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ASSET PRICING, HABIT MEMORY, AND THE LABOR MARKET

by Ivan Jaccard

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In 2010 all ECB publications feature a motif taken from the €500 banknote.

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CONTENTS

Abstract

This article studies the asset pricing and the business cycle implications of habit formation in a production economy with capital adjustment costs and endogenous labor supply. A specification of internal habit in the mix of consumption and leisure which minimizes the wealth effect on labor supply is introduced into an otherwise standard real business cycle model. This mechanism enhances the model's ability to explain asset pricing puzzles.

Keywords: Equity Premium Puzzle, Labor Supply, Adjustment Costs.

JEL: G12, E32, J22.

Non-technical Summary

In search of a model linking financial markets to the real economy, this article proposes to explore the nexus between endogenous labor supply and financial returns. Our main conclusion is that the joint explanation of asset pricing and macroeconomic facts requires the introduction of preferences which minimize the wealth effect on labor supply while permitting to increase the volatility and the persistence of marginal utility.

In a model with endogenous labor supply, the stochastic discount factor and the labor-leisure decision are jointly determined. Variations in marginal utility jointly affect the stochastic discount factor used to price assets and agents' willingness to supply labor. Introducing endogenous labor supply therefore enriches the set of predictions of asset pricing models. This information can then be exploited to refine our understanding of asset pricing puzzles by considering models with a richer set of empirical implications.

Assuming that habits are formed over a mix of consumption and leisure overcomes the difficulties arising in models with endogenous labor supply and provides a potential solution to the equity premium puzzle. Compared to a standard business cycle model with adjustment costs, the proposed mechanism only requires the introduction of one additional free parameter. Moreover, this specification of preferences is consistent with balanced growth.

The objective of this specification of preference is to capture the idea that agents get hooked to a certain standard of living and that abrupt changes in lifestyles are very costly. In our economy, this aversion gives rise to a strong willingness to avoid outcomes implying low levels of consumption and high work intensity. Working hard in boom periods when consumption and wages are high provides an insurance against such outcomes since it allows agents to reduce work intensity during periods of recession when wages and consumption are low.

1 Introduction

As pointed out by many studies, [Boldrin, Christiano and Fisher (2001), Danthine and Donaldson (2002), Uhlig (2006), Uhlig(2007), Guvenen (2009)] the endogeneity of labor market movements is a major obstacle when it comes to explaining the joint behavior of financial market and macroeconomic data. In this article, we propose to overcome this difficulty by considering an economy with a representative agent whose habits are formed over a mix of consumption and leisure.

In a model with endogenous labor supply, the stochastic discount factor and the labor-leisure decision are jointly determined. Variations in marginal utility jointly affect the stochastic discount factor used to price assets and agents' willingness to supply labor. Introducing endogenous labor supply therefore enriches the set of predictions of asset pricing models. This information can then be exploited to refine our understanding of asset pricing puzzles by considering models with a richer set of empirical implications.

As far as the labor market is concerned, the facts that hours worked are volatile and strongly procyclical and that real wages are sluggish are the key empirical regularities which must be explained. As emphasized by Uhlig (2006, 2007), the challenging task is to reproduce these empirical labor market regularities in a model also able to explain asset pricing puzzles. The key difficulty stems from the fact that the resolution of asset pricing puzzles requires the introduction of mechanisms producing volatile and countercyclical movements in marginal utility. While this feature is needed to generate plausible asset pricing predictions, in models with a labor-leisure decision, it comes at the cost of creating large wealth effects on labor supply.

Large wealth effects on labor supply induce agents to reduce labor effort in periods of economic booms and therefore impairs the ability of standard macroeconomic models to explain the positive correlation between hours worked and output observed in the data. Intuitively, a rise in the volatility of marginal utility, while needed to explain high risk premia, generates an increase in uncertainty which hurts the agents. In a model with endogenous labor supply, this additional margin can be used to offset the undesirable effects of an increase in uncertainty. This negative wealth effect leading to a counterfactual decline in the volatility of hours worked therefore captures that agents would rather choose to reduce labor effort in good times, since reducing (increasing) labor effort in periods of economic booms (recessions) helps to reduce the volatility of output. Such a reduction in the volatility of output makes the whole economy less risky and allows agents to better insure their consumption against shocks.

The concept of habit formation that we propose to introduce permits to successfully resolve this tension between the asset pricing and the business cycle implications of models with endogenous labor supply. As in any asset pricing model, in our economy, a rise in habit intensity increases the volatility of marginal utility. The key is that this increase in intensity also leads to a reduction in the wealth elasticity of labor supply which decreases the sensitivity of labor supply to movements in marginal utility. Our mechanism, which crucially relies on this joint impact of habit formation, appears to improve the ability of an otherwise standard real business cycle model to jointly explain asset pricing and business cycle facts.

As in a standard model, in our economy, a positive technology shock leads to an increase in consumption. The difference comes from the fact that our specification of habit formation induces a strong willingness to smooth the mix of consumption and leisure, which we refer to as the composite good. In response to a positive shock, this effect leads agents to take less leisure and to increase labor effort to prevent the composite good from rising too quickly. Intuitively, with this specification of internal habit formation, agents internalize that a simultaneous increase in both consumption and leisure would lead to an increase in their habit stock which would be quite costly. This particular smoothing motive generates a strong substitutability between consumption and leisure which allows the model to explain the positive comovement between hours worked and output.

The proposed mechanism relies on the idea that agents strongly dislike periods of hard work and reduced consumption. Working harder in periods of booms when consumption is high and increasing leisure in periods of recession allows agents to self-insure against such outcomes. In a standard model, agents would react to a fall in consumption by working harder and the resulting increase in labor supply would contribute to stabilize output. In our economy, the key difference is that households are reluctant to increase labor supply in periods of recessions because wages are too low. This cyclical behavior of labor supply, which amplifies output fluctuations, makes our economy a lot riskier.

