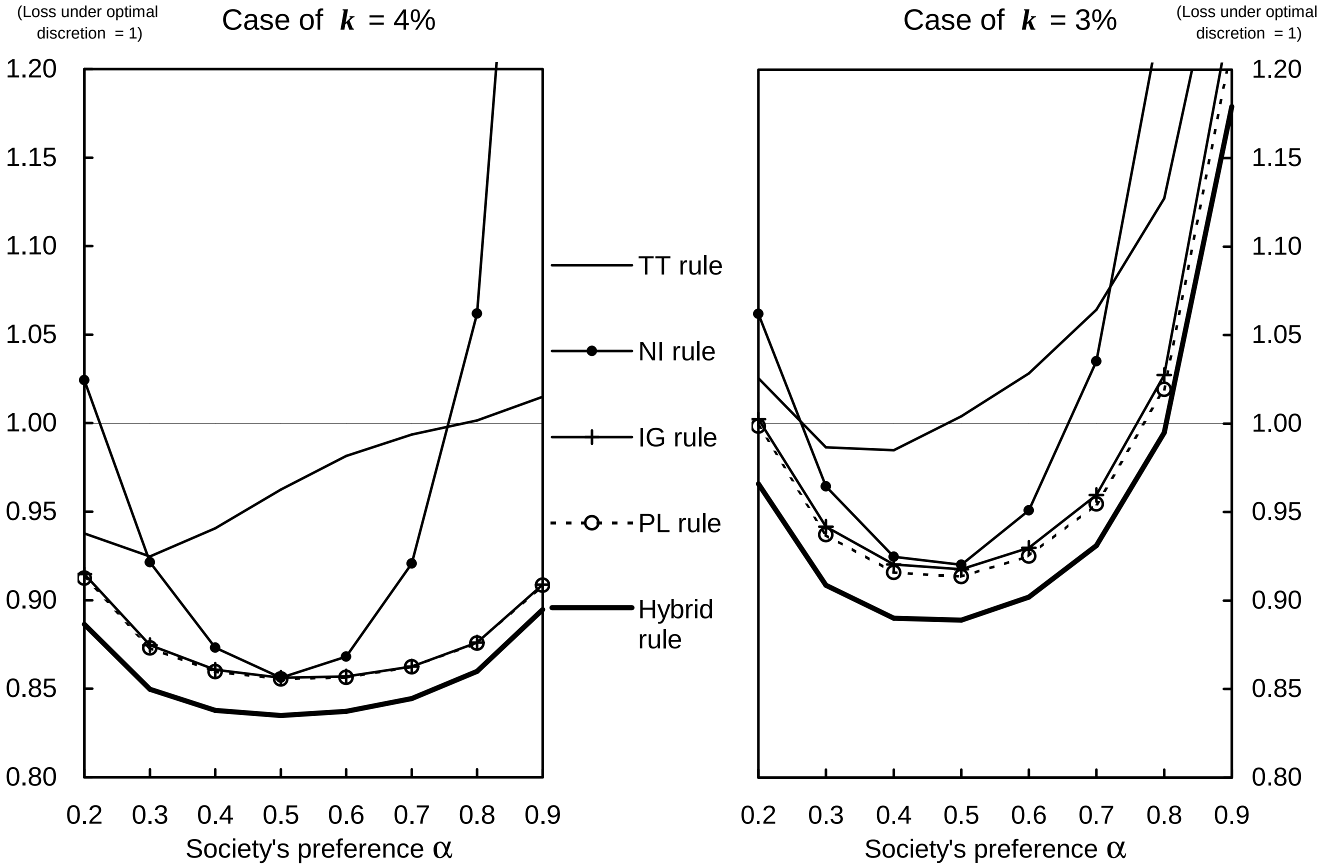
— Coefficient of the Lagged Interest Rate γ_i —



