

Assessing GMM Estimates of the Federal Reserve Reaction Function

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Abstract

Estimating a forward-looking monetary policy rule by the Generalized Method of Moments (GMM) has become a popular approach since the influential papers by Clarida, Galí, and Gertler (1998, 2000). We re-examine estimates of the Federal Reserve reaction function using several GMM estimators and a Maximum Likelihood (ML) estimator. First, we show that, over the baseline period 1979-2000, these alternative approaches yield substantially different parameter estimates. Using Monte-Carlo simulations, we show that the finite-sample GMM bias can only account for a small part of the discrepancy between estimates. We find that this discrepancy can more plausibly be rationalized by the serial correlation of the policy shock, causing mis-specification of GMM estimators through lack of instrument exogeneity. This correlation pattern is related to a shift in the reaction-function parameters in 1987. Re-estimating the reaction function over the 1987-2000 period produces GMM estimates which are very close to the ML estimate.

Keywords: Forward-looking model, monetary policy reaction function, GMM estimator, ML estimator, finite-sample properties.

JEL classification: E52, E58, F41.

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1 Introduction

According to the benchmark Taylor rule, central banks set the short-term interest rate in proportion of the inflation rate and the output gap. Since Taylor's (1993) prominent contribution, an abundant empirical as well as theoretical literature has claimed that central banks may have a forward-looking behavior (Clarida and Gertler, 1997, Clarida, Galí, and Gertler, 1998, 2000, Batini and Haldane, 1999). This assumption requires the use of an adequate estimation method to overcome the presence of expected inflation in the policy rule. Following the influential work by Clarida, Galí, and Gertler (1998), a large number of studies have thus use the Generalized Method of Moments (GMM) to estimate forward-looking reaction functions.¹

This paper re-examines the estimation of the Federal Reserve forward-looking policy rule and presents some original empirical results. While this topic has been yet widely studied, there are at least two motivations for our additional investigation. First, a large body of research over the last decade has analyzed the properties of GMM estimators. It has produced numerous results which have so far not been incorporated in the estimation of policy rules. In particular, a large number of papers have studied the small-sample properties of GMM estimators, in very different contexts (see the 1996 special issue of the JBES, e.g., Andersen and Sørensen, 1996, or Hansen, Heaton and Yaron, 1996). These papers provided evidence that GMM estimators may be strongly biased and widely dispersed in small samples. Fuhrer, Moore, and Schuh (1995) also pointed out the poor small-sample performances of GMM as compared to those of Maximum Likelihood (ML). In addition, several alternative GMM estimators have been proposed (Ferson and Foerster, 1994, and Hansen, Heaton, and Yaron, 1996), which are shown to have very different small-sample properties. As well, alternative computation procedures for the GMM weighting matrix are likely to provide contrasted results (e.g., Andrews and Monahan, 1992, and Newey and West, 1994). One of our aims is to re-examine estimates of the Federal Reserve reaction function in light of these developments. The forward-looking reaction function may be seen as an original field for investigating GMM properties.

A second strong motivation to investigate the Federal Reserve reaction function is the issue of parameter stability. The tenures of Paul Volcker (1979-87) and Alan Greenspan (since 1987) as chairman of the Board of Governors of the Federal Reserve System have been characterized by two very contrasted subperiods in terms of interest rate movements, but no consensus has so far emerged on whether these two eras represent a single policy regime. Some authors argued that there is no significant difference in the way monetary policy is conducted since Volcker was appointed chairman in 1979. In particular, Clarida, Galí, and Gertler (2000) (henceforth CGG) found that during the Volcker-Greenspan period, the Federal Reserve adopted a proactive stance toward controlling inflation. Other authors (Judd and Rudebusch, 1998, Rudebusch, 2001) concentrate on Greenspan's tenure. The response to expected inflation is typically found to be much lower over the recent period, whereas the output gap

¹Examples include Mehra (1999), Clarida, Galí, and Gertler (2000), Orphanides (2000, 2001), Rudebusch (2001) for US data, or Angeloni and Dedola (1999), Peersman and Smets (1999), Gerlach and Schnabel (2000), Nelson (2000), and Faust, Rogers, and Wright (2001) for European data.

becomes a significant determinant of monetary policy. Estimations in this paper are found to provide some insights on this issue.

Our approach and an outline of the paper are the following. First, we estimate the forward-looking reaction function using three alternative GMM estimators: the standard “two-step” GMM, and the more recent “iterative” and “continuous-updating” estimators. We consider different procedures for computing the GMM covariance matrix as well. GMM estimators are compared to the alternative ML estimator. The ML approach involves the estimation of a structural model of the economy, and has seldom been used in the present context.² For this purpose, we use a version of the Rudebusch and Svensson (1999) model. Over the 1979-2000 period, we find that these alternative estimation approaches provide very contrasted estimates of the reaction function (section 3). Then, the finite-sample properties of GMM and ML estimators of the reaction function are compared using Monte-Carlo simulations. We focus on explaining the observed discrepancy between parameter estimates. Two explanations are considered: finite-sample biases and model mis-specification. Our evidence suggests that the discrepancy between parameter estimates can be explained by the serial correlation of residuals, which induces lack of exogeneity of instruments (section 4). We further claim that serial correlation is related to a shift in the reaction-function parameters. We provide evidence of a structural change in the reaction-function parameters over the period. Estimates over the 1987-2000 period are found to be remarkably close one to each other. The output gap turns out to play a significant role in monetary policy rules over the recent period, providing a major source of structural change. In contrast, we obtain that the inflation parameter did not change over the whole period, and is estimated at a much lower value than claimed in some recent studies (section 5). We further show that these results are robust to the macroeconomic model chosen to estimate the reaction function (section 6). As a prelude to estimation results, section 2 reviews the specification of the monetary policy reaction function. Additional details on estimation procedures are relegated to an Appendix.

2 The forward-looking reaction function

According to the baseline policy rule proposed by Taylor (1993), the central bank is assumed to set the target for the nominal short-term interest rate (i_t^*) as a function of the (four-quarter) inflation rate ($\bar{\pi}_t$) and the output-gap measure (y_t):

$$i_t^* = i^* + \beta(\bar{\pi}_t - \pi^*) + \gamma y_t \quad (1)$$

where i^* is the long-run equilibrium nominal interest rate and π^* is the inflation target. (The output-gap target is assumed to be zero.) The response to expected inflation (β) is a crucial parameter, since in standard macroeconomic models policy rules with $\beta > 1$ will be stabilizing (Taylor, 1999b, and in the context of forward-looking models, Kerr and King, 1996, or Clarida, Galí, and Gertler, 2000).

²One notable exception is Fuhrer (1997), whose study nevertheless favors a backward-looking specification for the policy rule.

The Taylor rule has received a widespread attention in the empirical literature. In particular, it has been found to provide a rough description of US monetary policy during the Volcker and Greenspan tenures (Taylor, 1993, 1999a, Judd and Rudebusch, 1998). However, most empirical studies investigated “modified” Taylor rules. First, the Federal Reserve has been found to smooth changes in interest rates. Several motivations for such an interest-rate smoothing have been proposed (Woodford, 1999, or Sack and Wieland, 2000). For instance, facing uncertainty concerning the model’s parameters, it is optimal for the central bank to adjust interest rates only gradually. Therefore, we specify the policy rule as a partial-adjustment model, in which the short-term rate adjusts gradually to its target i_t^* , defined by equation (1):

$$i_t = \rho(L) i_{t-1} + (1 - \rho(1)) i_t^* + \eta_t \quad (2)$$

where η_t is a random policy shock and $\rho(L) = \rho_1 + \rho_2 L + \dots + \rho_n L^{n-1}$. $\rho(1)$ measures the degree of interest-rate smoothing. We consider below one or two lags in the interest-rate dynamics, depending on the estimation period.³

Second, some authors, following Clarida, Galí, and Gertler (1998, 2000), have adopted a forecast-based specification of the Taylor rule, in which the central bank sets the level of interest rate as a function of expected inflation and output gap. Several authors have claimed that such a forward-looking reaction function is consistent with the observed behavior of central banks over the recent period (Clarida and Gertler, 1997, Clarida, Galí, and Gertler, 1998, 2000, Mehra, 1999, Orphanides, 2001).⁴ Most central banks explicitly claim that they do not only consider past or current economic conditions, but they also include economic forecasts in their macroeconomic condition statement. In addition, from a theoretical viewpoint, that policy rules should be forward-looking has been advocated by Svensson (1997) and Batini and Haldane (1999). Therefore, a large number of recent studies estimate the following specification, which incorporates the expected inflation rate and output gap:

$$i_t = \rho(L) i_{t-1} + (1 - \rho(1)) (i_t^* + \beta (E_{t-1} \bar{\pi}_{t+4} - \pi^*) + \gamma E_{t-1} y_{t+1}) + \eta_t \quad (3)$$

where E_{t-1} denotes the expectation operator conditional on the information set available at date $t - 1$. The interest rate i_t is set on the basis of information available at date $t - 1$, to account for the fact that current inflation and output gap are not observed in real time by the central bank. The information set at date $t - 1$ contains the lagged values of the Funds rate, inflation, and the output gap up to $t - 1$.⁵ We assume that the central bank reacts to the annual inflation rate over the following

³Recently, Rudebusch (2001) has argued that an alternative representation of the reaction function is a non-inertial rule with serially correlated shocks. As we will show in section 4.2, the bias we obtain in parameter estimates is mainly related to serial correlation in the residuals of the partial-adjustment model.

⁴Fair (2001) strongly rejects the forward-looking specification for the Federal Reserve reaction function. However, he uses a different specification from the one considered in other studies.

⁵To some extent, even this assumption is questionable, since the output gap is measured precisely after several quarters only. In this vein, Orphanides (2000) claims that monetary policy during the tenure of Arthur Burns as Federal Reserve chairman appears non-optimal a posteriori essentially because estimates of the output gap have been dramatically revised since the end of the seventies.

four quarters, whereas it reacts to the output gap for the next quarter. Equation (3) is one of the baseline specifications estimated by Clarida, Galí, and Gertler (2000), Orphanides (2001), or Rudebusch (2001).

Estimating equation (3) provides estimates of the response to expected inflation and output gap in the monetary policy rule and the speed of adjustment to the target i_t^* . The long-run inflation target π^* is not identified, however, since the equation in regression form is written as

$$i_t = \rho(L) i_{t-1} + (1 - \rho(1)) (\beta E_{t-1} \bar{\pi}_{t+4} + \gamma E_{t-1} y_{t+1} + \alpha) + \eta_t \quad (4)$$

where the constant term is equal to $\alpha = i^* - \beta\pi^* = r^* + (1 - \beta)\pi^*$, with $r^* = i^* - \pi^*$ the equilibrium real rate. The constant α has no specific interpretation and no sign restriction on this parameter is binding. However, since β is larger than 1 in a stabilizing monetary policy rule, the constant term α is expected to be lower than the equilibrium real rate, provided the inflation target π^* is positive. Furthermore, though r^* and π^* are not separately identifiable in a single-equation approach, an estimate of the inflation target can be obtained if one assumes a value for r^* (for instance, the sample average real rate). This is the restriction imposed in most papers using GMM approach. Alternatively, an estimate of the equilibrium real rate can be obtained from an auxiliary model. For instance, in the context of our ML estimation (see section 3.3 below), the I-S curve can be used to estimate the equilibrium real rate.

3 The Federal Reserve reaction function from 1979 to 2000: evidence from alternative estimators

3.1 Data

We consider the Federal Reserve monetary policy over the period 1979:Q3–2000:Q3.⁶ We use quarterly data, drawn from the OECD databases BSDB and MEI. The Federal funds rate is used as the monetary policy instrument. Inflation is defined the rate of growth of the GDP deflator (denoted P_t), so that $\pi_t = 400 (\ln(P_t) - \ln(P_{t-1}))$ and $\bar{\pi}_t = \frac{1}{4} \sum_{i=0}^3 \pi_{t-i}$. Output gap is defined by the percent deviation of real GDP (Q_t) from potential GDP (Q_t^*), i.e. $y_t = 100 (\ln(Q_t) - \ln(Q_t^*))$. Following a number of recent studies (CGG, Rudebusch, 2001), we use the output-gap series constructed by the Congressional Budget Office (CBO).⁷ Note that, in line with most of the reaction-function literature, we maintain the assumption that the nominal rate and inflation are stationary. Although empirical evidence is not clear-cut, stationarity is an assumption of most theoretical models of monetary policy rules. In section 5, we introduce the possibility of a shift in the long-run equilibrium nominal interest rate and in the target inflation rate in 1987:Q3.

⁶While our data ends in 2001:Q3, our estimates end in 2000:Q3, because GMM estimates require four leads of the inflation rate.

⁷We also used an output-gap measure computed as the percent deviation of GDP from a linear trend. We obtained essentially the same results, except that the output-gap parameter turns out to be lower and less often significant than with the CBO measure.

Our sample period covers the tenures of Paul Volcker (1979:Q3-1987:Q2) and Alan Greenspan (1987:Q3 up to now). In this first stage, we assume that parameters of the reaction function are stable over this period. This assumption, however, is controversial. Statistical evidence in favor of stability of the reaction function within this sample is mixed: Estrella and Fuhrer (1999) find that stability of parameters cannot be rejected. In contrast, Judd and Rudebusch (1998) reject stability, though at the 7 percent significance level, while Rudebusch (2001) and Orphanides (2001) focus on Greenspan's tenure. Intermediate results are found by CGG, who report evidence of instability in the autoregressive parameter only. Also, over a very long sample, starting from the mid-fifties, Fair (2001) finds that the parameters of the Federal Reserve reaction function are stable when excluding the 1979-82 period. In section 5.1, we do address the issue of stability in the policy-rule parameters.