Introducing memory effects into the law of motion of the habit stock [see Constantinides (1990), Abel (1999), Cambpell and Cochrane (1999)] increases the intensity of habit formation. Intuitively, memory effects imply that the habit stock depreciates at a slower rate. The more persistent nature of habits amplifies the effects of current choices on future utility and reinforces agents' willingness to smooth variations in the composite good, consisting of these two different components. The role of memory is essentially to strengthen the mechanism generating this complementarity between consumption and hours worked via a reduction in the wealth elasticity of labor supply. As far as the asset pricing implications are concerned, the main contribution of memory is to increase the volatility and the persistence of marginal utility and to enhance the model's ability to generate the amount of precautionary savings needed to explain the risk-free rate puzzle.

This paper is related to a growing literature that builds models for jointly studying asset prices and business cycle fluctuations. As in Jermann (1998), the key mechanism inducing volatile movements in marginal utility relies on the combination of internal habit formation and adjustment costs. Compared to Jermann (1998) and Campanale, Castro and Clementi (2009), our study focuses on the asset pricing implications of endogenous labor supply. In Boldrin, Christiano and Fisher (2001) and Uhlig (2007), business cycle and asset pricing facts are explained in a representative agent model with endogenous labor supply and habit formation. In contrast to these two studies which emphasize the role of labor market frictions, our approach focuses on the combination of adjustment costs and internal habit formation. In Uhlig (2007), the endogeneity of labor market movements is overcome by introducing an exogenous law of motion for wages while the mechanism proposed by Boldrin, Christiano and Fisher (2001) relies on limited sectoral mobility. Finally, Guvenen (2009) develops a two-agent model with limited participation and heterogeneity in the elasticity of intertemporal substitution whereas Danthine, Donaldson and Siconolfi (2006) focus on the role of distribution risks in an economy with two types of agents.

This article is also related to the literature which emphasizes the role of labor supply in explaining the co-movement between output, consumption, investment and hours worked. As in Rebelo and Jaimovich (2009) the fact that our specification of habit formation minimizes the wealth effect on labor supply is a key ingredient [see also Schmitt-Grohe and Uribe (2009), Greenwood, Hercowitz and Huffmann (1988)].

The competitive equilibirum is presented in section 2. The model calibration is described in section 3 and the results are discussed in section 4. Sections 4.1 and 4.2 focus on the labor supply implications while the asset pricing implications are discussed in sections 4.3 and 4.4. Section 5 concludes.

2 The environment

Our specification of internal habit formation is introduced into the standard neoclassical growth model. The economy consists of a large number of identical and infinitely lived agents that derive utility from consumption and leisure. The important assumption is that agents form habits over the mix of consumption and leisure. The final output good is produced by a corporate sector which holds the stock of capital and finances investment via retained earnings.

2.1 Households

The representative agent maximizes expected lifetime utility subject to a sequential budget constraint and the habit stock accumulation equation. Along the balanced growth, the economy is growing at a constant rate $\frac{\Gamma_{t+1}}{\Gamma_t} = \gamma$ and detrended variables are denoted by small letters[1]. The problem of the representative households can be described by the following optimization program:

$$\text{Max } E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} \frac{\left[c_t v(L_t) - x_t\right]^{1-\sigma}}{1-\sigma} \right\}$$

such that:

$$W_t N_t + S_t(p_{Et} + d_t) + B_t = c_t + p_{Et} S_{t+1} + p_{Bt} B_{t+1}$$
$$\gamma x_{t+1} = a x_t + b\left[c_t v(L_t)\right]$$
$$L_t + N_t = 1$$

where β^* is the modified subjective rate of time discount[2], and σ is the curvature coefficient. Consumers choose consumption c_t, hours worked N_t, equity holding, S_{t+1} and bond holding B_{t+1}. Equity prices are denoted p_{Et} and the reference level, or habit stock, is denoted x_t. The risk-free rate is given by the inverse of the 1-period risk-free bond price $1/p_{Bt}$

When it comes to revenues, agents firstly receive a labor income, $W_t N_t$ from working in the firm, where W_t is the wage rate. A dividend income d_t is received each period from owning the firm. The total capital income from being a shareholder is given by the number of stocks held times the

[1] For instance $c_t = C_t/\Gamma_t$ denotes detrended consumption.
[2] $\beta^* = \beta\gamma^{1-\sigma}$

market value of the asset plus dividends, $S_t(p_{Et} + d_t)$. Finally, households face a (normalized) time constraint $1 = N_t + L_t$, with L_t representing leisure.

The law of motion of the habit stock depends on the composite good $c_t v(L_t)$, which reflects our central assumption that habits are formed over the mix of consumption and leisure. The memory parameter a captures the rate at which the stock of habit depreciates while b measures the sensitivity of the reference level with respect to changes in the composite good[3].

With this specification of internal habit formation, agents fully understand the harmful effects of a simultaneous increase in both consumption and leisure, which causes the composite good, $c_t v(L_t)$, to increase rapidly. Compared to a standard specification of habit formation, the introduction of leisure provides agents with an additional margin which can be used to control the evolution of the habit stock.

2.2 Firms

Each period, managers have to decide how much labor to hire, N_t, and how much to invest in business capital, i_t. Managers maximize the value of the firm to its owners, the representative agent, which is given by the present discounted value of all current and expected incomes d_t.

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^{*t} \frac{\lambda_t}{\lambda_0} d_t$$

where:

$$d_t = y_t - W_t N_t - i_t$$

and where $\beta^{*t}\lambda_t/\lambda_0$ is the stochastic discount factor. Total output is denoted by y_t and the firm's capital stock follows an intertemporal accumulation equation with adjustment costs:

$$(1-\delta)k_t + \Phi(\frac{i_t}{k_t})k_t = \gamma k_{t+1}$$

Production of the final output good, y_t, requires the use of labor, N_t, and capital, k_t. The good is produced via Cobb-Douglas production function:

$$y_t = A_t {k_t}^{\alpha} N_t^{1-\alpha}$$

[3] I am very grateful to Andy Abel for his suggestion of this specification of habit.