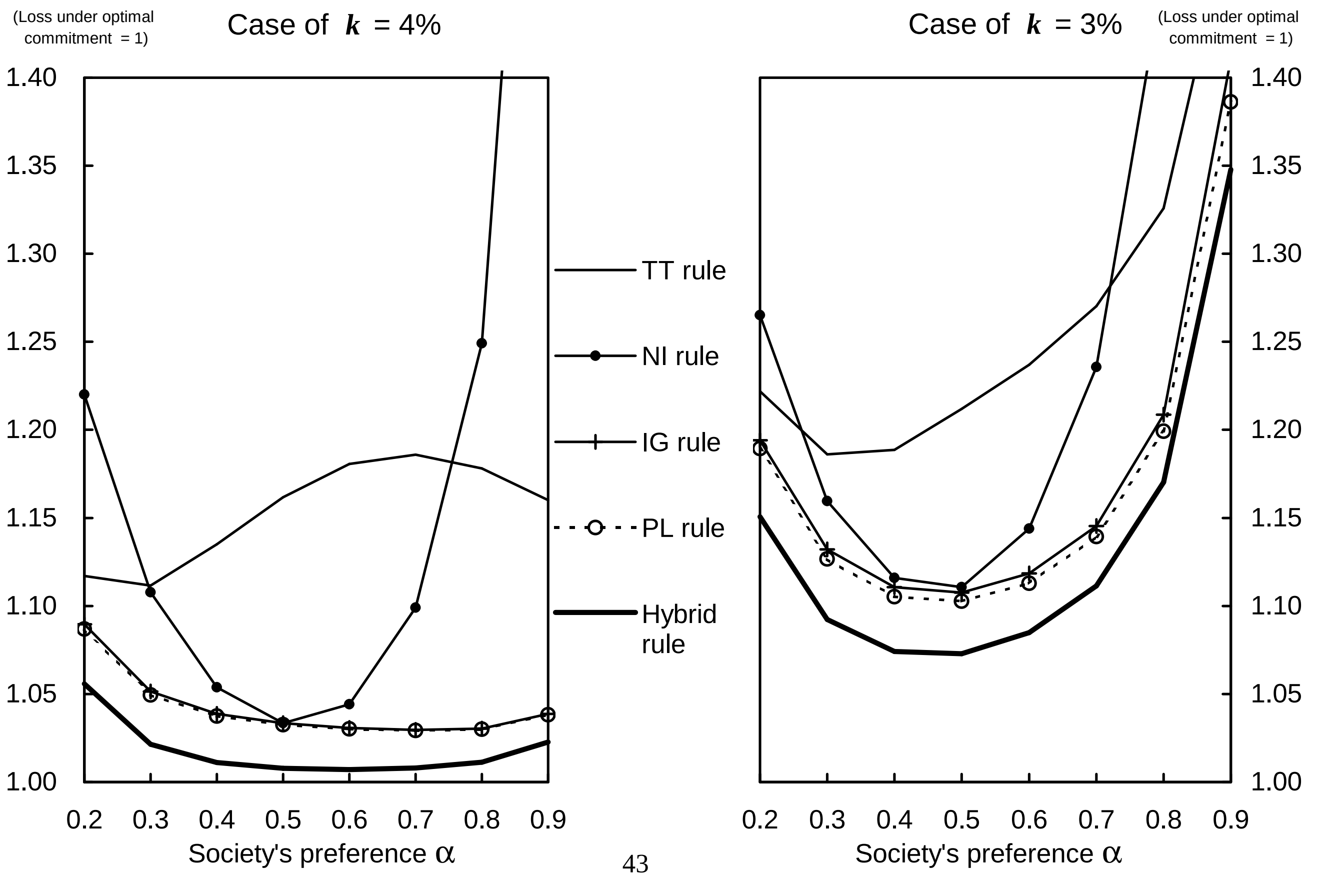


(Chart 9)