3.2 GMM estimates

Estimating the forward-looking reaction function with OLS, by substituting expected values of inflation and output gap with the actual values, would provide inconsistent estimators because of two problems.⁸ First, since the expected inflation (output gap) is measured with error by observed inflation (output gap), we face an error-in-variable problem. A bias in parameter estimates occurs because the error term $\varepsilon_t = \eta_t - (1 - \rho(1)) [\beta(\bar{\pi}_{t+4} - E_{t-1}\bar{\pi}_{t+4}) + \gamma(y_{t+1} - E_{t-1}y_{t+1})]$ is correlated with some explanatory variables, namely the future inflation rate and output gap. Second, since the current policy shock η_t is likely to affect future inflation and output gap, there is an endogeneity bias. Both problems can be overcome by the GMM. This technique only requires that the error term ε_t is orthogonal to a vector of instruments Z_{t-1} in the information set available at date $t - 1$, so that $E[\varepsilon_t Z_{t-1}] = E[g_t(\theta)] = 0$, where θ denotes the vector of unknown parameters. An efficient GMM estimator of θ is obtained by minimizing, with respect to θ , the expression

$$\bar{g}(\theta)' (S_T)^{-1} \bar{g}(\theta)$$

where $\bar{g}(\theta) = \frac{1}{T} \sum_{t=1}^T g_t(\theta)$ and S_T is a consistent estimator of the covariance matrix of $g_t(\theta)$. Provided instruments are correlated with endogenous regressors and uncorrelated with the error term, GMM estimators are strongly consistent, and asymptotically normal (Hansen, 1982). This technique has been applied to rational-expectation models along the lines of Cumby, Huizinga, and Obstfeld (1983), Hansen and Singleton (1982), and Hayashi and Sims (1983). In this context, the GMM approach is very appealing, because it only requires identifying relevant instrument variables and does not necessitate strong assumptions on the underlying model.

Several GMM estimators have been proposed in the theoretical literature. We consider three alternative estimators already studied, for instance by Hansen, Heaton, and Yaron (1996): the two-step GMM, the iterative GMM, and the continuous-updating GMM. These approaches differ in the way the parameter vector and the

⁸An alternative approach for models with expectations, not pursued here, is to use actual inflation and/or output-gap forecasts. McNees (1985, 1992) and Orphanides (2001) used the Board of Governors' staff forecasts presented at each FOMC meeting.

covariance matrix interact. To our knowledge, all existing estimations of the forward-looking reaction function based on GMM have relied on the two-step estimator. We also consider the way the covariance matrix is computed, investigating three different approaches. In the first one (estimator S_{1T}), the bandwidth is fixed chosen to be equal to 4, consistently with the assumed correlation structure of expectation errors (Newey and West, 1987). The two other estimators rely on prewhitening the moment conditions and on the computation of a data-dependent bandwidth. For estimator S_{2T} , the optimal bandwidth is determined parametrically, and the quadratic spectral kernel is used, following the procedure developed by Andrews and Monahan (1992). The last estimator, S_{3T} , is based on a non-parametric optimal bandwidth, as proposed by Newey and West (1994). The Appendix provides details on the GMM procedures.

Before proceeding, we address the issue of the choice of instruments. To reflect the central bank's information set at time $t - 1$, our instrument set includes four lagged values of the Funds rate, inflation, and the output gap: $i_{t-1}, \dots, i_{t-4}, \pi_{t-1}, \dots, \pi_{t-4}, y_{t-1}, \dots, y_{t-4}$. This set contrasts with most previous GMM estimates of the Federal Reserve reaction function, which include several additional instruments (such as lags of commodity price inflation, M2 growth, and the interest-rate spread, in CGG). There are two motivations for the choice of such a restricted information set. First, as underlined by an abundant literature, when a large number of instruments is selected, some instruments may be weakly relevant, thus deteriorating the finite-sample properties of GMM estimators. The second motivation is that the comparison of alternative GMM and ML estimators using Monte-Carlo simulations (see section 4) necessitates a plausible data generating process (DGP) for all instruments. We therefore intentionally reduce the information set to lags of the three variables used in the structural model discussed below. As a robustness check, we re-estimated equation (4) using the same instrument set as CGG, and our estimation results were not substantially altered. Investigating further the sensitivity of the reaction function to the choice of instruments is out of the scope of the present paper.

Table 1 reports parameter estimates of the forward-looking reaction function (4) obtained using the various GMM procedures. Estimates obtained using the two-step GMM, the iterative GMM, and the continuous-updating GMM are reported in Panel A, B, and C respectively. Over the 1979-2000 period, the second lag of interest rate is never significant, so we present estimates assuming $\rho_2 = 0$. First, we consider the two-step GMM with covariance-matrix estimator S_{1T} (with a fixed bandwidth $L = 4$ and the Bartlett kernel) (first rows of Panel A, Table 1). This case corresponds to the approach adopted by Orphanides (2001, Table 5) and CGG (Table 4, second row). The estimate of the response to expected inflation ($\beta = 2.63$) is significantly larger than the 1.5 coefficient originally proposed by Taylor (1993). The estimate of the output-gap parameter ($\gamma = 0.71$) is very close to the Taylor 0.5 coefficient, although it is only weakly significant.⁹ We re-estimate the model with two lags of the interest rate, and we obtain $\beta = 2.4$ and $\gamma = 0.6$, but the second lag of interest rate is found

⁹Results are very close to those obtained by CGG. Over the period 1979-96, they obtain for this specification $\beta = 2.62$ and $\gamma = 0.83$. Both parameters are found to be strongly significant. While they use a specification with two lags of the interest rate, their estimated degree of smoothing ($\rho(1) = 0.78$) is very close to our own estimate ($\rho_1 = 0.83$).

to be insignificant. When estimating the model with lagged output gap in place of expected output gap, we find the inflation parameter to be slightly lower ($\beta = 2.3$) and the output-gap parameter ($\gamma = 0.5$) to be insignificant. Broadly speaking, the standard, two-step GMM approach provides point estimates which are rather robust to slight changes in the specification.

The iterative and continuous-updating GMM (Panels B and C) yield even larger estimates for the inflation parameter. Estimate of β is as high as 3.59 for the iterative GMM and 3.62 for the continuous-updating GMM. The values are larger than most of those found in the empirical policy rule literature. The output-gap parameter is found to be lower than with two-step GMM and statistically insignificant (0.49 and 0.43 respectively).

We turn now to GMM estimation based on improved covariance matrix estimators. The broad picture suggested by our results is that the different covariance-matrix estimators provide widely dispersed point estimates. For instance, the inflation parameter estimated with continuous-updating GMM decreases from 3.62 with estimator S_{1T} to 2.72 with S_{2T} and 2.11 with S_{3T} . The iterative GMM estimate of β is as high as 6.00 with S_{2T} and 6.98 with S_{3T} . Interestingly, the two-step GMM provides more stable parameter estimates: The inflation parameter is $\beta = 3.07$ with covariance-matrix estimator S_{2T} and decreases to 2.56 with S_{3T} . In addition, Hansen's J-statistics are very large with covariance-matrix estimators S_{2T} and S_{3T} , so that the over-identifying restrictions imposed by instruments are rejected. This suggests that some instruments fail to satisfy the orthogonality conditions. This result is of importance, since it indicates that estimator S_{1T} may fail to detect model mis-specification.

To sum up, several empirical results are worth emphasizing. First, according to most GMM estimates, the weight of expected inflation in the reaction function is very large, while the weight of output gap is only weakly significant. Second, empirical results provided by iterative GMM and continuous-updating GMM contrast markedly with those provided by the usual two-step GMM approach. Note that, as suggested by Stock and Wright (2000, p. 1090), the large dispersion of estimates may reflect a specification problem or the presence of weak instruments. Last, for efficient covariance-matrix estimators, there are signs of rejection of over-identifying restrictions.

3.3 ML estimates

We focus now on the alternative ML estimation procedure. The ML approach requires that an auxiliary model is estimated for the forcing variables (here, the inflation rate and the output gap). The auxiliary model is used to forecast expected variables appearing in the reaction function, yielding cross-equation restrictions. The complete model is solved using the generalized saddlepath procedure developed by Anderson and Moore (1985). The ML estimation procedure is described in the Appendix. An appealing advantage of ML over GMM, in a forward-looking context, is that expectations obtained with ML estimation are fully model-consistent. Thus, the expected values of inflation and output gap, which appear in the reaction function (4), are consistent with the inflation and output-gap equations. The ML approach is of course

demanding, since a structural model has to be estimated. However, in the present case, the widely-used Phillips curve (PC)/I-S curve framework provides us with a reliable benchmark model of the inflation-output joint dynamics. We consider in this section the model proposed by Rudebusch and Svensson (1999), which embodies the main features of the standard macroeconomic paradigm. The key relationships of this model are:

$$\pi_t = \alpha_{\pi 1}\pi_{t-1} + \alpha_{\pi 2}\pi_{t-2} + \alpha_{\pi 3}\pi_{t-3} + \alpha_{\pi 4}\pi_{t-4} + \alpha_y y_{t-1} + u_t \quad (5)$$

$$y_t = \beta_{y1}y_{t-1} + \beta_{y2}y_{t-2} + \beta_r(\bar{r}_{t-1} - \bar{\pi}_{t-1} - \beta_0) + v_t \quad (6)$$

where $\bar{x}_t = \frac{1}{4} \sum_{i=0}^3 x_{t-i}$ denotes the four-quarter moving average of x_t . The PC (equation (5)) relates quarterly inflation (π_t) to its own lags and to lagged output gap. To be consistent with the “accelerationist” hypothesis, which precludes any inflation/output gap trade-off in the long run, we impose that the four autoregressive parameters sum to one, so that $\alpha_{\pi 4} = 1 - \alpha_{\pi 1} - \alpha_{\pi 2} - \alpha_{\pi 3}$. We also set the constant term to zero in this equation, so that the steady-state value of output gap is zero. Using the Likelihood-Ratio test, this joint restriction is not rejected in our ML estimation (with a p-value of 0.57). The I-S curve (equation (6)) relates the output gap to its own lags and to the four-quarter moving average of the short real rate. This last term is a proxy of the short real rate, as in Rudebusch and Svensson (1999). Parameter β_0 may be interpreted as the equilibrium real rate, since it is the value of real rate consistent with a steady-state output gap of zero. The backward-looking nature of this model can be pointed as a potential source of mis-specification. However, such a model has proved to be a robust representation of the US economy. Moreover, no compelling empirical forward-looking counterpart of the Rudebusch-Svensson model has so far emerged (see Estrella and Fuhrer, 1999). In section 6, we check the robustness of our ML estimation by exploring a hybrid model inspired by Rudebusch (2001).

Parameter estimates are reported in Table 2. The PC, the I-S curve, and the reaction function are estimated simultaneously, with a free covariance matrix of innovations. Computed standard errors are corrected for serial correlation and heteroskedasticity of residuals. To be consistent with the GMM estimates of the reaction function, we use the sample period 1979:Q3 to 2000:Q3.

The empirical PC/I-S model is very close to the estimates reported by Rudebusch and Svensson (1999) over the period 1961:Q1 to 1996:Q2. The effect of output gap on inflation is quite strong ($\alpha_y = 0.127$) and the response of output gap to the real rate is $\beta_r = -0.089$. These effects are slightly lower than those obtained by Rudebusch and Svensson (1999), but they have the right sign and are significantly different from zero.

Turning to the reaction-function parameters, we obtain that the estimate of the inflation parameter β is equal to 1.88. This is much lower than all point estimates obtained by GMM. The estimate of the output-gap parameter γ also markedly differs from the GMM estimates, since it is essentially zero and non significant. Finally, the smoothing parameter ρ_1 is found to be lower than the GMM estimate (0.71 vs. 0.83). Therefore, the two estimation procedures produce very contrasted estimates of the Federal Reserve reaction function. According to GMM, the Federal Reserve strongly

reacts both to inflation and to output gap. In contrast, according to ML, it reacts to inflation only, and in a rather moderate way.

Since GMM and ML estimators are asymptotically equivalent, such a discrepancy is likely to be explained by mis-specification. Evidence of mis-specification should be apparent from the properties of residuals. Exploring the properties of GMM residuals is not a promising way, however, since they include expectation errors, which are likely to be serially correlated. Consequently, we focus on ML residuals. First, we consider serial correlation. The Ljung-Box statistic, $Q(K)$, which tests the null hypothesis that the first K serial correlations of residuals are jointly zero, is distributed as a $\chi^2(K)$. $Q(4)$ does not reject the null of no correlation up to 4 lags, whereas $Q(8)$ rejects the null only marginally. As far as the PC and I-S curve are concerned, residuals are not found to be serially correlated. Turning to heteroskedasticity, we compute the Engle statistic, $R(K)$, which tests the null that the first K auto-correlations of squared residuals are jointly zero. The test statistic is distributed as a $\chi^2(K)$. We obtain that residuals of the reaction function as well as the I-S curve are strongly heteroskedastic. Finally, according to the Jarque-Bera statistic, the null hypothesis of normality is rejected for residuals of the reaction function and the I-S curve. Figure 1 displays reaction-function residuals. The vertical line corresponds to 1987:Q3. The figure suggests that residuals are very volatile over the first part of the sample, whereas they are strongly correlated over the second part.¹⁰

These results suggest that the statistical properties of the reaction-function residuals are likely to prevent consistency of GMM as well as ML estimators. For GMM, serial correlation may result in a lack of exogeneity of instruments. This problem is confirmed by a careful inspection of GMM estimates. As claimed in section 3.1, Hansen’s J-test strongly rejects over-identifying restrictions when efficient covariance-matrix estimators are used. Similarly, the ML estimator may be inconsistent, since serial correlation of residuals is not taken into account in the estimation. Estimating a model with serial correlation in monetary policy shocks would allow to obtain consistent estimators. Note, however, that we do not interpret residual serial-correlation in terms of persistent monetary policy shocks (as in Rudebusch, 2001). Instead, we claim that serial correlation is related to a structural shift in the reaction-function parameters over our sample period. This does not preclude persistency in monetary policy shocks found by Rudebusch (2001) over the 1987-2000 period. But, as pointed out by this author, there is “econometric near-observational equivalence of the partial-adjustment rule and the non-inertial rule with serially correlated shocks” (page 3).