The capital share is α and A_t is the standard random total technology shock variable that can be interpreted as a temporary displacement to total factor productivity.

2.3 Market clearing

In equilibrium, all produced goods are either consumed or invested and bonds are in zero net supply:

$$y_t = c_t + i_t$$

$$B_t = 0$$

2.4 First-order conditions

The labor supply equation, the asset pricing formula characterizing the dynamics of equity prices, and the risk-free rate can be derived using the first-order condition with respect to L_t, S_{t+1} and B_{t+1}.

L_t :

$$[c_t v(L_t) - x_t]^{-\sigma} c_t v'(L_t) + \varphi_t b c_t v'(L_t) = \lambda_t W_t$$

S_{t+1} :

$$p_{Et} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t} [d_{t+1} + p_{Et+1}]$$

B_{t+1} :

$$\frac{1}{1 + r_{ft}} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t}$$

where φ_t is the Lagrange multiplier associated to the habit accumulation equation and λ_t is marginal utility. As stated in the introduction, the challenge is to find a specification of marginal utility able to jointly explain asset pricing and labor market facts. The remaining first-order conditions are shown in the appendix.

3 Parameter selection and steady state

The calibration procedure is carried out in two steps. A first set of parameters is chosen based on National Income Account data, following the standard in business cycle literature. A second set of parameters, for which a priori knowledge is weak, is chosen to maximize the model's ability to replicate a set of business cycle and asset pricing moments.

3.1 Long-run behavior

A first set of parameters is chosen to match long-run model behavior. The quarterly trend growth rate is 1.005, and the constant capital share in the Cobb-Douglas production function, α, is 0.36. These are the standard values used in the real business cycle literature. The depreciation rate, δ is set to 0.0136. According to Davis and Heathcote (2005), this value corresponds to the depreciation rate for appropriately measured capital between 1948 to 2001. To maximize the model's ability to match the low risk-free rate, the subjective discount factor β is set to 0.997.

3.2 Driving process

Technology shocks are the only source of business cycle fluctuations and the exogenous process is given by an autoregressive exogenous process:

$$A_t = \rho A_{t-1} + \varepsilon_t$$

Following King and Rebelo (1999), the standard deviation of the shock innovation $std(\varepsilon_t)$ is set to 0.0072 percent and the persistence parameter, ρ, is set to 0.979.

3.3 Labor supply

The introduction of endogenous labor supply involves the calibration of two additional parameters controlling the curvature of $v(L)$. Following the business cycle literature, the fact that on average, households spend about 20 percent of their available time on professional activities [King and Rebelo (1999)] is used to calibrate the first elasticity parameter. This restriction, which implies that in the steady state $N = 0.2$, pins down $v'(L)/v(L)$.

Following Uhlig (2007), the Frish elasticity of labor supply is then used to fix the second curvature parameter $v''(L)/v'(L)$. As shown in the appendix, the first-order conditions with respect to c_t and L_t can be combined to derive a log-linearized labor supply equation. From this equation, the Frish elasticity of labor supply, $\partial \widehat{N}_t / \partial \widehat{w}_t$ can be derived and used to calibrate $v''(L)/v'(L)$. Following Uhlig (2007), the Frish elasticity is set to 3.

3.4 Coefficient of relative risk aversion and curvature parameter

While the curvature parameter, σ, is still related to risk aversion in consumption, this exact relationship breaks down in the case of non-separability between consumption and leisure. Given that our mechanism crucially relies on agents' willingness to smooth variations in $c_t v(L_t)$, as proposed by Rudebusch and Swanson (2008), the coefficient of relative risk aversion in the composite good could be computed to assess the relevance of our specification.

As shown in the appendix, the deterministic coefficient of relative risk aversion in the composite good is exactly equal to the curvature parameter σ. In a model with internal habit formation, relative risk aversion is independent of the two habit parameters a and b [see Constantinides (1990)]. To ensure that the conclusion of this study do not rely on an implausible curvature coefficient, we propose to pick a conservative value and to set σ to 3, as suggested by Kocherlakota (1996).

3.5 Habit formation and adjustment costs

Habit formation involves the calibration of the two parameters a and b. Given the absence of empirical evidence regarding plausible steady state values for the habit stock, we start by eliminating one degree of freedom by imposing the following restriction $b = 1 - a$, where $0 \leq a \leq 1$, which implies that the law of motion is now given by:

$$\gamma x_{t+1} = a x_t + (1-a) c_t v(L_t)$$

In the steady state, this restriction implies that:

$$\frac{cv(L) - x}{cv(L)} = \frac{\gamma - 1}{\gamma - a}$$

Given that $\gamma > 1$, this restriction ensures that in the steady state the nonnegativity condition $\frac{cv(L)-x}{cv(L)} > 0$ is always satisfied [Chapman (1998)]. We will come back to this issue in section 4.3 when the around steady state dynamics of the model will be studied.

As in Jermann (1998)[4], the parameters of the capital adjustment costs function $\Phi(\frac{i}{k})$ are set so that the model with adjustment costs has the same steady state as the model without adjustment costs. It is assumed that near the steady state point: $\Phi > 0$, $\Phi' > 0$ and $\Phi'' < 0$. The advantage of adopting this general specification is that the introduction of adjustments costs can be captured by one single elasticity parameter:

$$\epsilon = \frac{\Phi''\left(\frac{i}{k}\right)\frac{i}{k}}{\Phi'\left(\frac{i}{k}\right)}$$

where $1/\epsilon$ can be interpreted as the elasticity of the investment to capital ratio to changes in Tobin's Q[5], and where i/k is the steady state investment to capital ratio.