Social Welfare Loss Relative to Optimal Discretion Policy

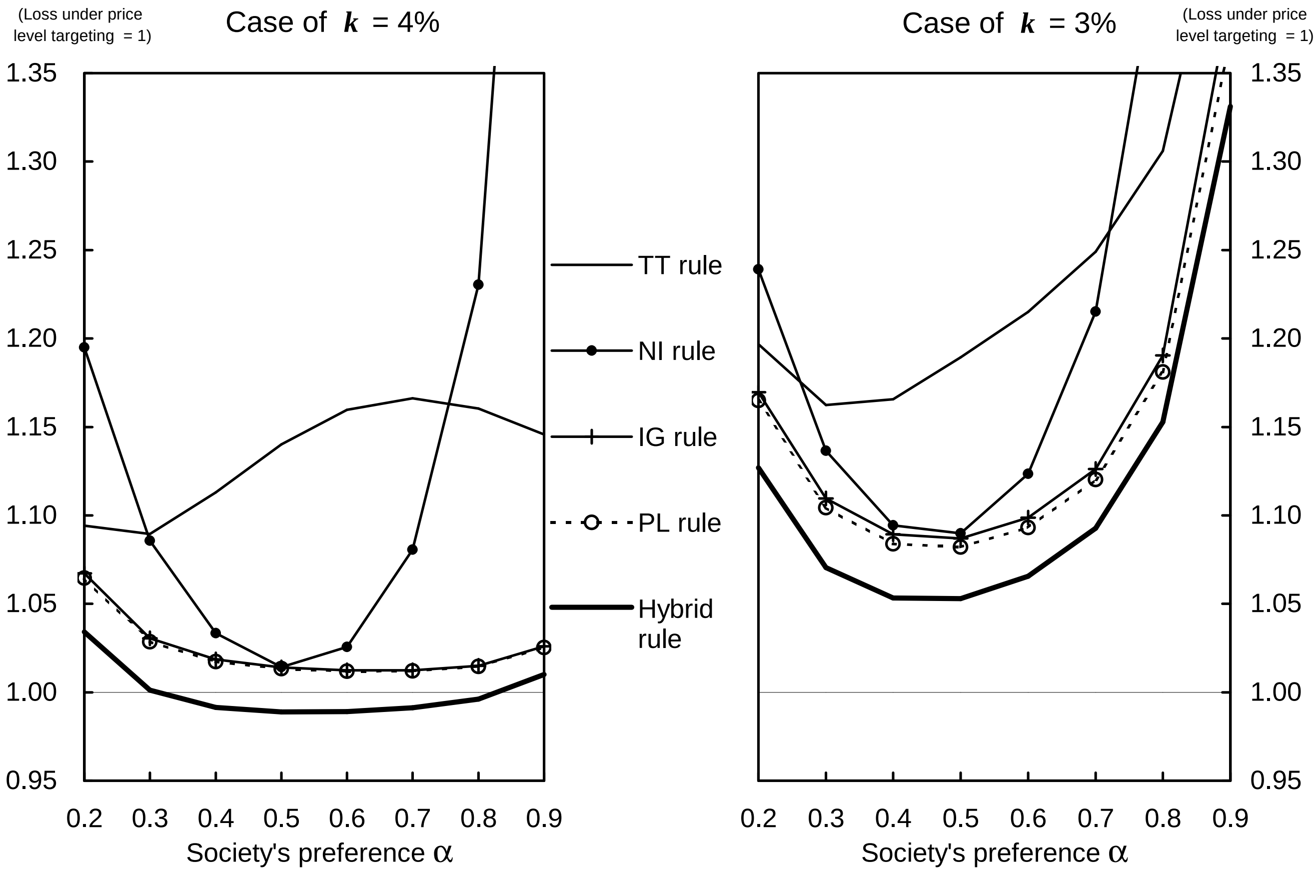


Social Welfare Loss Relative to Optimal Commitment Policy

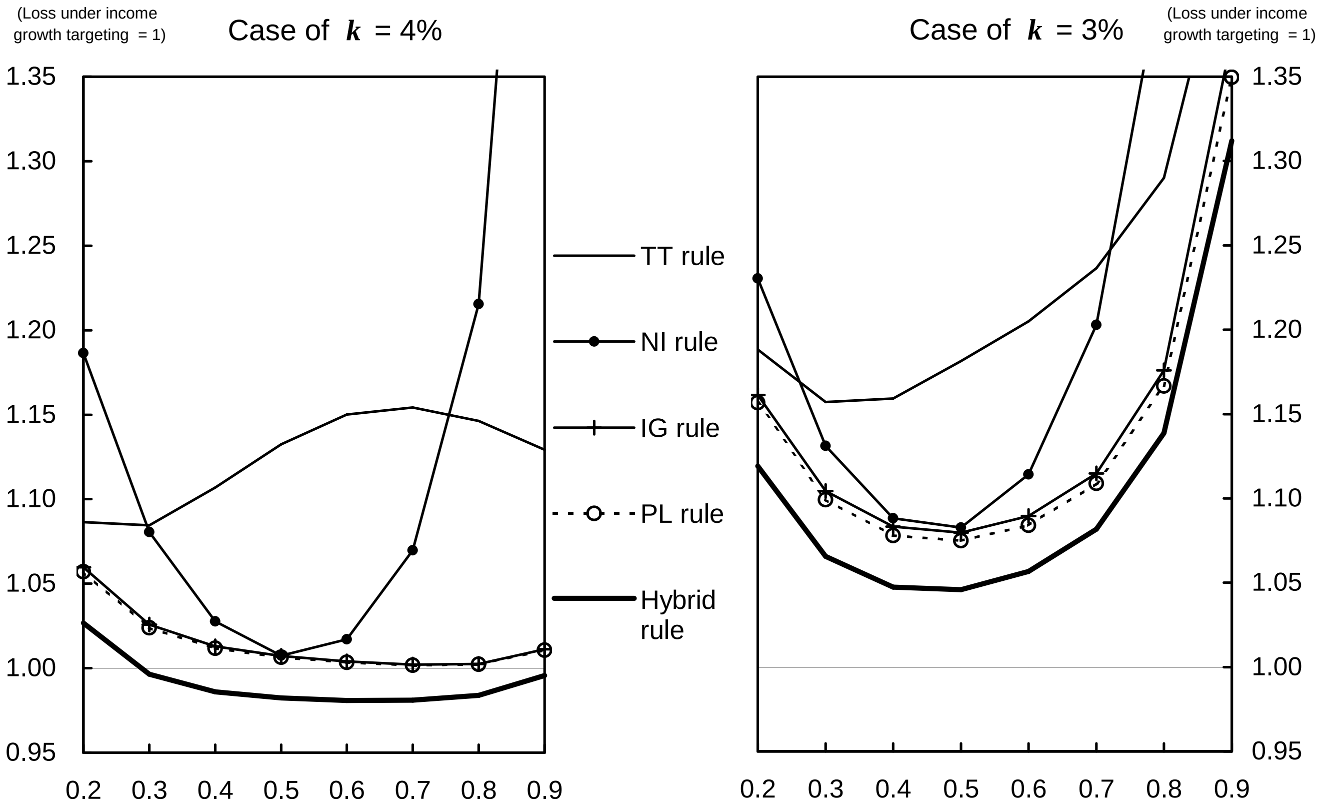


(Chart 10)