4 Investigating the discrepancy: Monte-Carlo evidence

In this section, we conduct Monte-Carlo experiments to investigate the possible sources of discrepancy between GMM and ML estimators. As underlined by Hall

¹⁰Serial correlation of second-period residuals decreases only slowly with the horizon. It is 0.74 at horizon 1, and still 0.38 at horizon 4.

and Rossana (1991) in a related context, three reasons are likely to explain the discrepancy between GMM and ML. (1) The downward bias on the autoregressive parameter, in partial-adjustment models. This bias occurs even when the model is correctly specified, and has been established analytically by Sawa (1978) in the case of iid innovations. When innovations are serially correlated, such a bias is likely to exist, even when estimators are designed so as to be immune to residual autocorrelation (see Hall and Rossana, 1991). (2) GMM small-sample bias, originating in weak instrument relevance. The finite-sample performance of the GMM estimator is very sensitive to the correlation between instruments and endogenous regressors. This low correlation case (weak instrument relevance) has been analyzed, among others, by Nelson and Startz (1990), Hall, Rudebusch, and Wilcox (1996), or Staiger and Stock (1997). (3) Mis-specification of the model, which results in inconsistency of GMM and ML estimators. In the case of GMM, mis-specification occurs when instruments are correlated with innovations, so that instruments fail to be exogenous with respect to parameters of interest. Such a problem is likely to occur in our model, since reaction-function residuals display serial correlation. Similarly, serially-correlated innovations provide inconsistent ML estimators.

4.1 Finite-sample biases

We first consider Monte-Carlo simulations to assess the finite-sample properties of GMM and ML estimators in our forward-looking reaction function set-up. The experiment is designed as follows. The DGP is given by the complete model constituted of equations (5), (6) together with the reaction function (4). Parameters are those obtained by ML (Table 2). The innovation covariance matrix, $\hat{\Sigma}$, is the sample covariance matrix of $(\hat{u}_t, \hat{v}_t, \hat{\eta}_t)$. For a given sample size T , a sequence of $T + 50$ random innovations are drawn from the Gaussian distribution $N(0, \hat{\Sigma})$ with no serial correlation. Two sample sizes are explored: $T = 85$ corresponds to our estimation sample; and $T = 200$ illustrates the effect of the sample size on the finite-sample bias. It is chosen to represent an upper bound to the number of observations available in actual macroeconomic database (say, 50 years of quarterly data). Initial conditions are set equal to the average values over the sample. The first 50 entries are discarded to reduce the effects of initial conditions on the solution path. For each artificial database, estimation is performed as follows: For GMM, the reaction function is estimated using four lags of (simulated) inflation, output gap, and interest rate as instruments. For ML, the complete model is estimated. The Monte-Carlo experiment is based on $N = 2000$ replications. Therefore, for each sample size and each estimator, we obtain 2000 parameter estimates, so that the empirical distribution of parameter estimates can be analyzed.¹¹

Simulations are performed using GAUSS version 3.2 on a Pentium III platform.

¹¹A large number of replications allows to obtain more precise estimates of the parameter vector. Such a precision is crucial, here, to obtain an accurate measure of finite-sample biases. Let $\hat{\theta}_i$ be the estimate of the parameter vector (say, θ) obtained for replication $i = 1, \dots, N$ and $\bar{\theta} = \frac{1}{N} \sum_{i=1}^N \hat{\theta}_i$ be the mean of the empirical distribution over the N replications. Then, the standard error of θ is σ_{θ}/\sqrt{N} .

Two-step and iterative GMM estimators are obtained by simple matrix computations. Continuously-updating GMM and ML estimators are obtained by a numerical optimization routine. We use the BFGS algorithm of the CO procedure for constrained optimization. We found no discrepancies when we used different algorithms. All estimations are performed using numerical derivatives.

In some experiments, for $T = 85$ observations, the continuous-updating GMM estimator failed to converge. For instance, the number of crashes is 5.3 percent of our samples with estimator S_{1T} . Hansen, Heaton, and Yaron (1996) and Smith (1999) also reported an important number of crashes and some difficulties to obtain reasonable parameter estimates with the continuous-updating GMM. Two kinds of problems occur. First, the numerical search for the minimizer sometimes fails. In addition, even when convergence is reached, some parameter estimates can have unrealistic values, so that the empirical distribution of estimates is severely distorted. For this reason, in Table 3a, two rows are devoted to the continuous-updating GMM estimator. In the first row, we report distribution statistic after we discarded only estimates which reached the maximum number of iterations (here, 200). In the second row, we select estimates satisfying the additional criterion that the smoothing parameter ρ_1 lies inside the interval $[-1; 1]$. In our Monte-Carlo experiment, 5.6 percent of estimations fall outside of this parameter space, with estimator S_{1T} .

The distribution of the alternative GMM and ML estimators is summarized in Table 3a for the small-sample size ($T = 85$). Since parameter α does not provide incremental insight on the finite-sample properties, we do not report results for this parameter. Figures 2a and 2b also displays the distribution of parameters ρ , β , and γ for various estimation approaches. The table reveals four main results concerning the finite-sample properties of the reaction-function parameters. First, the autoregressive parameter is found to be systematically biased toward zero whatever the estimation procedure. Although the sample size is small, the bias is not very large, however. The parameter is set to 0.71 in the DGP. The median parameter estimate is 0.66 for the ML procedure and 0.69 for the two-step GMM approach. The sign and size of the bias are consistent with the analytical results of Sawa (1978).

Second, as far as parameters for endogenous regressors (β and γ) are concerned, we obtain significant, yet economically small, bias. For β , the median bias is about 0.06 for GMM estimators, but as low as -0.006 with ML. In addition, parameter γ is under-estimated whatever the estimation approach. The sign and ordering of the biases in alternative estimators are in accordance with point estimates obtained in section 3. However, the discrepancy between parameter estimates cannot be explained by finite-sample biases only.

Third, the standard deviation of parameter estimates is much lower with ML than with GMM. For instance, for β , the standard deviation is 0.22 with ML, whereas it is 0.51 with two-step GMM, 0.55 with iterative GMM and 25.75 with continuous-updating GMM (with covariance-matrix estimator S_{1T}). The continuous-updating estimator provides very imprecise estimates, even though the median is only slightly upward biased. When “unreasonable” outcomes are excluded, the standard deviation of truncated continuous-updating estimator is still as high as 2.67. The dispersion of the iterative and continuous-updating GMM estimators is excessive as compared

with the two-step GMM estimator. Note, however, that the standard deviation is partially misleading in the case of the continuous-updating GMM, because of the occurrence of extreme outcomes, reflecting a fat-tailed distribution (see Figure 2b). Thus, discrepancies between GMM techniques are rather small in terms of parameter bias, but much larger in terms of precision of the estimation.

Fourth, the distribution of GMM estimators is asymmetric. This characteristic is apparent from Figures 2a and 2b, and is particularly pronounced for continuous-updating GMM and, to a lesser extent, for iterative GMM. As indicated by 10th and 90th percentiles, the autoregressive parameter is leftward skewed, whereas the inflation parameter is rightward skewed. These features provide a rationale for the very large β estimates obtained with iterative and continuous-updating GMM on the actual data.

The right-most columns of the table report the percentages of the 2000 replications in which Hansen's J-statistic exceeds the relevant critical value of the χ^2 distribution. Three results are worth noting. First, the two-step and iterative GMM tend to reject the over-identifying restrictions too often. Second, as claimed by Hansen, Heaton, and Yaron (1996), the continuous-updating GMM approach does not reject the null hypothesis too often. This result is also consistent with the recommendation of Stock and Wright (2000) to base inference on the continuous-updating criterion. Third, selecting covariance-matrix estimators S_{2T} and S_{3T} does not improve performances of the J-test.

Results for large sample ($T = 200$) are reported in Table 3b. As expected, biases of GMM and ML estimators have mostly disappeared, so that the large-sample distribution is well approximated by the asymptotic distribution. Note, however, that the standard deviation of the two-step and iterative GMM estimators remains slightly larger than the standard deviation of the ML estimator. Moreover, even after discarding unrealistic outcomes, the continuous-updating GMM estimator still display significant biases and excessive dispersion.

Our findings are broadly consistent with the existing literature on finite-sample properties of GMM estimators, as regards bias and dispersion (see, e.g., Tauchen, 1986, Nelson and Startz, 1990, Fuhrer, Moore, and Schuh, 1995, and papers in the 1996 special issue of the JBES). A noticeable feature of our results is the poor performance of the continuous-updating estimator. The finding that the continuous-updating GMM estimator has fat tails and yields a non-negligible proportion of implausible estimates was reported by Hansen, Heaton, and Yaron (1996). We also obtain that, for some parameters, the continuous-updating GMM is more widely biased than the two-step GMM. This is in contrast with some, but not all, of the Monte-Carlo results in Hansen, Heaton, and Yaron (1996) and with the analytical results in Newey and Smith (2000). The latter authors prove that the bias of the continuous-updating estimator is smaller than that of the two-step GMM, but this result is warranted when the number of moment restrictions is large. In contrast, Smith (1999) and Stock and Wright (2000) document the poor performance of the continuous-updating GMM in some of their estimates and Monte-Carlo experiments. As regards the iterative GMM, we find that this estimator has no specific advantage over the two-step estimator. Similar results are obtained by Smith (1999). Note that

the iterative estimator has been advocated by Kocherlakota (1990) and Ferson and Foerster (1994) on the basis of the rejection rates of the J-statistics rather than for its performance in terms of parameter estimates.

To explain the overall poor performance of continuous-updating GMM, we can point that estimation of the covariance matrix S_T plays a central role in this procedure. Two problems may then occur. First, the covariance matrix may be imprecisely estimated in small sample. This is outlined by Burnside and Eichenbaum (1996) as the major cause of poor finite-sample performance of GMM-based Wald tests. One reason is that the covariance-matrix estimator is a function of cross-moments between squared GMM residuals and squared instruments. Such empirical moments are likely to converge toward the true value only very slowly. Therefore, even though an efficient estimator of the covariance matrix is used, it is likely to be a poor estimator in finite sample. This problem is illustrated in Table 3a. When improved covariance-matrix estimators S_{2T} and S_{3T} are used in place of S_{1T} , the median bias in continuous-updating estimators decreases only very slightly. The second problem is related to the construction of the continuous-updating GMM objective function, $\bar{g}(\theta)'(S_T(\theta))^{-1}\bar{g}(\theta)$ in which the parameter vector and the covariance matrix are determined simultaneously (see the Appendix for details and notations). As pointed out by Hansen, Heaton, and Yaron (1996), this objective function may be minimized for a parameter value that produces a large value of the variance of moment condition $S_T(\theta)$, even if the average moment condition $\bar{g}(\theta)$ deviates from zero.

An additional issue is the presence of weakly relevant instruments in our simulation exercise. As it appears clearly from equations (4), (5), and (6), most variables in the information set are relevant instruments for the DGP chosen. Two exceptions are y_{t-3} and y_{t-4} , which are asymptotically uncorrelated with endogenous regressors, conditional upon other variables in the instrument set. This may partly explain the large standard deviation obtained for GMM estimators in Table 3. In order to address this issue, we compare the distribution of the GMM estimators when using our baseline instrument set and when using an instrument set excluding y_{t-3} and y_{t-4} . We focus on the two-step GMM estimator, and we replicate $N = 10'000$ samples of $T = 85$ observations to obtain precise estimates of the parameter standard deviation. Results (not reported to save space, but available from the authors upon request) indicate that median biases and parameter standard deviations are essentially unaffected by inclusion of y_{t-3} and y_{t-4} . More specifically, when those variables are included, the standard deviation of β slightly decreases, whereas the standard deviation of γ slightly increases. We may explain this result by arguing that, within sample, y_{t-3} and y_{t-4} are slightly correlated with endogenous regressors, so that their inclusion in the instrument set does not worsen parameter precision.

4.2 Mis-specification and endogeneity bias

Another route for explaining the discrepancy between estimators is that the model or moment restrictions used in implementing one or several of the estimators might be mis-specified. Under mis-specification, we expect the estimators to be inconsistent and to converge to different probability limits. In addition, mis-specification is likely

to affect differently the finite-sample behavior of GMM and ML estimators. In the case of GMM, mis-specification occurs as a consequence of lack of instrument exogeneity. Since our instruments are in the information set dated $t - 1$, the only source of endogeneity here is serial correlation of innovations.

To measure the effect of mis-specification on the finite-sample properties of estimators, we perform the following Monte-Carlo experiment. We simulate model (4), (5), and (6), with the parameter estimates obtained by ML, as in the previous experiment. But innovations are now simulated as follows: Over the first part of the sample (the first $0.35T$ observations), innovations are assumed to be uncorrelated Gaussian variates, with the covariance matrix obtained with residuals over the 1979-87 subperiod. Innovations over the second part of the sample (the last $0.65T$ observations) have the same covariance matrix as residuals over the 1987-2000 subperiod, yet the policy shock is serially correlated. We assume that the policy shock is well approximated by an AR(1) process, whose parameter has been obtained using sample residuals (the first-order correlation is estimated to 0.74). This simulation strategy allows to mimic the two characteristics of residuals, heteroskedasticity as well as serial-correlation of the policy shock.

Results of this simulation exercise are reported in Table 4a for sample size $T = 85$ and Table 4b for sample size $T = 200$. As expected, we obtain a significant bias for all estimators. The three parameters of interest (ρ_1 , β , and γ) are systematically over-estimated. Note that the mis-specification bias for the autoregressive parameter, facing serial correlation of the error term, is of opposite sign to the finite-sample bias obtained above, consistently with the theoretical result obtained by Sawa (1978). The median bias on ρ_1 is lower for the ML estimator than for the two-step GMM (0.04 vs. 0.11). Both approaches also differ by the bias on parameters pertaining to endogenous regressors. The median bias on β is as high as 0.33 for two-step GMM, whereas it is as low as 0.13 for ML. Moreover, parameter γ is very strongly over-estimated. Although the true parameter is essentially zero, the median estimator is as high as 0.49 for two-step GMM and 0.21 for ML. Note that iterative GMM estimators are close to two-step estimators, whereas continuous-updating estimators are generally lower. For instance, the bias of the continuous-updating estimator for parameter γ is as low as 0.26. This result suggests that this approach is less biased under mis-specification.

As far as the standard deviation of parameter estimates is concerned, we note that GMM procedures produce excessively large standard deviations. For instance, the standard deviation of β is 0.33 with ML and 0.79 with two-step GMM, 4.21 with iterative GMM, and 3.37 with truncated continuous-updating GMM. Using efficient covariance-matrix estimators does not allow to obtain more precise GMM estimators. Instead, it generally implies an increase in the standard deviation of the parameter distribution.