3.6 Empirical procedure

The parameters a and ϵ are picked, within a range of plausible values, to maximize the model's ability to match the equity premium and the risk-free rate. Let θ denote the vector of the 2 model parameters:

$$\theta = [a,\ \epsilon]$$

where $1/\epsilon$ stands for the elasticity of the investment capital ratio with respect to Tobin's Q, and where a is the habit formation parameter. θ is then chosen in order to minimize the following loss function:

$$L = [\mu - f(\theta)]'\Omega[\mu - f(\theta)]$$

μ is the vector of empirical moments to match, namely the equity premium and the risk-free rate, while $f(\theta)$ denotes the theoretical moments generated from the model, and Ω is the weighting matrix[6]. The loss function

[4] See also Baxter and Crucini (1993)

[5] So the case $1/\varepsilon = \infty$ corresponds to the case without adjustment costs while the case $1/\varepsilon = 0$ corresponds to the case with infinite adjustment costs.

[6] Since we have as many moments as parameters to estimate, we use the identity matrix.

L is computed for a grid of values for θ :

$$a = [0 : 1], \ 1/\epsilon = [0.16 : \infty]$$

4 Results

The loss function reaches a minimum at the following parameter values:

$$a = 0.955, 1/\epsilon = 0.24$$

Table 1 and 2 show the business cycle and the asset pricing implications of the model and the moments that are targeted are emphasized in bold.

Table 1: Business cycle statistics (HP-filtered data)[7]

Standard deviation

	σ_y	σ_c/σ_y	σ_i/σ_y	σ_N/σ_y	σ_w/σ_y	σ_{p_E}/σ_y
Data	1.66	0.76	3.39	1.07	0.56	6.02
Model	1.44	0.71	2.11	0.55	0.45	8.82

Correlation with output

	$\rho(y_t, y_t)$	$\rho(y_t, c_t)$	$\rho(y_t, i_t)$	$\rho(y_t, N_t)$	$\rho(y_t, w_t)$	$\rho(y_t, p_{Et})$
Data	1	0.76	0.82	0.86	0.16	0.46
Model	1	0.99	0.99	0.99	0.99	0.99

First-order autocorrelation

	$\rho(\Delta y_t)$	$\rho(\Delta c_t)$	$\rho(\Delta i_t)$	$\rho(\Delta N_t)$	$\rho(\Delta w_t)$	$\rho(\Delta p_{Et})$
Data	0.83	0.79	0.81	0.88	0.73	0.80
Model	0.72	0.72	0.72	0.72	0.72	0.72

[7] Equity prices are taken from the online database of R.Shiller. The remaining variables are taken fromt the online database of the Federal Reserve Bank of St-Louis. $\sigma_y, \sigma_c, \sigma_i, \sigma_N, \sigma_w$ and σ_{pE} denote the standard deviation of output, consumption, investment , hours worked, real wages and equity prices. Correlation of variable z_t with output is denoted $\rho(z_t, y_t)$ while the first order autocorrelation of variable z_t is denoted $\rho(\Delta z_t)$. Following the business cycle literature, the cyclical component of the variables are extracted using a HP-filtered and all the variables have been expressed in logs.

Table 2: Financial returns (Growth rates in annualized percent)[8]

Mean

	$E(r_m - r_f)$	$E(r_f)$
Data	**6.19**	**0.75**
Model	**6.19**	**0.75**

Standard deviation and correlation

	$\sigma(r_m)$	$\sigma(r_f)$	$\rho(\Delta r_m)$	$\rho(\Delta r_f)$	$\rho(r_m, r_f)$
Data	16.56	3.68	-0.06	0.73	0.20
Model	20.38	6.95	0.74	0.99	0.08

Compared to a general equilibrium business cycle model, the equity premium, the low risk-free rate and the high volatility of equity returns and equity prices can be explained in a model also able to capture the main business cycle regularities. Compared to a standard endowment economy model, the difference is that these asset pricing puzzles can be explained in a production economy model where labor supply and investment are endogenously determined.

Despite some improvements with respect to the existing literature, several implications of the model remain difficult to reconcile with the data. For instance, the fact the volatility of hours worked is only partially explained seems to indicate that introducing an extensive margin could help to improve the model's performance. Moreover, the model fails to explain the low correlation between real wages and output. The model also has trouble explaining the low risk-free rate volatility observed in the data. Compared to other studies with internal habit formation [Jermann (1998), Boldrin, Christiano and Fisher (2001)], the reduction that is obtained is however encouraging[9].

4.1 Substitutability between consumption and leisure

The main effect of our specification of habit formation is to induce agents to choose consumption and leisure so as to smooth the composite good, $c_t v(L_t)$.

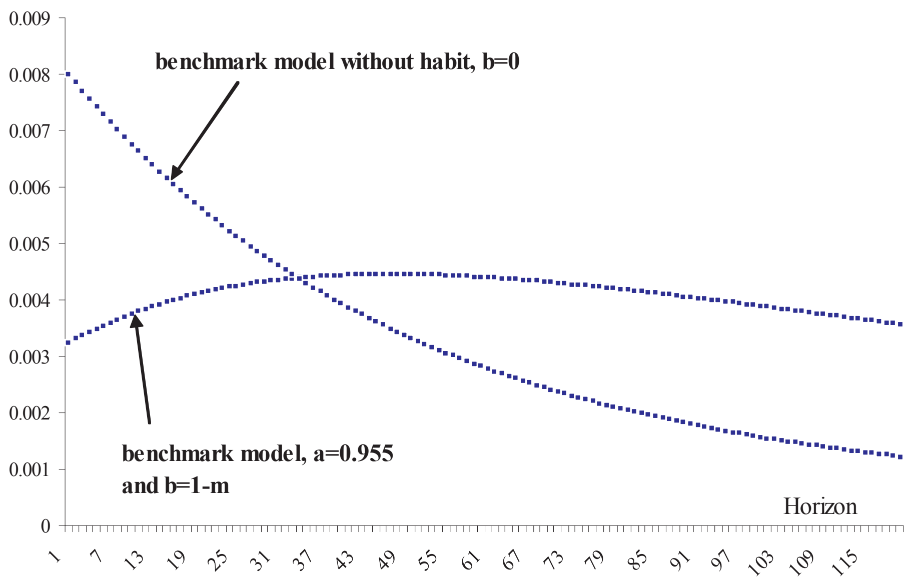
[8] The empirical facts are taken from Piazzesi, Schneider and Tuzel (2007). The mean equity premium, the mean risk-free, and the mean equity premium are denoted $E(r_m - r_f)$ and $E(r_f)$. The standard deviation of equity returns and of the risk-free rate are denoted $\sigma(r_m)$ and $\sigma(r_f)$. Their first-order autocorrelations are denoted $\rho(\Delta r_f)$ and $\rho(\Delta r_m)$ while $\rho(r_m, r_f)$ denotes the equity return risk-free rate correlation.