Social Welfare Loss Relative to Price Level Targeting

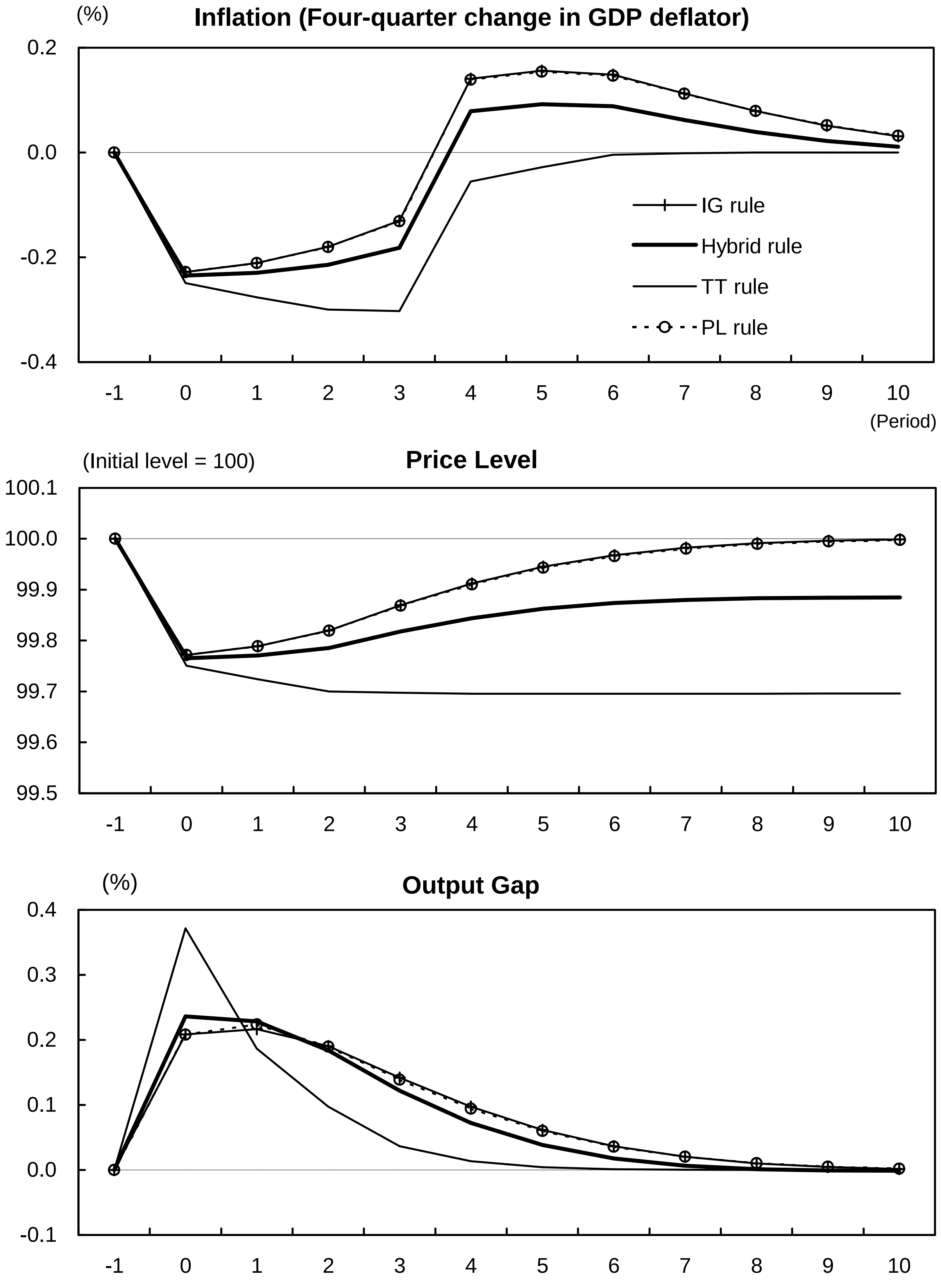


Social Welfare Loss Relative to Income Growth Targeting



(Chart 11)

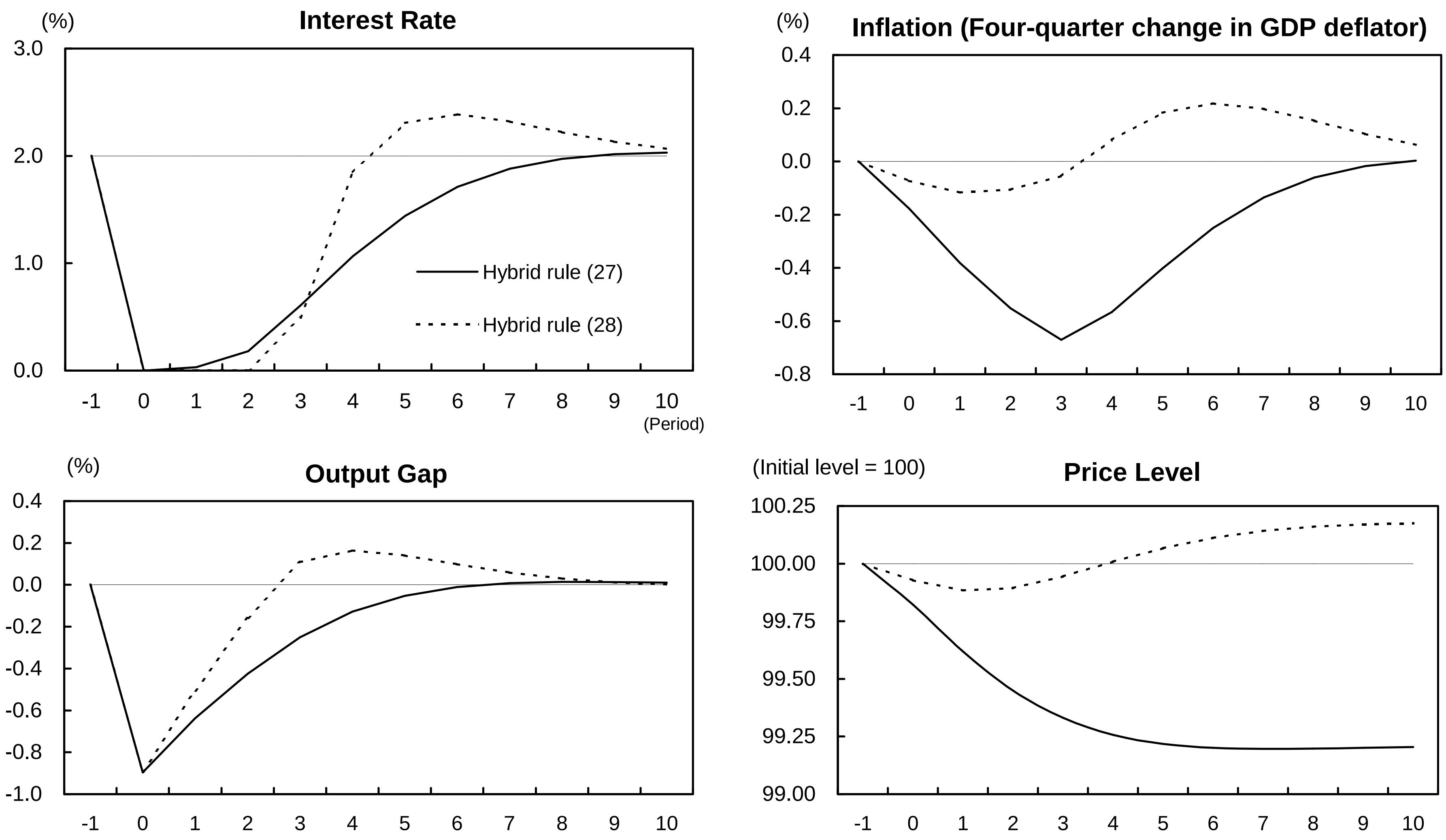
Dynamic Response to Price Shock under Optimal Simple Policy Rules



Note: The above figures plot the dynamic response of the endogenous variables following a one standard deviation negative realization of the price shock in period zero. Society's preference α is set to 0.5.

(Chart 12)

Dynamic Response to Negative Demand Shock with Zero Interest Rate Bound



Note: The above figures plot the dynamic response of the endogenous variables following a 1% negative demand shock in period zero. We assume that the equilibrium real interest rate is 2%. The coefficients of the hybrid rule are set to the optimal ones for the case of $\alpha = 0.5$ and $k = 3\%$.