Taken together, evidence on bias and standard deviation of estimators under mis-specification suggests that serial correlation is a plausible explanation for the discrepancy between estimators found in section 3. Note, however, that this simulation exercise does not explain the whole discrepancy between GMM and ML estimators. Instead, it provides a measure of the bias in the reaction-function parameters when simulated innovations display a dynamic similar to the one of observed residuals.

Within this framework, we also address the issue of why the J-test based on the covariance-matrix estimator S_{1T} does not reject the over-identifying restrictions, whereas J-tests based on estimators S_{2T} and S_{3T} strongly reject these restrictions. To do so, we estimate the J-statistics for all simulated samples and compute the frequency of rejection of the null hypothesis at the asymptotic 1%, 5%, and 10% critical values. This measures the power of the J-test against invalid moment restrictions due to serially correlated innovations. Tests performed using estimator S_{1T} reject the null in only 52 percent of the simulated samples at the theoretical 5 percent level. By contrast, with estimator S_{3T} , rejection is obtained in 92 percent of the samples. This result suggests that the low power of the J-statistic is a reason why the GMM approach with estimator S_{1T} is unable to detect the failure of GMM due to lack of exogeneity. We also observe that J-statistics computed with continuous-updating GMM have very low ability to detect lack of exogeneity of instruments. This result is consistent with empirical evidence provided by Table 1, where this approach is found to be unable to reject over-identifying restrictions, whatever the covariance-matrix estimator.

5 Estimation over the 1987-2000 period

5.1 Stability tests

In the preceding section, we have shown that the discrepancy between GMM and ML estimates can be partly explained by the serial correlation of the policy shock. We now interpret this serial correlation as reflecting a shift in the parameters of the reaction function. More precisely, we argue that constraining parameters to be constant over the sample period leads to an omitted-variable bias in the estimation. This claim is corroborated, for instance, by the finding that the output-gap parameter is significant over the 1987-2000 period (as shown, for instance, by Rudebusch, 2001), but not over the 1979-2000 period.

To investigate this issue, we perform parameter stability tests both on GMM and ML estimates. We assume that the shift occurs, if any, in 1987:Q3. Of course we do not claim the date of the shift to be fully uncontroversial, in particular because the shift in parameters may have occurred before this date.¹² But in our context, it is reasonable to treat this candidate break point as known, given the change in Chairman as well as the large number of studies that focus on the 1987-2000 sample. Therefore, we assume that the US reaction function remains stable during the tenure of the Federal Reserve chairmen.

Results of stability tests are presented in Table 5. We adopt the strategy developed by Andrews and Fair (1988) for known break point. For the GMM estima-

¹²In particular, over the period 1979:Q4–1982:Q4, the operating procedures of the Federal Reserve involved targeting non-borrowed reserves. This period, characterized by very volatile interest rates and a sharp disinflation episode, may strongly affect the estimation of the reaction function over the 1979–2000 period. For instance, Fair (2001) is not able to reject stability of the reaction-function parameters for his specification, but he claims that there is a large economic difference in the coefficient on inflation between his first (1954-79) and second (1982-99) subperiods.

tor (Panel A), we report the Wald test statistics for the two-step and the iterative GMM, for the three covariance-matrix estimators. We do not present results for the continuous-updating GMM estimator, because the algorithm failed to converge over both subperiods. Under the null of stability, the Wald test statistic is distributed as a $\chi^2(4)$. Therefore, we strongly reject the null of stability of the reaction-function parameters for each of the GMM estimators.

The test for stability in the ML framework offers the opportunity to consider the stability of the parameters in the reaction function as well as in the macroeconomic model. Tests are performed using a Likelihood-Ratio statistic. Under the alternative hypothesis, the model is estimated with a shift in all parameters in 1987:Q3. Under the null, the shift is assumed to occur only on the macroeconomic parameters or, alternatively, on the reaction-function parameters. Results reported in Table 5 confirm that the model with stability of the parameters is rejected at usual significance level. Moreover, we strongly reject the stability of the reaction-function parameters, while we are not able to reject the stability of macroeconomic parameters. This last result is consistent with the test performed by Rudebusch and Svensson (1999).

In sum, stability of the reaction-function parameters is rejected for GMM as well as ML estimation procedures. Our results are not inconsistent with those reported by CGG. These authors obtain evidence that, for the same specification, the autoregressive parameters significantly changed between Volcker and Greenspan tenures.

5.2 Estimation results

We consider now the estimation of the forward-looking reaction function over the Greenspan era (1987:Q3–2000:Q3). For GMM, we estimate (4) as before, yet while allowing a second lag in the dynamics of the interest rate. It has been found to be relevant over this sample by Rudebusch (2001). For ML, we adopt the following strategy. Since we wish to estimate the dynamics of the inflation rate and the output gap precisely, we maintain the estimation of the model over the period 1979–2000, taking advantage of the stability of the PC/I-S equations. But, we introduce a shift in the parameters of the reaction function in 1987:Q3. Note that the covariance matrix of innovations is also allowed to shift at this date. With such a strategy, macroeconomic parameters are estimated over a large sample, whereas reaction-function parameters are allowed to shift across Federal Reserve chairmen.

Table 6 reports parameter estimates obtained with the different GMM approaches. Several results are interesting to comment. First, the autoregressive component of the reaction function differs markedly from the one estimated over the 1979–2000 period. This confirms the result found by Rudebusch (2001) and provides a rationale for residual autocorrelation when the reaction-function parameters are assumed to be stable over the whole sample. Second, point estimates of the reaction-function parameters obtained by the various GMM procedures are now very close one to each other. For instance, estimates of parameter β range between 1.58 and 1.90. Omitting the continuous-updating estimator, the range is even narrower (between 1.64 and 1.72). We obtain a similar result for the other parameters. Third, using different covariance-matrix estimators only results in a change in standard error of

parameters, but not in a change in parameter estimates themselves. In most cases, standard errors decrease from covariance-matrix estimator S_{1T} to S_{2T} and to S_{3T} . Computing improved covariance matrices therefore yields in a greater precision in parameter estimates. Last, in no case does the J-statistic reject the over-identifying restrictions. In sum, the key insights obtained for the reaction function over the 1987:Q3–2000:Q3 period are robust to change in GMM options.

ML estimates of the model with a shift in the reaction-function parameters are displayed in Table 7. First of all, parameters of the PC and the I-S curve are not significantly altered by the shift in the reaction-function parameters. As far as the reaction function is concerned, we obtain two models with very typical features. Over the first period corresponding to Volcker’s tenure, the inflation parameter is $\beta = 1.49$, whereas the output-gap parameter is very close to zero and insignificant, so that it has been constrained to zero in the estimate reported in the table. Only one lag of interest rate is significant. Over the second subperiod corresponding to Greenspan’s tenure, the reaction function is essentially a Taylor rule, with $\beta = 1.52$ and $\gamma = 0.51$. The partial-adjustment model requires a second lag of interest rate to adjust to the data. These parameter estimates are very similar to those obtained by Rudebusch (2001) over almost the same sample.

Statistical properties of residuals of the reaction function estimated over each subperiod are also reported in the table. The main features are the following. The standard error of residuals is divided by 5 between the first and second subperiods. The null hypothesis of no serial correlation is not rejected at any significance level for both subperiods and the heteroskedasticity almost completely disappears over the two subperiods. These results suggest that the serial correlation and the heteroskedasticity obtained over the whole sample were to a great extent attributable to the shift in parameter. Figure 3 displays the reaction-function residuals when a shift is introduced in equation (4). It confirms that reaction-function residuals do not appear serially correlated anymore.

Overall, the discrepancy between GMM and ML estimates is fairly small over the 1987–2000 period. For instance, the baseline two-step GMM estimate of β is 1.65 whereas the ML estimate is 1.5. Similarly, the GMM estimate of γ is 0.65 whereas the ML estimate is 0.5. Over the 1979–87 period, the response to expected inflation is also estimated to 1.5, whereas the output-gap parameter is essentially zero.¹³ These estimates differ substantially from those obtained over the whole period, assuming parameter stability. We conclude that the assumption of a stable output-gap parameter and a stable autoregressive dynamics is responsible for the very large inflation parameter obtained by GMM over the 1979–2000 period. Interestingly, the induced bias is more pronounced on GMM estimators than on ML estimators. This appears to be related to the strong serial correlation of residuals which results in a lack of exogeneity of instruments.

¹³Two-step GMM estimation of the reaction function over the 1979–87 period (not reported, but available from the authors upon request) is as follows: $\rho_1 = 0.73$, $\beta = 1.63$, and $\gamma = -0.06$.

6 Robustness of ML estimates

Results obtained in previous sections provide support to the use of ML procedure when estimating reaction functions. However, simulations reported in section 4.1 are likely to provide an excessively optimistic view of ML, since the true DGP is assumed to be known when implementing the ML approach. In actual estimation settings, a major concern is of course that there might be uncertainty over the relevant DGP. Simulations in section 4.2 provide a first answer to this concern, since even when the estimated model is assumed to be mis-specified (due to serial correlation of the policy shock) the induced bias on ML estimators is lower than that on GMM estimators. This is also reflected by estimation results in section 3, where ML estimates over the 1979-2000 period for parameters on endogenous regressors appear to be more reasonable and less dispersed than alternative GMM estimates. To investigate further this concern, we assess now whether ML estimation is robust to the choice of the macroeconomic model.

First, we consider as a macroeconomic model which includes VAR-like equations for inflation and the output gap. As in previous section, parameters are assumed to be constant over the whole period while the monetary policy rule shifts in 1987:Q3. In order to obtain some reasonable long-run properties to the system, we impose the following restrictions: First, no long-run inflation-output trade-off is allowed, so that, in the inflation equation, lagged inflation parameters sum to one, the constant is set to zero, and the interest-rate parameters sum to zero. Second, the output gap is assumed to depend on lags of the real rate ($i_{t-k} - \bar{\pi}_{t-k}$) rather than on separate lags of i_t and $\bar{\pi}_t$. Since we introduce four lags in each equation, seven restrictions are thus imposed. Overall, this macroeconomic model has 19 parameters, while the Rudebusch-Svensson model has 8 parameters only. This represents both a plausible and quite general perturbation to the baseline model. Parameter estimates are obtained by estimating this model with the ML approach. We obtain the following reaction-function parameters (results for other equations are not reported to save space), with standard errors in parenthesis: $\rho = 0.451$ (0.204), $\beta = 1.468$ (0.249), $\alpha = 3.703$, (0.992) for the first-period reaction function and $\rho_1 = 1.325$ (0.108), $\rho_2 = -0.513$ (0.093), $\beta = 1.659$ (0.303), $\gamma = 0.613$ (0.230), and $\alpha = 1.841$ (0.765) for the second-period reaction function.

These parameters are very close to those obtained with the Rudebusch-Svensson model. However, it can be argued that the Rudebusch-Svensson model is itself a parsimonious, constrained VAR model which imposes additional, relevant constraints. To perform a more severe sensitivity analysis, we now consider a model which extends the Rudebusch-Svensson model to incorporate some forward-looking components in the PC and the I-S curve. We consider the hybrid model proposed by Rudebusch (2001). The empirical version of this model, suitable for quarterly data, is:

$$\pi_t = \mu_\pi E_{t-1} \bar{\pi}_{t+3} + (1 - \mu_\pi) \sum_{j=1}^4 \alpha_{\pi j} \pi_{t-j} + \alpha_y y_{t-1} + u_t \quad (7)$$

$$y_t = \mu_y E_{t-1} y_{t+1} + (1 - \mu_y) \sum_{j=1}^2 \beta_{y j} y_{t-j} + \beta_r (r_{t-1} - r^*) + v_t \quad (8)$$

where $r_{t-1} = \mu_r (E_{t-1}\bar{i}_{t+3} - E_{t-1}\bar{\pi}_{t+3}) + (1 - \mu_r) (i_{t-1} - \bar{\pi}_{t-1})$ is a weighted combination of an ex-ante 1-year rate and an ex-post 1-year rate. The case $\mu_\pi = \mu_y = \mu_r = 0$ corresponds to the backward-looking model of Rudebusch and Svensson (1999). In the other extreme model ($\mu_\pi = \mu_y = \mu_r = 1$), the PC and the I-S curve have a purely New-Keynesian forward-looking structure. In such a forward-looking macroeconomic model, an inertial monetary policy rule has been shown to be optimal (Woodford, 1999, Sack and Wieland, 2000).

Parameter estimates of the hybrid model are reported in Table 8. First, the degree of forward-lookingness of the PC and the I-S curve are 0.31 and 0.44 respectively. The estimate of μ_π is broadly consistent with some empirical evidence obtained, for instance, by Roberts (2000). The estimated value of μ_y is consistent with a rather high cost for adjusting output. Fuhrer (2000) obtains that μ_y is approximately equal to 0.3. The weight of ex-ante rate, μ_r , is rather high, but imprecisely estimated. The LR test for joint nullity of μ_π , μ_y , and μ_r rejects the null hypothesis (with a p-value equal to 1.9%), suggesting that the dynamics of inflation and output gap are indeed partially forward-looking.

Estimates of the reaction-function parameters are remarkably close to ML estimates obtained with the purely backward-looking macroeconomic model and to GMM estimates over the 1987-2000 period. The response to expected inflation is $\beta = 1.65$, whereas the response to expected output gap is $\gamma = 0.51$. Standard errors are also very close one to each other.

In summary, our previous ML estimates are broadly robust to a change in the macroeconomic model. Though a comprehensive analysis of GMM and ML under mis-specification is out of the scope of the present paper, our various results indicate that ML is a fairly reliable estimation procedure in our context.

7 Conclusion

This paper has re-examined the Federal Reserve reaction function, using a now standard dynamic forward-looking Taylor-rule specification, and implementing alternative GMM as well as ML estimation procedures.

We provide some original empirical results. First, over the baseline 1979-2000 period, the various GMM procedures yield very different estimates. Iterative and continuous-updating GMM, which have not often been considered in the reaction-function literature, produce particularly high and somewhat unrealistic inflation parameter within our sample. In addition, some covariance-matrix estimators lead to rejection of the over-identifying restrictions by the J-statistic. These results are likely to be explained by a mis-specification of moment conditions.