[9] In Jermann (1998) and Boldrin, Christiano and Fisher (2001), the risk-free rate volatility is 11.46%, and 24.6%.

This reduction in volatility is obtained by increasing the substitutability between consumption and leisure.

Figure 1 shows the impulse response of $c_t v(L_t)$ to a positive technology shock in the benchmark case $a = 0.955$, $b = 1 - a$, and in the case $b = 0$, which corresponds to the case where the habit formation channel is switched off. The more gradual and persistent response of the composite good which is induced by habit formation illustrates the main channel through which the mechanism is operating.

Figure 1: Response of $c_t v(L_t)$ to a positive technology shock[10]



In response to a positive technology shock, while consumption increases, the key is that agents' desire to smooth fluctuations in the composite good induce them to work harder. In good times, agents choose to take less leisure to prevent their habit stock from rising too quickly and consumption smoothing of the composite good is achieved by making c_t and l_t move in opposite directions. As shown in Table 3 below, this mechanism allows the model to reproduce the complementarity between consumption and hours worked observed in the data. As illustrated by the third column of Table 3, compared

[10] In percentage log-deviation from the steady state. The horizon indicates the number of quarters after the shock.

to the benchmark calibration, switching off the habit formation channel by setting b to 0 would compromise the model's ability to generate this positive co-movement:

Table 3: Impact of habit formation on the consumption hours worked correlation (HP-filtered data)

	Data	Benchmark	Without habit
$\rho(c_t, N_t)$	0.67	0.99	-0.99

4.2 Wealth effect on labor supply

The second main impact of this specification of internal habit formation is to reduce the wealth elasticity of labor supply. To illustrate how this mechanism works, the first-order conditions with respect to consumption and leisure are used to derive a log-linearized labor supply equation:

$$\widehat{N}_t = \kappa_w \widehat{W}_t + \kappa_\lambda \widehat{\lambda}_t + \kappa_\varphi \widehat{\varphi}_t - \kappa_x \widehat{x}_t$$

where $\widehat{N}_t, \widehat{W}_t, \widehat{\lambda}_t, \widehat{x}_t$ denote hours worked, the real wage, marginal utility, and the habit stock. $\widehat{\varphi}_t$ is the Lagrange multiplier attached to the habit accumulation constraint[11].

Compared to a standard labor supply equation, the main impact of our specification of habit formation is to modify the elasticity parameters $\kappa_w, \kappa_\lambda, \kappa_\varphi$ and κ_x. In particular, κ_λ, which represents the wealth elasticity of labor supply is directly affected by the two habit parameters a and b. An important implication of this specification of habit is to induce a negative relationship between κ_λ, and the intensity of habit formation.

Keeping a at its benchmark value, Figure 2 shows how a variation of b from 0 to 1-a affects the elasticity parameter κ_λ. The case $b = 0$ corresponds to the case where the habit channel is completely switched off while the case $b = 1 - m$ corresponds to the high intensity case. As shown by Figure 2, the wealth elasticity of labor supply coefficient κ_λ decreases as the habit intensity parameter, b, increases.

The fact that an increase in the intensity of habit formation reduces the wealth elasticity of labor supply is a central ingredient. In the data, the fact that hours worked are volatile and procyclical is difficult to reconcile with a

[11] See the appendix for a formal derivation of the log-linearized labor supply equation.

high wealth effect on labor supply. Our specification of habit formation which reduces the wealth elasticity of labor supply coefficient, κ_λ, while inducing an increase in the volatility of $\widehat{\lambda}_t$, allows to solve this tension between the asset pricing and the labor market implications of the model.

Figure 2: Impact of habit formation on the wealth elasticity of labor supply, κ_λ