Appendix

A. Numerical Solutions of Unique Rational Expectations Equilibrium under Optimal Commitment Policy and Optimal Simple Policy Rules

Consider the following linear rational expectations model:

$$\begin{bmatrix} z_{t+1}^b \\ E_t[z_{t+1}^f] \end{bmatrix} = M \begin{bmatrix} z_t^b \\ z_t^f \end{bmatrix} + \begin{bmatrix} e_{t+1} \\ 0_{n_f \times 1} \end{bmatrix}, \qquad \text{(A1)}$$

where z_t^b denotes a ($n_b \times 1$)-vector of predetermined (backward-looking) variables with their initial values z_0^b given, z_t^f a ($n_f \times 1$)-vector of non-predetermined (forward-looking) variables, and e_t a ($n_b \times 1$)-vector of mean-zero innovations to z_t^b with covariance matrix Σ. M is a $(n_b + n_f)\times(n_b + n_f)$-matrix whose elements depend on the monetary policy parameters (i.e. parameters on the first-order conditions or simple policy rules) as well as the structural parameters of the economy.

We compute a unique stationary rational expectations equilibrium of the model (A1) by using the Blanchard-Kahn method[1]. The reduced form representation of equilibrium processes is given by

$$z_t^f = M_f z_t^b, \quad z_{t+1}^b = M_b z_t^b + e_{t+1}. \qquad \text{(A2)}$$

The elements of the matrices M_f and M_b depend on those of matrix M.

We then conduct stochastic simulations of the reduced model (A2) to obtain the stochastic distributions of endogenous variables. Using the covariance matrix Σ of innovation e_t, we generate 100 sets of artificial normally-distributed shocks with 1,000 quarters of shocks in each set. We use these shocks to conduct stochastic simulations under alternative monetary policies, and calculate the variances of the endogenous variables (and thus the social welfare losses).

We can apply the above calculation method to solving a unique rational expectations equilibrium under the optimal commitment policy and simple policy rules.

Case 1: Optimal Commitment Policy

Necessary for calculating a unique rational expectations equilibrium under the optimal commitment policy are the following three equations:

$$\Delta p_t = q\Delta p_{t-1} + (1-q)E_t[\Delta p_{t+1}] + l(y_t - y_t^*) + m_t, \qquad (2)$$

$$m_{t+1} = rm_t + \hat{m}_{t+1}, \qquad (4)$$

$$l(1-a)\Delta p_t = -a\{y_t - y_t^* - (1-q)(y_{t-1} - y_{t-1}^*) - qE_t[y_{t+1} - y_{t+1}^*]\}, \qquad \text{(7b)}$$

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[1] See Blanchard and Kahn (1980), Söderlind (1999), and Klein (2000) for details of computation method.

$$\text{where} \quad y_{-1} - y_{-1}^{*} = 0 \,. \tag{7a}$$

These equations can be reduced to the state-space form (A1), where

$$z_t^b = \begin{bmatrix} m_t \\ \Delta p_{t-1} \\ y_{t-1} - y_{t-1}^{*} \end{bmatrix}, \quad z_0^b = \begin{bmatrix} \hat{m}_0 \\ 0 \\ 0 \end{bmatrix}, \quad z_t^f = \begin{bmatrix} \Delta p_t \\ y_t - y_t^{*} \end{bmatrix}, \quad e_t = \begin{bmatrix} \hat{m}_t \\ 0 \\ 0 \end{bmatrix},$$

$$M = \begin{bmatrix} r & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1/(1-q) & -q/(1-q) & 0 & 1/(1-q) & -l/(1-q) \\ 0 & 0 & -(1-q)/q & l(1-a)/aq & 1/q \end{bmatrix}.$$

Using the reduced form (A2) based on this state-space form, we calculate a unique rational expectations equilibrium to obtain unique optimal plans of the forward-looking variables, i.e. inflation and the output gap. Then, the interest rate implied by the IS curve (1) is determined, conditional on the unique optimal plans of inflation and the output gap.

Case 2: TT Rule

Necessary for calculating a unique rational expectations equilibrium under the TT rule are the following six equations.

$$y_t - y_t^{*} = f(y_{t-1} - y_{t-1}^{*}) + (1-f)\{E_t[y_{t+1} - y_{t+1}^{*}] - s\,(i_t - E_t[\Delta p_{t+1}] - s^{-1}E_t[\Delta y_{t+1}^{*}])\} + e_t\,, \tag{1}$$

$$\Delta p_t = q\Delta p_{t-1} + (1-q)E_t[\Delta p_{t+1}] + l(y_t - y_t^{*}) + m_t\,, \tag{2}$$

$$e_{t+1} = y\,e_t + \hat{e}_{t+1}\,, \tag{3}$$

$$m_{t+1} = r m_t + \hat{m}_{t+1}\,, \tag{4}$$

$$\Delta y_{t+1}^{*} = (1-w)t + w\Delta y_t^{*} + \hat{x}_{t+1}\,, \tag{5}$$

$$i_t = g i_{t-1} + (1-g)s^{-1}E_t[\Delta y_{t+1}^{*}] + g_p\Delta p_t + g_y(y_t - y_t^{*})\,. \tag{17}$$

Note that, unlike the optimal commitment policy, the central bank cannot offset the effects of the demand and productivity shocks on the economy completely when it adopts a simple policy rule which involves no explicit dependence on these shocks. Therefore, to calculate the equilibrium process for endogenous variables under the TT rule, we must include equations (1), (3), and (5) as well as (2) and (4) in the model.

The six equations above are written in state-space form (A1), where

$$z_t^b = \begin{bmatrix} e_t \\ m_t \\ \Delta y_t^{*} - t \\ \Delta p_{t-1} \\ y_{t-1} - y_{t-1}^{*} \\ i_{t-1} - t/s \end{bmatrix}, \quad z_0^b = \begin{bmatrix} \hat{e}_0 \\ \hat{m}_0 \\ \hat{x}_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad z_t^f = \begin{bmatrix} \Delta p_t \\ y_t - y_t^{*} \end{bmatrix}, \quad e_t = \begin{bmatrix} \hat{e}_t \\ \hat{m}_t \\ \hat{x}_t \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$
M = \begin{bmatrix}
y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & r & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & w & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & (1-g)w/s & 0 & 0 & g & g_p & g_y \\
0 & -1/(1-q) & 0 & -q/(1-q) & 0 & 0 & 1/(1-q) & -l/(1-q) \\
-1/(1-f) & s/(1-q) & -gw & -sq/(1-q) & -f/(1-f) & s\,g & M_{87} & M_{88}
\end{bmatrix},
$$

$$M_{87} = s\{g_p - 1/(1-q)\}, \quad M_{88} = 1/(1-f) + s\{g_y + l/(1-q)\}.$$

Then we obtain the reduced form representation (A2) of the unique rational expectations equilibrium of the model (A1), and choose the optimal coefficients on the TT rule by solving the following optimization problem:

$$\min_{g,\, g_p,\, g_y} \ (1-a)Var[\Delta p_t] + a Var[y_t - y_t^*] \tag{22}$$

$$\textit{subject to} \ \text{ the reduced form (A2) and } \sqrt{Var[i_t]} \le k$$

In the same way, we can compute rational expectations equilibrium and optimal coefficients under the alternative policy rules. In the following, we only show the state-space form (A1) under each policy rule.