Second, the ML estimate of the inflation parameter is much lower than GMM estimates, and more in line with the Taylor rule. Reaction-function residuals are also found to be strongly heteroskedastic and slightly autocorrelated. Further scrutiny of residuals suggests that mis-specification comes from a shift in the reaction function. First-period residuals are very volatile, whereas second-period residuals display a large serial correlation.

Third, we find that GMM finite-sample bias is not sufficient to account for the discrepancy between alternative estimators. Rather, the discrepancy is shown to be consistent with serial correlation of the policy shock over the second period, because it affects more seriously GMM estimates than ML estimates. We interpret such a serial correlation as a consequence of a significant shift in reaction-function parameters in 1987:Q3. Then, we show that, over the 1987-2000 period, parameter estimates are very stable across estimation procedures. Moreover, the response to expected inflation is much lower than over the 1979-2000 period, consistently with the value suggested initially by Taylor (1993).

In addition, we obtain several results on the properties of estimation procedures for forward-looking monetary policy rules. First, all GMM estimators exhibit large dispersion and suffer from finite-sample bias. In particular, they tend to overstate the degree to which interest rate responds to expected inflation. However, the size of this bias is rather limited. Our overall assessment of GMM in the case of the reaction function is thus less critical than that obtained by Fuhrer, Moore, and Schuh (1995) in the case of inventories.

Second, performances of the three GMM estimators considered are contrasted. The two-step GMM estimator exhibits a smaller bias, and a lower dispersion than other GMM estimators. This provides a rationale for using that simple approach, as is usually done in empirical studies of the reaction function. In contrast, the continuous-updating estimator is found to be more widely dispersed and fat-tailed, in our set-up.

Third, ML is a feasible alternative to GMM for estimating a forward-looking reaction function. A traditional drawback with ML is that it requires estimating a structural model for forcing variables. However, in the present context, a PC/I-S curve model, such as the Rudebusch-Svensson model, provides a fairly reliable model of the economy. In addition, estimation results show some robustness to the model used. Given the sample sizes typically available for estimating monetary policy rules, ML should be viewed as an attractive alternative to the GMM approach.

8 Appendix

8.1 GMM estimators

Let equation (4) be expressed in standard regression notation as

$$y = X\theta + \varepsilon$$

with y a $(T \times 1)$ vector and X a $(T \times n)$ matrix. $X_t = (x_{1t} \dots x_{nt})'$ is a vector of observations and θ is the $(n \times 1)$ vector of unknown parameters. Let $Z = (Z_1 \dots Z_T)'$ be a $(T \times q)$ matrix of instruments, with $q > n$. All the q instruments are assumed to be predetermined, in the sense that they are orthogonal to the current error term. For simplicity, instruments are assumed to be in the information set available at date $t - 1$, so that $E(\varepsilon_t Z_{it-1}) = 0, \forall t$ and $i = 1, \dots, q$. This can be written as

$$Eg_t(\theta) = 0$$

where $g_t(\theta) = (y_t - X_t'\theta) \cdot Z_{t-1} = \varepsilon_t \cdot Z_{t-1}$.

The GMM estimator, denoted $\hat{\theta}_{GMM}$, is the value of θ that minimizes the scalar

$$Q_T(\theta) = \bar{g}_T(\theta)' \left(S_T \left(\hat{\theta}_T^1 \right) \right)^{-1} \bar{g}_T(\theta) \quad (9)$$

where the $(q \times 1)$ vector $\bar{g}_T(\theta) = \frac{1}{T} \sum_{t=1}^T g_t(\theta)$ denotes the sample mean of $g_t(\theta)$. $S_T \left(\hat{\theta}_T^1 \right)$ is a consistent estimator of the $(q \times q)$ covariance matrix of $\sqrt{T} \bar{g}_T(\theta)$, obtained using $\hat{\theta}_T^1$ as a consistent estimator of θ . The GMM estimator is then defined by:

$$\hat{\theta}_T = \left(X'Z \left(S_T \left(\hat{\theta}_T^1 \right) \right)^{-1} Z'X \right)^{-1} X'Z \left(S_T \left(\hat{\theta}_T^1 \right) \right)^{-1} Z'y \quad (10)$$

with asymptotic covariance matrix $\hat{\Omega} = \left(X'Z \left(S_T \left(\hat{\theta}_T^1 \right) \right)^{-1} Z'X \right)^{-1}$.

In the paper, we implement three alternative GMM estimators already considered in the theoretical literature. In the first approach, the parameter vector is estimated with the two-step two-stage least squares, or “two-step GMM”, initially proposed by Hansen (1982), Hansen and Singleton (1982), Cumby, Huizinga, and Obstfeld (1983), and Hayashi and Sims (1983). Assuming an initial guess for the covariance matrix, such as $S_T^{(0)} = \frac{1}{T} \sum_{t=1}^T Z_t Z_t'$, a first estimate of the parameter vector, $\hat{\theta}_T^{(1)}$, is obtained using $S_T^{(0)}$ to weight the moment conditions, so that $\hat{\theta}_T^{(1)} = \left(X'Z (Z'Z)^{-1} Z'X \right)^{-1} X'Z (Z'Z)^{-1} Z'y$. Then, the covariance matrix $S_T \left(\hat{\theta}_T^{(1)} \right)$ is estimated with $\hat{\varepsilon}_t = y_t - X_t' \hat{\theta}_T^{(1)}$ using the procedure described below. Last, the two-step GMM estimator, denoted $\hat{\theta}_T^{(2)}$, is obtained by minimizing equation (9).

The second approach, suggested by Ferson and Foerster (1994) or Hansen, Heaton, and Yaron (1996), relies on estimating parameters and the covariance matrix recursively. Beginning with the two-step estimator, at each step j , $S_T \left(\hat{\theta}_T^{(j-1)} \right)$ is used to construct the new parameter vector $\hat{\theta}_T^{(j)}$. The “iterative GMM” estimator, denoted $\hat{\theta}_T^{(\infty)}$, is obtained when convergence of the parameter vector is reached or when j attains a maximal number of iterations.¹⁴

In the last approach, called “continuous-updating GMM”, developed by Hansen, Heaton, and Yaron (1996) and studied in Stock and Wright (2000) and Newey and Smith (2000), the covariance matrix and the parameter vector are simultaneously determined in the minimization. Therefore, the continuous-updating estimator is the solution of the following problem

$$\min_{\{\theta\}} \bar{g}_T(\theta)' (S_T(\theta))^{-1} \bar{g}_T(\theta).$$

¹⁴We adopt $\max_{\{i\}} \left(\hat{\theta}_{iT}^{(j)} - \hat{\theta}_{iT}^{(j-1)} \right) < 10^{-5}$ as the convergence criterion, where $\hat{\theta}_{iT}^{(j)}$ denotes the i th element of $\hat{\theta}_T^{(j)}$, the estimate of the parameter vector at the j th iteration. The maximal number of iterations is chosen equal to 1000.

GMM estimates are justified on asymptotic grounds. Small-samples properties of the GMM procedure have been studied in a number of papers. The general result is that the asymptotic theory provides a poor approximation in finite samples. Using Monte-Carlo experiments, Tauchen (1986), Kocherlakota (1990), and Andersen and Sørensen (1996) provided evidence that GMM estimates can be strongly biased in small samples. The alternative GMM estimators have the same asymptotic distribution. Nevertheless, the continuous-updating GMM offers the advantage over the two-step and iterative GMM that estimates are invariant with respect to the initial covariance matrix for S_T , and with respect to the normalization of the equation of interest. The iterative GMM approach has been advocated by Kocherlakota (1990), Ferson and Foerster (1994). Based on Monte-Carlo simulations, they find iterative GMM to have superior finite-sample properties as compared with the two-step GMM. Hansen, Heaton, and Yaron (1996) also compare the finite-sample properties of these alternative GMM estimators. In some cases, the two-step and the iterative estimators are found to be more widely median biased than the continuous-updating estimator. But, the distribution of the continuous-updating estimator has much fatter tails.

Estimating the covariance matrix S_T has been widely discussed in the theoretical literature. An asymptotically efficient estimator is obtained by choosing a consistent estimator of $V = E(g_t(\theta)g_t(\theta)') = E(\varepsilon_t^2 Z_t Z_t')$ the covariance matrix of $g_t(\theta)$. When innovations are likely to be heteroskedastic and serially correlated, the covariance matrix can be consistently estimated by the estimator proposed by Newey and West (1987):

$$\hat{S}_T(\hat{\theta}_T) = S_0 + \sum_{l=1}^L w(l)(S_l + S_l') \quad \text{with} \quad S_l = \frac{1}{T} \sum_{t=l+1}^T \hat{\varepsilon}_t \hat{\varepsilon}_{t-l} (Z_{t-1} Z_{t-1-l}')$$

where $\hat{\varepsilon}_t = y_t - X_t' \hat{\theta}_T$ and $w(l) = 1 - \frac{l}{L+1}$ denotes the Bartlett kernel. The bandwidth L is determined by the correlation structure of moment conditions which is known a priori in some application (Newey and West, 1987). Andrews and Monahan (1992) and Newey and West (1994) also proposed improved techniques to estimate the covariance matrix. In contrast to Newey and West (1987), both approaches suggest to prewhiten the moment conditions and to use a data-dependent bandwidth. In Andrews and Monahan (1992), the optimal bandwidth is computed assuming an AR(1) process for the moment conditions, whereas the bandwidth is computed non-parametrically in Newey and West (1994). In addition, Andrews and Monahan (1992) suggest using the quadratic spectral kernel rather than the Bartlett kernel (see these two papers for additional details of the procedures).

We consider three variants of the covariance-matrix estimator: (1) S_{1T} is the estimator proposed by Newey and West (1987) with $L = 4$. (2) S_{2T} is the estimator proposed by Andrews and Monahan (1992). (3) S_{3T} is the estimator suggested by Newey and West (1994).

Another important feature of GMM estimation is that the information set may contain more instruments than unknown parameters (provided $q > n$). In such a case, when the model is correctly specified, all the elements of the sample moments $\bar{g}_T(\hat{\theta}_T)$ are close to zero, but they cannot be set to zero exactly. It turns out that if

the covariance matrix S_T is chosen optimally, then the minimized distance

$$J_T = T \bar{g}_T (\hat{\theta}_T)' (S_T)^{-1} \bar{g}_T (\hat{\theta}_T)$$

is asymptotically distributed as a χ^2 with $q - n$ degrees of freedom. This provides us with Hansen's test of the over-identifying restrictions (Hansen, 1982). A rejection of these restrictions would indicate that some variables in the information set fail to satisfy the orthogonality conditions.

8.2 The ML approach

ML estimation of the reaction function, together with the PC and the I-S curve is implemented using the procedure developed by Anderson and Moore (1985), which computes the reduced form of any linear forward-looking model. Our forward-looking model can be written in the format

$$\sum_{j=-\tau}^0 H_j x_{t+j} + \sum_{j=1}^{\theta} H_j E_t(x_{t+j}) = e_t \quad (11)$$

where $x_t = (\pi_t, y_t, i_t)'$ and H_j are conformable square matrices containing the model's parameters. The innovations e_t are assumed to be iid with zero mean and covariance matrix Σ . τ and θ denote leads and lags respectively. In our empirical application, we have $\tau = \theta = 4$. For instance, forward-looking terms are inflation as well as output gap leads in the reaction function, in the case of the purely backward-looking macroeconomic model.

Using the generalized saddlepath procedure of Anderson and Moore (1985), the expectation of future terms in equation (11) is expressed as a function of expectations of lagged terms:

$$E_t(x_{t+k}) = \sum_{j=-\tau}^{-1} B_j E_t(x_{t+k+j}) \quad k > 0. \quad (12)$$

Then, equation (12) is used to derive the expectation of future terms as a function of the present and past terms. Substituting expectations into equation (11) gives the so-called observable structure

$$\sum_{j=-\tau}^0 S_j x_{t+j} = e_t. \quad (13)$$

The procedure proposed by Anderson and Moore (1985) is very efficient and can be applied to a wide range of applications. It has been widely used in the empirical literature (see, e.g., Fuhrer, Moore, and Schuh, 1995, Fuhrer and Moore, 1995a and b). Note that given the recursive structure of our model, S_0 is equal to identity.

Finally, the concentrated log-likelihood function is computed using the observable structure (13):

$$\ln L = -\frac{1}{2} nT [1 + \ln(2\pi)] - \frac{T}{2} \ln |\hat{\Sigma}|$$

where $\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T \hat{e}_t \hat{e}_t'$ is the estimated covariance matrix of residuals. The log-likelihood function is maximized using the BFGS algorithm of the GAUSS constrained optimization procedure. For Monte-Carlo simulations, the maximum number of iterations is chosen equal to 200. Note that, in the procedure used, pre-sample initial values of the variables of interest are treated as deterministic. The parameter covariance matrix is computed using the inverse of the Hessian of the log-likelihood function.

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Captions

Table 1: The table reports parameter estimates of the reaction function (equation (4)) by various GMM procedures over the 1979–2000:Q3 period. Two-step, iterative, and continuous-updating estimators are displayed in Panel A, B, and C respectively. Details on these estimators and on the covariance-matrix estimators S_{1T} , S_{2T} , and S_{3T} are provided in the Appendix. The instruments set includes, in addition to the constant term, four lags of the Funds rate, inflation, and the output gap.

Table 2: The table reports parameter estimates and residual summary statistics for the ML joint estimation of equations (4), (5), and (6) over the 1979–2000:Q3 period. $Q(K)$ is the Ljung-Box statistic, which tests the null that the first K serial correlations of residuals are jointly zero. $R(K)$ is the Engle statistic, which tests the null that the first K serial correlations of squared residuals are jointly zero. Under the null, these statistics are distributed as a $\chi^2(K)$. J-B is the Jarque-Bera statistic, which tests the null of normality. Under the null, it is distributed as a $\chi^2(2)$. se denotes the standard error of residual estimates. $\log\text{-L}$ denotes the sample log-likelihood.