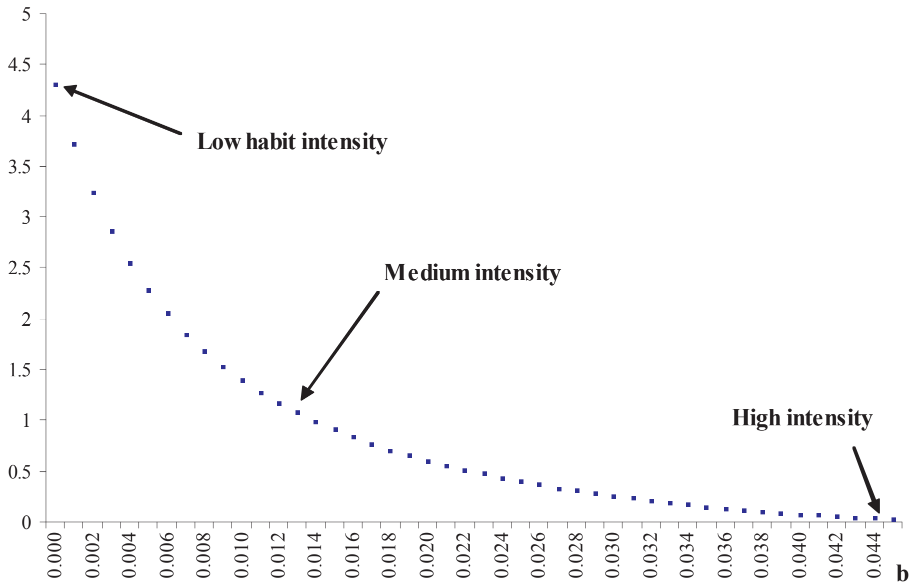


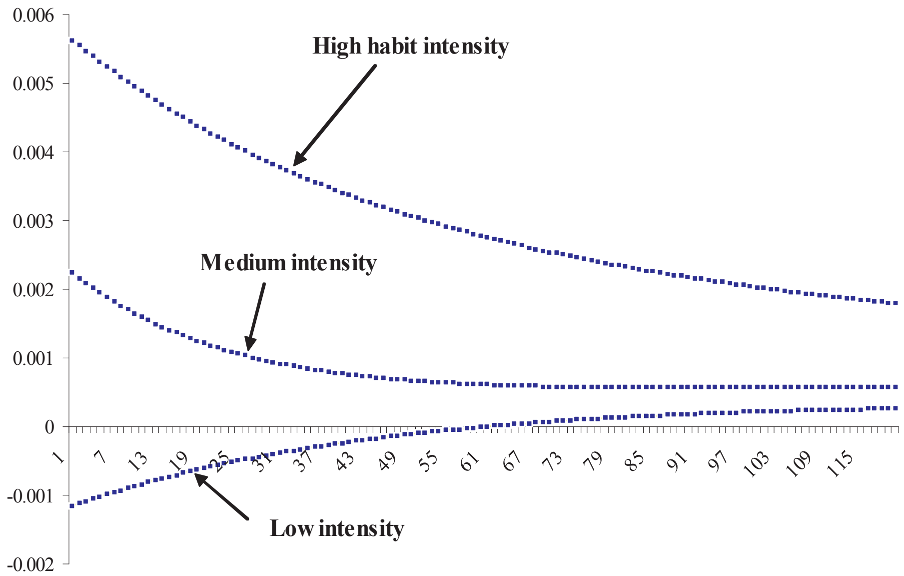
Figure 3 shows how habit formation affects the response of hours worked to technology shocks. While a positive technology shock increases labor demand, the equilibrium response of hours worked crucially depends on the reaction of labor supply.

In response to a positive technology shock, the wealth effect captured by the decline in $\widehat{\lambda}_t$ shifts the labor supply curve to the left and generates a counterfactual reduction in the volatility of hours worked. When habit intensity is low for instance, the elasticity parameter κ_λ is so large that the strength of the wealth effect leads to a fall in hours worked. As habit intensity rises, the wealth elasticity of labor supply coefficient decreases and the model's ability to generate procyclical movements in hours worked improves.

As illustrated by Tables 4 and 5 in the annex, removing habit formation would not only impair the model's ability to explain asset pricing facts but also its ability to explain business cycle fluctuations. Without a strong response of hours worked, business cycle models with high adjustment costs can only explain a small fraction of output fluctuations and are unable to generate plausible asset pricing implications.

In a model with internal habit formation, labor supply is also affected by the Lagrange multiplier on the habit accumulation equation $\widehat{\varphi}_t$. The introduction of habit formation has a similar impact on this coefficient which decreases as the intensity of habit rises. The habit stock being predetermined, movements in $\widehat{x}_t$ only have a limited impact on the short-run dynamics on labor market variables. Finally, the Frish elasticity of labor supply, κ_w, captures the strength of the substitution effect induced by variations in the real wage. While κ_w is also affected by the two habit parameters a and b, as explained in section 3.3, $v''(L)/v'(L)$ is calibrated to fix the Frish elasticity at 3.

Figure 3: Response of hours worked to a positive technology shock



4.3 Net utility over the business cycle

As pointed out by Chapman (1998), the introduction of habit formation raises the issue of the nonnegativity of $c_t v(L_t) - x_t$. As discussed in section

3.5, the restriction imposed on the two habit parameter a and b ensures that this nonnegativity condition is always satisfied in the steady state. To assess whether this condition still holds in the around steady state dynamics, the empirical distribution of $c_t v(L_t) - x_t$ is derived by simulating 300'000 observations. As shown by Figure 4 (see annex), over the period considered which corresponds to 75'000 years of data, the case $c_t v(L_t) - x_t < 0$ was never observed[12].

This result can be explained by the fact that consumption, leisure, and the habit stock are endogenously determined in this model. Given that agents have full control over all components of utility, paths for consumption and leisure ensuring that the case $c_t v(L_t) - x_t < 0$ is never encountered can be chosen.

An increase in the intensity of habit formation reduces the distance between the composite good and the habit stock but also make fluctuations in $c_t v(L_t)$ more costly. The reduction in the volatility of the composite good which is induced decreases the probability of observing a sharp drop in $c_t v(L_t)$. Moreover, compared to an external specification, the fact that the habit stock is chosen by the agents reduces the risk of a sudden jump in x_t.

4.4 Risk-free rate volatility

To illustrate the impact of habit memory on the risk-free rate volatility, the usual standard log-normal approximation is used to decompose the risk-free rate into two components [see Jermann (1998)]:

$$r_{ft} = (1/\beta^*) \exp \left[\log \lambda_t - E_t \log \lambda_{t+1} - \frac{1}{2} var_t(\log \lambda_{t+1}) \right]$$

The first term captures the impact of intertemporal marginal rate of substitution on equilibrium interest rates while the variance term captures the precautionary effects. Compared to a model with complete depreciation [Jermann (1998), Boldrin, Christiano and Fisher (2001)], the smaller risk-free rate volatility that is obtained is due to the increase in the persistence of marginal utility induced by memory.

To illustrate this point, let us assume that $\log \lambda_t$ can be described by a autoregressive process of order one:

[12] The mean is 0.0297 and the lowest observed value is 0.0144

$$\log \lambda_t = \rho_\lambda \log \lambda_{t-1} + \varepsilon_{\lambda t}$$

and that the standard deviation of $\log \lambda_t$ is therefore given by:

$$\sigma_\lambda = \frac{\sigma_\varepsilon}{(1-\rho_\lambda^2)^{1/2}}$$

Clearly, increasing the persistence parameter, ρ_λ, enables to increase the volatility of marginal utility and to reduce movements in $\log \lambda_t - E_t \log \lambda_{t+1}$. Smaller movements in this first term permits to avoid the generation of excessive risk-free rate variation while increasing the variance of marginal utility helps to generate the amount of precautionary savings needed to solve the risk-free rate puzzle.