Case 3 : PL rule

$$i_t = g i_{t-1} + (1-g)s^{-1}E_t[\Delta y_{t+1}^*] + g_p(p_t - p^*) + g_y(y_t - y_t^*), \tag{18}$$

$$
z_t^b = \begin{bmatrix} e_t \\ m_t \\ \Delta y_t^* - t \\ p_{t-1} - p^* \\ p_{t-2} - p^* \\ y_{t-1} - y_{t-1}^* \\ i_{t-1} - t/s \end{bmatrix}, \quad
z_0^b = \begin{bmatrix} \hat{e}_0 \\ \hat{m}_0 \\ \hat{x}_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad
z_t^f = \begin{bmatrix} p_t - p^* \\ y_t - y_t^* \end{bmatrix}, \quad
e_t = \begin{bmatrix} \hat{e}_t \\ \hat{m}_t \\ \hat{x}_t \\ 0 \\ 0 \\ 0 \end{bmatrix},
$$

$$
M = \begin{bmatrix}
y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & w & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & (1-g_i)w/s & 0 & 0 & 0 & g_i & g_p & g_y \\
0 & M_{82} & 0 & M_{84} & M_{85} & 0 & 0 & M_{88} & M_{89} \\
-1/(1-f) & M_{92} & -g_i w & M_{94} & M_{95} & -f/(1-f) & g_i s & M_{98} & M_{99}
\end{bmatrix},
$$

$$M_{82} = -1/\{(1-q)\}, \quad M_{84} = -(1+q)/(1-q), \quad M_{85} = q/(1-q), \quad M_{88} = (2-q)/(1-q),$$

$$M_{89} = -l/(1-q), \quad M_{92} = s/(1-q), \quad M_{94} = s(1+q)/(1-q), \quad M_{95} = -sq/(1-q),$$

$$M_{98} = s\{ g_p - 1/(1-q)\}, \quad M_{99} = 1/(1-f) + s[g_y + l/(1-q)].$$

Case 4 : IG rule and NI rule

$$i_t = g_i i_{t-1} + (1-g_i) s^{-1} E_t[\Delta y^*_{t+1}] + g_p \Delta p_t + g_y(\Delta y_t - \Delta y^*_t), \tag{19}$$

$$i_t = g_i i_{t-1} + (1-g_i) s^{-1} E_t[\Delta y^*_{t+1}] + g_p[\Delta p_t + \Delta y_t - \Delta y^*_t], \tag{20}$$

$$z^b_t = \begin{bmatrix} e_t \\ m_t \\ \Delta y^*_t - t \\ \Delta p_{t-1} \\ y_{t-1} - y^*_{t-1} \\ i_{t-1} - t/s \end{bmatrix}, \quad z^b_0 = \begin{bmatrix} \hat{e}_0 \\ \hat{m}_0 \\ \hat{x}_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad z^f_t = \begin{bmatrix} \Delta p_t \\ y_t - y^*_t \end{bmatrix}, \quad e_t = \begin{bmatrix} \hat{e}_t \\ \hat{m}_t \\ \hat{x}_t \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$M = \begin{bmatrix} y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & (1-g_i)w/s & 0 & -g_y & g_i & g_p & g_y \\ 0 & -1/(1-q) & 0 & -q/(1-q) & 0 & 0 & 1/(1-q) & -l/(1-q) \\ -1/(1-f) & s/(1-q) & -g_i w & sq/(1-q) & M_{85} & s g_i & M_{87} & M_{88} \end{bmatrix},$$

$$M_{85} = -(f/(1-f) + s g_y), \quad M_{87} = s\{ g_p - 1/(1-q)\}, \quad M_{88} = 1/(1-f) + s\{ g_y + l/(1-q)\}.$$

When we impose $g_p = g_y$ on the elements of matrix M, the state-space form (A1) corresponds to the NI rule.

Case 5 : Hybrid rule

$$i_t = g_i i_{t-1} + (1-g_i) s^{-1} E_t[\Delta y^*_{t+1}] + g_p \Delta p_t + g_y(y_t - y^*_t) + g_y(y_{t-1} - y^*_{t-1}), \tag{21}$$

$$z^b_t = \begin{bmatrix} e_t \\ m_t \\ \Delta y^*_t - t \\ \Delta p_{t-1} \\ y_{t-1} - y^*_{t-1} \\ i_{t-1} - t/s \end{bmatrix}, \quad z^b_0 = \begin{bmatrix} \hat{e}_0 \\ \hat{m}_0 \\ \hat{x}_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad z^f_t = \begin{bmatrix} \Delta p_t \\ y_t - y^*_t \end{bmatrix}, \quad e_t = \begin{bmatrix} \hat{e}_t \\ \hat{m}_t \\ \hat{x}_t \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$M = \begin{bmatrix} y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & (1-g_i)w/s & 0 & g_y & g_i & g_p & g_y \\ 0 & -1/(1-q) & 0 & -q/(1-q) & 0 & 0 & 1/(1-q) & -l/(1-q) \\ -1/(1-f) & s/(1-q) & -g_i w & sq/(1-q) & M_{85} & s g_i & M_{87} & M_{88} \end{bmatrix}$$

$$M_{85} = -(f/(1-f) + s\,g_y)\,,\quad M_{87} = s\{g_p - 1/(1-q)\}\,,\quad M_{88} = 1/(1-f) + s\{g_y + g_y + l/(1-q)\}\,.$$