Table 3: This table displays the distribution of alternative GMM and ML estimators of the reaction-function parameters. The DGP is given by equations (4), (5), and (6), while parameters are those obtained by ML (Table 2). The sample size is $T = 85$ (Table 3a) or $T = 200$ (Table 3b). For each parameter, the mean, standard deviation, median, 10th percentile and 90th percentile of the distribution are reported. Truncated continuous-updating corresponds to estimates whose smoothing parameter ρ_1 lies inside the interval $[-1, 1]$. Rejection rates are the percentages of the 2000 replications in which the Hansen’s J-statistic exceeds the relevant critical value of the χ^2 distribution. For the continuous-updating GMM approach, figures in parenthesis indicate the frequency of iterations which were discarded before computing summary statistics.

Table 4: This table displays the distribution of alternative GMM and ML estimators of the reaction-function parameters. The DGP is given by equations (4), (5), and (6), while parameters are those obtained by ML (Table 2). Over the first subperiod ($0.35T$ first observations), the policy shock is assumed to be iid, whereas over the second subperiod the policy shock has a first-order correlation of 0.74. The sample size is $T = 85$ (Table 4a) or $T = 200$ (Table 4b). For each parameter, the mean, standard deviation, median, 10th percentile and 90th percentile of the distribution are reported. Truncated continuous-updating corresponds to estimates whose smoothing parameter ρ_1 lies inside the interval $[-1, 1]$. Rejection rates are the percentages of the 2000 replications in which the Hansen’s J-statistic exceeds the relevant critical value of the χ^2 distribution. For the continuous-updating GMM approach, figures in parenthesis indicate the frequency of iterations which were discarded before computing summary statistics.

Table 5: This table reports tests for stability of reaction-function parameters. The shift is assumed to occur, if any, in 1987:Q3. For GMM, the test is based on the Wald statistic, which is distributed as a $\chi^2(4)$. Results for the continuous-updating estimator are not reported, because the algorithm failed to converge over both sub-periods. For ML, the test is based on the Likelihood-Ratio statistic. The following null hypotheses are considered: stability of the reaction-function parameters, stability of macroeconomic parameters, and stability of all parameters. The test statistics are distributed as a χ^2 with 5, 8, and 13 parameters respectively. log-L denotes the sample log-likelihood. dof denotes the degree of freedom of the test statistic.

Table 6: The table reports parameter estimates of the reaction function (equation (4)) by various GMM procedures over the 1987:Q3–2000:Q3 period. Two-step, iterative, and continuous-updating estimators are displayed in Panel A, B, and C respectively. Details on these estimators and on the covariance-matrix estimators S_{1T} , S_{2T} , and S_{3T} are provided in the Appendix. The instruments set includes, in addition to the constant term, four lags of the Funds rate, inflation, and the output gap.

Table 7: The table reports parameter estimates and residual summary statistics for the ML joint estimation of equations (4), (5), and (6) over the 1979:Q3–2000:Q3 period with a shift in the reaction-function parameters in 1987:Q3. $Q(K)$ is the Ljung-Box statistic, which tests the null that the first K serial correlations of residuals are jointly zero. $R(K)$ is the Engle statistic, which tests the null that the first K serial correlations of squared residuals are jointly zero. Under the null, these statistics are distributed as a $\chi^2(K)$. J-B is the Jarque-Bera statistic, which tests the null of normality. Under the null, it is distributed as a $\chi^2(2)$. se denotes the standard error of residual estimates. log-L denotes the sample log-likelihood.

Table 8: The table reports parameter estimates and residual summary statistics for the ML joint estimation of equations (4), (7), and (8) over the 1979:Q3–2000:Q3 period with a shift in the reaction-function parameters in 1987:Q3. $Q(K)$ is the Ljung-Box statistic, which tests the null that the first K serial correlations of residuals are jointly zero. $R(K)$ is the Engle statistic, which tests the null that the first K serial correlations of squared residuals are jointly zero. Under the null, these statistics are distributed as a $\chi^2(K)$. J-B is the Jarque-Bera statistic, which tests the null of normality. Under the null, it is distributed as a $\chi^2(2)$. se denotes the standard error of residual estimates. log-L denotes the sample log-likelihood.

Figure 1: This figure displays the reaction-function residuals for the ML joint estimation of equations (4), (5), and (6) over the 1979:Q3–2000:Q3 period. The model is assumed to have stable parameters.

Figure 2a: This figure displays the distribution of ML and two-step GMM estimators of ρ , β , and γ . The DGP is given by equations (4), (5), and (6), while parameters are those obtained by ML (Table 2). The sample size is $T = 85$.

Figure 2b: This figure displays the distribution of ML and alternative GMM estimators of β . The DGP is given by equations (4), (5), and (6), while parameters are those obtained by ML (Table 2). The sample size is $T = 85$.

Figure 3: This figure displays the reaction-function residuals for the ML joint estimation of equations (4), (5), and (6) over the 1979:Q3–2000:Q3 period. The model is assumed to have a shift in the reaction-function parameters in 1987:Q3.

Table 1: GMM estimates (1979:Q3-2000:Q3)

| | Panel A | | Panel B | | Panel C | |
|--|---------------------|----------|----------------------|----------|--------------------------------|----------|
| | Two-step GMM | | Iterative GMM | | Continuous-updating GMM | |
| | Estimate | Std err. | Estimate | Std err. | Estimate | Std err. |
| Covariance-matrix estimator S_{1T} | | | | | | |
| ρ_1 | 0.831 | 0.043 | 0.782 | 0.061 | 0.766 | 0.053 |
| β | 2.631 | 0.486 | 3.591 | 0.566 | 3.619 | 0.385 |
| γ | 0.712 | 0.388 | 0.489 | 0.372 | 0.435 | 0.309 |
| α | -0.585 | 1.465 | -2.836 | 2.479 | -2.982 | 2.277 |
| J-statistic (stat. / p-val.) | 9.765 | 0.370 | 6.371 | 0.702 | 6.180 | 0.722 |
| Covariance-matrix estimator S_{2T} | | | | | | |
| ρ_1 | 0.875 | 0.031 | 0.938 | 0.030 | 0.837 | 0.014 |
| β | 3.025 | 0.449 | 6.000 | 2.296 | 2.725 | 0.141 |
| γ | 1.377 | 0.261 | 1.725 | 1.101 | 1.279 | 0.337 |
| α | -1.249 | 1.188 | -8.117 | 6.833 | -0.369 | 0.313 |
| J-statistic (stat. / p-val.) | 23.393 | 0.005 | 14.006 | 0.122 | 13.152 | 0.156 |
| Covariance-matrix estimator S_{3T} | | | | | | |
| ρ_1 | 0.848 | 0.025 | 0.932 | 0.024 | 0.804 | 0.018 |
| β | 2.568 | 0.248 | 6.979 | 2.302 | 2.107 | 0.412 |
| γ | 0.937 | 0.126 | 2.312 | 1.036 | 1.566 | 0.420 |
| α | -0.208 | 0.811 | -11.25 | 6.854 | 1.755 | 0.802 |
| J-statistic (stat. / p-val.) | 28.798 | 0.001 | 29.057 | 0.001 | 13.33 | 0.148 |

Table 2: ML estimates (1979:Q3-2000:Q3)

| Reaction function | | | Phillips curve | | | I-S curve | | |
|-------------------|-----------|----------|------------------|-----------|----------|--------------|-----------|----------|
| | Estimate | Std err. | | Estimate | Std err. | | Estimate | Std err. |
| ρ_1 | 0.713 | 0.060 | $\alpha_{\pi 1}$ | 0.501 | 0.120 | β_{y1} | 1.113 | 0.111 |
| β | 1.879 | 0.272 | $\alpha_{\pi 2}$ | 0.022 | 0.150 | β_{y2} | -0.192 | 0.101 |
| γ | 0.020 | 0.271 | $\alpha_{\pi 3}$ | 0.401 | 0.152 | β_r | -0.089 | 0.056 |
| α | 1.368 | 1.312 | $\alpha_{\pi 4}$ | 0.076 | - | β_0 | 3.193 | 1.227 |
| | | | α_y | 0.127 | 0.043 | | | |
| | Statistic | p-value | | Statistic | p-value | | Statistic | p-value |
| Q(4) | 5.364 | 0.252 | Q(4) | 0.869 | 0.929 | Q(4) | 5.337 | 0.254 |
| Q(8) | 15.973 | 0.043 | Q(8) | 7.070 | 0.529 | Q(8) | 8.909 | 0.350 |
| R(4) | 36.567 | 0.000 | R(4) | 4.539 | 0.338 | R(4) | 19.543 | 0.001 |
| R(8) | 47.681 | 0.000 | R(8) | 10.394 | 0.238 | R(8) | 36.886 | 0.000 |
| J-B | 125.284 | 0.000 | J-B | 1.405 | 0.495 | J-B | 25.104 | 0.000 |
| see | 1.016 | | see | 0.793 | | see | 0.695 | |
| log-L | -295.214 | | | | | | | |

Table 3a: Monte-Carlo simulation of the ML model (sample size: $T=85$)

| | ρ_1 (true value: 0.71) | | | | | β (true value: 1.88) | | | | | γ (true value: 0.02) | | | | | Rejection rate | | |
|--|-----------------------------|---------|------|--------|------|----------------------------|---------|------|--------|------|-----------------------------|---------|-------|--------|------|----------------|------|------|
| | Mean | Std dev | 10% | Median | 90% | Mean | Std dev | 10% | Median | 90% | Mean | Std dev | 10% | Median | 90% | 1% | 5% | 10% |
| GMM | | | | | | | | | | | | | | | | | | |
| Covariance-matrix estimator S_{1T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.68 | 0.11 | 0.53 | 0.69 | 0.80 | 2.00 | 0.51 | 1.58 | 1.95 | 2.47 | 0.03 | 0.50 | -0.39 | 0.00 | 0.49 | 0.29 | 0.47 | 0.57 |
| Iterative | 0.66 | 0.12 | 0.49 | 0.67 | 0.80 | 2.00 | 0.55 | 1.55 | 1.94 | 2.49 | 0.00 | 0.42 | -0.44 | -0.03 | 0.50 | 0.08 | 0.21 | 0.32 |
| Cont.-updating (5.3%) | 0.55 | 0.23 | 0.00 | 0.61 | 0.79 | 1.21 | 25.75 | 1.21 | 1.97 | 2.76 | -1.62 | 32.29 | -1.06 | -0.12 | 0.50 | 0.01 | 0.06 | 0.14 |
| Truncated CU (10.9%) | 0.58 | 0.20 | 0.28 | 0.62 | 0.79 | 2.12 | 2.67 | 1.46 | 1.97 | 2.74 | -0.11 | 1.54 | -0.73 | -0.11 | 0.50 | 0.01 | 0.05 | 0.12 |
| Covariance-matrix estimator S_{2T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.68 | 0.11 | 0.53 | 0.69 | 0.80 | 1.97 | 0.51 | 1.54 | 1.92 | 2.44 | 0.02 | 0.51 | -0.45 | -0.02 | 0.52 | 0.34 | 0.52 | 0.62 |
| Iterative | 0.64 | 0.16 | 0.44 | 0.67 | 0.80 | 1.97 | 0.53 | 1.49 | 1.92 | 2.48 | -0.02 | 0.56 | -0.51 | -0.05 | 0.50 | 0.12 | 0.26 | 0.37 |
| Cont.-updating (0.1%) | 0.60 | 0.18 | 0.25 | 0.64 | 0.80 | 5.41 | 93.02 | 1.08 | 1.93 | 2.71 | -2.82 | 83.16 | -1.08 | -0.12 | 0.52 | 0.05 | 0.16 | 0.27 |
| Truncated CU (2.4%) | 0.60 | 0.17 | 0.36 | 0.64 | 0.79 | 2.01 | 1.15 | 1.41 | 1.93 | 2.69 | -0.14 | 1.65 | -0.73 | -0.12 | 0.51 | 0.05 | 0.16 | 0.27 |
| Covariance-matrix estimator S_{3T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.67 | 0.12 | 0.51 | 0.68 | 0.81 | 1.98 | 0.49 | 1.53 | 1.93 | 2.50 | 0.02 | 0.44 | -0.45 | -0.01 | 0.51 | 0.51 | 0.66 | 0.74 |
| Iterative | 0.62 | 0.17 | 0.40 | 0.65 | 0.80 | 1.97 | 0.57 | 1.46 | 1.91 | 2.54 | -0.03 | 0.62 | -0.54 | -0.05 | 0.49 | 0.20 | 0.34 | 0.44 |
| Cont.-updating (6.6%) | 0.59 | 0.19 | 0.23 | 0.62 | 0.80 | 3.31 | 67.80 | 0.88 | 1.93 | 2.82 | -1.68 | 61.53 | -1.23 | -0.09 | 0.57 | 0.09 | 0.20 | 0.30 |
| Truncated CU (8.1%) | 0.59 | 0.18 | 0.34 | 0.62 | 0.79 | 1.79 | 6.15 | 1.33 | 1.93 | 2.80 | -0.16 | 5.24 | -0.79 | -0.09 | 0.57 | 0.06 | 0.17 | 0.27 |
| ML | 0.65 | 0.08 | 0.55 | 0.66 | 0.75 | 1.88 | 0.22 | 1.62 | 1.87 | 2.15 | -0.01 | 0.27 | -0.32 | -0.01 | 0.34 | - | - | - |