A model without memory, $a = 0$, but with a high sensitivity parameter, b, would also be able to generate the volatility of marginal utility needed to explain the low mean risk-free rate. A specification with habit formation but without memory effects does however not permit to generate an increase in the persistence of marginal utility that is sufficient to reduce movements in this first component and tend to generate excessive risk-free rate variations.

5 Conclusion

In search of a model linking financial markets to the real economy, this article has proposed to explore the nexus between endogenous labor supply and financial returns. Our main conclusion is that the joint explanation of asset pricing and macroeconomic facts requires the introduction of preferences which minimize the wealth effect on labor supply while permitting to increase the volatility and the persistence of marginal utility.

Assuming that habits are formed over a mix of consumption and leisure overcomes the difficulties arising in models with endogenous labor supply and provides a potential solution to several asset pricing puzzles. Compared to a standard business cycle model with adjustment costs, the proposed mechanism only requires the introduction of one additional free parameter. Finally, our specification of preferences with habit formation in the mix of consumption and leisure is consistent with balanced growth.

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7 Annex 1: Benchmark model without habit formation

Table 4: Business cycle statistics (HP-filtered data)

Standard deviation

	σ_y	σ_c/σ_y	σ_i/σ_y	σ_N/σ_y	σ_w/σ_y	σ_{p_E}/σ_y
Data	1.66	0.76	3.39	1.07	0.56	6.02
Model	0.83	1.10	0.62	0.19	1.19	2.59

Correlation with output

	$\rho(y_t, y_t)$	$\rho(y_t, c_t)$	$\rho(y_t, i_t)$	$\rho(y_t, N_t)$	$\rho(y_t, w_t)$	$\rho(y_t, p_{Et})$
Data	1	0.76	0.82	0.86	0.16	0.46
Model	1	1	0.99	-0.99	1	0.99

First-order autocorrelation

	$\rho(\Delta y_t)$	$\rho(\Delta c_t)$	$\rho(\Delta i_t)$	$\rho(\Delta N_t)$	$\rho(\Delta w_t)$	$\rho(\Delta p_{Et})$
Data	0.83	0.79	0.81	0.88	0.73	0.80
Model	0.72	0.72	0.72	0.72	0.72	0.72

Table 5: Financial returns (Growth rates in annualized percent)

Mean

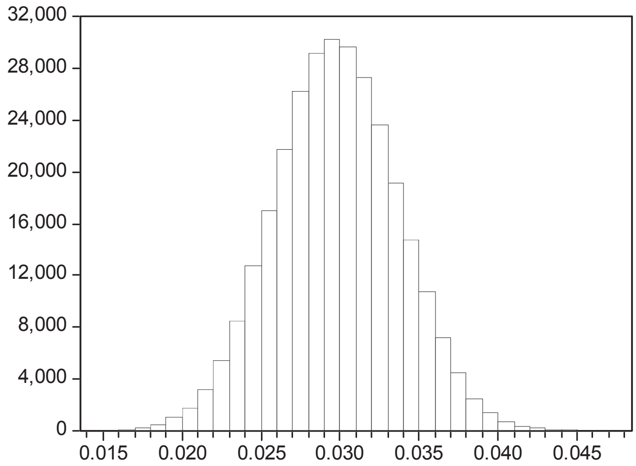
	$E(r_m - r_f)$	$E(r_f)$
Data	**6.19**	**0.75**
Model	**0.16**	**5.33**

Standard deviation and correlation

	$\sigma(r_m)$	$\sigma(r_f)$	$\rho(\Delta r_m)$	$\rho(\Delta r_f)$	$\rho(r_m, r_f)$
Data	16.56	3.68	-0.06	0.73	0.20
Model	3.46	0.88	0.74	0.99	0.005

8 Annex 2: Nonnegativity condition

Figure 4: Simulated distribution of $c_t v(L_t) - x_t$



9 Technical appendix

The problem of the social planner and the competitive equilibrium are equivalent in this economy. The model is solved using dynare[13] and by taking a second-order linear approximation.

9.1 Optimization program and first-order conditions

$$L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} \frac{[c_t v(L_t) - x_t]^{1-\sigma}}{1-\sigma} \right.$$

$$+ \sum_{t=0}^{\infty} \beta^{*t} \lambda_t \left[A_t k_t^{\alpha} N_t^{1-\alpha} - c_t - i_t \right]$$

$$+ \sum_{t=0}^{\infty} \beta^{*t} \mu_t \left[(1-\delta) k_t + \Phi(\frac{i_t}{k_t}) k_t - \gamma k_{t+1} \right]$$

[13] http://www.cepremap.cnrs.fr/dynare/

$$+\sum_{t=0}^{\infty}\beta^{*t}\varphi_t\left[ax_t+b\left\{c_t v(L_t)\right\}-\gamma x_{t+1}\right]\Bigg\}$$

First-order conditions[14]:

c_t :

$$\left[c_t v(L_t)-x_t\right]^{-\sigma}v(L_t)+\varphi_t b v(L_t)=\lambda_t$$

L_t :

$$\left[c_t v(L_t)-x_t\right]^{-\sigma}c_t v'(L_t)+\varphi_t b c_t v'(L_t)=\lambda_t(1-\alpha)\frac{y_t}{N_t}$$

i_t :

$$\lambda_t=\Phi'(\frac{i_t}{k_t})\mu_t$$

k_{t+1} :

$$\mu_t=\widetilde{\beta}E_t\mu_{t+1}\left\{(1-\delta)+\Phi'(\frac{i_{t+1}}{k_{t+1}})-\Phi'(\frac{i_{t+1}}{k_{t+1}})\frac{i_{t+1}}{k_{t+1}}\right\}$$

$$+\widetilde{\beta}E_t\lambda_{t+1}\alpha\frac{y_{t+1}}{k_{t+1}}$$

x_{t+1} :

$$\varphi_t=\widetilde{\beta}E_t\varphi_{t+1}a-\widetilde{\beta}E_t\left[c_{t+1}v(L_{t+1})-x_{t+1}\right]^{-\sigma}$$