B. Numerical Solution of Optimal Discretionary Equilibrium under Targeting Regimes

The central bank's decision problem under discretion can be written as

$$\min_{\{u_{t+j}\}} \; E_t\left[\sum_{j=0}^{\infty} b^j g'_{t+j}\Lambda g_{t+j}\right] \tag{B1}$$

$$\textit{subject to}\quad g_t = G_b z_t^b + G_f z_t^f + G_u u_t \tag{B2}$$

$$\begin{bmatrix} z_{t+1}^b \\ E_t[z_{t+1}^f] \end{bmatrix} = \begin{bmatrix} A_{bb} & A_{bf} \\ A_{fb} & A_{ff} \end{bmatrix}\begin{bmatrix} z_t^b \\ z_t^f \end{bmatrix} + \begin{bmatrix} B_b \\ B_f \end{bmatrix} u_t + \begin{bmatrix} e_{t+1} \\ 0_{n_f \times 1} \end{bmatrix} \tag{B3}$$

$$z_t^f = M_f z_t^b\,, \tag{B4}$$

where g_t denotes a ($n_g \times 1$)-vector of the central bank's target variables, Λ a symmetric ($n_g \times n_g$)-matrix of the central bank's preference for g_t, u_t a ($n_u \times 1$)-vector of policy instruments, and M_f a matrix of the undetermined coefficients. (B3) represents the law of motion of the economy. Given the linear quadratic structure of the problem, the equilibrium processes of the forward-looking variables take the form (B4). Recognizing that private agents form their expectations according to (B4), the central bank conducts monetary policy under optimal discretion.

Since the decision problem (B1)-(B4) can be reduced to a conventional dynamic programming one, we can solve optimal discretionary equilibrium using Bellman's equation[2]. As a result, the optimal feedback rule under discretion is given by

$$u_t = F z_t^b\,, \tag{B5}$$

and z_t^b and z_t^f evolve, given z_0^b, according to

$$z_t^f = M_f z_t^b\,,\quad z_{t+1}^b = M_b z_t^b + e_{t+1}\,. \tag{B6}$$

The elements of matrices F, M_f and M_b depend on the central bank's preference as well as the structural parameters of the economy. We then conduct simulations of the reduced form (B5)-(B6) to obtain the stochastic distributions of the endogenous variables, using the covariance matrix Σ of innovation e_t.

Case 1 : Inflation Targeting

The central bank's decision problem under inflation targeting is as follows:

[2] See Flodén (1996) and Söderlind (1999) for details of the computation method.

$$\min_{\{y_{t+j}-y^*_{t+j}\}} E_t\left[\sum_{j=0}^{\infty} b^j\{(1-a_c)(\Delta p_{t+j})^2 + a_c(y_{t+j}-y^*_{t+j})^2\}\right] \quad (12)$$

subject to $\Delta p_t = q\Delta p_{t-1} + (1-q)E_t[\Delta p_{t+1}] + l(y_t - y^*_t) + m_t$ (2)

$m_{t+1} = rm_t + \hat{m}_{t+1}$ (4)

This problem can be reduced to the state-space form (B1)-(B3), where

$$z^b_t = \begin{bmatrix} m_t \\ \Delta p_{t-1} \end{bmatrix},\ z^b_0 = \begin{bmatrix} \hat{m}_0 \\ 0 \end{bmatrix},\ z^f_t = \Delta p_t,\ u_t = y_t - y^*_t,\ e_t = \begin{bmatrix} \hat{m}_t \\ 0 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 1-a_c & 0 \\ 0 & a_c \end{bmatrix},\ G_b = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ G_f = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ G_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ A_{bb} = \begin{bmatrix} r & 0 \\ 0 & 0 \end{bmatrix},\ A_{bf} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$A_{fb} = [-1/(1-q) \quad -q/(1-q)],\ A_{ff} = 1/(1-q),\ B_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ B_f = -l/(1-q).$$

Solving this dynamic programming problem, we obtain the following equilibrium processes for inflation and the output gap:

$$\Delta p_t = a_p m_t + b_p \Delta p_{t-1},$$
$$y_t - y^*_t = a_y m_t + b_y \Delta p_{t-1}.$$

Given these equilibrium processes, it follows from the IS curve (2) that the equilibrium process for the interest rate is

$$i_t = ri^*_t + (b_p + b_y/s)\Delta p_t - 1/\{s(1-f)\}(y_t - y^*_t)$$
$$+ f/\{s(1-f)\}(y_{t-1} - y^*_{t-1}) + r(a_p + a_y/s)m_t + 1/\{s(1-f)\}e_t.$$

Case 2 : Price Level Targeting

The central bank's decision problem under price level targeting is as follows:

$$\min_{\{y_{t+j}-y^*_{t+j}\}} E_t\left[\sum_{j=0}^{\infty} b^j[(1-a_c)(p_{t+j} - p^*)^2 + a_c(y_{t+j}-y^*_{t+j})^2]\right] \quad (13)$$

subject to $\Delta p_t = q\Delta p_{t-1} + (1-q)E_t[\Delta p_{t+1}] + l(y_t - y^*_t) + m_t$ (2)

$m_{t+1} = rm_t + \hat{m}_{t+1}$ (4)

This problem can be reduced to the state-space form (B1)-(B3), where

$$z^b_t = \begin{bmatrix} m_t \\ p_{t-1} - p^* \\ p_{t-2} - p^* \end{bmatrix},\ z^b_0 = \begin{bmatrix} \hat{m}_0 \\ 0 \\ 0 \end{bmatrix},\ z^f_t = p_t - p^*,\ u_t = y_t - y^*_t,\ e_t = \begin{bmatrix} \hat{m}_t \\ 0 \\ 0 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 1-a_c & 0 \\ 0 & a_c \end{bmatrix},\ G_b = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ G_f = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ G_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ A_{bb} = \begin{bmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ A_{bf} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$A_{fb} = [-1/(1-q) \quad -(1+q)/(1-q) \quad q/(1-q)],\ A_{ff} = (2-q)/(1-q),\ B_b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$B_f = -l/(1-q).$$