Table 3b: Monte-Carlo simulation of the ML model (sample size: $T=200$)

| | ρ_1 (true value: 0.71) | | | | | β (true value: 1.88) | | | | | γ (true value: 0.02) | | | | | Rejection rate | | |
|--|-----------------------------|---------|------|--------|------|----------------------------|---------|------|--------|------|-----------------------------|---------|-------|--------|------|----------------|------|------|
| | Mean | Std dev | 10% | Median | 90% | Mean | Std dev | 10% | Median | 90% | Mean | Std dev | 10% | Median | 90% | 1% | 5% | 10% |
| GMM | | | | | | | | | | | | | | | | | | |
| Covariance-matrix estimator S_{1T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.69 | 0.06 | 0.62 | 0.70 | 0.77 | 1.93 | 0.18 | 1.73 | 1.91 | 2.15 | 0.00 | 0.18 | -0.21 | 0.01 | 0.23 | 0.07 | 0.18 | 0.27 |
| Iterative | 0.69 | 0.06 | 0.61 | 0.70 | 0.76 | 1.93 | 0.18 | 1.73 | 1.91 | 2.16 | 0.00 | 0.18 | -0.22 | 0.00 | 0.22 | 0.03 | 0.12 | 0.20 |
| Cont.-updating (0.1%) | 0.66 | 0.10 | 0.50 | 0.68 | 0.75 | -3.63 | 249.12 | 1.65 | 1.91 | 2.19 | 1.10 | 51.33 | -0.37 | -0.03 | 0.21 | 0.01 | 0.09 | 0.18 |
| Truncated CU (0.5%) | 0.66 | 0.09 | 0.56 | 0.68 | 0.75 | 1.94 | 0.27 | 1.72 | 1.91 | 2.19 | -0.05 | 0.56 | -0.28 | -0.03 | 0.21 | 0.01 | 0.09 | 0.19 |
| Covariance-matrix estimator S_{2T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.69 | 0.06 | 0.61 | 0.70 | 0.77 | 1.92 | 0.17 | 1.73 | 1.91 | 2.13 | 0.01 | 0.19 | -0.22 | 0.01 | 0.23 | 0.08 | 0.20 | 0.28 |
| Iterative | 0.69 | 0.07 | 0.60 | 0.70 | 0.76 | 1.92 | 0.17 | 1.72 | 1.90 | 2.13 | 0.00 | 0.19 | -0.23 | 0.00 | 0.23 | 0.05 | 0.13 | 0.21 |
| Cont.-updating (0.1%) | 0.66 | 0.10 | 0.47 | 0.68 | 0.76 | 1.93 | 0.25 | 1.65 | 1.91 | 2.18 | -0.04 | 0.24 | -0.41 | -0.04 | 0.21 | 0.03 | 0.11 | 0.18 |
| Truncated CU (0.3%) | 0.66 | 0.10 | 0.54 | 0.68 | 0.76 | 1.93 | 0.25 | 1.71 | 1.91 | 2.18 | -0.04 | 0.24 | -0.29 | -0.04 | 0.21 | 0.03 | 0.11 | 0.18 |
| Covariance-matrix estimator S_{3T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.69 | 0.07 | 0.61 | 0.70 | 0.77 | 1.93 | 0.19 | 1.73 | 1.92 | 2.17 | 0.01 | 0.20 | -0.23 | 0.00 | 0.25 | 0.18 | 0.30 | 0.41 |
| Iterative | 0.68 | 0.07 | 0.59 | 0.69 | 0.77 | 1.93 | 0.19 | 1.72 | 1.91 | 2.17 | 0.00 | 0.20 | -0.24 | -0.01 | 0.25 | 0.09 | 0.21 | 0.29 |
| Cont.-updating (0.1%) | 0.66 | 0.11 | 0.44 | 0.68 | 0.76 | 2.40 | 25.23 | 1.60 | 1.92 | 2.23 | -0.77 | 21.47 | -0.46 | -0.04 | 0.27 | 0.05 | 0.14 | 0.23 |
| Truncated CU (0.3%) | 0.66 | 0.11 | 0.52 | 0.68 | 0.76 | 1.94 | 0.43 | 1.69 | 1.92 | 2.23 | -0.07 | 1.44 | -0.32 | -0.04 | 0.27 | 0.04 | 0.13 | 0.22 |
| ML | 0.69 | 0.05 | 0.63 | 0.69 | 0.74 | 1.88 | 0.11 | 1.75 | 1.88 | 2.02 | 0.01 | 0.16 | -0.19 | 0.01 | 0.21 | - | - | - |

Table 4a: Monte-Carlo simulation of the ML model with serially-correlated monetary policy shock (sample size: T=85)

| | ρ_1 (true value: 0.71) | | | | | β (true value: 1.88) | | | | | γ (true value: 0.02) | | | | | Rejection rate | | |
|--|-----------------------------|---------|------|--------|------|----------------------------|---------|-------|--------|------|-----------------------------|---------|-------|--------|------|----------------|------|------|
| | Mean | Std dev | 10% | Median | 90% | Mean | Std dev | 10% | Median | 90% | Mean | Std dev | 10% | Median | 90% | 1% | 5% | 10% |
| GMM | | | | | | | | | | | | | | | | | | |
| Covariance-matrix estimator S_{1T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.81 | 0.09 | 0.69 | 0.82 | 0.91 | 2.34 | 0.79 | 1.70 | 2.21 | 3.11 | 0.75 | 1.40 | -0.08 | 0.51 | 1.53 | 0.30 | 0.52 | 0.63 |
| Iterative | 0.80 | 0.12 | 0.64 | 0.83 | 0.93 | 2.60 | 4.21 | 1.60 | 2.27 | 3.52 | 1.63 | 7.50 | -0.16 | 0.54 | 2.29 | 0.03 | 0.12 | 0.23 |
| Cont.-updating (6.7%) | 0.70 | 0.23 | 0.18 | 0.77 | 0.93 | -0.56 | 127.22 | 0.86 | 2.24 | 4.24 | 5.85 | 44.11 | -0.84 | 0.28 | 2.48 | 0.00 | 0.01 | 0.03 |
| Truncated CU (12.8%) | 0.72 | 0.19 | 0.46 | 0.77 | 0.91 | 2.69 | 3.37 | 1.51 | 2.24 | 4.09 | 0.66 | 3.34 | -0.49 | 0.28 | 2.04 | 0.00 | 0.01 | 0.02 |
| Covariance-matrix estimator S_{2T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.81 | 0.10 | 0.68 | 0.82 | 0.92 | 2.42 | 2.34 | 1.62 | 2.20 | 3.31 | 0.90 | 2.97 | -0.15 | 0.48 | 1.98 | 0.74 | 0.87 | 0.92 |
| Iterative | 0.77 | 0.19 | 0.55 | 0.82 | 0.94 | 2.79 | 4.77 | 1.41 | 2.21 | 3.91 | 1.65 | 9.37 | -0.33 | 0.45 | 2.66 | 0.25 | 0.40 | 0.52 |
| Cont.-updating (3.7%) | 0.73 | 0.20 | 0.31 | 0.77 | 0.94 | 9.01 | 84.49 | 0.34 | 2.17 | 4.29 | 9.72 | 132.86 | -1.20 | 0.31 | 3.25 | 0.04 | 0.11 | 0.17 |
| Truncated CU (7.2%) | 0.73 | 0.18 | 0.49 | 0.77 | 0.93 | 2.54 | 5.10 | 1.29 | 2.16 | 3.96 | 0.84 | 7.46 | -0.62 | 0.30 | 2.66 | 0.04 | 0.11 | 0.17 |
| Covariance-matrix estimator S_{3T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.81 | 0.11 | 0.67 | 0.82 | 0.92 | 2.48 | 2.83 | 1.58 | 2.17 | 3.26 | 1.13 | 8.11 | -0.13 | 0.50 | 1.89 | 0.82 | 0.92 | 0.95 |
| Iterative | 0.75 | 0.21 | 0.51 | 0.81 | 0.94 | 2.58 | 3.83 | 1.26 | 2.16 | 3.83 | 1.28 | 4.29 | -0.36 | 0.47 | 2.80 | 0.30 | 0.46 | 0.58 |
| Cont.-updating (7.9%) | 0.72 | 0.21 | 0.29 | 0.76 | 0.95 | 2.81 | 117.43 | -1.21 | 2.14 | 4.15 | 12.51 | 124.50 | -1.75 | 0.34 | 2.97 | 0.06 | 0.13 | 0.18 |
| Truncated CU (12.7%) | 0.71 | 0.20 | 0.43 | 0.76 | 0.92 | 2.34 | 6.18 | 1.04 | 2.14 | 3.86 | 1.25 | 8.81 | -0.78 | 0.32 | 2.41 | 0.04 | 0.11 | 0.16 |
| ML | 0.75 | 0.08 | 0.64 | 0.75 | 0.84 | 2.04 | 0.33 | 1.68 | 2.01 | 2.45 | 0.30 | 0.45 | -0.16 | 0.23 | 0.80 | - | - | - |

Table 4b: Monte-Carlo simulation of the ML model with serially-correlated monetary policy shock (sample size: $T=200$)

| | ρ_1 (true value: 0.71) | | | | | β (true value: 1.88) | | | | | γ (true value: 0.02) | | | | | Rejection rate | | |
|--|-----------------------------|---------|------|--------|------|----------------------------|---------|------|--------|------|-----------------------------|---------|-------|--------|------|----------------|------|------|
| | Mean | Std dev | 10% | Median | 90% | Mean | Std dev | 10% | Median | 90% | Mean | Std dev | 10% | Median | 90% | 1% | 5% | 10% |
| GMM | | | | | | | | | | | | | | | | | | |
| Covariance-matrix estimator S_{1T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.80 | 0.06 | 0.72 | 0.81 | 0.87 | 2.11 | 0.25 | 1.83 | 2.08 | 2.44 | 0.35 | 0.32 | -0.01 | 0.32 | 0.76 | 0.11 | 0.28 | 0.40 |
| Iterative | 0.80 | 0.06 | 0.72 | 0.80 | 0.87 | 2.14 | 0.29 | 1.82 | 2.09 | 2.50 | 0.38 | 0.38 | -0.03 | 0.32 | 0.85 | 0.03 | 0.13 | 0.23 |
| Cont.-updating (0.5%) | 0.76 | 0.11 | 0.58 | 0.78 | 0.87 | -3.31 | 246.22 | 1.68 | 2.09 | 2.60 | -2.05 | 77.12 | -0.26 | 0.22 | 0.81 | 0.00 | 0.04 | 0.09 |
| Truncated CU (1.2%) | 0.77 | 0.10 | 0.65 | 0.78 | 0.87 | 4.21 | 63.87 | 1.77 | 2.08 | 2.59 | 0.20 | 4.16 | -0.15 | 0.22 | 0.81 | 0.00 | 0.04 | 0.09 |
| Covariance-matrix estimator S_{2T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.80 | 0.06 | 0.72 | 0.80 | 0.87 | 2.13 | 0.31 | 1.82 | 2.08 | 2.48 | 0.40 | 0.42 | 0.00 | 0.34 | 0.86 | 0.26 | 0.45 | 0.56 |
| Iterative | 0.80 | 0.07 | 0.71 | 0.80 | 0.88 | 2.18 | 0.77 | 1.79 | 2.10 | 2.63 | 0.50 | 1.02 | -0.03 | 0.34 | 0.99 | 0.09 | 0.24 | 0.36 |
| Cont.-updating (0.5%) | 0.74 | 0.15 | 0.43 | 0.78 | 0.87 | 2.56 | 12.61 | 1.60 | 2.09 | 2.71 | -0.52 | 41.30 | -0.47 | 0.24 | 0.88 | 0.02 | 0.07 | 0.13 |
| Truncated CU (1.8%) | 0.75 | 0.14 | 0.57 | 0.78 | 0.87 | 2.15 | 1.31 | 1.71 | 2.09 | 2.69 | 0.32 | 1.24 | -0.22 | 0.24 | 0.86 | 0.02 | 0.08 | 0.15 |
| Covariance-matrix estimator S_{3T} | | | | | | | | | | | | | | | | | | |
| Two-step | 0.81 | 0.06 | 0.73 | 0.81 | 0.88 | 2.16 | 0.30 | 1.84 | 2.10 | 2.54 | 0.47 | 0.46 | 0.02 | 0.39 | 0.98 | 0.39 | 0.59 | 0.68 |
| Iterative | 0.81 | 0.08 | 0.71 | 0.82 | 0.90 | 2.32 | 1.14 | 1.82 | 2.14 | 2.78 | 0.74 | 1.79 | -0.03 | 0.41 | 1.34 | 0.15 | 0.29 | 0.40 |
| Cont.-updating (4.9%) | 0.75 | 0.14 | 0.48 | 0.78 | 0.88 | -0.14 | 73.62 | 1.43 | 2.11 | 2.84 | 4.36 | 77.37 | -0.59 | 0.27 | 1.06 | 0.02 | 0.07 | 0.13 |
| Truncated CU (6.0%) | 0.75 | 0.14 | 0.59 | 0.78 | 0.87 | 2.10 | 2.85 | 1.70 | 2.11 | 2.83 | 0.40 | 2.82 | -0.27 | 0.27 | 1.02 | 0.01 | 0.06 | 0.13 |
| ML | 0.77 | 0.05 | 0.71 | 0.77 | 0.83 | 2.00 | 0.17 | 1.81 | 2.00 | 2.21 | 0.22 | 0.23 | -0.03 | 0.20 | 0.51 | - | - | - |

Table 5: Stability tests

| Panel A: GMM estimates (Wald test) | | | | |
|---|---------------------|---------|----------------------|---------|
| | Two-step GMM | | Iterative GMM | |
| | statistic | p-value | statistic | p-value |
| Covariance-matrix estimator S_{1T} | 28.67 | 0.00 | 20.03 | 0.00 |
| Covariance-matrix estimator S_{2T} | 132.94 | 0.00 | 86.60 | 0.00 |
| Covariance-matrix estimator S_{3T} | 133.84 | 0.00 | 90.72 | 0.00 |

| Panel B: ML estimates (LR test) | | | | | |
|--|---|---------|--------------|---------|-----|
| Model | Null hypothesis | log-L | LR statistic | p-value | dof |
| Shift in all parameters | - | -223.57 | | | |
| Shift in macroeconomic parameters | Stability of reaction-function parameters | -232.24 | 17.34 | 0.00 | 5 |
| Shift in reaction-function parameters | Stability of macroeconomic parameters | -227.87 | 8.60 | 0.38 | 8 |
| No shift | Stability of all parameters | -295.15 | 143.17 | 0.00 | 13 |