λ_t :

$$A_t k_t^{\alpha}N_t^{1-\alpha}-c_t-i_t=0$$

μ_t :

$$(1-\delta)k_t+\Phi(\frac{i_t}{k_t})k_t-\gamma k_{t+1}=0$$

φ_t :

$$ax_t+bc_t v(L_t)-\gamma x_{t+1}=0$$

[14]where $\widetilde{\beta}=\beta\gamma^{-\sigma}$

10 Relative risk aversion and habit memory

Define: $c_{Mt} = c_t v(L_t)$

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} \frac{[c_{Mt} - x_t]^{1-\sigma}}{1-\sigma} + \sum_{t=0}^{\infty} \beta^{*t} \varphi_t \left[a x_t + b c_{Mt} - \gamma x_{t+1} \right] \right\}$$

The derivative of the utility function with respect to c_{Mt} is given by:

$$\Lambda_{c_{Mt}} = [c_{Mt} - x_t]^{-\sigma} + b\varphi_t \tag{1}$$

Differentiating this condition again with respect to c_{Mt}, we obtain that:

$$\Lambda_{c_{Mt},c_{Mt}} = -\sigma \left[c_{Mt} - x_t\right]^{-\sigma-1} + \sigma \left[c_{Mt} - x_t\right]^{-\sigma-1} \frac{\partial x_t}{\partial c_{Mt}} + b \frac{\partial \varphi_t}{\partial c_{Mt}} \tag{2}$$

Taking the first-order conditions with respect to φ_t and x_{t+1}, we have that:

$$\varphi_t = a\widetilde{\beta} E_t \varphi_{t+1} - [c_{Mt+1} - x_{t+1}]^{-\sigma} \tag{3}$$

and:

$$a x_t + b c_{Mt} - \gamma x_{t+1} \tag{4}$$

In the deterministic steady state, the coefficient of relative risk aversion is given by:

$$RR_a = -c_M \frac{\Lambda_{c_M,c_M}}{\Lambda_{c_M}} \tag{5}$$

In the steady state, equation (3) and equation (4) imply that:

$$x = \frac{b c_M}{\gamma - a} \tag{4S'}$$

$$\varphi = -\frac{c_M^{-\sigma} \left(1 - \frac{b}{\gamma - a}\right)^{-\sigma}}{(1 - a\widetilde{\beta})} \tag{3S'}$$

implying that:

$$\frac{\partial \varphi}{\partial c_M} = \sigma \frac{c_M^{-\sigma-1}\left(1-\frac{b}{\gamma-a}\right)^{-\sigma}}{(1-a\widetilde{\beta})} \quad \text{(3S")}$$

$$\frac{\partial x}{\partial c_M} = \frac{b}{\gamma-a} \quad \text{(4S")}$$

Substituting (4S'), (3S") and (4S") into (5), we obtain after several manipulations that:

$$RR_a = \sigma$$

10.1 Labor supply

The following functional form for $v(L)$ is used to solve the model:

$$v(L) = (\psi + L_t^{\upsilon})$$

In the steady state, the first-order condition with respect to L_t pins down $v'(L)/v(L)$. In the case that is considered, this condition pins down the parameter ψ :

$$\psi = L^{\upsilon}\left(\frac{N}{1-N}\frac{\upsilon}{(1-\alpha)}\frac{c}{y} - 1\right)$$

The second free parameter υ is set by fixing the Frish elasticity of labor supply at 3.

To derive the labor supply curve, we firstly derive equation (1L'), which is obtained by manipulating the linearized first-order condition with respect to c_t :

$$\widehat{c}_t = \omega_N \widehat{N}_t + \omega_x \widehat{x}_t - \omega_\varphi \widehat{\varphi}_t - \omega_\lambda \widehat{\lambda}_t \quad \text{(1L')}$$

The labor supply curve can be derived by combining equations (1L') with the linearized first-order condition with respect to L_t :

$$\widehat{N}_t = \frac{1}{\xi_c\omega_N - \xi_N}\widehat{w}_t + \frac{1+\xi_c\omega_\lambda}{\xi_c\omega_N - \xi_N}\widehat{\lambda}_t + \frac{\xi_c\omega_\varphi + \xi_\varphi}{\xi_c\omega_N - \xi_N}\widehat{\varphi}_t - \frac{\xi_c\omega_x}{\xi_c\omega_N - \xi_N}\widehat{x}_t$$

where:

$$\Gamma = 1 - \frac{b}{\gamma - a}, \;\; \Omega = 1 - \frac{b\widetilde{\beta}}{1 - \widetilde{\beta} a}$$

$$\omega_N = \frac{\upsilon L^{\upsilon}}{(\psi + L^{\upsilon})} \left\{ 1 - \frac{\Gamma \Omega_c}{\sigma} \right\} \frac{N}{1 - N}, \omega_x = \frac{b}{(\gamma - a)}, \;\; \omega_{\varphi} = \frac{\Gamma}{\sigma} \frac{b\widetilde{\beta}}{(1 - \widetilde{\beta} a)}, \;\; \omega_{\lambda} = \frac{\Gamma \Omega_c}{\sigma}$$

$$\xi_c = \left\{ 1 - \frac{\sigma}{\Omega_c \Gamma} \right\}, \;\; \xi_N = \left\{ (\upsilon - 1) - \frac{\upsilon L^{\upsilon}}{(\psi + L^{\upsilon})} \frac{\sigma}{\Gamma \Omega_c} \right\} \frac{N}{1 - N}, \;\; \xi_{\varphi} = \frac{b\widetilde{\beta}}{(1 - \widetilde{\beta} a)} \frac{1}{\Omega_c}$$

The Frish elasticity of labor supply is given by:

$$\frac{\partial \widehat{N}_t}{\partial \widehat{w}_t} = \frac{1}{\xi_c \omega_N - \xi_N}$$