Solving this dynamic programming problem, we obtain the following equilibrium processes for inflation and the output gap:

$$p_t - p^* = a_p m_t + b_p (p_{t-1} - p^*) + c_p (p_{t-2} - p^*) ,$$
$$y_t - y_t^* = a_y m_t + b_y (p_{t-1} - p^*) + c_y (p_{t-2} - p^*) .$$

Given these equilibrium processes, it follows from the IS curve (2) that the equilibrium process for the interest rate is

$$i_t = ri_t^* + (b_p + b_y / s)(p_t - p^*) + (c_p + c_y / s - 1)(p_{t-1} - p^*) - 1/\{s(1-f)\}(y_t - y_t^*)$$
$$+ f/\{s(1-f)\}(y_{t-1} - y_{t-1}^*) + r(a_p + a_y / s)m_t + 1/\{s(1-f)\}e_t \qquad .$$

Case 3 : Income Growth Targeting

The central bank's decision problem under income growth targeting is as follows:

$$\min_{\{y_{t+j} - y_{t+j}^*\}} E_t \left[\sum_{j=0}^{\infty} b^j \{(1-a_c)(\Delta p_{t+j})^2 + a_c (\Delta y_{t+j} - \Delta y_{t+j}^*)^2\} \right] \tag{14}$$

subject to $\Delta p_t = q\Delta p_{t-1} + (1-q)E_t[\Delta p_{t+1}] + l(y_t - y_t^*) + m_t$ (2)

$m_{t+1} = rm_t + \hat{m}_{t+1}$ (4)

This problem can be reduced to the state-space form (B1)-(B3), where

$$z_t^b = \begin{bmatrix} m_t \\ \Delta p_{t-1} \\ y_{t-1} - y_{t-1}^* \end{bmatrix}, \quad z_0^b = \begin{bmatrix} \hat{m}_0 \\ 0 \\ 0 \end{bmatrix}, \quad z_t^f = \Delta p_t, \quad u_t = y_t - y_t^*, \quad e_t = \begin{bmatrix} \hat{m}_t \\ 0 \\ 0 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 1-a_c & 0 & 0 \\ 0 & a_c & -a_c \\ 0 & -a_c & a_c \end{bmatrix}, \quad G_b = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G_f = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad G_u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad A_{bb} = \begin{bmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A_{bf} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad A_{fb} = \begin{bmatrix} -1/(1-q) & -q/(1-q) & 0 \end{bmatrix}, \quad A_{ff} = 1/(1-q), \quad B_b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad B_f = -l/(1-q).$$

Solving this dynamic programming problem, we obtain the following equilibrium processes for inflation and the output gap:

$$\Delta p_t = a_p m_t + b_p \Delta p_{t-1} + c_p (y_{t-1} - y_{t-1}^*) ,$$
$$y_t - y_t^* = a_y m_t + b_y \Delta p_{t-1} + c_y (y_{t-1} - y_{t-1}^*) .$$

Given these equilibrium processes, it follows from the IS curve (2) that the equilibrium process for the interest rate is

$$i_t = ri_t^* + (b_p + b_y / s)\Delta p_t + [c_p + c_y / s - 1/\{s(1-f)\}](y_t - y_t^*)$$
$$+ f/\{s(1-f)\}(y_{t-1} - y_{t-1}^*) + r(a_p + a_y / s)m_t + 1/\{s(1-f)\}e_t .$$

Case 4 : Nominal Income Growth Targeting

The central bank's decision problem under nominal income growth targeting is as follows:

$$\min_{\{y_{t+j}-y^*_{t+j}\}} E_t\left[\sum_{j=0}^{\infty} b^j (\Delta p_{t+j} + \Delta y_{t+j} - \Delta y^*_{t+j})^2\right] \quad (15)$$

subject to $\Delta p_t = q\Delta p_{t-1} + (1-q)E_t[\Delta p_{t+1}] + l(y_t - y^*_t) + m_t$ (2)

$m_{t+1} = rm_t + \hat{m}_{t+1}$ (4)

This problem can be reduced to the state-space form (B1)-(B3), where

$$z^b_t = \begin{bmatrix} m_t \\ \Delta p_{t-1} \\ y_{t-1} - y^*_{t-1} \end{bmatrix}, \quad z^b_0 = \begin{bmatrix} \hat{m}_0 \\ 0 \\ 0 \end{bmatrix}, \quad z^f_t = \Delta p_t, \quad u_t = y_t - y^*_t, \quad e_t = \begin{bmatrix} \hat{m}_t \\ 0 \\ 0 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}, \quad G_b = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G_f = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad G_u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad A_{bb} = \begin{bmatrix} r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_{bf} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$A_{fb} = \begin{bmatrix} -1/(1-q) & -q/(1-q) & 0 \end{bmatrix}, \quad A_{ff} = 1/(1-q), \quad B_b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad B_f = -l/(1-q).$$

Solving this dynamic programming problem, we obtain the following equilibrium processes for inflation and the output gap:

$$\Delta p_t = a_p m_t + b_p \Delta p_{t-1} + c_p (y_{t-1} - y^*_{t-1}),$$
$$y_t - y^*_t = a_y m_t + b_y \Delta p_{t-1} + c_y (y_{t-1} - y^*_{t-1}).$$

Given these equilibrium processes, it follows from the IS curve (2) that the equilibrium process for the interest rate is

$$i_t = ri^*_t + (b_p + b_y/s)\Delta p_t + [c_p + c_y/s - 1/\{s(1-f)\}](y_t - y^*_t)$$
$$+ f/\{s(1-f)\}(y_{t-1} - y^*_{t-1}) + r(a_p + a_y/s)m_t + 1/\{s(1-f)\}e_t.$$

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