Table 6: GMM estimates (1987:Q3-2000:Q3)

| | Panel A | | Panel B | | Panel C | |
|--|--------------|----------|---------------|----------|-------------------------|----------|
| | Two-step GMM | | Iterative GMM | | Continuous-updating GMM | |
| | Estimate | Std err. | Estimate | Std err. | Estimate | Std err. |
| Covariance-matrix estimator S_{1T} | | | | | | |
| ρ_1 | 1.327 | 0.097 | 1.321 | 0.093 | 1.305 | 0.067 |
| ρ_2 | -0.519 | 0.092 | -0.512 | 0.091 | -0.504 | 0.064 |
| β | 1.661 | 0.337 | 1.696 | 0.346 | 1.755 | 0.247 |
| γ | 0.647 | 0.159 | 0.669 | 0.156 | 0.684 | 0.136 |
| α | 1.807 | 0.883 | 1.741 | 0.907 | 1.536 | 0.768 |
| J-statistic (stat. / p-val.) | 3.361 | 0.910 | 2.627 | 0.956 | 2.440 | 0.965 |
| Covariance-matrix estimator S_{2T} | | | | | | |
| ρ_1 | 1.336 | 0.074 | 1.330 | 0.075 | 1.321 | 0.059 |
| ρ_2 | -0.530 | 0.080 | -0.522 | 0.082 | -0.520 | 0.064 |
| β | 1.648 | 0.295 | 1.686 | 0.303 | 1.581 | 0.196 |
| γ | 0.689 | 0.151 | 0.737 | 0.145 | 0.896 | 0.132 |
| α | 1.826 | 0.776 | 1.765 | 0.788 | 2.243 | 0.626 |
| J-statistic (stat. / p-val.) | 7.014 | 0.535 | 4.060 | 0.852 | 3.337 | 0.911 |
| Covariance-matrix estimator S_{3T} | | | | | | |
| ρ_1 | 1.333 | 0.056 | 1.310 | 0.051 | 1.297 | 0.041 |
| ρ_2 | -0.527 | 0.062 | -0.494 | 0.059 | -0.504 | 0.043 |
| β | 1.662 | 0.291 | 1.717 | 0.308 | 1.897 | 0.261 |
| γ | 0.647 | 0.154 | 0.784 | 0.153 | 0.698 | 0.110 |
| α | 1.807 | 0.847 | 1.671 | 0.909 | 1.097 | 0.846 |
| J-statistic (stat. / p-val.) | 7.306 | 0.504 | 5.779 | 0.672 | 4.232 | 0.836 |

Table 7: ML estimates of the backward-looking model with a shift in the reaction-function parameters (1979:Q3-2000:Q3)

Table 8: ML estimates of the hybrid model with a shift in the reaction-function parameters (1979:Q3-2000:Q3)

Figure 1: Reaction—function residuals
Estimation over 1979–2000 without a shift

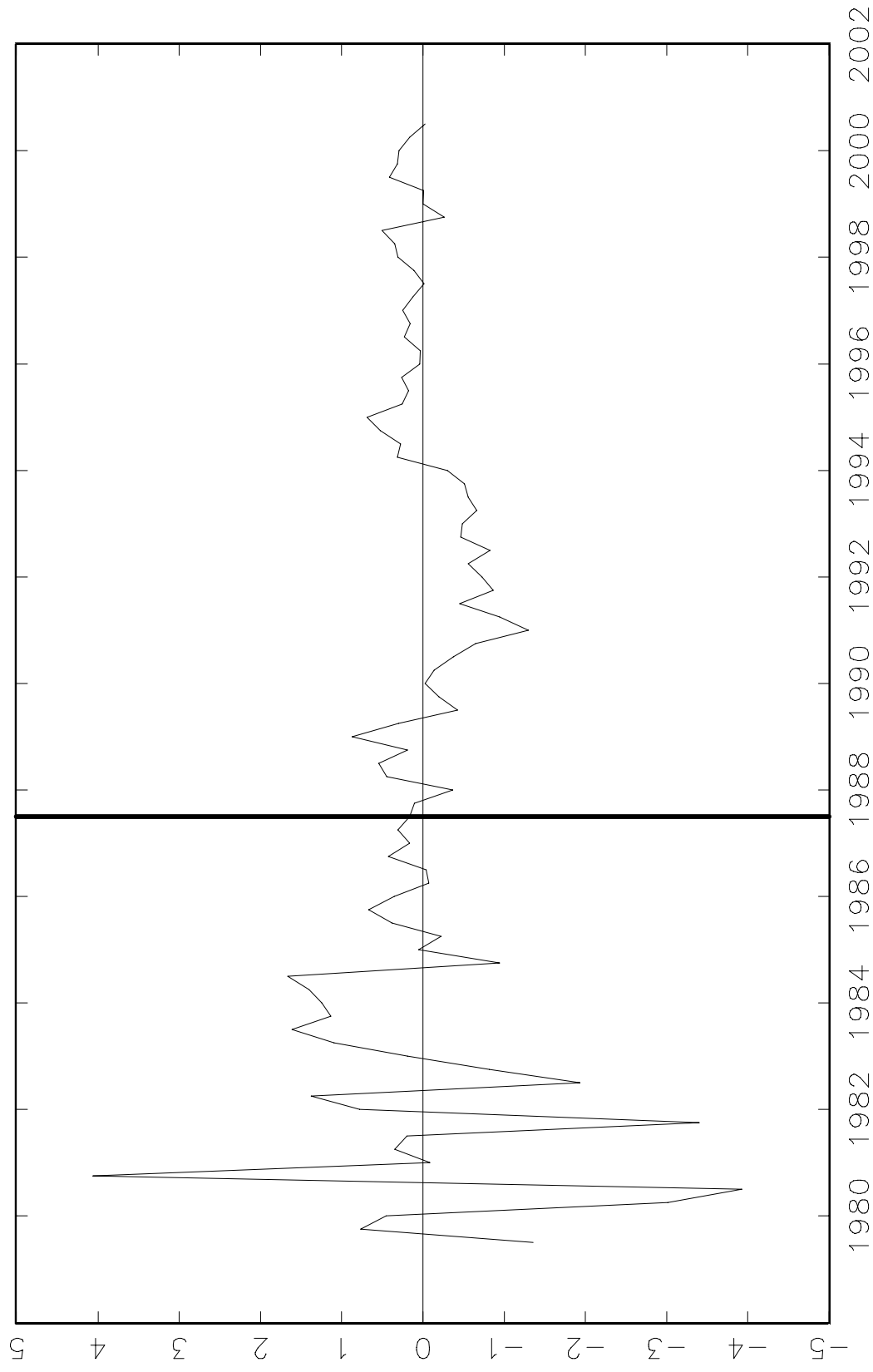


Figure 2a: Distribution of parameter estimators

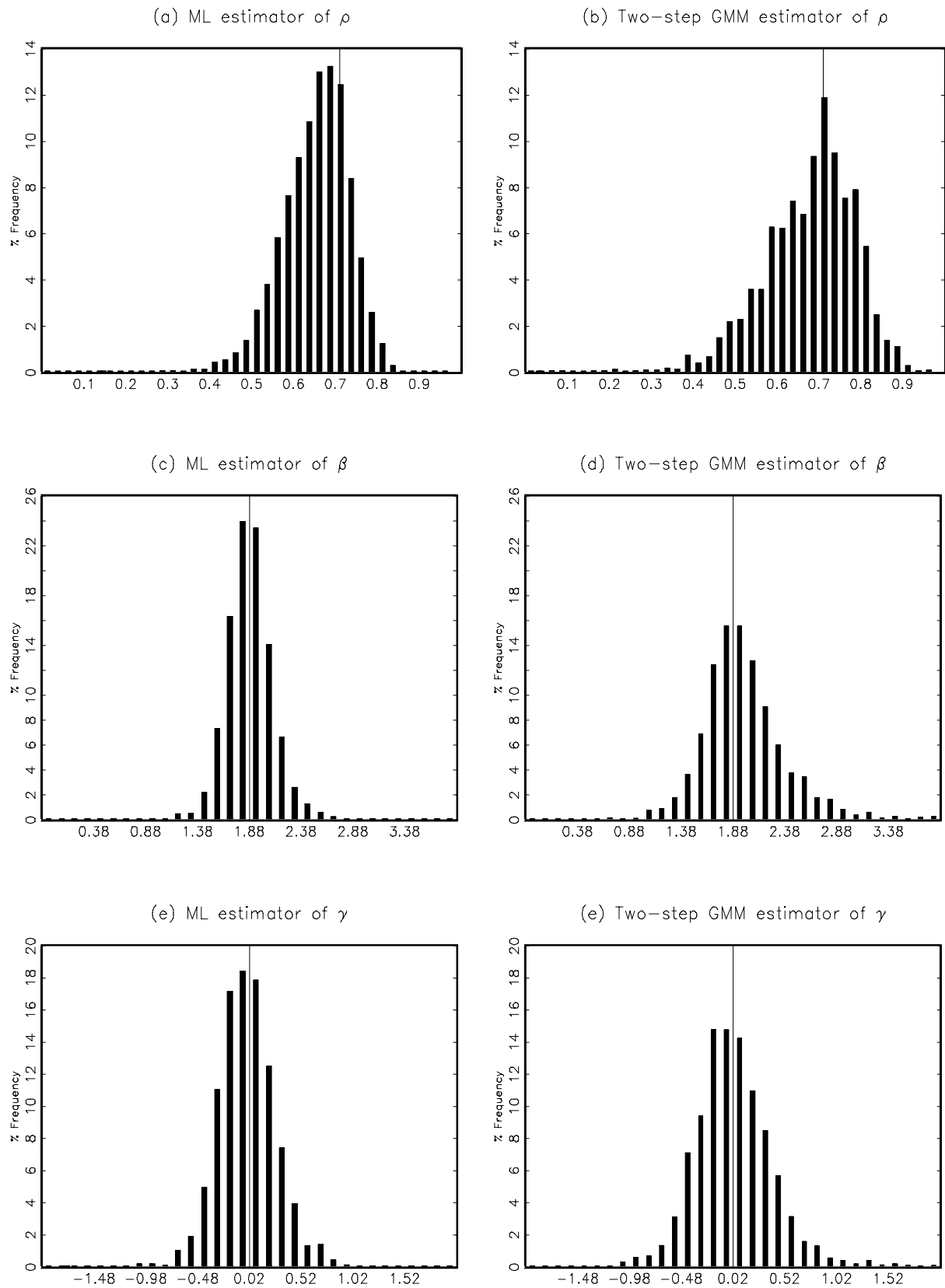


Figure 2b: Distribution of estimators of β

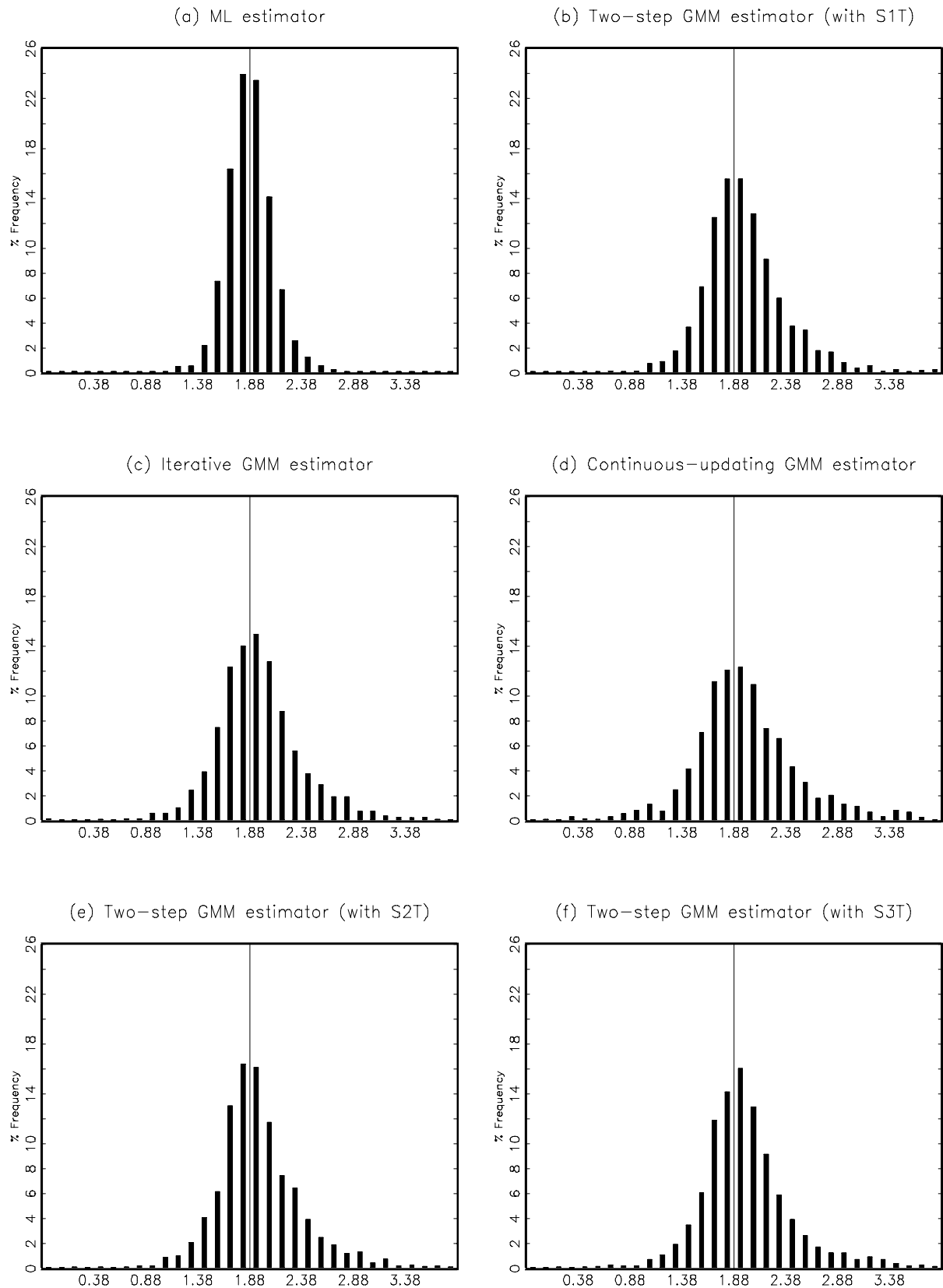


Figure 3: Reaction—function residuals
Estimation over 1979–2000 with a shift in 1987:Q3

